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Citation for published version (APA):

Document status and date:
Published: 17/03/2015

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

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Download date: 04. Apr. 2020
Mosaic Drawings and Cartograms

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Figure 1: US election 2012, electoral college votes. Diffusion cartogram by M. Newman (Univ. Michigan), square mosaic cartogram (Wikipedia), square and hexagonal mosaic cartograms computed by our algorithm.

1 Introduction

Cartograms visualize quantitative data about a set of regions such as countries or states by scaling each region such that its area is proportional to its data value. There are several different types of cartograms and some algorithms to construct them automatically exist. The most common cartograms are contiguous area cartograms [8] (Fig. 1 left). Here the regions are deformed in such a way that adjacencies are kept. Contiguous area cartograms perform well if the data values are positively correlated to the land areas of the input regions, but producing good cartograms if this is not the case remains a challenge.

The size of regions in contiguous area cartograms is generally hard to judge. To remedy this situation several types of cartograms depict regions using simple geometric shapes like disks, squares or rectangles. When using rectangles, adjacencies and relative positions can be maintained [2, 10]. However, the rectangular shape is not very recognizable and hence Mumford et al. [4, 5] initiated the study of rectilinear cartograms where each region is represented by a rectilinear polygon. But the areas of general rectilinear polygons are again difficult to estimate and compare. This is much easier if the polygons are composed of a small number of unit squares (Fig. 1 middle two). We focus on this type of cartograms, or more generally cartograms which use multiples of simple tiles – usually squares or hexagons (Fig. 1 right) – to represent regions. In absence of a dedicated name in the literature we call such cartograms mosaic cartograms.

Mosaic cartograms using squares have been popularized by the New York Times, usually in the context of the US elections, but also to show changing demographics. Mosaic cartograms using hexagons are less frequent, examples are the “Indices of Deprivation 2010” by the Leicestershire County Council.

Mosaic cartograms communicate data well that consist of, or can be cast into, small integer units (for example, electoral college votes), they allow users to accurately compare regions, and they can often maintain a (schematized) version of the input regions’ shapes. We propose the first method to construct mosaic cartograms fully automatically. To do so, we first introduce mosaic drawings of triangulated planar graphs. We then modify mosaic drawings into mosaic cartograms with zero cartographic error while maintaining correct adjacencies between regions.

Quality criteria. There are several quality criteria for (mosaic) cartograms. One of the most important ones is the cartographic error, which is defined for each region as \( \frac{A_c - A_s}{A_s} \), where \( A_c \) is the area of the region in the cartogram and \( A_s \) is the specified area depending on the data value to be shown. In a mosaic cartogram a region is represented by an edge-connected set of tiles, which we call a configuration. Each configuration must be simple, that is, contain no holes. The mosaic resolution measures the (maximum and average) number of tiles used per region. We consider the following quality criteria in this paper:

- Average and maximum cartographic error
- Correct adjacencies of configurations: two configurations are adjacent if and only the corresponding input regions are adjacent
- Shape of the regions
- Relative positions of regions
- Mosaic resolution

It is generally challenging to simultaneously satisfy all criteria well. We decided to enforce correct adjacencies in our algorithm, that is, we produce only mosaic...
cartagoms which have exactly the same configuration adjacencies as the corresponding regions of the input map. There is a clear trade-off between mosaic resolution and recognizability of the map. A low mosaic resolution, that is a small to medium number of tiles per region, allows users to explicitly count tiles and compare regions. A high mosaic resolution makes it easier to preserve shapes and relative positions and to achieve zero cartographic error. Fortunately our method can create mosaic cartogram with zero cartographic error for relatively low mosaic resolutions.

Most mosaic cartograms which are made by hand do neither preserve the adjacencies of all input regions nor their shapes. A common semi-automated approach is to take a map or a contiguous area cartogram and to overlay it with a suitable grid. This requires a rather high mosaic resolution, since otherwise rounding errors will easily destroy or create adjacencies. Furthermore, mosaic cartograms created in this way usually do not have zero cartographic error.

2 Mosaic Drawings

We define mosaic drawings for plane triangulated graphs, that is, planar graphs with a given embedding where every interior face is a triangle. Mosaic drawings are drawn on a tiling of the plane. Of particular interest are the uniform, and especially the regular tilings, although other types of tilings might also result in intriguing drawings. There are three types of regular tilings: the triangular, the square, and the hexagonal tiling. The triangular tiling uses two different rotations of the basic triangular shape and hence is visually a little more complex than the two different rotations of the basic triangular shape and the hexagonal tiling. The triangular tiling uses two different rotations of the basic triangular shape and hence is visually a little more complex than the two different rotations of the basic triangular shape and the hexagonal tiling. The triangular tiling uses two different rotations of the basic triangular shape and hence is visually a little more complex than the two different rotations of the basic triangular shape and the hexagonal tiling.

We call a set of edge-connected tiles of a tiling \( T \) a configuration. We say that a configuration \( C \) is simple if its tiles are simply connected (\( C \) has no holes). Two configurations \( C_1 \) and \( C_2 \) are adjacent if and only if there is at least one tile \( t_1 \in C_1 \) and at least one tile \( t_2 \in C_2 \) such that \( t_1 \) and \( t_2 \) are edge-connected. A mosaic drawing \( D_T(G) \) of plane triangulated graph \( G = (V,E) \) on \( T \) represents every vertex \( v \in V \) by a simple configuration \( C(v) \) of edge-adjacent tiles from \( T \) in such a way that two vertices \( v \) and \( u \) of \( G \) are connected by an edge \( e = (v,u) \) if and only if the configurations \( C(v) \) and \( C(u) \) are adjacent (see Fig. 2).

We say that a mosaic drawing is simple if the union of all configurations is simply connected, that is, it has no holes. Mosaic drawings are a type of contact representation, since they do not draw edges explicitly, but imply them by the contact of the configurations.

For which tilings do mosaic drawings exist? Below we show – with the help of results from the Graph Drawing and VLSI layout literature – that simple mosaic drawings exist for both square and hexagonal tilings. For a given (style of) mosaic drawings we can then consider various quality criteria, such as the size (number of tiles) of the drawing and the area (number of tiles inside a bounding box).

**Square and hexagonal tilings.** We show that any plane triangulation has a mosaic drawing on a square and hexagonal tiling via orthogonal (grid) drawings [6] of a graphs. Here the vertices are placed on an integer grid and the edges are routed along the grid.

Given a plane triangulation we can obtain a mosaic drawing in the following way as illustrated in Fig. 3: We first take the weak dual of the triangulation, which has a vertex for each triangle of the triangulation, and an edge for any pair of adjacent triangles. The dual of a triangulation has a maximum degree of three and can be laid out orthogonally on a grid. To obtain a mosaic drawing on a square tiling, we represent every grid point of the orthogonal drawing by four squares of the tiling and consistently distribute them to the configurations of the triangulation. On a hexagonal grid we can proceed in exactly the same way but shear the orthogonal drawing; alternatively we can directly embed the dual on a hexagonal grid [9].

The construction above shows that we can obtain mosaic drawings that are linear in the size of the corresponding orthogonal drawings. This allows us to derive complexity results for mosaic drawings from existing results for orthogonal drawings. Orthogonal drawings on small-area grids have been studied extensively [6, 12]. For instance, if the triangulation induces an outerplanar graph, we can obtain a mosaic drawing of relatively small area by making use of
the fact that the dual is a binary tree [7]. However, minimizing the size (area, length, etc.) of an orthogonal drawing is NP-hard even if the orientations of the edges used are already given [1, 13], which makes it unlikely that we can efficiently minimize the area of a mosaic drawing.

3 Mosaic cartograms

Our input is now a triangulated planar graph $G = (V, E)$ together with integer weights $w(v)$ for each vertex $v \in V$. A mosaic cartogram for $G$ is a simple mosaic drawing of $G$ where each configuration $C(v)$ consists of exactly $w(v)$ tiles for each vertex $v \in V$. The construction of mosaic drawings from orthogonal drawings of the dual as sketched above does not necessarily produce simple mosaic drawings. While we could extend this construction and fill faces in a consistent way, we instead choose an alternative method which is based on orderly spanning trees [3] and produces compact simple mosaic drawings (see Fig. 4).

Orderly spanning trees are spanning trees with certain order relations between the nodes. Chiang et al. [3] show how to compute an orderly spanning tree $ST$ for a planar triangulation $G$. They then construct a (vertical) visibility drawing for $ST$, which is stretched into a 2-visibility drawing of $G$. They grow horizontal branches to fill up gaps and finally shrink thick branches to height 1. The result is a square mosaic drawing of $G$. Since at most three regions meet at each intersection, we can directly shear the same drawing onto a hexagonal grid and so obtain a hexagonal mosaic drawing with the same complexity.

To compute mosaic cartograms, we start with a mosaic drawing of the (augmented) dual of the input map. Unfortunately the orderly spanning trees created by Chiang et al. [3] have a tendency to “curl inwards”. The resulting mosaic drawings often do not retain any of the relative positions of the input nodes and are hence a rather poor starting point for mosaic cartograms (see Fig. 5 left). However, the spanning trees induced by a Schnyder labeling are also orderly spanning trees [11] and lead to mosaic drawings which do capture a significant part of the relative positions of the input (see Fig. 5 right top).

We use an iterative method to grow (or shrink) the configurations according to the input data. While doing so, we maintain the correct adjacencies at all times. We use so-called guiding shapes – configurations which represent the desired final state of each configuration $C(v)$ – to slowly nudge each configuration towards the correct number of tiles and the correct shape. Guiding shapes are iteratively moved using a force-directed approach to reduce overlap while moving the configurations into appropriate relative positions. Note that the configurations never overlap, only their guiding shapes might. We maintain a proper mosaic drawing at all times, although it might
not always be simple. Each configuration, however, is always simple. This process stops as soon as no more progress can be made. Now most configurations have a good shape and the correct number of tiles, a few might have some tiles too many or too few. The final step of our algorithm uses a minimum cost flow formulation to assign the correct number of tiles to each configuration. This process maintains the correct adjacencies and disturbs the shapes as little as possible. It also removes any holes in the drawing which might have arisen during the iterative resizing and moving.

4 Future Work

An obvious direction for future work are other types of tilings, most notably the triangular tiling. Our method in principle extends to any uniform tiling, if one interprets it as a square or hexagonal tiling by grouping adjacent tiles appropriately (Fig. 7). However, more direct approaches and corresponding size and area bounds would be of interest.

![Hexagonal and Triangular Tiling](image)

Figure 7: Interpreting the hexagonal tiling as a square tiling and the trihexagonal tiling as a hexagonal tiling.

References


