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$\mathcal{H}_2$ -norm-based Multi-pulse Diesel Fuel Injection Control with Minimal Cyclic Combustion Variation

Xi Luo, M.C.F. Donkers, Bram de Jager and Frank Willems

Abstract—Due to varying in-cylinder conditions, cyclic combustion variation leads to fluctuating and deteriorated diesel engine performance. In this paper, we study both the deterministic and the stochastic cyclic variation of a closed-loop controlled combustion process with a cycle-to-cycle fuel injection controller using in-cylinder pressure information. A controller design method is proposed that yields a stabilizing controller with fast dynamical performance, while minimizing the unavoidable amplification of the stochastic cyclic variation. Following the design method with different design parameters, multiple controllers are designed and experimentally tested using a single-cylinder engine test setup. The reference tracking results illustrate the trade-off between the controllers’ capability to reduce the deterministic or to avoid amplification of the stochastic cyclic combustion variation.

Index Terms—Engine control; Stochastic systems; LMIs

I. INTRODUCTION

CYCLIC combustion variation for diesel engines represents changes between consecutive combustion cycles due to varying in-cylinder conditions. On a combustion cycle basis, the varying heat release process leads to fluctuating engine performance, i.e., varying engine torque and engine-out emission levels. These fluctuations do not only lead to vibrations and deteriorated overall engine performance, but also hampers the possibility for further performance optimization. The cyclic combustion variation can be both deterministic and stochastic [1]. The deterministic component may result from varying in-cylinder temperature and pressure, residual gas amount, fuel injection pressure, etc. The stochastic cyclic variation represents the random changes of the combustion process on a cycle-to-cycle basis [2]. These variations become more significant for chemically driven combustion processes with longer ignition delay, which results from early fuel injection combined with high dilution with exhaust gas or from using fuel with a lower cetane number.

Accurate control of engine’s operating conditions is an effective way to reduce the combustion variations, e.g., using advanced air-path control [3] and fuel injection pressure control [4]. However, these methods have limited effects in reducing the cyclic variation, due to their slow dynamical response. Alternatively, it is considered effective to reduce the deterministic cyclic combustion variation with closed-loop fuel injection control, due to its ability of modifying the fuel

injection profile on a combustion cycle basis. For example, the cyclic variation caused by a varying amount of residual gas can be reduced by controlling the fuel injection timing [5]. Due to different in-cylinder conditions, wall temperature and injector fueling accuracy, cylinder-to-cylinder variation can be greatly reduced by manipulating the fuel injection timing and duration for each cylinder individually [6]–[8]. In previous work [9], [10], we have demonstrated the feasibility of compensating for both air- and fuel-path disturbances by modifying the fuel injection profile. These robustness results indicate the potential of controlling the fuel injection profile to reduce the deterministic cyclic combustion variations.

While cycle-to-cycle fuel injection control is effective in reducing the deterministic combustion variation, when compared to traditional open-loop fuel injection control, it unavoidably increases the stochastic cyclic variation, which leads to larger variations in engine performance. This is because the stochastic variation of the current combustion process is superposed with the uncorrelated random variation of the previous one via the feedback loop. This side effect has been observed in [6]. Although it is possible to reduce the stochastic cyclic variation by changing the operating conditions, e.g., reducing the exhaust gas recirculation (EGR) rate, this approach is not preferable since it potentially leads to suboptimal engine performance. Hence, it is of interest to have a cycle-to-cycle fuel injection controller, which reduces the deterministic cyclic combustion variation without significantly amplifying the stochastic cyclic variation, which is not explicitly considered in the existing controller design methods [6]–[10].

To achieve minimal cyclic combustion variation via cycle-to-cycle fuel injection control, we propose a novel and systematic controller design method in this paper based on an optimization problem involving linear matrix inequalities (LMIs). A stabilizing controller can be computed with fast dynamical performance and a minimal amplification of the stochastic cyclic combustion variation. We focus in this work on multi pulse fuel injection control in diesel engines.

The outline of this paper is as follows. First, the control problem is formulated in Section II. Section III discusses the fuel injection controller synthesis method, which considers closed-loop stability, the controller’s dynamical performance and the steady-state stochastic cyclic variation. Following the design method with different design parameters, multiple fuel injection controllers are designed and experimentally tested in Section IV. Conclusions and directions for future research are stated in Section V.

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II. PROBLEM FORMULATION

Before analyzing the cyclic variation of the closed-loop controlled combustion process, we formulate the cycle-to-cycle fuel injection control problem. In this section, we first introduce a general description of the combustion process. Then, the closed-loop fuel injection control scheme is discussed.

A. System Description

We consider the multi-pulse fuel injection profile for each combustion process, which is parametrized with the injection timing and duration of each pulse. With $m \in \mathbb{N}$ pulses in one combustion cycle, the injection parameter $u \in \mathbb{R}^{2m}$ for a multi-pulse fuel injection profile is defined as

$$u = [\text{SOI}_1, \text{DOI}_1, \dots, \text{SOI}_m, \text{DOI}_m]^\top,$$

where SOI_i and DOI_i with $i = 1, \dots, m$ are the start and duration of the i -th injection pulse, respectively. The development from assigning the injection parameters till the actual fuel injection involves a hydraulic process. The fuel is injected via nozzles under a certain injection pressure after the injector's needle has been lifted. Even if the same injection parameters are assigned, we may have varying fueling profiles for different combustion cycles due to varying hydraulic dynamics under different injection pressure. We regard this variation as an additive uncertainty of the fuel injection parameters, which is denoted by $d_u \in \mathbb{R}^{2m}$.

The air-path actuation of a diesel engine includes the valve positions for the exhaust gas recirculation (EGR) and variable geometry turbocharger (VGT), which is denoted by $v = [v_{\text{EGR}}, v_{\text{VGT}}]^\top$. The engine performance, i.e., engine torque, brake specific fuel consumption (BSFC) and engine-out NOx and particulate matter emissions, is denoted by z . The engine performance is influenced by disturbances d_e , such as the variations of the intake or exhaust manifold pressure and temperature. Due to the difficulty in directly measuring the engine performance in real time, combustion metrics are used as the feedback signals for cycle-to-cycle fuel injection control. These metrics, denoted by $y \in \mathbb{R}^n$ with $n \in \mathbb{N}$, are computed based on the cylinder pressure measurement after each combustion cycle for individual cylinders. The variation of the feedback combustion metrics due to measurement noise is captured by the additive disturbance / noise $w \in \mathbb{R}^n$. Common choices of the feedback combustion metrics include the net indicated mean effective pressure (IMEP) and the crank angle degree where 50% of the total accumulated heat release is reached (CA50). The actual selection of y depends on the control objective as well as the number of closed-loop controlled fuel injection parameters.

B. Cycle-to-cycle Fuel Injection Control

The scheme of cycle-to-cycle fuel injection control is shown in Fig. 1. At an operating point $o.p.$ described by engine torque and speed, the engine map determines the nominal air-path actuations $\hat{v}$, the nominal fuel injection parameters $\hat{u}$ and the reference combustion metrics y_r . With possible disturbances, the executed air-path actuations and fuel injection parameters

Fig. 1. Block diagram of closed-loop fuel injection control.

become v_e and u_e , respectively. Subject to disturbances d_e , the combustion process takes place and produces engine performance z as well as the feedback combustion metrics y , which is superposed with the noise w . Based on the reference tracking error

$$e = y_r - y, \quad (1)$$

the controller computes the modifications for the fuel injection parameters, such that the executed fuel injection parameters become $u_e = \hat{u} + u + d_u$. Since we focus on cycle-to-cycle fuel injection control, the air-path control is omitted from consideration and is depicted as a dashed line in Fig. 1. Simple feedback controllers are often applied for cycle-to-cycle fuel injection control due to limited on-board computation power [7], [8], [11], [12]. As the fuel injection parameters are modified on a cyclic basis, we describe the feedback law as

$$u_{k+1} = u_k + K e_k, \quad (2)$$

where K is the controller matrix and the combustion cycle index is denoted by $k \in \mathbb{N}$. It should be noted that the control law (2) is an integral controller on a combustion cycle basis.

Based on the scheme shown in Fig. 1, the objective of cycle-to-cycle fuel injection control is to minimize the reference tracking error e by manipulating the fuel injection profile with the modification u , such that the combustion process becomes consistent and the deterioration of the engine performance z is reduced subject to disturbances d_u and d_e . Since the deterministic components of d_u and d_e usually change slowly from cycle to cycle, the deterministic cyclic combustion variation can be greatly reduced by using a fuel injection controller with sufficiently fast dynamical performance. On the other hand, d_u , d_e and w contain stochastic components, such that the combustion process varies randomly on a cyclic basis. Minimal cyclic combustion variation can be achieved via cycle-to-cycle fuel injection control, by balancing between reduced deterministic combustion variation and amplified stochastic variation. Therefore, it is desired to have a stabilizing fuel injection controller, which has sufficiently fast dynamical performance and a minimal amplification of the stochastic cyclic variation.

III. MAIN RESULTS

To fulfill the controller design objective of minimizing the reference tracking error (1), while at the same time having minimal cyclic combustion variation, it is necessary to consider both the closed-loop reference tracking dynamics and the stochastic cyclic combustion variation. In this section, a control-oriented combustion model is briefly discussed.

Based on this model, we analyze the closed-loop reference tracking dynamics and the stochastic cyclic variation of the feedback combustion metrics. The controller design method is introduced at the end of this section.

A. Control-oriented Combustion Model

To achieve minimal cyclic variation and stable tracking error dynamics, a control-oriented model is needed to analyze the cyclic combustion behavior. As discussed and validated in [9], [10], the combustion process can be described on a cycle-to-cycle basis using the algebraic function

$$y_k = f(u_{e,k}, v_{e,k}, d_{e,k}) + w_k. \quad (3)$$

The validity of the model is based on the assumption that the diesel combustion has a negligible amount of internal residual gas, such that each combustion cycle is independent. With certain operating conditions, (3) can be locally approximated with an affine function, which is also validated in [9], [10]. The locally linearized model becomes

$$y_k - y_r = G(u_{e,k} - \hat{u}) + Hd_{e,k} + w_k, \quad (4)$$

where

$$G = \frac{\partial f}{\partial u}(\hat{u}, \hat{v}, 0), \quad H = \frac{\partial f}{\partial d}(\hat{u}, \hat{v}, 0), \quad (5)$$

and we assume $v_{e,k} = \hat{v}$. Based on (1), the reference tracking error becomes

$$e_k = -G(u_k + d_{u,k}) - Hd_{e,k} - w_k. \quad (6)$$

Both d_e and d_u have a deterministic and a stochastic part. To simplify the following analysis, we assume that they only contain deterministic disturbances that are constant or, at most, vary slowly on a combustion cycle basis. Their stochastic parts are considered to be included in the measurement noise w . We assume the measurement noise is white with $\mathbb{E}\{w\} = 0$ and $\mathbb{E}\{ww^\top\} = \mathcal{W}$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator.

B. Closed-loop Stability

The expression for the reference tracking error (6) enables us to analyze the dynamical reference tracking error in the closed loop. The error difference between two consecutive combustion cycles is

$$e_{k+1} - e_k = -G(u_{k+1} - u_k) - w_{k+1} + w_k, \quad (7)$$

where we assume $d_{u,k} = d_u$ and $d_{e,k} = d_e$ are constant (i.e., they are quasi-static on a combustion cycle basis). By substituting the feedback law (2), the closed-loop dynamics of the reference tracking error becomes

$$e_{k+1} = (I - GK)e_k - w_{k+1} + w_k. \quad (8)$$

Using the Lyapunov function $V(e_k) = e_k^\top \mathcal{P} e_k$ with $\mathcal{P} = \mathcal{P}^\top \succ 0$, the nominal error dynamics ($w = 0$) is exponentially stable for a given K , i.e., the error dynamics satisfy $\|e_k\| < \mathcal{C}\alpha^k \|e_0\|$ for some $\mathcal{C} > 0$, if we can find a matrix $\mathcal{P}$, which satisfies

$$(I - GK)^\top \mathcal{P} (I - GK) \prec \alpha \mathcal{P}, \quad (9)$$

where $\alpha \in (0, 1]$ is an upper bound of the exponential convergence rate, which indicates the controller's dynamical performance.

C. Stochastic Cyclic Variation

Assuming we have a stabilizing controller, the nominal reference tracking error (i.e., for $w_k = 0$) converges to zero. When considering (6), this means that $\mathbb{E}\{e_k\} = \mathbb{E}\{w_k\} = 0$ and that $\mathbb{E}\{y_k\} = y_r$. The variance matrix of the feedback signals is related to the reference tracking error variance

$$\mathcal{Y}_k = \mathbb{E}\{(y_k - y_r)(y_k - y_r)^\top\} = \mathbb{E}\{e_k e_k^\top\} = \mathcal{E}_k. \quad (10)$$

It is difficult to analyze the evolution of $\mathcal{E}_k$ over combustion cycles, due to non-zero cross terms when computing $\mathcal{E}_{k+1}$ using (8), e.g., $\mathbb{E}\{e_k w_k^\top\} = -\mathcal{W}$. As an alternative, we study the evolution of $\mathcal{E}_k$ based on the variance matrix of the fuel injection parameters. This method is justified by considering that the system is linearized as indicated by (6), such that the variation of e can be studied based on the variation of u .

To study the stochastic variation of u_k , we again assume $d_{u,k} = d_u$ and $d_{e,k} = d_e$ to be constant and substitute (6) into (2), leading to

$$u_{k+1} = u_k - K(G(u_k + d_u) + Hd_e + w_k). \quad (11)$$

Furthermore, we define u^* satisfying $u^* = u^* - K(G(u^* + d_u) + Hd_e)$, implying that $G(u^* + d_u) + Hd_e = 0$, which means that system (11) is in equilibrium and that stochastic disturbances absent. Note that u^* exists if the system (8) is exponentially stable (for $w_k = 0$). The variable u^* allows us to rewrite (11) as

$$u_{k+1} - u^* = (I - KG)(u_k - u^*) - Kw_k, \quad (12)$$

meaning that the evolution of the variance matrix $\mathcal{U}_k = \mathbb{E}\{(u_k - u^*)(u_k - u^*)^\top\}$ can be written as

$$\mathcal{U}_{k+1} = (I - KG)\mathcal{U}_k(I - KG)^\top + K\mathcal{W}K^\top. \quad (13)$$

Now because (6) holds, we have that

$$\mathcal{E}_k = G\mathcal{U}_k G^\top + \mathcal{W}, \quad (14)$$

and the steady-state variance matrix of the feedback signals is given by

$$\mathcal{Y} = \mathcal{E} = G\mathcal{U}G^\top + \mathcal{W}, \quad (15)$$

where $\mathcal{U}$ is given by

$$\mathcal{U} - (I - KG)\mathcal{U}(I - KG)^\top - K\mathcal{W}K^\top = 0. \quad (16)$$

Since $\mathcal{U}$ is a function of K and $\mathcal{W}$, it is clear that the variance matrix of the feedback combustion metrics depends on the measurement noise as well as the closed-loop controller. Moreover, having closed-loop fuel injection control does not decrease the (stochastic) variance of combustion metrics since $\mathcal{Y} \succeq \mathcal{W}$, which is indicated by (15).

D. Controller Synthesis Method

By analyzing the reference tracking error dynamics with a given cycle-to-cycle fuel injection controller, we are able to estimate its closed-loop stability, convergence speed and the stochastic cyclic combustion variation. This allows us to design a stabilizing controller with fast dynamical performance and with a minimal amplification of the stochastic cyclic combustion variation.

To evaluate the stochastic combustion variation, we apply a selection and weighting matrix S to y , since the steady-state variances of some feedback combustion metrics are of less interest. The weighted variation becomes

$$\text{trace}(SY S^\top). \quad (17)$$

Based on the controller synthesis method with guaranteed $\mathcal{H}_2$ norm [13], which is the deterministic counterpart of having a minimal variance, we propose a fuel injection controller design method based on an optimization problem. It computes a stabilizing controller with minimal stochastic combustion variation subject to a certain dynamical performance requirement. The controller design method is presented in the following theorem, which is tailored for the discussed control problem.

Theorem 1: For a given system with G , $\mathcal{W}$, S and desired convergence rate $\alpha \in (0, 1]$, a stabilizing cycle-to-cycle fuel injection controller, that satisfies the dynamical performance requirement with minimal stochastic cyclic variation of feedback combustion metrics, can be computed by solving the optimization problem

$$\min_{\mathcal{H}, \mathcal{Q}, \mathcal{Z}} \text{trace}(\mathcal{H}) \quad (18a)$$

$$s.t. \begin{bmatrix} \mathcal{H} & SG \\ G^\top S^\top & \mathcal{Q} \end{bmatrix} \succ 0, \quad (18b)$$

$$\begin{bmatrix} \mathcal{Q} & \mathcal{Q} - \mathcal{Z}G & \mathcal{Z} \\ \mathcal{Q} - G^\top \mathcal{Z}^\top & \alpha \mathcal{Q} & 0 \\ \mathcal{Z}^\top & 0 & \mathcal{W}^{-1} \end{bmatrix} \succ 0. \quad (18c)$$

The controller is given by $K = \mathcal{Q}^{-1}\mathcal{Z}$.

Proof: The proof is based on showing that (18) implies that (an upper bound on) (17) is minimized, while at the same time (9) holds, allowing us to conclude exponential stability of the tracking error dynamics.

To show the first statement, we substitute $\mathcal{Q} = \mathcal{U}^{-1}$, $\mathcal{Z} = \mathcal{U}^{-1}K$ into (18c), pre- and postmultiply the first row and column of the resulting matrix with $\mathcal{U}$, and apply a Schur complement. This yields that (18c) is equivalent to

$$\mathcal{U} - \frac{1}{\alpha}(I - KG)\mathcal{U}(I - KG)^\top - K\mathcal{W}K^\top \succ 0. \quad (19)$$

Satisfaction of (19) for some $\alpha \in (0, 1]$, implies that the inequality is also satisfied for $\alpha = 1$. Now also using a Schur complement, (18b) can be written as

$$\mathcal{H} \succ SGUG^\top S^\top. \quad (20)$$

The upper bound on the weighted stochastic cyclic combustion variation, as in (17), is minimized by solving (18a), since (20) and (15) hold, and since (19) is satisfied for $\alpha = 1$, which bounds the steady-state covariance (16).

To prove that (9) holds, we pre- and post-multiply (19) with $\sqrt{\alpha}G$ and $\sqrt{\alpha}G^\top$, respectively, leading to

$$\alpha GUG^\top - (I - GK)GUG^\top(I - GK)^\top - \alpha GK\mathcal{W}K^\top G^\top \succ 0. \quad (21)$$

Now since $\mathcal{W} \succ 0$, this expression implies that (9) holds with $\mathcal{P} = GUG^\top \succ 0$. Hence, the stability and dynamical performance requirement with the convergence rate less than α are satisfied. This completes the proof. ■

TABLE I
NOMINAL ENGINE OPERATING CONDITIONS

Engine speed	1200 [rpm]	IMEP	8.1 [bar]
Intake manifold pressure	1.63 [bar]	SOI ₁	-12 [°CA ATDC]
Intake manifold temp.	44 [°C]	DOI ₁	250 [μs]
Exhaust manifold pressure	1.93 [bar]	SOI ₂	-5 [°CA ATDC]
Rail pressure	1630 [bar]	DOI ₂	700 [μs]
EGR fraction	26 [%]	SOI ₃	13 [°CA ATDC]
Air-fuel ratio (λ)	1.6 [-]	DOI ₃	250 [μs]

IV. EXPERIMENTAL RESULTS

Minimal cyclic combustion variation can be achieved by implementing a fuel injection controller that has sufficiently fast reference tracking performance and leads to a minimal amplification of the stochastic cyclic variation. In this section, we experimentally validate the proposed controller design method and illustrate its potential to achieve minimal cyclic combustion variation. We focus on showing the trade-off between the controller's dynamical performance and the stochastic cyclic variation, and not on deterministic variations, such as the rejection of (slowly varying) disturbances. The engine test bench is introduced first. Then, we identify the local combustion model. Finally, multiple controllers are designed with different settings, and their reference tracking performance and the resulting stochastic cyclic variation are compared.

A. Engine Test Bench

Experiments are conducted using the engine test bench CYCLOPS, which is a single cylinder test bench based on a 12.6 [L] six cylinder heavy-duty engine [14]. The isolated exhaust gas recirculation system for the test cylinder enables independent control of the air-path conditions. An uncooled AVL GU21C sensor is used to measure the in-cylinder pressure signal of the test cylinder. Cylinder pressure-based heat release computation, fuel injection controller implementation and data acquisition are carried out in real-time using a FPGA/CPU-based Speedgoat rapid control prototyping system.

B. Combustion Model Identification

We chose an Euro VI type of operating condition for model identification and controller tests. A triple-injection fueling profile is adopted, which contains a pilot, main and post injection pulse. The operating conditions and the fuel injection parameters are listed in Table I. Instead of individually controlling each pulse, as considered in previous work [9], [10], we simplify the control problem and demonstrate the potential of our methodology by only controlling the injection timing (SOI₂) and duration (DOI₂) of the main pulse. The injection timing of the pilot (SOI₁) and post injection pulse (SOI₃) are shifted according to the change of SOI₂. The durations of the pilot and post injection pulse stay unchanged. We consider CA50 and net IMEP as the feedback signal for combustion control. Hence, the controlled fuel injection parameters and the feedback combustion metrics become

$$u = [\text{SOI}_2, \text{DOI}_2]^\top, \quad y = [\text{CA50}, \text{IMEP}]^\top.$$

Fig. 2. Variations of CA50 and IMEP due to the local change of SOI_2 and DOI_2 . Steady-state measurement data is plotted in circles. Polynomial fitting results are plotted in lines. The error bars indicate the variation bound ($\pm 3\sigma$) of each measurement point.

According to Theorem 1, the controller synthesis requires both the local combustion behavior G and the variance matrix of the measurement noise $\mathcal{W}$. To acquire G , we perturb the controlled fuel injection parameters and observe the variations of feedback combustion metrics in steady state. We vary SOI_2 and DOI_2 around their nominal settings listed in Table I, with the variation range of ± 3 [°CA] and ± 100 [μs], respectively. Other engine control parameters, i.e., EGR and VGT valve positions, are kept constant. The resulting variations of CA50 and IMEP in steady state are shown in Fig. 2. For each varied fuel injection profile, the steady-state value of individual feedback combustion metric is computed by averaging its measurements over 100 cycles. The error bars indicate the steady-state variation bound ($\pm 3\sigma$) at each measurement point. As shown in Fig. 2, the variation of SOI_2 mainly influences CA50, while the change of DOI_2 has larger impact on IMEP. The correlations in the off-diagonal plots are weak, which is anticipated. Due to the linear correlations, we fit the steady-state measurement data of individual combustion metric with a first-order polynomial using SOI_2 and DOI_2 as independent variables, which results in two 3-dimensional graphs. Similar observations and methods are included in [9], [10]. The fitting quality is considered satisfying since the coefficient of determination (R^2) for CA50 and IMEP is 0.99 and 0.91, respectively. We also include the intersection between the fitted polynomials and the plotted plane in Fig. 2, which agrees with the data points. The matrix G can be computed by taking derivatives of the fitted polynomials

$$G = \begin{bmatrix} 1.1529 & 0.4411 \\ -0.0389 & 0.8304 \end{bmatrix}.$$

Since SOI_2 and DOI_2 have different dimensions, their variations are normalized with respect to the nominal settings in Table I, so that the entries of G have similar magnitudes.

We assume that d_u and d_e only contain deterministic variations, such that the stochastic variation of feedback combustion metrics originates from the measurement noise. Hence, we

have $\mathcal{Y} = \mathcal{W}$ in steady state when fuel injection parameters are fixed (cf. (15) for $u_k = 0$). Based on the steady-state measurement data over 550 cycles in the nominal operating conditions as listed in Table I, we compute the (co)variance (σ^2) of the feedback combustion metrics $\mathcal{Y} = \mathcal{W}$, using a measured sequence of y_k , leading to

$$\mathcal{W} = \begin{bmatrix} 0.0811 & -0.0207 \\ -0.0207 & 0.0685 \end{bmatrix}.$$

C. Fuel Injection Controller Design and Validation

Using the identified G and $\mathcal{W}$, we design multiple cycle-to-cycle fuel injection controllers to study and compare their dynamical performance and the steady-state cyclic variation. As a proof of concept, we impose S as an identity matrix and choose α equal to 0.2, 0.5 and 0.9. Three different controllers are computed using Theorem 1, which are denoted with K_1 , K_2 and K_3 , respectively. The largest closed-loop eigenvalue for each controller equals the corresponding α . All designed controllers are experimentally tested running the engine test bench in the operating conditions as listed in Table I. Their closed-loop stability and dynamical performance are compared by tracking the same reference signals. The step reference changes of CA50 and IMEP are ± 2 [°CA] and ± 1 [bar], respectively. This results in negligible change of the engine operating conditions. As shown in Fig. 3, all controllers remain stable and quickly track the reference changes by manipulating the fuel injection parameters. It is difficult to distinguish the dynamical performance of K_1 and K_2 in Fig. 3, while the performance of K_3 is significantly slower. However, among all controllers, K_3 leads to the smallest steady-state variations of both controlled fuel injection parameters and feedback combustion metrics. As we did not vary the operating conditions in steady state, the cyclic combustion variation is purely stochastic. Hence, the measurement results indicate that smaller α leads to faster dynamical performance and larger stochastic variation.

We further analyze the measurement results shown in Fig. 3 to quantify the trade-off between fast dynamical performance and small stochastic cyclic combustion variation. We denote the controller's dynamical performance with the settling time, which is defined as the number of combustion cycles it takes to reach the steady-state value by following a step reference change. The settling time for each controller is computed by averaging the settling time for an 8-step reference change as shown in Fig. 3. The stochastic cyclic combustion variation for each controller is computed according to (17) based on the steady-state measurement data (with zero reference change) over 800 cycles. The dynamical performance and the stochastic cyclic variation of different controllers are compared in Table II. With larger α , the settling time increases while the stochastic cyclic variation decreases. This indicates the trade-off between fast dynamical performance and small stochastic cyclic combustion variation. Hence, it is necessary to properly select α to balance these two performance criteria, such that the overall cyclic combustion variation is minimized. We also include the estimated upper bound of the stochastic cyclic variation in Table II, which is computed using (17).

Fig. 3. Reference tracking behavior for K_1 , K_2 and K_3 to illustrate capabilities to reduce cyclic variation.

It is noticed that the measured cyclic variations exceed the estimations. This difference results from unconsidered system dynamics, e.g., a delayed feedback loop. Nevertheless, they both decrease with larger α . By computing $\text{trace}(SW S^T)$, the stochastic cyclic variation with open-loop control is 0.1496, which is lower than the measured cyclic variations listed in Table II. This confirms the discussion in Section III-C that the closed-loop stochastic cyclic variation is larger than the open-loop one.

V. CONCLUSIONS

In this paper, we have analyzed both the deterministic and stochastic cyclic combustion variation with cycle-to-cycle fuel injection control for diesel engines with multi-pulse fuel injection using in-cylinder pressure measurement. To achieve minimal overall cyclic combustion variation, a novel and systematic controller design method has been proposed in the form of an optimization problem. The solution of this optimization problem yields a stabilizing controller with sufficiently fast dynamical performance and leads to a minimal amplification of the stochastic cyclic variation. Using

TABLE II
CONTROLLER PERFORMANCE COMPARISON

Performance criterion	K_1	K_2	K_3
Convergence rate upper bound (α)	0.2	0.5	0.9
Settling time [cycles]	5	10	30
Measured stochastic cyclic variation	0.4354	0.3322	0.2096
Estimated sto. variation upper bound	0.3544	0.2776	0.1751

a single cylinder engine test setup, we have experimentally demonstrated the feasibility of the proposed controller design method by comparing the reference tracking behavior of multiple controllers with different design parameters. The potential to achieve minimal cyclic combustion variation has been illustrated with the trade-off between the controller's fast dynamical performance and small stochastic cyclic variation. Based on the promising results of this paper, it is of further interest to consider dynamical engine operating conditions in the controller design method.

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