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Original Research Paper

Heat transfer by nanofluids in wavy microchannels

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ABSTRACT

Pumping coolants through microchannels with well-defined structures along micro-electronic devices is a typical approach to remove the heat. It has been found recently that so-called nanofluids, i.e. dilute water-Cu or water- Al_2O_3 suspensions with a particle diameter of 100–150 nm, are highly efficient coolants. Numerical simulations can help to optimize the microchannel structures and typically homogenous single-phase models are applied. However, these underestimate the experimental results. An alternative approach is two-phase models based on an Eulerian approach for the base fluid and a Lagrangian description of the suspended particles. In this paper we follow that route and solve the three-dimensional governing equations including continuity, Navier-Stokes and energy equations with the well-known SIMPLE method. The governing equations for particles are solved by a 4th order Runge-Kutta algorithm. We focus on a wavy microchannel structure and demonstrate that the disagreement between the two simulation approaches is due to the non-homogeneous particle distribution in the domain. We also find that the Nusselt number increases with the increase in volume fraction and the decrease in particle diameter and that it is about three times higher for a nanofluid in a wavy microchannels as compared to water in a straight microchannels.

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1. Introduction

Adding metal nanoparticles to a fluid improves the thermal conductivity coefficient of the working fluid. However, the exact relation between fluid and particle properties, as well as parameters like particle volume concentration, the exact flow field, or the impact of geometrical constraints is still not fully understood [1,2]. Numerous approximate relations are available in the literature to predict the properties of a nanofluid such as its thermal conductivity and viscosity [1,2]. To date, none of them allows a precise prediction of the behavior of nanofluids for a wide range of parameters and flowing in arbitrarily shaped microchannel geometries. In particular, the theoretical results underestimate the experimentally found heat transfer characteristics of nanofluids [3].

We use a two-phase Eulerian-Lagrangian model to study. By treating the nanoparticles and the fluid separately, they can be described using their own governing equations and no assumptions about the combined system, the nanofluid, are needed. This

is a strong advantage as compared to single phase models which assume that the particles are homogeneously distributed and that the particle velocity and temperature are identical to the fluid velocity and temperature. These advantages come at the price of increased computing costs making an efficient parallelization and the use of parallel computers mandatory.

The current paper focuses on nanofluids in wavy microchannels. General flow and heat transfer characteristics in microchannels with wavy walls were numerically studied by Gong et al. [4]. They showed that the performance of microchannels increases with increasing Reynolds number and that channel with symmetric wavy walls perform better than asymmetric ones. They further expressed that wavy microchannels can be attractive candidates for cooling high heat flux micro-electronic devices in the future. Sui and Teo [5] studied fluid mixing and heat transfer enhancement in three-dimensional wavy microchannels. They used water as a coolant and concluded that the increase in mass flow rate increases the Nusselt number. Mohammed et al. [6] simulated heat transfer enhancement in heat sinks consisting of wavy microchannels. They demonstrated that wavy microchannels allow to enhance the heat transfer with only a small pressure drop and reported the optimum amplitude of the wavy structure to obtain

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Nomenclature

a	wall amplitude (m)	t	wall thickness (m)
A_c	cross sectional area	T	nondimensional temperature
b	power of volume fraction in Eq. (24)	$U.V.W$	velocity component in x,y,z directions (m s^{-1})
c	power of particle diameter in Eq. (24)	$u.v.w$	nondimensional velocity component in x,y,z-direction
C_p	heat capacity (kJ/kg K)	$X.Y.Z$	x,y,z coordinate (m)
D_h	hydraulic diameter (m)	$x.y.z$	nondimensional X, Y, Z
D_p	particle diameter (m)		
d_p	nondimensional particle diameter		
f	normalized pressure drop	Greek symbols	
F_b	buoyancy force (N)	ψ	general variable (u, v, w) in Eq. (6)
F_{br}	brownian force (N)	ϕ	particle volume fraction
F_D	drag force (N)	λ	wall wavelength (m)
F_g	gravity force (N)	μ	viscosity (Pa s)
F_L	Saffman's lift force (N)	ρ	density (kg m^{-3})
F_p	pressure gradient force (N)	θ	temperature (K)
F_T	thermophoretic force (N)	τ	time (s)
H	height of the microchannel (m)	$\xi.\eta.\zeta$	curvilinear coordinate components
J	jacobian of the coordinate transformation	δV	volume of Eulerian cells
k	thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)		
$L_x.L_y.L_z$	micro device dimensions (m)	Subscripts	
n	normal direction on the walls	B	fluid bulk value
Nu	Nusselt number	f	fluid
$Nu.A$	Nusselt number and area Multiplication	int	solid-fluid interface
np	number of particles	nf	nanofluid
$P.p$	dimensional and nondimensional pressure (Pa)	p	particle
Pe	Peclet number	r	ratio of Nusselt number of nanofluid to pure water
Pr	Prandtl number	s	solid
q''	heat flux at the bottom of the micro-device (W m^{-2})	sf	solid to fluid ratio
$q_{11}.q_{22}.q_{33}$	parameters defined in Eq. (8)		
Re	Reynolds number	Superscripts	
S	width of the microchannel (m)	c	contravariant velocities

the maximum cooling performance. Rostami et al. [7] studied conjugated heat transfer in wavy microchannels to optimize the geometry for the maximum Nusselt number. They found that the intensity of secondary flows and the size of recirculation zones depend on geometrical parameters, so that there is an optimum geometry which has the maximum Nusselt number.

Another way to increase the heat transfer is the change in fluid properties. Adding nano-sized metal powder to the base fluid improves heat transfer properties of the working fluid, i.e. known that using nanofluids instead of traditional fluids in heat transfer applications results in an increase of the Nusselt number [9,10,12,13].

Jung et al. [13] experimentally studied forced convective heat transfer of nanofluid in a single microchannel. Together with the two base fluids water and Ethylene glycol, Al_2O_3 nanoparticles with a diameter of 170 nm and volume concentration between 0.6 and 1.8% were used. They found larger heat transfer coefficients for smaller channel cross sections and reported a 32% increase in the measured heat transfer coefficient for nanofluids with 1.8% particle volume fraction.

As stated above, single-phase simulations are based on empirical relations for properties such as the thermal conductivity or effective viscosity. In addition, they are known to under predict the experimentally measured enhancement of the heat transfer properties of nanofluids [8,9]. Therefore, two-phase models are a better choice. Kalteh et al. [8,9] performed two-dimensional simulations of heat transfer enhancement in a microchannel heat sink utilizing nanofluid. They modeled the nanofluid by the Eulerian-Eulerian method, where the particles and the fluid are described by continuous Eulerian fields. They found that two-phase modeling

is suited to reproduce experimental data. Several authors have applied the two-phase Eulerian-Lagrangian model, where the fluid is described on an Eulerian grid and the particles follow Lagrangian trajectories, in circular microchannels using the FLUENT software package [10–12]. Bianco et al. [10] investigated two-phase nanofluid forced convection in a two-dimensional circular tube. The nanofluid was water with Al_2O_3 particles of 100 nm diameter. The maximum difference in the obtained heat transfer coefficient between single-phase and two-phase models is about 11% and the heat transfer coefficient for two-phase model was larger than that of the base fluid. Wen et al. [11] and He et al. [12] have done similar studies in pipe flow.

Most previous articles are limited to two dimensional problems, while there are only few publications on three-dimensional heat transfer problems. For example, three-dimensional natural convection heat transfer by nanofluids in an enclosure has been investigated by Purusothaman et al. [14,15].

Rebay et al. [16] and Bianco et al. [17] introduce the microscale and nanoscale heat transfer characteristics due to the presence of nanofluids numerically and experimentally.

In the case of nanofluid in wavy channels, Rashidi et al. [18] studied different methods including Eulerian-Eulerian, VOF and mixture methods in two phase models. They showed that the Eulerian-Eulerian method underestimates the heat transfer coefficient in comparison with other two phase models.

To the best of the author's knowledge, there is no study of two-phase modeling by means of an Eulerian-Lagrangian method of nanofluids in wavy microchannels available in the literature. We close this gap by studying three-dimensional conjugate heat transfer in nanofluid-filled wavy microchannels using the Eulerian-

Lagrangian two-phase model and predict the Nusselt number in wavy microchannel heat sinks.

2. Governing equations and boundary conditions

Fig. 1 shows the geometry of the wavy micro-channels on a micro-device. We choose the dimensions of the micro-device to be $10 \times 10 \times 0.5 \text{ mm}^3$. The flow is laminar, in, steady state, three dimensional, incompressible and periodic. A channel is defined by its wavelength (λ), amplitude (a), height (H), wall thickness (t) and width (S). In our example, the wavelength of the 100 parallel microchannels is $\lambda = 200 \mu\text{m}$. The hydraulic diameter is $D_h = 90 \mu\text{m}$ and other parameters are $S = 72 \mu\text{m}$, $H = 120 \mu\text{m}$ and $t = 28 \mu\text{m}$. These parameters are selected based on our previous study [7] as a geometry with maximum Nusselt number. For the simulations, it is sufficient to investigate the flow in a unit cell as defined in Fig. 1.

Another characteristic of the wavy channels is the critical Reynolds number to transition from laminar flow to turbulence. In the wavy walls channel the critical Reynolds number depends on $2a/\lambda$ and is less than the critical Reynolds number ($Re_c = 2300$) for straight channels [19]. Tolentino et al. [20] have shown that the critical Reynolds number is even less than 250 for $2a/\lambda = 0.18$. Therefore, to be sure that the flow remains laminar and to avoid a large pressure drop, we assure $Re < 250$ and $2a/\lambda = 0.2$.

Our base fluid has the properties of water, the nano-particle parameters correspond to Cu and Al_2O_3 and the body is made of silicon.

We neglect the interactions between particles, because the particle diameter is small and the volume fraction is less than 0.02. A

two-way coupling between the fluid and particle phases is employed and appears in the momentum and energy equations of the fluid as source terms.

Due to the wavy boundaries, we use a body fitted mesh. Then, the governing equations are transformed from the Cartesian coordinate system (x, y, z) to the boundary-fitted coordinate system (ξ, η, ζ) by introducing the following non-dimensional parameters:

$$\begin{aligned} x_i &= \frac{X_i}{D_h} \cdot \lambda^* = \frac{\lambda}{D_h} \cdot d_p = \frac{D_p}{D_h} \cdot k_{sf} = \frac{k_s}{k_f} \cdot u_i = \frac{U_i}{U_B} \cdot p = \frac{P}{\rho_f U_B^2} \\ t^* &= \frac{\tau U_B}{D_h} \cdot T = \frac{k_f \theta}{q'' D_h} \cdot Re = \frac{\rho_f U_B D_h}{\mu} \cdot Pr = \frac{\mu C p_f}{k_f} \cdot Pe = Re \cdot Pr \end{aligned} \quad (1)$$

The governing equations including continuity, momentum and energy in a general non-orthogonal curvilinear coordinate framework with (ξ, η, ζ) can be expressed as follows [21,22]:

Continuity:

$$\frac{\partial(1-\phi)u^c}{\partial \xi} + \frac{\partial(1-\phi)v^c}{\partial \eta} + \frac{\partial(1-\phi)w^c}{\partial \zeta} = 0 \quad (2)$$

with the definitions

$$\begin{aligned} u^c &= J(\xi_x + \xi_y + \xi_z) \\ v^c &= J(\eta_x + \eta_y + \eta_z) \\ w^c &= J(\zeta_x + \zeta_y + \zeta_z) \end{aligned} \quad (3)$$

where “J” is the Jacobean of the coordinate transformation and is equal to the volume of each Eulerian cell (δV).

$$J = \delta V = x_\xi y_\eta z_\zeta + x_\zeta y_\xi z_\eta + x_\eta y_\zeta z_\xi - x_\xi y_\zeta z_\eta - x_\zeta y_\eta z_\xi - x_\eta y_\xi z_\zeta \quad (4)$$

Fig. 1. Geometry of the wavy microchannel and computational domain.

$$J \begin{bmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \varsigma_x & \varsigma_y & \varsigma_z \end{bmatrix} = \begin{bmatrix} Y_\eta Z_\varsigma - Y_\varsigma Z_\eta & Z_\eta X_\varsigma - Z_\varsigma X_\eta & X_\eta Y_\varsigma - X_\varsigma Y_\eta \\ Y_\varsigma Z_\xi - Y_\xi Z_\varsigma & Z_\varsigma X_\xi - Z_\xi X_\varsigma & X_\varsigma Y_\xi - X_\xi Y_\varsigma \\ Y_\xi Z_\eta - Y_\eta Z_\xi & Z_\xi X_\eta - Z_\eta X_\xi & X_\xi Y_\eta - X_\eta Y_\xi \end{bmatrix} \quad (5)$$

Momentum:

$$\begin{aligned} \frac{\partial(1-\phi)u^c\psi}{\partial\xi} + \frac{\partial(1-\phi)v^c\psi}{\partial\eta} + \frac{\partial(1-\phi)w^c\psi}{\partial\varsigma} &= -\frac{\pi}{6}d_p^3 \frac{\rho_p}{\rho_f} \sum_{j=1}^{np} \frac{d\psi_{pj}}{d\tau} + \\ \frac{\partial}{\partial\xi} \left[\frac{(1-\phi)Jq_{11}}{Re} \psi_\xi \right] + \frac{\partial}{\partial\xi} \left[\frac{(1-\phi)J}{Re} (q_{12}\psi_\eta + q_{13}\psi_\varsigma) \right] + \\ \frac{\partial}{\partial\eta} \left[\frac{(1-\phi)Jq_{22}}{Re} \psi_\eta \right] + \frac{\partial}{\partial\eta} \left[\frac{(1-\phi)J}{Re} (q_{12}\psi_\xi + q_{23}\psi_\varsigma) \right] + \\ \frac{\partial}{\partial\varsigma} \left[\frac{(1-\phi)Jq_{33}}{Re} \psi_\varsigma \right] + \frac{\partial}{\partial\varsigma} \left[\frac{(1-\phi)J}{Re} (q_{13}\psi_\xi + q_{23}\psi_\eta) \right] + (1-\phi)JS_p^\psi. \end{aligned} \quad (6)$$

where ψ is u , v and w and

$$\begin{aligned} JS_p^u &= -J(p_\xi \xi_x + p_\eta \eta_x + p_\varsigma \varsigma_x) \\ JS_p^v &= -J(p_\xi \xi_y + p_\eta \eta_y + p_\varsigma \varsigma_y) \\ JS_p^w &= -J(p_\xi \xi_z + p_\eta \eta_z + p_\varsigma \varsigma_z) \end{aligned} \quad (7)$$

Also:

$$\begin{aligned} q_{11} &= \xi_x \xi_x + \xi_y \xi_y + \xi_z \xi_z, q_{22} = \eta_x \eta_x + \eta_y \eta_y + \eta_z \eta_z \\ q_{33} &= \varsigma_x \varsigma_x + \varsigma_y \varsigma_y + \varsigma_z \varsigma_z, q_{12} = \xi_x \eta_x + \xi_y \eta_y + \xi_z \eta_z \\ q_{13} &= \xi_x \varsigma_x + \xi_y \varsigma_y + \xi_z \varsigma_z, q_{23} = \eta_x \varsigma_x + \eta_y \varsigma_y + \eta_z \varsigma_z \end{aligned} \quad (8)$$

Energy in fluid:

$$\begin{aligned} \frac{\partial(1-\phi)u^cT}{\partial\xi} + \frac{\partial(1-\phi)v^cT}{\partial\eta} + \frac{\partial(1-\phi)w^cT}{\partial\varsigma} &= -\frac{\pi}{6}d_p^3 \frac{(\rho Cp)_p}{(\rho Cp)_f} \sum_{j=1}^{np} \frac{dT_{pj}}{d\tau} + \\ \frac{\partial}{\partial\xi} \left[\frac{(1-\phi)Jq_{11}}{Pe} T_\xi \right] + \frac{\partial}{\partial\xi} \left[\frac{(1-\phi)J}{Pe} (q_{12}T_\eta + q_{13}T_\varsigma) \right] + \\ \frac{\partial}{\partial\eta} \left[\frac{J(1-\phi)q_{22}}{Pe} T_\eta \right] + \frac{\partial}{\partial\eta} \left[\frac{(1-\phi)J}{Pe} (q_{12}T_\xi + q_{23}T_\varsigma) \right] + \\ \frac{\partial}{\partial\varsigma} \left[\frac{(1-\phi)Jq_{33}}{Pe} T_\varsigma \right] + \frac{\partial}{\partial\varsigma} \left[\frac{(1-\phi)J}{Pe} (q_{13}T_\xi + q_{23}T_\eta) \right]. \end{aligned} \quad (9)$$

Energy in solid:

$$\begin{aligned} \frac{\partial}{\partial\xi} (Jq_{11}T_\xi) + \frac{\partial}{\partial\xi} (Jq_{12}T_\eta + Jq_{13}T_\varsigma) + \\ \frac{\partial}{\partial\eta} (Jq_{22}T_\eta) + \frac{\partial}{\partial\eta} (Jq_{12}T_\xi + Jq_{23}T_\varsigma) + \\ \frac{\partial}{\partial\varsigma} (Jq_{33}T_\varsigma) + \frac{\partial}{\partial\varsigma} (Jq_{13}T_\xi + Jq_{23}T_\eta) = 0 \end{aligned} \quad (10)$$

where the second term in the momentum and the first term in the energy equations are the source terms due to the presence of nanoparticles as defined by [23]. Also, “ δV ” is the volume of the Eulerian cells and “ np ” is the number of nano-particles in the Eulerian cell.

The Knudsen number is less than 0.001. This allows us to assume no slip conditions for the velocity ($u = v = w = 0$) and no jump of the temperature at wall boundaries. At the inlet and outlet, periodic boundary conditions are applied:

$$\begin{aligned} u(0,y,z) &= u(0.5\lambda^*,s-y,z) \\ v(0,y,z) &= -v(0.5\lambda^*,s-y,z) \\ w(0,y,z) &= w(0.5\lambda^*,s-y,z). \end{aligned} \quad (11)$$

For the temperature field, there are four boundary conditions:

1. At the inlet and outlet boundaries, periodic boundary conditions are used.

$$\begin{aligned} T(0,y,z) &= T(0.5\lambda^*,s-y,z) - \Delta T_B \\ T_s(0,y,z) &= T_s(0.5\lambda^*,s-y,z) - \Delta T_B \\ \Delta T_B &= \frac{(0.5\lambda)(t+S)}{A_c Pe} A_c = S.H \end{aligned} \quad (12)$$

where ΔT_B is the rise in fluid bulk temperature from ($x = 0$) to ($x = 0.5\lambda^*$) and “ $t + S$ ” is the pitch of the microchannel which corresponds to 100 μm . In addition, $(0.5\lambda)(t + S)$ is the part of the bottom area of the micro-device where the heat flux is applied to.

2. In the “ y ” direction for one pitch, the boundary condition is also periodic because of the geometry.

3. The top wall is insulated and a constant heat flux is applied to the bottom walls:

$$q'' = -k_s \frac{\partial \theta_s}{\partial Z} \Big|_{bottom} \rightarrow \frac{\partial \theta_s}{\partial Z} \Big|_{bottom} = -\frac{1}{k_{sf}} \quad (13)$$

4. On the interface between solid and fluid, the conjugate boundary condition is applied. This condition is based on continuous flux and temperature at the interface and defined as follows:

$$\begin{aligned} \theta_s &= \theta_f \\ k_s \frac{\partial \theta}{\partial N} \Big|_s &= k_{nf} \frac{\partial \theta}{\partial N} \Big|_{nf} \rightarrow k_s \frac{\partial T}{\partial n} \Big|_s = k_{nf} \frac{\partial T}{\partial n} \Big|_{nf} \end{aligned} \quad (14)$$

where “ n ” is the direction normal to the surface and T_{nf} is the temperature of the nanofluid in each cell based on Eq. (15). By applying the conjugate boundary condition, a given temperature on the interface is achieved.

$$T_{nf} = \frac{(1-\phi)(\rho Cp)_f T + \sum_{np} (\rho Cp)_p T_p}{(1-\phi)(\rho Cp)_f + \phi(\rho Cp)_p} \quad (15)$$

All of the boundary conditions for temperature based on Fig. 1 are tabulated as follows: Table 1

Governing equations for particles:

The particle motion is studied in the Lagrangian view and their governing equations are as follows:

Particle positions:

$$\frac{dX_p}{d\tau} = U_p. \quad (16)$$

Momentum:

Forces are summed up in the momentum equation as [11].

$$m_p \frac{dU_p}{d\tau} = F_D + F_L + F_g + F_b + F_{br} + F_p + F_T. \quad (17)$$

The forces are the drag force, the Saffman lift, the gravity force, buoyancy, forces causing Brownian motion, the fluid pressure gradient and thermo-phoretic forces respectively (see [24]).

Energy:

The energy equation for particles is [25]

$$\rho_p Cp_p \frac{d\theta_p}{d\tau} = \frac{6h_p}{D_p} (\theta - \theta_p) \quad (18)$$

where “ h_p ” is defined as [26]

$$Nu_p = \frac{h_p d_p}{k_f} = 2 + 1.1 Re_p^{0.6} Pr^{0.333} \quad (19)$$

To define the nanofluid bulk temperature, θ_B is evaluated at constant ξ and used in the Nusselt number calculation as:

Table 1
Boundary conditions for temperature.

	Inlet/outlet	Interface	Right (z direction)	Left (z direction)	Up/down (y direction)
Nanofluid	Periodic	Conjugate	Insulated	Conjugate	Conjugate
Body	Periodic	Conjugate	Insulated	Constant Heat Flux	Periodic

$$Nu_x = \frac{k_{nf}}{k_f} \frac{\left. \frac{\partial \theta}{\partial n} \right|_{int}}{(\theta_w - \theta_B)} \quad (20)$$

where the subscript “int” denotes the solid-fluid interface,

$$\theta_B = \frac{(\rho C_p)_f \int (1 - \phi) U \theta d(\delta V) + \frac{\pi}{6} D_p^3 (\rho C_p)_p \sum_{j=1}^{np0} U_{pj} \theta_{pj}}{\rho_f C_{p_{nf}} \int (1 - \phi) U d(\delta V) + \frac{\pi}{6} D_p^3 \rho_p C_{p_{nf}} \sum_{j=1}^{np0} U_{pj}} \quad (21)$$

where “np0” is the number of the particles in the cells in the same section.

3. Numerical solution

The governing equations for the fluid and the solid body are solved using the finite volume technique on a collocated grid. In this arrangement, all parameters such as velocity components (u, v, w), pressure and temperature are located at the same nodes. The HYBRID differencing [27] is used for the approximation of the convective terms. For velocity and pressure the SIMPLE algorithm [28] employed. The grid points are generated in a non-uniform manner with a higher concentration of points close to the walls.

To eliminate the checkerboard pressure due to using a collocated grid, the Rhie and Chow [29] interpolation is used in the pressure correction equation. To ensure that the results of the numerical study are independent of the computational grid, a grid sensitivity analysis is carried out and the mesh with $45 \times 36 \times 56$ grid points is selected [24] Table 2.

After solving the fluid, the equations for the particles are solved by a 4th order Runge-Kutta method. Then, velocities, temperature and positions of each particle obtained and the source terms to be used in the fluid governing equations are calculated. After that, we solve again for the fluid using the updated source terms and iterate this procedure until the Nusselt number converges. The minimum number of the particles is 4.1×10^6 for $D_p = 150$ nm. $\phi = 0.0025$ and the maximum number is 3.3×10^7 particles for $D_p = 100$ nm. $\phi = 0.02$. The solver for the governing equations is parallelized using MPI and depending on the number of particles 16–256 CPUs of a state of the art Linux cluster are used.

4. Results and discussion

Results are carried out for a range of volume fractions from 0 to 2% and particle diameters of 100–150 nm for two types of nanoparticles namely Cu and Al_2O_3 . We choose these two different materials because of their different thermal conductivities. In order to verify the accuracy of the present numerical study, the results in

a fully developed region are validated against the analytical results for forced convection flow in straight square ducts by Tunc and Bayazitoglu [30]. In the case of the wavy channel, the results are compared with data obtained by Muley et al. [31]. The present results are in good agreement with [30,31]. In the case of nanofluid, results are compared with experimental results of Jung et al. [13]. The agreement between the present method and the results of [13,30,31] has been shown in our previous work [24]. Also, the physics of the flow and temperature field in the wavy walls microchannels has been illustrated in our previous articles [7,24].

It was proposed by Heris et. al. [3], Pak and Cho [32], Xuan and Li [33] and Xuan and Roetzel [34], that convective heat transfer enhancement is mainly due to the dispersion of the suspended nanoparticles. Then, the main source of disagreement between the experimental data and the single-phase homogeneous model is that Brownian forces are neglected in those simulations. However, in the current study we show that the main force which affects the Nusselt number is indeed the drag force and not the Brownian force. To assure this, we run simulations including all forces given in Eq. (17) and compare the results to simulations where we remove all contributing forces except the Brownian and drag force. In this case, the Nusselt number changes by more than 0.5%. If we also, neglect the Brownian force, the Nusselt number differs by less than 0.1%. This subject has been claimed by Bongiorno [35]. He studied heat transfer in the nanofluids by order of magnitude analysis without solving the governing equations in a certain domain. He showed that the Brownian force doesn't affect the Nusselt number. Thus, the question to be asked is what is the source of difference between the homogeneous model and experimental data? There are three main assumptions in the single-phase model. First, the velocity of the particle is equal to the velocity of its surrounding fluid. Second, the temperature of a particle is equal to the temperature of its surrounding fluid. Third, the particles are distributed homogeneously in the domain. These assumptions are not required for the two-phase model used in this study and we investigate the validity of λ^* these assumptions in Figs. 2–4.

Fig. 2. Velocity of a randomly selected particle and its surrounding fluid.

Table 2
Grid independency.

Grid	No. of nodes	Nu
$36 \times 26 \times 42$	39,312	10.013
$40 \times 30 \times 46$	55,200	9.823
$44 \times 32 \times 50$	70,400	9.748
$48 \times 36 \times 56$	96,768	9.630
$52 \times 38 \times 60$	118,560	9.613

Fig. 3. Temperature of a randomly selected particle and its surrounding fluid.

Fig. 2 shows the velocity of a randomly selected particle and the velocity of its surrounding fluid. It can be seen that the velocity of the particle fluctuates due to the Brownian motion. However, the average velocity of the particle is equal to the velocity of the surrounding fluid, which justifies the first assumption of the homogeneous single-phase model.

Fig. 3 depicts the temperature of a randomly selected particle and the temperature of its surrounding fluid. The data clearly demonstrates the similarity of the particle temperature and the temperature of the fluid. Thus, we confirm that the second assumption of the homogeneous single-phase model is correct.

Fig. 4 shows the time averaged particle distribution in the fluid. Fig. 4(a) depicts the volume fraction at the outlet section ($x = 0.5$) and Fig. 4(b) in the flow direction (xy -plane). It can be seen that the particles are not homogeneously distributed. The particle distribution strongly depends on the fluid velocity and the upstream flow as shown and reported by Mirzaei et al. [36]. A higher concentration of particles in some parts of the domain affects the flow and heat transfer fields and has significant effects on the local Nusselt number. Following his argument that the Brownian motion does not affect the Nusselt number, Boungiorno [30] claimed that if we know the particle distribution we can solve the single-phase model

equations. However, the main problem is the knowledge of the particle distribution. If it is non-homogeneous, it might be the main source of difference between the results of single and two-phase models. With the two-phase Eulerian-Lagrangian model at hand we do have direct access to the particle distribution and are able to investigate this effect.

In Figs. 2–4 we discuss the three main assumptions of the homogeneous single-phase model. However, using Figs. 5, 6 we justify the higher pressure drop of Cu in comparison with Al_2O_3 . The velocity profile at the exit section of the channel is shown in Fig. 5 for different nanofluids at $\phi = 0.02$. It can be seen, that water-Cu shows a slightly larger velocity gradient near the wall. The density of Cu is higher than that of Al_2O_3 and obviously pure water. Thus, the impact of inertia plays a larger role for Cu, i.e. inertial forces are stronger in the u -momentum equation, resulting in a higher velocity gradient near the wall for Cu in comparison with Al_2O_3 .

According to Fig. 5, the pressure drop of nanofluids is illustrated in Fig. 6. The pressure drop increases with ϕ . It is clear that due to the higher velocity gradient near the wall the water-Cu nanofluid

Fig. 5. u -profile for $\text{Re} = 200$ at $x = 0.5 \lambda^*$ and $Z = -0.5H$ at the $\phi = 0.02$.

(a)

(b)

Fig. 4. Particle distribution for $\text{Re} = 200$ at $Z = -0.5H$ for $\phi = 0.02$ (a) at $x = 0.5 \lambda^*$ (b) at the “ xy ” plane.

Fig. 6. Pressure drop versus ϕ .

shows a higher pressure drop in comparison to water- Al_2O_3 . Also, the difference between the pressure drop of both nanofluids increases with ϕ . The pressure drop for water-Cu with $D_p = 100 \text{ nm}$, $\phi = 0.02$ is about 10.6% more than the pressure drop for pure water.

As shown in detail in our previous papers [7,24], due to secondary flows and recirculation zones in the wavy microchannels, the Nusselt number is increased in comparison with straight micro-channels. Also, by adding nano-sized metal particle to the base fluid the thermal conductivity of the working fluid increases the Nusselt number. Using nanofluid with insignificant increase in pressure drop and large increase in Nusselt number is a suitable way to enhance heat transfer in the wavy microchannels. The mean Nusselt number is shown in Fig. 7 versus ϕ for water- Al_2O_3 . It can be seen that it increases with ϕ . At the same volume fraction, but with decreased particle diameter and increased particle number, the mean Nusselt number increases. The mean Nusselt number for pure water is 9.58 but it is 10.75 for water- Al_2O_3 with $D_p = 100 \text{ nm}$, $\phi = 0.02$. In other words, the Nusselt number increment is 12.2% in comparison with pure water.

Fig. 7. Mean Nusselt number for water- Al_2O_3 versus ϕ .

Fig. 8 shows the mean Nusselt number versus ϕ for water-Cu. It can be seen that the Nusselt number linearly increases with ϕ . Also, the obtained Nusselt numbers are larger for smaller particles. The Nusselt number is 12.55 for $D_p = 100 \text{ nm}$, $\phi = 0.02$. This corresponds to an increase of about 31% in comparison with pure water. This might be due to larger thermal conduction coefficient of Cu in comparison to Al_2O_3 .

The relevant parameters determining the heat transfer rate are the Nusselt number and the interfacial area between the fluid and the body. Then the multiplication of the Nusselt number and the interfacial area for nanofluid in the wavy microchannel can be compared with pure water in a straight microchannel:

$$Nu.A_r = \frac{(Nu.A)_{nf}^{wavy}}{(Nu.A)_{water}^{straight}} \quad (22)$$

This results in a $Nu.A_r$ for water- Al_2O_3 of 2.83 and of 3.3 for water-Cu.

Fig. 9 illustrates the ratio of nanofluid and base fluid Nusselt numbers for different Reynolds numbers. It is obvious that the maximum ratio is obtained at $Re = 225$. In other words, adding metal particles to the base fluid is more significant in laminar flow. Adding metal particles to the fluid increases the thermal conductivity and this property appears in the diffusion terms of the energy equation. It is clear that increasing the Reynolds number (convective terms) is a better way to increase the Nusselt number. But, adding nanoparticles increases the “k” in diffusive terms of the energy equation. The diffusive terms are more effective near the wall, where the thermal boundary condition affects the domain. Also, an increase of “k” is more sensible at low Reynolds numbers, where the diffusive terms play more sensible role in compare with the convective terms in the energy equation of the fluid.

According to the obtained Nusselt number for both nanofluid in Figs. 7 and 8 a correlation including Reynolds number, particle diameter and volume fraction can be obtained:

$$Nu_0 = 1.63 \times 10^{-4}(Re + 50)^{1.9} + 3.7. \quad \text{Pure water} \quad (23)$$

$$Nu = Nu_0 \left(1 + C_N \frac{\phi^b}{D_p^a} \right) \quad \text{Nanofluid.} \quad (24)$$

where a, b and C_N are presented in Table 3.

Fig. 8. Mean Nusselt number for water-Cu versus ϕ .

Fig. 9. Nusselt number ratio versus Re in two different ϕ and (a) $D_p = 150$ nm for water- Al_2O_3 (b) $D_p = 120$ nm for water-Cu.

Table 3
Parameters in Eq. (24).

Particle	b	c	C_N
Cu	1	1.08	$0.01 - 7.48 \times 10^{-7}(Re - 210)^2$
Al_2O_3	0.75	0.3375	$0.2428 - 1.113 \times 10^{-5}(Re - 230)^2$

5. Conclusions

In this paper, conjugate heat transfer in a wavy microchannel using water-Cu and water- Al_2O_3 nanofluid has been studied with two-phase Eulerian-Lagrangian simulation. The three-dimensional governing equations in the fluid (Eulerian), the solid domain (Eulerian) and for the particles (Lagrangian) have been solved by the appropriate methods. The most important result of this paper is the explanation of the origin of the disagreement between the homogeneous single-phase model and two-phase data. It is not due to neglecting Brownian forces in nanofluid properties, but due to an incorrectly assumed homogeneous particle distribution. Due to the presence of recirculation zones and secondary flows, the area-heat transfer rate and pressure drop are increased in a wavy microchannel as compared to a straight microchannel by 174% and 195.7%, respectively. In comparison with pure water, the area-Nusselt number increases by about 33.2% for water-Cu with $D_p = 100$ nm, $\phi = 0.02$, whereas the increase in pressure drop is less than 10.6%. Thus, using nanofluid is an attractive way to increase the heat transfer rate in wavy microchannels without a significant increase of the pressure drop.

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