A shortest path algorithm for the departure time and speed optimization problem

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A Shortest-Path Algorithm for the Departure Time and Speed Optimization Problem

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Abstract. We present a shortest-path algorithm for the departure time and speed optimization problem under traffic congestion. The objective of the problem is to determine an optimal schedule for a vehicle visiting a fixed sequence of customer locations to minimize a total cost function encompassing emissions cost and labor cost. We account for the presence of traffic congestion, which limits the vehicle speed during peak hours. We show how to cast this problem as a shortest-path problem by exploiting some structural results of the optimal solution. We illustrate the solution method and discuss some properties of the problem.

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Keywords: speed optimization • shortest path • scheduling

1. Introduction

In recent years, there has been a growing interest in models and algorithms for routing and scheduling problems in which fuel consumption and related CO_{2e} emissions are taken into account. This line of research is rooted in the works of Fagerholt, Laporte, and Norstad (2010); Psaraftis and Kontovas (2010); Norstad, Fagerholt, and Laporte (2011); Kontovas and Psaraftis (2011); Cariou (2011); Lindstad, Asbjørnslett, and Strømman (2011); and Hvattum et al. (2013) in the field of maritime transportation, and in the works of Bektaş and Laporte (2011) and Demir, Bektaş, and Laporte (2012) in the field of road transportation. In particular, Kontovas and Psaraftis (2011) studied speed reduction as a strategy to reduce fuel consumption in maritime transportation. Cariou (2011) investigated the impact of slow steaming strategies for reducing the amount of liner shipping CO_{2e} emissions. Similarly, Lindstad, Asbjørnslett, and Strømman (2011) presented a strategic analysis of the effects of speed reduction on the costs of maritime transport, including CO_{2e} emissions. Psaraftis and Kontovas (2010) investigated the impact of speed reduction on modal split. Hvattum et al. (2013) proposed an exact algorithm to optimize the cruise speed of a vessel visiting a set of ports on a path subject to arrival time restrictions to minimize the fuel consumption while ensuring that ships arrive within given time windows. They referred to this problem as the *speed optimization problem* (SOP). For a comprehensive survey on ship speed

optimization studies, we refer to Psaraftis and Kontovas (2013, 2014, 2016).

Another set of papers focuses on a related problem: the routing and scheduling of a vehicle that must serve a set of customers on a path with hard time windows. In this context, Demir, Bektaş, and Laporte (2012) adapted the algorithm proposed by Hvattum et al. (2013) to optimize the travel speed of a vehicle on each arc of the path. More recently, Kramer et al. (2015) developed a quadratic-time algorithm to solve this problem to optimality. Franceschetti et al. (2013) extended the model of Demir, Bektaş, and Laporte (2012) to allow for traffic congestion, which limits the vehicle speed during peak hours. They referred to the augmented problem as the *departure time and speed optimization problem* (DSOP). The consideration of traffic congestion greatly complicates the analysis since it may now be optimal for the vehicle to wait idly at the depot or at customer locations to mitigate the negative impact of slow congestion speed on emissions costs. The authors developed a polynomial-time heuristic algorithm that builds on the closed-form solution they obtained for the single-arc version of the problem. Finally, Fukasawa et al. (2016) put forward a new set partitioning formulation for the joint routing and speed optimization problem, where the objective is the minimization of the total cost of fuel, emissions, and labor.

In this paper, we develop a shortest-path (SP) based algorithm for the DSOP, which is exact and efficient.

Figure 1. (Color online) Sequence of Events at Customer Location i

Hence, we extend the work of Kramer et al. (2015) by considering traffic congestion and the work of Franceschetti et al. (2013) by providing an optimal solution. Note that our algorithm can be used as a subroutine to optimize individual routes in multivehicle routing and scheduling problems with speed considerations.

The remainder of the article is organized as follows. In Section 2 we present our model and discuss feasibility conditions. In Section 3 we establish key structural results of an optimal solution, which we exploit to build our shortest-path formulations, presented in Section 4. We illustrate our solution method in Section 5 and conclude with a summary of our contributions in Section 6. All proofs are presented in the online appendix.

2. Model

A single vehicle departs from an origin location called the *depot* to visit a number of customer locations according to a fixed route. For example, a delivery vehicle leaves from a central warehouse to visit retail locations and deliver merchandise, or a plumber leaves her office to visit customer homes to make repairs. Let 0 denote the depot, let locations 1 to n represent customers, and let $n + 1$ denote a copy of the original location, which represents the return of the vehicle to its starting point.¹ Hence, the fixed route is $(0, 1, \dots, n + 1)$. We refer to arc $(i, i + 1)$ as arc i for $i = 0, \dots, n$. Let d_i denote the distance on arc i and let h_i denote the service time at location i for $i = 0, \dots, n$. In the delivery vehicle example, the service time at the depot would correspond to the time spent loading the vehicle with merchandise and the service time at each customer location would be the unloading and delivery time. Each customer location $i \in \{1, \dots, n\}$ has a hard time window $[l_i, u_i]$ within which service must start. The vehicle may arrive at customer i earlier than l_i but service may not start until that time. Let μ_i be the arrival time of the vehicle at location i for $i = 0, \dots, n + 1$. We set $\mu_0 = 0$, which corresponds to the start of the planning horizon. Note that this value does not necessarily correspond to the time at which the driver arrives at the depot. We set $l_0 = l_{n+1} = 0$ and $u_{n+1} = \infty$ for notational convenience.

As in the classical vehicle routing problem with time windows (VRPTW), a vehicle is allowed to wait at the depot and at the customer locations, but not between locations. Without loss of generality, we assume that service at a customer location starts exactly at time l_i if the vehicle has arrived by then; otherwise it starts immediately upon arrival. This assumption is without loss of generality as the labor cost is computed on the total travel time of the vehicle, therefore there is no difference if the waiting time takes place preservice or postservice at a customer location. Upon completion of service at a location, the vehicle may also wait before leaving; let z_i denote the *postservice waiting time* at location $i = 0, \dots, n$. In other words, there are potentially two periods of waiting at customer location $i = 1, \dots, n$: a *preservice waiting time* of $(l_i - \mu_i)^+$ and a *postservice waiting time* of z_i . Note that we allow for the vehicle to wait at the depot before (and after) the start of service: y denotes the time elapsed between the start of service at the depot and the start of the planning horizon. The trip ends with the arrival at location $n + 1$, that is, with the return to the depot. Finally, let ρ_i denote the departure time from location i for $i = 0, \dots, n$. We have $\rho_0 = y + h_0 + z_0$ and $\rho_i = \max\{\mu_i, l_i\} + h_i + z_i$ for $i = 1, \dots, n$.

Figure 1 depicts the sequence of events at customer i . In this example, the vehicle arrives before l_i so that there is a positive preservice waiting time. Because service does not start immediately at l_i , there is a positive voluntary preservice waiting time and the vehicle does not leave the location immediately after the completion of service, so there is also a positive postservice waiting time.

We assume that the planning horizon starts with an initial period of traffic congestion lasting a time units during which the vehicle travels at a constant speed v_{con} . After this period of traffic congestion has ended, the driver can choose its vehicle's speed between the limits $v^{\min}$ and $v^{\max}$, where $v_{\text{con}} \leq v^{\min}$. We refer to this period as the *free-flow period*. Let v_i denote the chosen free-flow speed on arc i for $i = 0, \dots, n$. Note that v_i is only defined if location $i + 1$ is reached past the end of the congestion period, that is, if $\mu_{i+1} > a$.

Let $T_i(\rho_i, v_i)$ denote the total travel time to traverse arc i if the vehicle leaves location i at time ρ_i and

Table 1. Main Notation

Cost parameters	
A, B, C	Emissions cost parameters
D	Driver wage
Congestion parameters	
a	Duration of the congestion period
v_{con}	Congestion speed
v^{min}	Minimum free-flow speed
v^{max}	Maximum free-flow speed
Location parameters	
d_i	Arc i distance
h_i	Service time at location i
l_i	Lower time window limit at location i
u_i	Upper time window limit at location i
U_i	Effective upper time window limit at location i (see Section 2.3)
μ_i	Earliest possible arrival time at location i (see Section 2.3)
$\hat{\mu}_i$	Earliest possible arrival time at location $i > 1$ given $\mu_1 = U_1$
Decision variables	
y	Preservice waiting time at the depot
v_i	Free-flow speed on arc i
z_i	Postservice waiting time at location i
Other variables	
μ_i	Arrival time of the vehicle at location i
ρ_i	Departure time from location i

chooses a free-flow speed v_i (if applicable). We divide this time between the time spent driving in congestion and the time spent driving in free-flow as follows: we set $T_i(\rho_i, v_i) = T_i^{\text{con}}(\rho_i) + T_i^{\text{free}}(\rho_i, v_i)$, where $T_i^{\text{con}}(\rho_i) = \min\{(a - \rho_i)^+, d_i/v_{\text{con}}\}$ is the time spent traveling in congestion on arc i and $T_i^{\text{free}}(\rho_i, v_i) = [d_i - (a - \rho_i)^+ v_{\text{con}}]^+ / v_i$ is the time spent traveling at the free-flow speed v_i on arc i when departing at time ρ_i from location i . The arrival time at location $i + 1$ is then calculated as $\mu_{i+1} = \rho_i + T_i(\rho_i, v_i)$.

The decision maker makes two types of decisions: how fast to drive on each arc during the free-flow period and how long to wait at each location. Hence an optimal schedule is fully characterized by (i) the free-flow speed vector $\mathbf{v} = (v_0, \dots, v_n)$, (ii) the preservice waiting time at the depot y , and (iii) the postservice waiting time vector $\mathbf{z} = (z_0, \dots, z_n)$. Table 1 summarizes our notation.

2.1. Objective Function

The decision maker aims at minimizing total cost, which is the sum of (i) the labor cost, that is, the cost of paying the driver of the vehicle, and (ii) the cost of carbon dioxide equivalent (CO_{2e}) emissions, which are directly proportional to the amount of fuel used during the trip.

We consider two ways of calculating the total time for which the driver is paid. In the *early policy*, the driver is paid starting from time zero until the arrival time at location $n + 1$; in the *late policy*, the driver is paid starting from the start of the service at the depot until the arrival time at location $n + 1$. Under the *early*

policy, the driver reports to the depot at the start of the planning horizon, which coincides with the start of a typical work day, but may have to wait before starting her service and driving duties (during this time she may be asked to perform some administrative duties). Under the *late policy*, the driver arrives at the depot right on time to start service.

We assume that the labor cost on arc i is measured from the arrival time at customer location i to the arrival time at location $i + 1$, that is, from μ_i to μ_{i+1} . Formally, the labor cost, L_i , for traversing arc i , given an arrival time of μ_i at customer location i , a postservice waiting z_i , and a free-flow speed v_i is given by

$$L_i(\mu_i, z_i, v_i) = D[(l_i - \mu_i)^+ + h_i + z_i + T_i(\max\{\mu_i, l_i\} + h_i + z_i, v_i)], \quad (1)$$

where D is the labor cost. For arc 0, the labor cost is calculated differently depending on the driver wage policy. Under the early policy, we have

$$L_0(y, z_0, v_0) = D[y + h_0 + z_0 + T_0(y + h_0 + z_0, v_0)]. \quad (2)$$

Otherwise, under the late policy, we have

$$L_0(y, z_0, v_0) = D[h_0 + z_0 + T_0(y + h_0 + z_0, v_0)]. \quad (3)$$

The difference between the two driver wage policies is the preservice waiting time at the depot y , which is paid under the early policy but not under the late policy. Therefore, under the late policy, any waiting time should be concentrated before the start of service, with the vehicle leaving immediately upon service completion (that is, setting $y \geq 0$ and $z_0 = 0$) so as to save on labor costs. By contrast, under the early policy, waiting at the depot is equally costly whether it takes place preservice or postservice, therefore concentrating all of the waiting after the service has ended (that is, setting $y = 0$ and $z_0 \geq 0$) does not affect total costs. As a result, the decision maker only needs to determine $\mathbf{v}$, y , and $\mathbf{z}$ under the late policy and $\mathbf{v}$ and $\mathbf{z}$ under the early policy.

To measure the emissions cost, we use the model of Barth, Younglove, and Scora (2005) and Scora and Barth (2006) who assume that the amount of CO_{2e} emissions produced by a vehicle is directly proportional to the amount of fuel consumed. According to their model, the emissions costs E_i , for traversing arc i given a chosen free-flow speed v_i and a departure time from location i of ρ_i , is given by

$$E_i(\rho_i, v_i) = Ad_i + BT_i(\rho_i, v_i) + C[T_i^{\text{con}}(\rho_i)(v_{\text{con}})^3 + T_i^{\text{free}}(\rho_i, v_i)v_i^3], \quad (4)$$

where A , B , and C are nonnegative constants that depend on features of the vehicle such as frontal surface area and curb weight as well as on the road conditions (see Franceschetti et al. 2013 for the computation of these values).

The total cost of traversing arc i , measured from the arrival time at customer location i to the arrival time at location $i + 1$, that is, from μ_i to μ_{i+1} , is denoted by g_i and is given by

$$g_i(\mu_i, z_i, v_i) = E_i(\max\{\mu_i, l_i\} + h_i + z_i, v_i) + L_i(\mu_i, z_i, v_i). \quad (5)$$

The total cost of traversing arc 0, measured from the start of the planning horizon to the arrival time at location 1, is given by

$$g_0(y, z_0, v_0) = E_0(y + h_0 + z_0, v_0) + L_0(y, z_0, v_0). \quad (6)$$

The decision-maker's problem is to determine the speed vector $\mathbf{v}$, the preservice waiting time at the depot y , and the postservice waiting time vector $\mathbf{z}$ to minimize the total cost $\mathcal{C}$ incurred over the entire vehicle trip. The problem can be written as follows:

$$\begin{aligned} \text{minimize } \mathcal{C}(\mathbf{v}, \mathbf{z}, y) &= g_0(y, z_0, v_0) + \sum_{i=1}^n g_i(\mu_i, z_i, v_i) \\ \text{subject to} \end{aligned} \quad (7)$$

$$\mu_1 = y + h_0 + z_0 + T_0(y + h_0 + z_0, v_0), \quad (8)$$

$$\begin{aligned} \mu_i &= \max\{l_{i-1}, \mu_{i-1}\} + h_{i-1} + z_{i-1} \\ &\quad + T_{i-1}(\max\{l_{i-1}, \mu_{i-1}\} + h_{i-1} + z_{i-1}, v_{i-1}) \\ &\quad \text{for } i = 2, \dots, n+1, \end{aligned} \quad (9)$$

$$\mu_i \leq u_i \quad \text{for } i = 1, \dots, n, \quad (10)$$

$$v^{\min} \leq v_i \leq v^{\max} \quad \text{for } i = 0, \dots, n. \quad (11)$$

The first two sets of constraints follow from the definition of the arrival time into a location. The third set corresponds to the upper time windows constraints and the last set is where we impose the minimum and maximum free-flow speed constraints.

Note that, for fixed $\mathbf{v}$, y , and $\mathbf{z}$, the total cost under the early policy is higher than under the late policy by a quantity equal to Dy , which corresponds to the cost of paying the driver during the preservice waiting time at the depot.

2.2. Important Speed Values

Given a fixed departure time ρ_i from node i , such that $\rho_i \geq a$, the free-flow speed that minimizes the emissions costs on arc i , that is, $E_i(\rho_i, v_i)$, is equal to $\bar{v} = (B/(2C))^{1/3}$. Observe that this value does not depend on ρ_i . This is because the departure time is past the end of the congestion period and there are no restrictions on the arrival time at location $i + 1$. Similarly, given fixed values for μ_i , y , and z_i , the speed value that minimizes the total cost of traversing arc i (i.e., $g_i(\mu_i, z_i, v_i)$ for $i > 0$ and $g_0(y, z_0, v_0)$ for $i = 0$) is equal to $\bar{v} = ((B + D)/(2C))^{1/3}$. We always have $\bar{v} \leq \bar{v}$ and we further assume that $v^{\min} \leq \bar{v}$ and $\bar{v} \leq v^{\max}$. These two speed values will be useful in our analysis and have previously been identified by Demir, Bektaş, and Laporte (2012).

2.3. Feasibility Conditions and Effective Upper Time Window Limits

In this section we establish the conditions under which the problem is feasible and we introduce the concept of *effective* upper time window limits, which is useful in our subsequent derivations. Let μ_i denote the earliest possible arrival time at location i , which is calculated by assuming that the vehicle drives at the maximum free-flow speed whenever possible until reaching location i and never waits at any location except when arriving earlier than a lower time window, that is, setting $v_j = v^{\max}$, $y = 0$ and $z_j = 0$ for $j = 0, \dots, i - 1$ for $j = 0, \dots, i - 1$. We obtain μ_i for $i = 0, \dots, n + 1$, recursively starting from location 0 as follows:

$$\mu_0 = 0, \quad (12)$$

$$\begin{aligned} \mu_i &= \max\{\mu_{i-1}, l_{i-1}\} + h_{i-1} \\ &\quad + T_{i-1}(\max\{\mu_{i-1}, l_{i-1}\} + h_{i-1}, v^{\max}) \\ &\quad \text{for } i = 1, \dots, n + 1. \end{aligned} \quad (13)$$

The problem is feasible if $\mu_i \leq u_i$ for $i = 1, \dots, n$. In what follows we assume that these conditions are satisfied. For $i = 1, \dots, n + 1$, let U_i denote the latest possible arrival time at location i so that it is still possible to meet all of the time windows at locations $i + 1, \dots, n + 1$. By construction, we always have $U_i \leq u_i$. We call U_i the *effective* upper time window limit at location i : if the vehicle was to arrive at location i between U_i and u_i , then it would not violate the upper time window limit of location i , but it would certainly violate the upper time window limit of at least one of the subsequent locations. We set $U_{n+1} = u_{n+1} = \infty$ and calculate $U_1, \dots, U_n$ recursively starting from location n , assuming maximum speed and no postservice waiting time, as follows:

$$U_n = u_n, \quad (14)$$

$$\begin{aligned} U_i &= \min\{u_i, U_{i+1} - \min\{(U_{i+1} - a)^+, d_i/v^{\max}\} \\ &\quad - [d_i - (U_{i+1} - a)^+ v^{\max}]^+/v_{\text{con}} - h_i\} \\ &\quad \text{for } i = n - 1, \dots, 1. \end{aligned} \quad (15)$$

The second term in the minimum expression in (15) is the time at which the driver must start service at location i so as to start service at location $i + 1$ exactly at time U_{i+1} , leaving location i right upon service completion and driving on arc i at free-flow speed $v^{\max}$, whenever possible. Note that the feasibility of the problem implies that $l_i \leq U_i$ for $i = 1, \dots, n$. We use these effective upper time window limits U_i , rather than u_i , throughout our analysis, and we refer to $[l_i, U_i]$ as the *effective* time window at location $i \in \{1, \dots, n + 1\}$.

3. Structural Results

In this section we present some structural properties of the optimal solution to the DSOP. When there is

Figure 2. (Color online) Example of Scheduling Satisfying the Conditions of Proposition 1

traffic congestion, the optimal driving schedule may be such that it is optimal for the vehicle to wait idly at the depot or at customer locations because doing so can reduce the driving time in congestion, thereby possibly reducing CO_{2e} emissions. However, waiting may increase labor costs: the idle time at a customer location is always costly, while the idle time at the depot is only costly if the driver is paid from the start of the planning horizon (early policy).

Any solution to the DSOP can be seen as a subsequence $(j_1, \dots, j_m)$ of the location indices $\{0, 1, \dots, n+1\}$ with $0 \leq j_1 \leq \dots \leq j_m = n+1$ as follows. Arcs 0 to $j_1 - 1$ are entirely traversed during the congestion period, that is, $\mu_i \leq a$ for $i = 1, \dots, j_1$, and arcs j_k to $j_{k+1} - 1$ are traversed at the same free-flow speed for $k = 1, \dots, m$, that is, $v_{j_k} = v_{j_k+1} = \dots = v_{j_{k+1}-1}$. In other words, the set of arcs is partitioned into adjacent segments over which the driver keeps a constant free-flow speed: the first segment consists of arcs 0 to $j_1 - 1$, the second one consists of arcs j_1 to $j_2 - 1$, and the last segment consists of arcs j_{m-1} to n . Also, arc j_1 is the first arc to be traversed during the free-flow period (if all of the arcs are traversed during the congestion period, we have $m = 1$). Our first result, which holds under both driver wage policies, uses this notation to establish important properties of an optimal solution.

Proposition 1. *There exists an optimal solution such that*

- (i) *all of the postservice waiting (if there is any) takes place at the last location that is reached before the end of the congestion period, that is, $z_{j_1} \geq 0$ and $z_i = 0$ for $i \neq j_1$, where j_1 is the largest index in $\{1, \dots, n+1\}$ such that $\mu_{j_1} \leq a$;*
- (ii) *if the vehicle reaches a customer location $i > j_1$ strictly within its time window, that is, $\mu_i \in (l_i, U_i)$, then the free-flow speed on arcs $i-1$ and i must be the same.*

Proposition 1(i) implies that there exists an optimal solution such that the vehicle does not wait idly postservice after the end of the congestion period, which is intuitive since doing so would increase labor costs without reducing CO_{2e} emissions. Proposition 1(ii) implies that changes in the optimal free-flow speed can only occur when the vehicle reaches a customer location by its lower time window, that is, $\mu_i \leq l_i$, or exactly at its effective upper time window, that is, $\mu_i = U_i$. In other words, the speed on two adjacent arcs must be the same unless the start of service at the common location is exactly equal to its lower or upper time window limit.

Figure 2 depicts a schedule that satisfies the conditions of Proposition 1, where the x -axis represents the geographical distance and the y -axis represents time. The schedule is represented by a discontinuous black

line, where the points of discontinuity occur at the customer locations. The slope of the increasing portions corresponds to the time elapsed per unit of distance, which is the inverse of vehicle speed, while the vertical portions of the line correspond to periods of service and waiting (with waiting being represented by dotted lines). Time windows limits for each location are marked on the y -axis. In this example, the vehicle travels on arc 0 during the congestion period (therefore at a speed of v_{con}), then waits at location 1 following the completion of service. The congestion period ends while it is traversing arc 1, so the driver switches to a free-flow speed, which we denote by v^1 . The vehicle keeps the same speed on arc 2, arriving at location 3 exactly at time U_3 . Finally it drives on arcs 3 and 4 at a constant speed, denoted by v^2 . Formally, in this example, we have $m = 3$, $j_1 = 1$, $j_2 = 3$, and $j_3 = 5$. This schedule satisfies Proposition 1(i) since the only postservice waiting occurs at location 1, which is the last customer location that is reached before the end of the congestion period. It also satisfies Proposition 1(ii) since the vehicle arrives at locations 2 and 4 strictly within their time window and keeps the same free-flow speed before and after reaching these locations. Furthermore, at location 3, which marks a change in free-flow speed, the vehicle's arrival time exactly matches the effective upper time window limit.

Given Proposition 1, there exists an optimal sequence $\{j_1, \dots, j_{m-1}, n+1\}$ of locations such that service starts at locations $j_2, \dots, j_{m-1}$ exactly at their lower or upper time window limit. Therefore we can obtain an optimal schedule by considering all possible sequences in combination with the two possible values for start of service at locations that mark a speed change. So suppose we fix a sequence $\{j_1, \dots, j_{m-1}, n+1\}$ and fix the start of service at locations $j_2, \dots, j_{m-1}$. Optimizing the preservice and postservice waiting times and vehicle speeds can be done independently for each segment of locations: 0 to j_1 , j_1 to j_2 , etc. By definition, we know the free-flow speed is constant on each such segment and by Proposition 1, we know that the arrival time at the intermediate nodes is strictly within their time window. Lemma 1 further establishes that the optimization of each segment amounts to solving a single-arc DSOP problem, which implies that there is no waiting time at the intermediate nodes.

Lemma 1. *The problem of minimizing the total cost of traversing a sequence of arcs $i, \dots, j$ where (i) the free-flow speed, when applicable, is constant; (ii) the service at location j must start exactly at time η with $\eta \in \{l_j, U_j\}$; and (iii) the time window restrictions at the intermediate locations $i+1, \dots, j-1$ are relaxed is equivalent to minimizing the total cost of traversing a single arc with distance $d = \sum_{k=i}^{j-1} d_k$ where the service at the destination location must start exactly at time $\eta - \sum_{k=i+1}^{j-1} h_k$.*

In Section EC.1 in the online appendix, we show how to solve a single-arc problem with a fixed start of service time. Note that the results are different from those of Franceschetti et al. (2013) because they do not assume a fixed start of service time at the final location.

Because the final arc represents the return to the depot and there is no time window at location $n+1$ (i.e., $l_{n+1} = 0$ and $u_{n+1} = \infty$), the last segment must be dealt with differently. In this case the results from Franceschetti et al. (2013) can be used directly to show that if the vehicle drives at least partially in free-flow, then the last part of the trip should be driven at speed $\bar{v}$ as defined in Section 2.2.

Lemma 2. *If the optimal solution is such that the vehicle drives at least part of the trip in free-flow (i.e., $m > 1$), then it should drive at speed $\bar{v}$ on the last arcs, that is, $v_{j_{m-1}} = v_{j_{m-1}+1} = \dots = v_n = \bar{v}$.*

When the driver is paid from the start of service at the depot (late policy), we have the following extra result.

Proposition 2. *Consider the case where the driver is paid from the start of service at the depot (late policy). If at least one customer location has a finite upper time window, then there exists a pivot location $\hat{j} \in \{1, \dots, n\}$ defined as follows: $\hat{j}$ is the lowest index such that $\mu_{\hat{j}} \in \{l_{\hat{j}}, U_{\hat{j}}\}$, and $\mu_i \in (l_i, U_i)$ for $i = 1, \dots, \hat{j} - 1$. Furthermore, if $\mu_{\hat{j}} \leq a$ and $\mu_j = U_j$ then $\hat{j} = 1$. Also, the optimal solution is such that $z_0 = \dots = z_{\hat{j}-1} = 0$.*

When at least one customer location has a finite upper time window, Proposition 2 establishes that under the late policy, there always exists a customer location, called the *pivot location* $\hat{j}$, that is reached exactly at its lower or effective upper time window without any postservice waiting time at any of the preceding locations. Intuitively, the role of the pivot location is to determine the optimal amount of preservice waiting time at the depot. When the driver is paid from the start of service at the depot, postponing service at the depot is an effective way to reduce the total cost as it reduces labor costs. Given a fixed arrival time at the pivot location, one can optimize the preservice waiting time at the depot and the (constant) free-flow speed on arcs 0 to $\hat{j} - 1$ by treating the problem as a single-arc DSOP with fixed arrival time (see Section EC.1 in the online appendix for how to solve such a problem under the late policy).

Note that the pivot location can be reached either before or after the end of the congestion period. When it is reached after congestion had ended, that is, $\mu_{\hat{j}} > a$, Proposition 2 together with Proposition 1(i) imply that there is no postservice waiting time at any location. By contrast, if the pivot location is reached during the congestion period, that is, $\mu_{\hat{j}} \leq a$, there could be postservice waiting time at one of the customer locations

(either location $\hat{j}$ or a later one). Furthermore, if the arrival time at the pivot location matches its effective upper time window and occurs before congestion ends, then the pivot location is location 1.

By contrast, when there are no upper time windows and the driver is paid from the start of service at the depot (late policy), the decision maker can postpone service indefinitely without bearing any cost consequences, and therefore it makes sense to wait until the congestion period has ended so as to avoid driving at the congestion speed. Furthermore, it also makes sense to wait long enough to ensure the vehicle never arrives at any customer location earlier than its lower time window limit, thereby avoiding preservice wait times. As a result, the total time for which the driver is paid for is equal to the sum of the service time and travel time, which is minimized when driving at free-flow speed $\bar{v}$ defined in Section 2.2 on every arc. In this case, the total cost is equal to $\mathcal{C} = A \sum_{i=0}^n d_i + (B + C\bar{v}^2 + D)(\sum_{i=0}^n d_i/\bar{v}) + D \sum_{i=0}^n h_i$, which is a lower bound on all possible values achievable.

The structural properties of the optimal solution established in Propositions 1 and 2 allow us to recast the DSOP as a shortest-path problem, which we present in detail in Section 4.

4. Shortest-Path Formulation

In this section we show how to model the DSOP as a shortest-path problem. To this end, we exploit the two main results proven in Section 3. First, we use the property that an optimal schedule can be broken down into segments of arcs where the vehicle keeps a constant free-flow speed, arrives at each intermediate location strictly within its locations time windows limits, and only waits postservice (if at all) at the starting point location of the segment. Second, we exploit the property that service at the locations that mark a change in speed starts either exactly at their lower or effective upper time window limits. The design of the shortest-path network differs for the two driver wage policies, so we present the two in Sections 4.1 and 4.2.

Note that the design of the shortest-path network and its optimization under either policy can be achieved in $O(n^2)$ time, which can be established using the same argument as in Franceschetti (2015, Corollary 5.2). This is the same complexity as that of the heuristic from Franceschetti et al. (2013). However, the shortest-path based algorithm presented in this paper is exact, while the heuristic of Franceschetti et al. (2013) was not.

4.1. Shortest-Path Formulation Under the Early Policy

Proposition 1(i) implies that, when the driver is paid from the start of the planning horizon, locations 0

to j_1 are reached as early as possible given the congestion speed, that is, $\mu_i = \mu_i$ for $i = 0, \dots, j_1$, where μ_i is defined in Section 2.3. This is because there is no preservice or postservice waiting at the depot ($y^* = z_0^* = 0$) and no postservice waiting at locations 1 to $j_1 - 1$ ($z_1^* = \dots = z_{j_1-1}^* = 0$). Hence, under the early policy, the start of service time at location j_i for $i = 1, \dots, m - 1$ can only take three possible values: $\underline{\mu}_{j_i}$, l_{j_i} , or U_{j_i} . Note that the arrival time at location j_i can be earlier than l_{j_i} but the start of service time cannot be.

For use in the design of the shortest path, we define location $k \in \{0, \dots, n + 1\}$ as the largest index location that can be reached before the end of the congestion period (assuming no waiting), that is, k is the largest value of i such that $\mu_i \leq a$ (we set $k = 0$ if it is impossible to reach location 1 before the end of the congestion period, that is, if $h_0 + d_0/v_c > a$). Observe that $j_1 \leq k$. Specifically, if there is no waiting time at every location before k , that is, $y = 0$ and $z_0, \dots, z_{k-1} = 0$, then $j_1 = k$. If, instead, there is some waiting time at any location before k , then it is $j_1 < k$.

From the original network with $n + 1$ arcs and $n + 2$ nodes, we construct a shortest-path network with $2n + k + 2$ nodes as follows: node 0 (corresponding to the depot), nodes $\underline{i}$ for $i = 1, \dots, k$ (i.e., one such node for each of the first k customer locations), nodes i_l and i_u for $i = 1, \dots, n$ (i.e., two such nodes for each customer location), and location $n + 1$ (corresponding to the return to the depot). Let V denote the set of nodes in the shortest-path network, that is, $V = \{0, \underline{1}, \dots, \underline{k}, 1_l, \dots, n_l, 1_u, \dots, n_u, n + 1\}$. For $i = 1, \dots, k$, location $\underline{i}$ corresponds to the event “the vehicle starts service at time $\max\{\mu_i, l_i\}$.” For $i = 1, \dots, n$, node i_l corresponds to the event “the vehicle arrives at location i at the latest at the lower time window limit l_i ” (and therefore starts service at time l_i), and node i_u corresponds to the event “the vehicle arrives at location i exactly at the effective upper time window limit U_i .” Finally, node $n + 1$ corresponds to the event “the vehicle returns to the depot.” Each arc in the shortest-path network corresponds to a segment of adjacent arcs in the original network. For example, the arc in the shortest-path network from location 0 to location $\underline{i}$ (or i_l or i_u) for $i = 1, \dots, n$ corresponds to arc 0 to $i - 1$ in the original network. The length of each arc is equal to the minimum cost of traversing the corresponding segment given constraints on the arrival time at each location. More specifically, for each arc, the origin node determines the earliest possible departure time from the segment and the destination node determines the service starting time at the destination. To calculate the arc length, we optimize the free-flow speed and the waiting time at the depot by considering the segment as a single-arc DSOP as explained in Lemma 1. If the solution is feasible, that is, if the time window restrictions at the intermediate locations are met, then

the length of the arc is equal to the minimum cost of traversing the segment, otherwise it is set to infinity.

More precisely, the length of the arcs in the shortest-path network are set as follows:

- The length of arcs $(0, i)$ for $i = 1, \dots, k$ is set equal to the (minimum) cost of driving in congestion from the depot to location i , without any waiting at locations 0 to $k - 1$.
- The length of arcs $(0, j_l)$ (respectively, $(0, j_u)$) for $j = 1, \dots, n$ is set equal to the minimum cost of leaving the depot at or past time h_0 and starting service at location j at time l_j (respectively, U_j), while reaching locations $i + 1, \dots, j - 1$ strictly within their time window (and is infinite if this is not possible).
- The length of arcs (i_l, j_l) (respectively, (i_l, j_u) , (i_u, j_l) , (i_u, j_u)) for $1 \leq i < j \leq n$ is set equal to the minimum cost of leaving location i at or past time $l_i + h_i$ (respectively, $U_i + h_i$, $l_i + h_i$, $U_i + h_i$) and starting service at location j at time l_j (respectively, U_j , l_j , U_j), while reaching locations $i + 1, \dots, j - 1$ strictly within their time window (and is infinite if this is not possible).
- The length of arcs $(\bar{i}, j_l)$ (respectively, $(\bar{i}, j_u)$) for $1 \leq i < j \leq n$ is set equal to the minimum cost of leaving location i at or past time $\bar{\mu}_i + h_i$ and starting service at location j at time l_j (respectively, U_j), while reaching locations $i + 1, \dots, j - 1$ strictly within their time window (and is infinite if this is not possible).
- The length of arcs $(i_l, n + 1)$ (respectively, $(i_u, n + 1)$ and $(\bar{i}, n + 1)$) for $1 \leq i \leq \min\{k, n\}$ is set equal to the minimum cost of leaving location i at or past time $l_i + h_i$ (respectively, $U_i + h_i$ and $\bar{\mu}_i + h_i$) and driving to location $n + 1$, while reaching locations $i + 1, \dots, j - 1$ strictly within their time window (and is infinite if this is not possible).
- The length of arc $(0, n + 1)$ is set equal to the minimum cost of leaving the depot at or past time h_0 reaching locations 1 to n within their time window and returning back to the depot (and is infinite if this is not possible). Unlike for the other arcs, to calculate the length of this arc, we use the results from Franceschetti et al. (2013) for solving a single arc DSOP on arc of length $\sum_{i=0}^n d_i$, as stated in Lemma 2.

- If $k = n + 1$, the length of arc $(\underline{n+1}, n + 1)$ is set equal to zero.

In particular, all arcs from an “ i -node” (i.e., either i_l or i_u) to a “ j -node” (i.e., either j_l , j_u or $\bar{j}$) with $j < i$ have infinite length. So do arcs $(\bar{i}_l, \bar{j})$, $(i_u, \bar{j})$, and $(\bar{i}, \bar{j})$ for $j > i$.

From this, it follows that the problem of finding the optimal vectors $\mathbf{v}$, $\mathbf{z}$, and $\mathbf{y}$ when the driver is paid from the start of the planning horizon reduces to a shortest-path from node 0 to node $n + 1$ as described above.

Figure 3. (Color online) Shortest-Path Network for an Example with $n = 2$ and $k = 2$

4.2. Shortest-Path Formulation Under the Late Policy

When the driver is paid from the start of service at the depot, we know that it may be optimal for the decision maker to postpone the start of service at the depot to save on labor costs, that is, set $y^* > 0$. Furthermore, there can be some additional waiting, which, if it exists, takes place postservice at the last location visited before the end of the congestion period (by Proposition 1(i)). This will be the case if the pivot location (i.e., the $\hat{j}$ index defined in Proposition 2) is visited before the end of the congestion period. When the pivot location is visited exactly at its upper time window, Proposition 2 establishes that it must be the first location; in practice this means that the driver postpones the start of service at the depot so as to arrive at the first customer location exactly at time U_1 . In that case, the arrival time at subsequent locations $i = 2, \dots, j_1$, denoted by $\hat{\mu}_i$, are calculated recursively as

$$\hat{\mu}_1 = U_1, \quad (16)$$

$$\begin{aligned} \hat{\mu}_i = \max\{\hat{\mu}_{i-1}, l_{i-1}\} + h_{i-1} \\ + T_{i-1}(\max\{\hat{\mu}_{i-1}, l_{i-1}\} + h_{i-1}, v^{\max}) \end{aligned} \quad \text{for } i = 2, \dots, j_1. \quad (17)$$

By contrast, when arrival at the pivot location is at its lower time window and occurs before the end of the congestion period, the arrival time at subsequent locations $\hat{j} + 1, \dots, j_1$ is given by $\bar{\mu}_i$ since the vehicle will start service at location $\hat{j}$ at the earliest possible time and there is no waiting until possibly at location j_1 . Hence, under the late policy, the start of service time at location j_i for $i = 1, \dots, m - 1$ can only take four possible values: $\hat{\mu}_{j_i}$, $\bar{\mu}_{j_i}$, l_{j_i} , or U_{j_i} . In other words, compared to the early policy, there is one more possible value for the arrival time at a location that marks the end of a segment. We exploit this structure to modify the shortest-path formulation presented in Section 4.1.

As in Section 4.1, we define $k \in \{0, \dots, n + 1\}$ to be the largest index location that can be reached before the

Figure 4. (Color online) Example of a Route with 4 Customer Locations

end of the congestion period (assuming no waiting), that is, k is the largest value of i such that $\mu_i \leq a$ (we set $k = 0$ if even location i cannot be reached before the end of the congestion period). Furthermore, we also define $p \in \{0, \dots, n+1\}$ to be the highest index location that can be reached before the end of the congestion period, when the vehicle waits preservice at the depot so as to arrive at location 1 exactly at time U_1 , that is, p is the largest value of i such that $\hat{\mu}_i \leq a$ (we set $p = 0$ if $U_1 > a$). Note that we always have $k \geq p$ since $\mu_i \leq \hat{\mu}_i$ for $i = 0, \dots, n+1$.

From the original network with $n+1$ arcs and $n+2$ locations, we construct an SP network with $2n+k+2+p$ nodes. The first $2n+k+2$ nodes are the same as in the SP network from Section 4.1 and have the same interpretation. The extra p nodes are labeled $\hat{i}$ for $i = 1, \dots, p$ and correspond to the event “the vehicle arrives at location i at time $\hat{\mu}_i$.” All of the arcs that exist in the SP network from Section 4.1 continue to exist and have the same length. In addition to these, we have the following extra arcs:

- $(0, \hat{i})$ for $i = 1, \dots, p$.

The length of arc $(0, \hat{i})$ is set equal to the (minimum) cost of driving from the depot to location i when the vehicle waits at the depot so as to arrive at location 1 exactly at time U_1 and does not wait at locations $1, \dots, p-1$;

- $(\hat{i}, j_l), (\hat{i}, j_u)$ for $1 \leq i < j \leq n$.

The length of arc $(\hat{i}, j_l)$ (respectively, $(\hat{i}, j_u)$) for $1 \leq i < j \leq n$ is set equal to the minimum cost of starting service at location i at time $\hat{\mu}_i$ and starting service at location j at time l_j (respectively, U_j), while reaching locations $i+1, \dots, j-1$ strictly within their time window (and is infinite if this is not possible);

- $(\hat{i}, n+1)$ for $1 \leq i \leq \min\{p, n\}$.

The length of arc $(\hat{i}, n+1)$ for $1 \leq i < j \leq n$ is set equal to the minimum cost of starting service at location i at time $\hat{\mu}_i$ and driving to location $n+1$, while reaching locations $i+1, \dots, j-1$ strictly within their time window (and is infinite if this is not possible);

- $(n+1, n+1)$ if $p = n+1$.

The length of this arc is set equal to zero.

From this, it follows that the problem of finding the optimal vectors $\mathbf{v}$, $\mathbf{z}$, and $\mathbf{y}$ when the driver is paid from the start of service at the depot reduces to a shortest path from node 0 to node $n+1$ as described above.

5. Numerical Illustrations

In this section we present the optimal solution for a numerical example under both driver wage policies. We do so to illustrate the workings of the shortest path and to present some structural properties of the optimal solution. Consider the route depicted in Figure 4 with four customer locations.

All distances reported in Figure 4 are in kilometers (km) and times are in seconds. We assume a congestion period a of 10,000 seconds, a congestion speed v_{con} of 10 km/h, a minimum free-flow speed $v^{\min}$ of 10 km/h and a maximum free-flow speed $v^{\max}$ of 90 km/h, which is representative of an intercity setting. Regarding the cost parameters, we use the same values as in Franceschetti et al. (2013), namely, $A = 7.47047 \times 10^{-5}$, $B = 0.0014$, $C = 1.977 \times 10^{-7}$, and $D = 0.0022$.

Using (14)–(15) we calculate the effective upper time window limits for each customer location and see that it matches the actual upper time window limit for each of them, except for location 2, where $U_2 = 10,280 < u_2$. Given the cost parameters, we have $\bar{v} = 55.19$ km/h and $\bar{v} = 75.48$ km/h. The furthest location that can be reached before the end of the congestion period is location 2, that is, we have $k = 2$ where k is defined in Section 4.1. Also, the furthest location that can be reached before the end of the congestion period when the vehicle arrives at location 1 at time U_1 is location 1, that is, we have $p = 1$ where p is defined in Section 4.2. Figures 5 and 6 depict the SP graph under the early and late policy, respectively. Both figures only show the arcs that have finite length, that is, for which the problem of finding the minimum cost of traversing the corresponding segment given the constraints was feasible. The missing arcs have infinite length. The shortest path on each graph is marked in bold. The optimal scheduling under the early and late policies are also depicted in Figures 7 and 8, respectively.

From Figure 5, we see that the shortest path from 0 to 5 under the early policy goes through nodes 2 and 4. This means that the optimal solution has three segments with speed changes occurring upon reaching locations 2 and 4, that is, we have $m = 3$, $j_1 = 2$, $j_2 = 4$, and $j_3 = 5$. It also means that the optimal solution is such that the vehicle drives in congestion on arcs 0 and 1, arrives at location 2 at time μ_2 , and starts service at location 4 at time l_4 . Figure 7 provides further information about the optimal schedule: we see that

Figure 5. (Color online) SP Graph Under the Early Policy

Figure 6. (Color online) SP Graph Under the Late Policy

Figure 7. (Color online) Optimal Schedule Under the Early Policy **Figure 8.** (Color online) Optimal Schedule Under the Late Policy

Table 2. Times (in Seconds) and Costs for the Optimal Solutions

	Driver wage policy	
	Early policy	Late policy
Driving time in congestion	5,399.99	3,600
Driving time in free-flow	1,456.27	1,373.38
Total driving time	6,856.26	4,973.38
Paid waiting time	2,621.5	1,000
Total labor time	13,477.77	9,973.38
Return to depot time	13,477.77	14,373.38
Labor cost	29.65	21.94
Emissions cost	13.63	10.73
Total cost	43.28	32.67

Table 3. Optimal Speeds Values (in km/h)

	Driver wage policy	
	Early policy	Late policy
v_0^*	10	10
v_1^*	10	81
v_2^*	55.19	81
v_3^*	55.19	75.48
v_4^*	75.48	75.48

the vehicle waits postservice at location 2 until the end of the congestion period, then drives on arcs 2 and 3 at speed $\bar{v}$, reaching location 4 strictly before its lower time window limit, and causing some mandatory pre-service waiting time. Upon completion of service at location 4, the vehicle returns to the depot driving at a speed of $\bar{v}$. The total cost of this solution is 43.99, which is equal to the sum of the arc lengths on the shortest path.

From Figure 6, we see that the shortest path from 0 to 5 under the late policy goes through nodes $\hat{1}$ and 3_u . This means that the optimal solution has three segments with speed changes occurring when reaching locations 1 and 3, that is, we have $m = 3$, $j_1 = 1$, $j_2 = 3$, and $j_3 = 5$. It also means that the optimal solution is such that the vehicle drives in congestion on arc 0, arrives at location 1 at time U_1 (making location 1 the pivot location), and at location 3 at time U_3 . From Figure 8 we further learn that service at the depot is delayed until after 4,400 seconds. Also, we see that the vehicle waits postservice at location 1 until the end of the congestion period, then drives on arcs 1 and 2 at the required speed to arrive at location 3 exactly at time U_3 . After completing the service at location 3, the vehicle drives on arcs 3 and 4 at speed $\bar{v}$. The total cost of this solution is 32.67, which is equal to the sum of the arc lengths on the shortest path.

Tables 2–4, respectively, display a comparison of the optimal cost values, speed values, and waiting times between the two driver wage policies.

Table 4. Optimal Preservice and Postservice Waiting Times (in Seconds)

Location	Driver policy			
	Early policy		Late policy	
	Preservice (y)	Postservice (z_i)	Preservice (y)	Postservice (z_i)
0	0	0	4,400	0
1	—	0	—	1,000
2	—	2,621.5	—	0
3	—	0	—	0
4	—	0	—	0

We see that both the labor and emission costs are lower under the late policy. This can be explained by the following factors. First, because service time at the depot is delayed under the late policy, the vehicle spends significantly more time driving in congestion under the early policy than under the late policy. Second, the optimal vehicle speed is (weakly) larger on each arc under the late policy. Third, the driver is paid for significantly more waiting time under the early policy (specifically an extra 3,490.90 minutes). In other words, by delaying the arrival (and therefore the payment) of the driver at the depot under the late policy, the decision maker is able to mitigate the negative impact of congestion, increase driving speed, and reduce in-transit waiting times. Note that the driver returns sooner to the depot under the early policy than under the late policy.

6. Conclusions

We have studied the DSOP, which consists of computing the optimal schedule for a vehicle visiting a fixed sequence of customer locations to minimize the sum of labor and emissions costs. We have provided an efficient shortest-path formulation to solve this problem optimally in polynomial time. Because traffic congestion creates incentives for voluntary waiting times in the schedule, existing methodologies for the DSOP without traffic congestion cannot be directly applied or easily adapted. Besides being efficient, our solution methodology provides an intuitive visual representation of the optimal solution through the shortest-path network, which represents the breakdown of the optimal schedule into adjacent segments of arcs on which the optimal speed is kept constant. In our model formulation, we have considered two different driver wage policies and we have illustrated through a numerical example how their optimal solutions differ. Our solution methodology can be embedded as a subroutine within an algorithm to solve more general vehicle routing and scheduling problems such as the pollution-routing problem with traffic congestion.

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Endnote

¹ All of our results also apply with minor modifications if the last node is also a customer node and there is no return to the depot or if the vehicle must return to the depot within a certain time period.

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