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Uncertainty in advance scheduling problem in operating room planning

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ABSTRACT

In this paper, we consider the Advance Scheduling Problem, in which candidate patients from the patients' waiting list have to be selected and allocated to the available operating room blocks in a short-term schedule, taking into consideration a number of restrictions. We determine the available blocks by a modified block scheduling strategy. We propose multiple objectives, including minimization of the patients waiting time, tardiness, cancellation, block overtime, and the number of surgery days of each surgeon within the planning horizon. In our problem, uncertainty is taken into account via stochastic surgery durations, but also a deterministic version is considered for reasons of comparison. We approach the problem with different formulations of two-stage stochastic programming and two-stage chance-constrained stochastic programming. We solve the proposed models through the sample average approximation (SAA) method and Benders decomposition technique. Based on examining the approaches on real hospital data, we illustrate that deterministic solutions do not fit properly in the stochastic environment for this problem. We also demonstrate that using a chance-constrained approach decreases the considerable variation between the deterministic and stochastic solution. Finally, we show that the SAA method and Benders decomposition algorithm can generate effective solutions in a relatively reasonable computational time in medium-sized problems.

1. Introduction

Usually, the problem of the planning and scheduling of operating rooms is considered in a three-level decision making procedure: strategic, tactical and operational level. At the strategic level, the capacity of the ORs is determined. At the tactical level, distributing this capacity among different surgery groups is concerned, and a cyclic timetable which is known as *Master Surgical Schedule* (MSS) is set up. At the operational level, we can consider two stages. The problem/stage of selecting candidate patients from a patients' waiting list and assigning them a date and an operating room, which is called *Advance Scheduling Problem* (ASP); and the problem/stage of daily scheduling and sequencing of patients in different operating rooms which is called *Allocation Scheduling Problem* (Cardoen, Demeulemeester, & Beliën, 2010; Guerriero & Guido, 2011).

Before establishing the surgery schedule, a decision regarding the patients booking policy has to be made. Generally, hospitals operate one of the following three policies. In a policy known as *Open Scheduling Policy*, there is no specific classification on different specialty groups, and the planning and scheduling of patients are determined based on their priority (e.g., based on the order of entry of the patients, usually first-come-first-service rule). In another policy which is known as *Block*

Scheduling Policy, the available capacity of operating rooms is divided into pre-determined time intervals (blocks) and each block is dedicated to a certain specialty group based on the MSS. Finally, there is also a third policy known as *Modified Block Scheduling Policy*, which is based on the combination of the two previous policies: some blocks are assigned to certain specialties, while others remain open. Furthermore, at a later moment, the unused capacity of blocks could be released for other specialties (Fei, Meskens, & Chu, 2009; Guerriero & Guido, 2011).

The problem of planning and scheduling of operating rooms has been surveyed extensively in the literature. Magerlein and Martin (1978), Cardoen et al. (2010), Guerriero and Guido (2011), van Riet and Demeulemeester (2015), and Samudra et al. (2016) surveyed a comprehensive review on the subject. As the problem investigated in this study focuses on ASP, in the following we concentrate on the ASP literature. Ozkarahan (2000) developed a goal programming formulation with the objectives of minimizing the idle time and overtime and maximizing the satisfaction of surgeons, staff, and patients. They considered restrictions on requests for a particular time block, OR utilization, surgeons preferences, and capacity of ICU in patient admission. Fei, Chu, Meskens, and Artiba (2008) proposed an integer programming formulation with the objective of minimizing the overtime and idle-time costs. A consolidated solution approach of a branch and bound and

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column generation method is developed, and the solution approach is assessed on randomly generated data. Thereafter, they (Fei, Meskens, & Chu, 2010) considered a two-phase advance and allocation scheduling problem in an open scheduling strategy (in contrast with Fei et al., 2008) in a multi-functional operating theater. New restrictions on cases deadlines, surgeons' availability and ORs opening hours are added to the problem of Fei et al. (2008). In the first phase, for the advance scheduling problem, set partitioning concept is employed to model the problem and a column generation method is developed to solve the model; and in the second phase, for the allocation scheduling problem, a hybrid genetic algorithm is proposed as the solution method. Molina and Framinan (2009) aimed at maximizing the service quality in their block scheduling strategy ASP. They considered two different patient assignment policies: (1) assigning a room to a patient and then assigning a surgeon to the patient-room set, and (2) scheduling a portion of patients based on policy 1, whereas others are assigned to a surgeon in the first step, and successively, a room is allocated to the patient-surgeon set. Real data is used to evaluate the solution quality of both policies. Roshanaei, Luong, Aleman, and Urbach (2017a) developed novel logic-based Benders' decomposition (LBBD) approaches in a network of hospitals, i.e., distributed operating room scheduling. They proposed a new Benders' feasibility cut and a cut propagation mechanism in their LBBD approach. Implementing computational experiments on real hospital data, they investigated the advantages of their approach regarding the solution method and considering such a hospitals network. Their LBBD is demonstrated to be significantly faster than commercial solvers. They extend their work in Roshanaei, Luong, Aleman, and Urbach (2017b) considering a pool of patients, surgeons, and ORs in a strategic hospital network. They also proposed a game theoretic approach to stabilize the network and to fairly redistribute the payoffs to the network.

As there is intrinsic uncertainty in several parameters of the planning and scheduling of operating rooms, it might be better to take uncertainty into account. Pham and Klinkert (2008) considered add-on patients along with elective patients. They proposed a multi-mode blocking job shop model for elective patients and a multi-mode blocking job insertion problem for add-on patients both with deterministic surgery times. They used CPLEX to solve the developed MIP. The model efficiency is tested on real and randomly generated data. Hans, Wullink, Van Houdenhoven, and Kazemier (2008) formulated a robust ASP as a bin packing problem with different bin sizes. Then, the problem is approached as a stochastic knapsack problem. The objective function is considered as minimizing the risk of overtime and cancellation and improving OR utilization. A constructive and local search heuristic mechanism is developed as the solution method and tested on real data. Lamiri, Xie, Dolgui, and Grimaud (2008a) investigated the ASP with uncertain demands for emergency surgery. Total OR time is assumed as a random variable with a known probability distribution. To optimize the elective patients' related costs and expected costs of overtime in ORs, a Monte Carlo sampling technique and a column generation procedure are used. To solve the model more efficiently and to achieve more reliable results, they extended their work and covered some drawbacks in Lamiri, Grimaud, and Xie (2009) and Lamiri, Xie and Zhang (2008b).

Shylo, Prokopyev, and Schaefer (2013) investigated the ASP in a block scheduling strategy with a different approach, based on a normal approximation of cumulative surgery durations. A chance-constrained programming formulation is used to control the level of overtime in ORs with high-volume specialties. Real data is extracted to evaluate the proposed framework. They examined their approach with a discrete event simulation model. Gul, Denton, and Fowler (2015) proposed a multistage stochastic MIP formulation for their ASP. They aimed at minimizing the expected cost of cancellation, waiting time and overtime in their model. A progressive hedging algorithm is applied as a solution method, and numerical experiments based on real data are carried out to evaluate the approach. Bruni, Beraldi, and Conforti

(2015) considered the uncertainty of arrivals of emergency patients and surgery durations in their ASP. Three recourse strategies are proposed characterizing different reactive scheduling policies. They developed tailored heuristic solution strategies as a solution approach in their randomly generated test problems. Salemi Parizi and Ghate (2016) investigated a multi-class, multi-resource advance scheduling with no-shows, cancellation, and overbooking. A Markov decision process framework is formulated. According to intractability, approximate dynamic programming rooted in Lagrangian relaxation, affine value function approximation, and constraint generation are developed as the solution procedure. An extended analysis is conducted to show the efficiency of their proposed approach. Addis, Carello, and Tanfani (2014) proposed a robust optimization model for their ASP with uncertain surgery durations in a block scheduling strategy. A penalty function over patients' waiting time and tardiness is suggested. The proposed model is implemented with AMPL and solved with CPLEX on real-based instances and randomly generated scenarios. They improved their work in Addis, Carello, Grosso, and Tanfani (2016), and considered a rolling horizon in their investigation. Newly arrived patients are taken into consideration in the rolling horizon approach, and mid-term scheduling is accessible in a look-ahead perspective. The same objective function as used in Addis et al. (2014) is employed. Wang, Roshanaei, Aleman, and Urbach (2016), applying discrete event simulation, surveyed the impact of uncertain surgical durations, emergency arrivals, and limited downstream resources on hospital performance indicators in a collaborating hospital network. They showed that dynamic arrivals of emergency patients and stochastic surgical duration are not overly disruptive to deterministically scheduled ORs. Jebali and Diabat (2017) developed a two-stage chance-constrained stochastic programming formulation for elective and emergency cases in the advance scheduling problem with consideration of ICU bed restrictions. Surgery duration and patient length of stay in ICU are considered as random variables. Sample average approximation algorithm is employed to solve the model, and numerical experiments are carried out to evaluate its performance.

From the above investigation, the following drawbacks and/or research gaps can be inferred:

- In most papers, researchers only have considered hospitals with open or block scheduling policy. It is hard to find a research paper in the field of OR planning and scheduling using modified scheduling policy.
- Most papers looked at the problem and the objective function from hospital managers' perspective or patients' perspective, or both at best. Surgeons' perspective has been usually neglected, whereas, in a lot of hospitals, surgeons are the ones whose preferences are in priority.
- Less attention has been paid to the arrival of emergency patients.
- Less attention has been paid to the uncertainty of surgery duration.
- The chance-constrained concept is rarely considered in the literature of OR planning and scheduling, while in the real world, hard constraints are often violated. Therefore, fulfilling some constraints with a high degree of confidence is a better approach.

Accordingly, with the aim of covering the mentioned drawbacks and/or research gaps, the main contribution of this paper is to address the advance scheduling problem with modified block scheduling policy, which seems to be the policy with the appropriate flexibility and competency in OR scheduling. Surgery durations are considered as stochastic/uncertain parameters, and emergency patient arrivals are taken into account. Surgeons' limitations are concerned through restrictions over the maximum number of surgeries per day allowed to be accomplished by a surgeon, and their preferences are concerned by minimizing the total number of days the surgeons are planned to come to the hospital to accomplish their surgeries. Further realistic features are considered in the problem by taking into account the advantages of

chance-constrained concept. It is enforced that the risk of blocks/rooms overtime violation is less than a given threshold. Thus, two-stage stochastic and two-stage chance-constrained programming frameworks are employed to formulate the proposed problem. Other contributions stand on solving the proposed formulations. In doing so, we apply two approaches of sample average approximation (SAA) method and Benders decomposition (BD) algorithm, and examine the efficiency of the developed solution approaches through test problems. Conclusively and to the best of our knowledge, this is the first time that ASP is considered with such mentioned features.

The remainder of the paper is organized as follows: In Section 2, we explain the problem in detail and present a MIP formulation. In Section 3, we formulate the two-stage stochastic formulation and two-stage chance-constrained programming counterpart of the proposed models. In Section 4, we introduce the solution methods employed to solve the proposed formulations. In Section 5, we investigate the advantages of the constructed formulations by means of solving some test problems and real data experiments. Finally, we discuss the conclusions and future study directions in Section 6.

2. Problem statement

We consider the model proposed in Addis et al. (2014) as a base, and carry out some required modifications to the model to make it more practical. The additional assumptions, prerequisites, features, and constraints to the model are listed below:

- In terms of the objective function, next to the overall penalty which takes into account urgency and waiting time of patients (Tanfani & Testi, 2010), and the penalty for tardiness and due date violation (Addis et al., 2014), we consider some new aspects. The number of unscheduled patients (the ones not admitted within the planning horizon), the overtime of blocks/rooms, and the number of days each surgeon has to come to the hospital to accomplish his/her surgery are other objectives which we consider to be minimized in this study.
- In terms of constraints, we add maximum blocks and ORs overtime, the maximum number of surgeries allowed to be operated by a surgeon to the formulation. We also include the violation of the overtime restrictions to the problem by using the chance-constrained concept.
- In terms of the policy, we consider an operating theatre with some operating rooms with a modified block scheduling policy in which each block is allocated to a pre-determined specialty group. But it is still possible to assign some patients of a specialty group to some other blocks with other specialties, or even to keep a block open for some patients of different specialty groups (e.g., emergency patients).
- And finally, in terms of uncertainty, we prefer stochastic solution methods to robust approaches.

These modifications and extensions make our model more realistic and applicable. Accordingly, it is assumed we have R operating rooms ($r = 1, \dots, R$) and a set of patients, $q = 1, \dots, Q$, have to be scheduled in a set of blocks, $b = 1, \dots, B$, in a set of days, $d = 1, \dots, D$, in a set of weeks, $w = 1, \dots, W$. All patients have a priority coefficient U_q which is used to determine the sum of weighted waiting times and tardiness over all patients. The priority coefficient along with the patient's due time helps in distinguishing between inpatient and emergency patients. Let A_q be the release time for the surgery of each patient and D_q be the due time for the surgery of each patient. The clinical condition of the patient will descend if he/she does not receive surgical services and a patient is considered tardy if he/she does not receive surgical services before $D_q - A_q$. In comparison with elective patients, the tardiness of emergency patients will impose a bigger penalty on the objective function. We predict the arrival of emergency patients in advance, i.e., $q \in P \cup I$, and

devise the required resources. Similar to Addis et al. (2016), we consider an occupational parameter, γ_{bdw} , to leave a certain fraction of the total block capacity time in day d in week w to be unutilized to manage new emergency arrivals and/or other possible disruptions. By means of a proper definition of U_q , emergency patients are certainly operated in the day of referring to the hospital; otherwise, they will be treated according to their clinical priority. We also assume that emergency patients could be operated in all blocks (in contrast with elective patients). The allocation of surgeons to the (elective) patients is already known. We suppose MSS as given. In other words, scheduling and assigning each specialty group to blocks of each OR in each day of the week is known a priori; and the planning horizon is constructed from the repetition of the same schedule of blocks (MSS) in a set of W weeks. The ASP attempted in this study is selecting candidate patients from the patients' waiting list and allocating them a date and a proper operating room block in a modified block scheduling strategy, taking into consideration all elements from the objective function. We initially consider the problem deterministically, but later, we take into account uncertainty via stochastic surgery durations and via the possibility of violation of the maximum permitted overtimes in blocks and operating rooms. Fig. 1 illustrates the proposed problem schematically.

2.1. Notations

The following notations are applied throughout this paper.

Indices:

- p index for elective patients ($p = 1, \dots, P$, where P is the number of elective patients)
- i index for non-elective patients ($i = 1, \dots, I$, where I is the number of non-elective patients)
- q index for elective and non-elective patients ($q = 1, \dots, Q$, where $Q (= P + I)$ is the number of all patients)
- r index for operating rooms ($r = 1, \dots, R$, where R is the number of operating rooms)
- b index for blocks ($b = 1, \dots, B$, where B is the total number of blocks in all ORs)
- s index for surgeons ($s = 1, \dots, S$, where S is the number of surgeons)
- e index for expertise (specialty groups) ($e = 1, \dots, E$, where E is the number of specialty groups)
- d index for days in a week ($d = 1, \dots, D$, where D is the number of days)
- w index for weeks ($w = 1, \dots, W$, where W is the number of weeks)

Parameters:

- C_{bdw} capacity of block b in day d in week w
- γ_{bdw} occupation parameter of block b in day d in week w
- $\tilde{t}_q$ expected surgery duration of patient q
- A_q release time/date for the surgery of patient q
- D_q due time/date for the surgery of patient q
- U_q clinical priority coefficient of patient q
- m_j weight of term j ($j = 1, \dots, 7$) in the objective function
- B_{edw}^E set of blocks which are assigned to expertise e in day d in week w
- E_q^Q expertise which patient q need ($E_q^Q = 1, \dots, E$)
- B_{rdw}^R set of blocks which are assigned to room r in day d in week w
- S_p^P surgeon who should/will operate patient p ($S_p^P = 1, \dots, S$)
- $B_{qd w}^Q$ set of blocks which patient q could be operated in day d in week w (due to the head of surgery department direction) in addition to the block allocated to the needed expertise E_q^Q
- $O_b^{\max}$ maximum overtime allowed by each block
- $O_r^{\max}$ maximum overtime allowed by each operating room

Fig. 1. An illustration of the ASP problem.

$N_s^{\max}$ maximum number of surgeries allowed to be accomplished in a day by a surgeon

D_{dw} is calculated as $D \times (w-1) + d$, total number of waiting days by a patient during the planning horizon ($D \times W$)

Decision variables:

x_{qbdw} equals 1 if patient q is scheduled to be operated in block b in day d in week w ; 0 otherwise

o_{bdw} amount of overtime of block b in day d in week w

n_{sdw} equals 1 if surgeon s is scheduled for surgery in day d in week w ; 0 otherwise

2.2. Deterministic mathematical formulation

In the following, a deterministic formulation of the proposed problem constructed based on the mean value of the surgery durations is presented.

$$\begin{aligned}
 Obj_D = & \min \left(\sum_q \sum_b \sum_d \sum_w [m_1(D_{dw} - A_q) + m_2(D_{dw} - D_q)^+] x_{qbdw} U_q \right) \\
 & + \left(\sum_{q: D_q \leq D \times W} [m_3(D_q - A_q) + m_4((D \times W + 1) - D_q)] (1 - \sum_b \sum_d \sum_w x_{qbdw}) U_q \right) + \\
 & m_5 \left(Q - \sum_q \sum_b \sum_d \sum_w x_{qbdw} \right) + m_6 \left(\sum_s \sum_d \sum_w n_{sdw} \right) + m_7 \left(\sum_b \sum_d \sum_w o_{bdw} \right)
 \end{aligned} \quad (D1)$$

Subject to:

$$\sum_w \sum_d \sum_{b \in \{B_{edw}^E | E_q^Q = e\} \cup B_{qdw}^Q} x_{qbdw} \leq 1 \quad \forall q \quad (D2)$$

$$\sum_q \bar{t}_q x_{qbdw} \leq (\gamma_{bdw} C_{bdw}) + o_{bdw} \quad \forall b, d, w \quad (D3)$$

$$o_{bdw} \leq O_b^{\max} \quad \forall b, d, w \quad (D4)$$

$$\sum_q \sum_{b \in B_{rdw}^R} \bar{t}_q x_{qbdw} \leq \sum_{b \in B_{rdw}^R} [(\gamma_{bdw} C_{bdw}) + o_{bdw}] \quad \forall d, w, r \quad (D5)$$

$$\sum_{b \in B_{rdw}^R} o_{bdw} \leq O_r^{\max} \quad \forall d, w, r \quad (D6)$$

$$D_{dw} \geq A_q \sum_b x_{qbdw} \quad \forall q, d, w \quad (D7)$$

$$\sum_b x_{pbdw} \leq n_{sdw} \quad \forall p: S_p^P = s, \forall s, d, w \quad (D8)$$

$$\sum_{p: S_p^P = s} \sum_b x_{pbdw} \leq N_s^{\max} \quad \forall s, d, w \quad (D9)$$

$$x_{qbdw} \in \{0, 1\} \quad \forall q, b, d, w \quad (D10)$$

$$n_{sdw}: \text{integer} \quad \forall s, d, w \quad (D11)$$

$$o_{bdw} \geq 0 \quad \forall b, d, w \quad (D12)$$

In our proposed model, objective function (D1) is made up of seven terms. The first and the second terms are respectively the penalties of the weighted sum of waiting time and tardiness for the scheduled patients. To get rid of the max operator in the second term of the objective function (D1), $(D_{dw} - D_q)^+ = \max\{D_{dw} - D_q, 0\}$, some pre-calculations are needed. The third and the fourth terms are respectively the penalties of the weighted sum of waiting times and tardiness for the unscheduled patients whose due times are within the planning horizon. It is worth

noting that as the real tardiness of unscheduled patients cannot be calculated, it is assumed that all unscheduled patients are scheduled right the day after the end of the planning horizon ($D \times W + 1$). To give preference to the urgent patients, the terms are weighted by the patients' clinical priority coefficient U_q . The fifth term is the penalty for the number of unscheduled patients. The sixth term penalizes the total number of days the surgeons should come to the hospital to accomplish their surgeries, and finally, the seventh term minimizes the total blocks overtime during the planning horizon. Different terms in the objective function have been homogenized by a coefficient $m_j, j = 1, \dots, 7$. Constraint (D2) guarantees that each patient receives surgical services at most once. Constraints (D3) and (D4) calculate the overtime in each block in each day of several weeks and force it to be not more than the maximum permitted overtime in each block, respectively. Constraints (D5) and (D6) undertake the same with (D3) and (D4) for the overtime in each room. Constraint (D7) enforces the surgery date to be scheduled after the release time of the surgery (A_q). Constraint (D8) makes n_{sdw} to get 1 when a surgeon has surgery/surgeries in the blocks of a day and a week. Constraint (D9) guarantees that each surgeon obeys the maximum permitted number of surgeries in each day. Finally, Constraints (D10)–(D12) indicate the domain of variables.

3. Stochastic/uncertain mathematical formulation

The inherent stochastic nature of the planning and scheduling of ORs makes it essential to take uncertainty into account. Among the various uncertainty sources, the arrival of emergency patients and surgery durations are the two most important and discussed uncertain parameters, especially in cases that the planning horizon is shorter than the maximum desired waiting time of elective patients. In this study, therefore, we concern these two sources of uncertainty with the problem assumptions and formulation. We model the uncertainty in surgery durations via stochastic programming approaches. In the following subsections, we apply two different approaches to stochastic programming (i.e., two-stage stochastic programming, and two-stage chance-constrained programming formulations).

3.1. Two-stage stochastic mathematical formulation

Two-stage stochastic programming is the most widely applied and studied in the territory of stochastic programming models where the decisions are made in two stages. Some strategic decisions are made in the first stage, and after the realization of the uncertain parameters, which affect the outcome of the first-stage decision, the recourse operational/tactical decisions can be made in the second stage to make up the first-stage decisions (Ruszczynski & Shapiro, 2003; Shapiro & Philpott, 2007).

We formulate the two-stage mathematical formulation of the proposed problem (TSS) using the same indices, parameters, and sets described for the deterministic mathematical formulation. Additionally, we use the following notations.

Index:

ω index for scenarios ($\omega = 1, \dots, \Omega$, where Ω is the number of scenarios)

Parameters:

$P_{[\omega]}$ probability of scenario ω
 $t_{q[\omega]}$ surgery duration of patient q in scenario ω

Decision variable:

$o_{bdw[\omega]}$ amount of overtime of block b in day d in week w in scenario ω

$$Obj_S = \min \sum_{\omega} P_{[\omega]} \left\{ \begin{aligned} & \left(\sum_q \sum_b \sum_d \sum_w [m_1(D_{dw}-A_q) + m_2(D_{dw}-D_q)^+] x_{qbdw} U_q \right) + \\ & \left(\sum_{q:D_q \leq D \times W} [m_3(D_q-A_q) + m_4((D \times W + 1)-D_q)] (1 - \sum_b \sum_d \sum_w x_{qbdw}) U_q \right) + \\ & m_5(Q - \sum_q \sum_b \sum_d \sum_w x_{qbdw}) + m_6(\sum_s \sum_d \sum_w n_{sdw}) \\ & + m_7(\sum_b \sum_d \sum_w o_{bdw[\omega]}) \end{aligned} \right\} \quad (S1)$$

Subject to: (D2), (D7)–(D11)

$$\sum_q t_{q[\omega]} x_{qbdw} \leq (\gamma_{bdw} C_{bdw}) + o_{bdw[\omega]} \quad \forall b, d, w, \omega \quad (S2)$$

$$o_{bdw[\omega]} \leq O_b^{\max} \quad \forall b, d, w, \omega \quad (S3)$$

$$\sum_q \sum_{b \in B_{rdw}^R} t_{q[\omega]} x_{qbdw} \leq \sum_{b \in B_{rdw}^R} [(\gamma_{bdw} C_{bdw}) + o_{bdw[\omega]}] \quad \forall r, d, w, \omega \quad (S4)$$

$$\sum_{b \in B_{rdw}^R} o_{bdw[\omega]} \leq O_r^{\max} \quad \forall r, d, w, \omega \quad (S5)$$

$$o_{bdw[\omega]} \geq 0 \quad \forall q, b, d, w, \omega \quad (S6)$$

According to (S3) and (S5), it is ruled that for all scenarios the block and room overtime should be restricted to pre-defined amounts of $O_b^{\max}$ and $O_r^{\max}$ respectively. In reality, however, this rule is sometimes violated in some critical or emergency situations. To be more realistic, we propose another formulation (two-stage chance-constrained Programming Formulation (TSCC)) in the next subsection to formulate this feature.

3.2. Chance-constrained programming formulation

Stochastic programming formulations guarantee that constraints are considered for all possible realizations of the uncertain parameters. But some constraints could be violated in some critical/emergency cases. To be more precise, some soft constraints can be violated at some penalty or for a maximum percentage of the realizations. One of the approaches used in optimization problems with uncertain parameters in such situations is the chance-constrained programming approach. Although some important issues such as computational intractability, difficulty in evaluating the probabilistic feasibility and non-convexity in the feasible region make this approach so challenging (Klein Haneveld & van der Vlerk, 1999, 2006; Luedtke, Ahmed, & Nemhauser, 2010), but in the absence of such complications, chance-constrained formulations can be reformulated in a deterministic manner. In most cases, complications convince us to use estimation and/or approximation methods while applying a chance-constrained approach. Usually Monte Carlo-based with discrete distribution chance-constrained problem approximation methods are suitable for the approximation approach; and the SAA method is one of the most applied and fitting methods for this purpose (Pagnoncelli, Ahmed, & Shapiro, 2009), in spite of the fact that SAA methods usually lead to the curse of dimensionality too (Luedtke & Ahmed, 2008).

For surgery durations, lognormal, gamma and normal distributions are considered and validated for surgery times in the literature (Strum, May, & Vargas, 2000; Stepaniak, Heij, & de Vries, 2010; Wiens, 1999), but, in case you need to calculate overtime in each block/room, the problem becomes more challenging since it is needed to sum surgery durations, and generally the convolution of different random variables will be difficult to determine analytically. Therefore, usually, approximation methods are used in the case. Monte Carlo simulation-based or scenario-based approximation methods are the most common approaches applied to solve the chance-constrained formulations where the continuous random variable distribution is approximated with a finite discrete distribution or with scenarios (Hans et al., 2008; Denton,

Miller, Balasubramanian, & Huschka, 2010; Min and Yih, 2010; Shylo et al., 2013).

In this study, the constraints that we should consider in probabilistic fashion are constraints (D4) and (D6) which are reformulated as (S'3) and (S'5).

$$P(\tilde{O}_{bdw} \leq O_b^{\max}) \geq 1 - \alpha^B \quad \forall b, d, w \quad (S'3)$$

$$P\left(\sum_{b \in B_{rdw}^R} \tilde{O}_{bdw} \leq O_r^{\max}\right) \geq 1 - \alpha^R \quad \forall r, d, w, \quad (S'5)$$

where α^B , α^R are the chance/risk level of violation of the blocks and ORs overtime. According to Klein Haneveld and van der Vlerk (1999, 2006), when the random variables are approximated with finite discrete distributions, ω , the convexity is lost, but it is still possible to rewrite the chance constraints in a scenario-based mixed-integer linear program as follows.

$$O_{bdw[\omega]} - O_b^{\max} \leq H_{[\omega]}^B \quad \forall b, d, w, \omega \quad (S''3)$$

$$\sum_{b \in B_{rdw}^R} O_{bdw[\omega]} - O_r^{\max} \leq H_{[\omega]}^R \quad \forall d, w, r, \omega, \quad (S''5)$$

where $H_{[\omega]}^B$ and $H_{[\omega]}^R$ are the extra blocks and ORs overtime. In other words, the overtime in blocks and rooms are allowed to be violated up to extra amounts of $H_{[\omega]}^B$ and $H_{[\omega]}^R$ in critical/emergency situations for each scenario $\omega = 1, \dots, \Omega$. Constraints (S''3) and (S''5) should be feasible for all values of x_{qbdw} and $O_{bdw[\omega]}$. Therefore, the set of constraints below guarantees constraints (S'3) and (S'5), and could be replaced with them.

$$O_{bdw[\omega]} - O_b^{\max} \leq H_{[\omega]}^B h_{b[\omega]} \quad \forall b, d, w, \omega \quad (S'''3)$$

$$\sum_{\omega} P_{[\omega]} h_{b[\omega]} \leq \alpha^B \quad \forall b \quad (S7)$$

$$\sum_{b \in B_{rdw}^R} O_{bdw[\omega]} - O_r^{\max} \leq H_{[\omega]}^R h_{r[\omega]} \quad \forall d, w, r, \omega \quad (S'''5)$$

$$\sum_{\omega} P_{[\omega]} h_{r[\omega]} \leq \alpha^R \quad \forall r \quad (S8)$$

$$h_{b[\omega]}, h_{r[\omega]} \in \{0, 1\} \quad \forall \omega \quad (S9)$$

Therefore, we formulate the deterministic counterpart formulation of the two-stage chance-constrained formulation of the proposed problem (TSCC) as follows.

$$Obj_S = \min \sum_{\omega} P_{[\omega]} \left\{ \begin{aligned} & \left(\sum_q \sum_b \sum_d \sum_w [m_1(D_{dw} - A_q) + m_2(D_{dw} - D_q)^+] x_{qbdw} U_q \right) + \\ & \left(\sum_{q: D_q \leq D \times W} [m_3(D_q - A_q) + m_4((D \times W + 1) - D_q)] (1 - \sum_b \sum_d \sum_w x_{qbdw} U_q) \right) + \\ & m_5 \left(Q - \sum_q \sum_b \sum_d \sum_w x_{qbdw} \right) + m_6 \left(\sum_s \sum_d \sum_w n_{sdw} \right) \\ & + m_7 \left(\sum_b \sum_d \sum_w O_{bdw[\omega]} \right) \end{aligned} \right\} \quad (S1)$$

Subject to: (D2), (D7)–(D11), (S2), (S''3), (S4), (S''5), (S6)–(S9).

4. Solution method for the stochastic/Uncertain mathematical formulations

To tackle uncertainty and solve stochastic mathematical models, and/or according to the problematic issue of recognizing the distribution function of the stochastic parameters, resolving the stochastic

parameter with different scenarios or discretizing the distribution function into finite discrete values (scenarios) is a prevalent approach. For this purpose, we need to guarantee that the original problem could be solved with reasonable accuracy in this approach. Therefore, we need a satisfactory number and distribution of scenarios to represent the stochastic parameter. In this regard, as a first approach, the Monte Carlo simulation technique could be applicable. Different researchers applied this technique under different names, remarkably Sample Average Approximation (SAA) method (Shapiro, 2001). In this method, the expected value of the objective function is approximated by the sample average estimate, stemming from a random sample. The attained problem then can be solved by deterministic optimization methods. Repeating the process iteratively with different samples provides estimates for the solution and the objective of the stochastic model. Finally, statistically valid lower and upper bounds can be constructed over the true optimal solution. In doing so, the two-stage model is solved to optimality M times based on N iid random samples each time. Let $\hat{v}_N^1, \dots, \hat{v}_N^M$ be the optimal values of the two-stage model. Therefore, $\bar{v}_{N,M} = \frac{1}{M} \sum_j \hat{v}_N^j$ would be an unbiased estimate of $[v_N]$. A lower bound, with a $100(1 - \alpha)\%$ approximation, for $[v_N]$ can be obtained by

$$L_{N,M} = \bar{v}_{N,M} - t_{\alpha,v} \hat{\sigma}_{N,M},$$

where $t_{\alpha,v}$ is the α -critical value of a v degree of freedom t-student distribution ($v = M - 1$) and $\hat{\sigma}_{N,M}$ is an estimation of the standard deviation of $\hat{v}_N^1, \dots, \hat{v}_N^M$. On the other hand, for a feasible solution $\hat{x}$, $\hat{v}_{N'}(\hat{x}) = c^T \hat{x} + \frac{1}{N'} \sum_{n=1}^{N'} Q(\hat{x}, \omega^n)$ is trivially an upper bound for the optimal solution of the two-stage model, where $\omega^1, \dots, \omega^{N'}$ is an iid random sample. N' is typically a fairly large number ($N' \gg N$). Accordingly, an upper bound, with a $100(1 - \alpha)\%$ approximation, for the two-stage problem could be obtained by

$$U_{N'}(\hat{x}) = \hat{v}_{N'}(\hat{x}) + z_{\alpha} \hat{\sigma}_{N'}(\hat{x}),$$

where z_{α} is the α -critical value of the standard normal distribution and $\hat{\sigma}_{N'}(\hat{x})$ is an estimation of the standard deviation of the sample N' according to $\hat{x}$. For the stepwise and detailed descriptions of the SAA method, please refer to Verweij, Ahmed, Kleywegt, Nemhauser, and Shapiro (2003), Shapiro & Philpott, (2007), and Mahootchi, Forghani, and Kamran (2017).

As the second approach, we take benefit from Benders decomposition technique to solve the proposed two-stage mathematical model with a large satisfactory number of scenarios. Benders' Decomposition (Benders, 1962) is a solution technique for solving certain large-scale optimization problems such as stochastic programming problems (in the realm of the stochastic programming it is also known as *L-shaped method*). In this technique, the main mathematical model is decomposed into two (or multiple) smaller formulations which are well-known as master and sub-problems, and instead of solving the main model, these smaller models are solved in an iterative approach which is more efficient (see Birge and Louveaux (1997) for the stepwise and detailed descriptions). Five decades after the introduction of the method, the classic approach and its extensions have been widely applied in various applications including ORs management problems (Denton and Gupta, 2003; Denton, Viapiano, & Voglet, 2007, 2010; Batun, Denton, Huschka, & Schaefer, 2011; Xiao, van Jaarsveld, Dong, & van de Klundert, 2016; Riise, Mannino, & Lamorgese, 2016; Roshanaei et al., 2017a,b; etc.). In this study, we apply a Benders' decomposition technique enriched with some valid inequalities, proposed in the literature to augment the relation between master and sub-

problem and to improve the performance of the technique, as a solution approach. Mathematical formulation and details about the proposed method are presented in [Appendix A](#).

5. Computational results

In this section, firstly, we survey illustrative numerical examples and carry out sensitivity analyses to examine the behavior of the solution regarding the different features of the problem. Secondly, we investigate the approach on real data in a hospital. We code the models and the solution approaches in the GAMS 24.5, and solve them with CPLEX solver. We run all the tests on a computer with Core (TM) 2 Duo CPU with 2.50 GHz speed and 4 GB of RAM.

5.1. Illustrative test problems

We consider a hospital with 3 operating rooms each with 2 blocks of specialty groups to examine the SAA method on the two-stage (TSS) and the two-stage chance-constrained (TSCC) formulations. In our illustrative test problems, each block is assigned 5hrs and therefore, each OR is allocated 10hrs working time a day. Each block is permitted up to 1.4hrs overtime, $O_b^{\max}$, and each OR is allowed up to overall 2hrs overtime, $O_r^{\max}$. It is assumed that each block is dedicated to a specialty group and patients should be operated in their assigned blocks; unless the head of the surgery department determines a surgery could be accomplished in some other blocks in addition to the block allocated to the needed expertise. In the test problems, it is assumed that 10% of surgeries could be operated in some blocks other than the dedicated ones. Also, it is supposed that emergency surgeries could be carried out on all blocks. All data according to patients of each specialty group are generated randomly. Surgery durations are supposed to be uniformly distributed with the mean 1.6 hrs over the interval $[1.6 - 0.5, 1.6 + 0.5]$. A clinical priority coefficient is assigned to each patient based on his/her release and due time. The weights of the terms in the objective function are determined based on various sets of parameter values, partly based on a priori preferences. The performance outcomes depending on these sets are compared, and the best choice is designated to be 1, 3, 1, 3, 10, 0.5, and 4, respectively. Of course, hospitals might have different preferences; thus, in real applications, the weights should be discussed and determined in cooperation with the hospital management. N and N' are assumed to be 15 and 2000 respectively, and finally, the SAA is allowed to iterate for $M = 20$ iterations at confidence level 95% ($\alpha = 0.05$). The same configurations are considered for the TSCC test problems. Besides, the extra overtime of the blocks and ORs, $H_{[\omega]}^B$ and $H_{[\omega]}^R$, are considered 1hr respectively and the chance/risk of violation of the blocks and ORs overtime, α^B and α^R , are set to 0.5. [Table 2](#) shows the summary of the results of the SAA in both TSS and TSCC formulations. Also, to figure out the influence of emergency patients, we solve the models with the consideration of emergency patients and compare the results with the situation that just takes inpatients into account (see [Table 3](#)). [Table 1](#) represents the respective definition used in [Table 2](#) and [3](#).

From [Tables 2](#) and [3](#), it can be inferred that for almost all cases of TSS, the solution of the deterministic model is infeasible for the second stage of the stochastic model and it enforces a big amount of infeasible overtime (the overtime exceeding the allowed amount). Therefore, it seems not wise to plan and schedule the surgeries based on the mean value approach. Whereas, in TSCC, regarding the chance-constrained feature, with a high violation risk level, $\alpha^B, \alpha^R = 0.5$, the deterministic model solution is much closer to a good feasible solution, and there exists a remarkable difference between the TSS and TSCC results (see

the results in Big Scenario column in [Tables 2](#) and [3](#)). Besides, it can be seen that by increasing the size of the problem, the CPU time increases severely (it is worth noting that a time limit of 200 s is used for solving each TSS/TSCC model in each SAA iterations); and the absolute optimality gap increases with the growth of the problem size. Two-stage chance-constrained approach and consideration of emergency patients make the problems more complicated and raises the CPU time drastically and deteriorates the optimality gap criterion. [Figs. 2–6](#) depict these trends more clearly.

As can be seen in [Figs. 2](#) and [3](#), the solution of the deterministic model and the ones in TSCC are the same in terms of N.R.P. and N.S.D.; while in the solution obtained via TSS-Best these terms have variations from the deterministic model. In [Fig. 4](#), feasible total block overtime in TSCC and TSCC-Best, with respect to the extra blocks and ORs overtime of $H_{[\omega]}^B$ and $H_{[\omega]}^R$, are greater than the ones in TSS and the deterministic model. A reverse trend exists regarding the infeasible total block overtime. [Fig. 6](#) integrates these behaviors in the objective function and consolidates the insights obtained from analyzing the trends. By comparing different results obtained via TSS and TSS-Best, it becomes clear that it is not effective to use the solutions of deterministic ASP in a stochastic environment. But, the results show that when there exist some considerable flexibilities regarding blocks/rooms overtime, i.e., the concept of chance-constrained approach is applicable with a relatively high level of risk, then the solution of the deterministic situation sounds partly rational in a stochastic environment. It can be conceived that by increasing the violation risk level, the solution and the results of the chance-constrained programming model tend from two-stage programming to deterministic programming. To discover the effect of the chance/risk level of violation on the TSCC results, we solve one of the test problems with different amount for the risk of violation of the blocks and ORs overtime (i.e., 0.1, 0.2, 0.30, 0.40, 0.5, 0.75 and 1). [Fig. 7](#) shows the effect more clearly (for more details on the results see [Table 4](#)).

From [Fig. 7](#) and the results reported in [Tables 2](#) and [4](#), it can be seen that a low risk level of violations (e.g., 0.1) does not significantly influence the results of TSS and there is a meaningful difference between the deterministic and stochastic solution. But when the risk level of violation increases gradually, it plays a decisive role in influencing the results and on decreasing the considerable variation between the deterministic and stochastic solution. This could be an interpretation of the value of the stochastic solution (VSS) which is one of the important criteria in the evaluation of stochastic programming (Birge & Louveaux, 1997). Besides, from the patients' point of view, the more the risk level increases, the more the patients' utility increases and fewer patients are rejected.

As mentioned above, the other solution method attempted in our study for solving the proposed (large-scale) stochastic optimization problems is benders decomposition technique (BD). To be capable of comparing two solution methods, we solve the same test problems with BD algorithm. The number of scenarios is supposed to be 1000. The allowed CPU time (Reslim) is set to 1000 and 2000 due to problem size. Mathematical formulation of the master and the sub-problems of the BD are presented in [Appendix A](#). [Table 5](#) shows a summary of the results of the BD in both approaches of with and without consideration of emergency patients in the two-stage formulation and [Figs. 8](#) and [9](#) provide a schematic comparison between the two proposed solution approaches of SAA and BD.

It is comprehensible from [Fig. 9](#) that BD outperforms SAA as the solution approach in moderate size problems. In terms of optimality gap, the gaps obtained from BD are tighter than the SAA's and in terms of CPU time, BD drastically outperforms the SAA approach.

Table 1

Definition of the notations used in Tables 2 and 3.

W.T.S.	The penalty of the sum of waiting time and tardiness for the scheduled patients (1st and 2nd terms in the objective function)
W.T.US.	The penalty of the sum of waiting time and tardiness for the unscheduled patients (3rd and 4th terms in the objective function)
N.R.P.	Number of unscheduled patients
N.S.D.	Total number of days all the surgeons should come to the hospital to accomplish their surgeries
B.Ov.	Total blocks overtime during the planning horizon (feasible overtime)
B.Ov.inf.	Total blocks infeasible overtime during the planning horizon (overtime which exceeds the allowed overtime (i.e., violation of (S3) in TSS and (S''3 & S7) in TSCC))
R.Ov.inf.	Total ORs infeasible overtime during the planning horizon (overtime which exceeds the allowed overtime (i.e., violation of (S5) in TSS and (S''5 & S8) in TSCC))

Table 2

Summary of the results of the SAA approach in TSS and TSCC test problems.

Problem	Variables	Deterministic	Big Scenario*		Big Scenario (Best)*		SAA*				
		TSS/TSCC	TSS	TSCC	TSS	TSCC		TSS		TSCC	
$P = 20$	Obj. Func.	19.20	48.84	23.21	31.80	23.21	$\bar{v}_{N,M}$	31.52	31.52	23	23
$I = 0$	W.T.S.	0	0	0	0	0	$\hat{v}_{N'}(\hat{x})$	31.80	31.80	23.21	23.21
$Q = 20$	W.T.US.	0	0	0	2	0	$\hat{o}_{N,M}$	0.09	0.09	0.32	0.32
$D = 1$	N.R.P.	0	0	0	2	0	$\hat{o}_{N'}(\hat{x})$	0.06	0.06	0.35	0.35
$W = 1$	N.S.D.	16	16	16	14	16	Abs. Gap	0.59	0.59	1.55	1.55
	B.Ov.	2.8	2.97	3.24	0.7	3.24					
	B.Ov.inf.	–	0.47	0.05	0	0.05	CPU time (s)	37		943	
	R.Ov.inf.	–	0.11	0	0	0					
$P = 40$	Obj. Func.	41.97	88.12	51.70	63.07	51.70	$\bar{v}_{N,M}$	62.48	62.48	51.6	51.6
$I = 0$	W.T.S.	1.17	1.17	1.17	1.42	1.17	$\hat{v}_{N'}(\hat{x})$	63.07	63.07	51.7	51.7
$Q = 40$	W.T.US.	1	1	1	4	1	$\hat{o}_{N,M}$	0.15	0.15	0.79	0.79
$D = 2$	N.R.P.	1	1	1	4	1	$\hat{o}_{N'}(\hat{x})$	0.13	0.13	0.59	0.59
$W = 1$	N.S.D.	26	26	26	24	26	Abs. Gap	1.16	1.16	2.91	2.91
	B.Ov.	4.2	4.79	5.18	1.41	5.18					
	B.Ov.inf.	–	0.71	0.12	0	0.12	CPU time (s)	89		820	
	R.Ov.inf.	–	0.17	0	0	0					
$P = 60$	Obj. Func.	81.79	185	110.93	124.5	110.93	$\bar{v}_{N,M}$	122.32	122.30	104.95	97.7
$I = 0$	W.T.S.	0.58	0.58	0.58	1.5	0.58	$\hat{v}_{N'}(\hat{x})$	124.46	124.46	110.93	110.93
$Q = 60$	W.T.US.	2	2	2	9.33	2	$\hat{o}_{N,M}$	0.23	0.30	1.11	0.81
$D = 3$	N.R.P.	2	2	2	9	2	$\hat{o}_{N'}(\hat{x})$	0.17	0.17	1.53	1.53
$W = 1$	N.S.D.	40	40	40	32	40	Abs. Gap	3.00	3.12	11.30	17.91
	B.Ov.	9.8	9.75	10.61	1.91	10.61					
	B.Ov.inf.	–	1.70	0.49	0	0.49	CPU time (s)	2374		4531	
	R.Ov.inf.	–	0.37	0.03	0	0.03					
$P = 80$	Obj. Func.	141.9	222.5	164.60	179.08	164.60	$\bar{v}_{N,M}$	176.93	176.93	159.87	155.09
$I = 0$	W.T.S.	2.42	2.42	2.42	2.28	2.42	$\hat{v}_{N'}(\hat{x})$	179.08	179.08	164.6	164.6
$Q = 80$	W.T.US.	8.5	8.5	8.5	15.67	8.5	$\hat{o}_{N,M}$	0.33	0.33	0.96	0.81
$D = 4$	N.R.P.	8	8	8	13	8	$\hat{o}_{N'}(\hat{x})$	0.20	0.20	1.13	1.13
$W = 1$	N.S.D.	46	46	46	44	46	Abs. Gap	3.24	3.24	8.96	13.42
	B.Ov.	7	8.17	8.78	2.28	8.78					
	B.Ov.inf.	–	1.22	0.30	0	0.30	CPU time (s)	782		4498	
	R.Ov.inf.	–	0.30	0.01	0	0.01					
$P = 100$	Obj. Func.	105.7	219	142.67	156.04	142.67	$\bar{v}_{N,M}$	153.50	138.12	134.76	115.49
$I = 0$	W.T.S.	3.74	3.74	3.74	3.73	3.74	$\hat{v}_{N'}(\hat{x})$	156.04	156.04	142.67	142.67
$Q = 100$	W.T.US.	3.75	3.75	3.75	11	3.75	$\hat{o}_{N,M}$	0.47	0.85	1.50	0.88
$D = 5$	N.R.P.	3	3	3	10	3	$\hat{o}_{N'}(\hat{x})$	0.32	0.32	1.70	1.70
$W = 1$	N.S.D.	58	58	58	54	58	Abs. Gap	4.18	20.36	14.38	32.37
	B.Ov.	9.8	11.48	12.29	3.33	12.29					
	B.Ov.inf.	–	1.7	0.50	0	0.50	CPU time (s)	4342		4735	
	R.Ov.inf.	–	0.43	0.04	0	0.04					

* Big scenario model results based on the decision variable values obtained from the expected value model (deterministic model).

† The big scenario (Best) model results the best response based on the decision variable values obtained from the SAA. In fact, these two columns, Big Scenario and Big Scenario (Best), compare what happens if we use the decision variable results obtained via the deterministic model and stochastic model in an uncertain real-world situation.

‡ The second column shows the results obtained from the SAA algorithm in the allowed CPU time (Reslim = 200) and the third column shows the results obtained from the solver; for example in the first example, the results show that the objective lower bound obtained in the allowed CPU time from the SAA algorithm and in the solver are the same (31.52). It is trivial that the solver's lower bound is more conservative than the SAA's, in other words, $\bar{v}_{N,M}$ of the SAA algorithm is always greater than or equal to solver's $\bar{v}_{N,M}$ and therefore according to the fact that $\hat{v}_{N'}(\hat{x})$ remains unchanged in both cases, the Gap of the SAA algorithm is always little than or equal to the solver's Gap. The same occurs in the two-stage chance-constrained formulation.

Table 3

Summary of the results of the SAA approach in TSS and TSCC test problems with consideration of emergency patients.

Problem	Variables	Deterministic	Big Scenario ⁺		Big Scenario (Best) [†]		SAA [‡]				
		TSS/TSCC	TSS	TSCC	TSS	TSCC		TSS		TSCC	
$P = 18$	Obj. Func.	17.20	47.9	21.38	29.79	21.31	$\bar{v}_{N,M}$	29.64	29.64	21.28	21.28
$I = 2$	W.T.S.	0	0	0	0	0	$\hat{v}_{N'}(\hat{x})$	29.79	29.79	21.31	21.31
$Q = 20$	W.T.US.	0	0	0	2	0	$\hat{\sigma}_{N,M}$	0.08	0.08	0.54	0.54
$D = 1$	N.R.P.	0	0	0	2	0	$\hat{\sigma}_{N'}(\hat{x})$	0.06	0.06	0.35	0.35
$W = 1$	N.S.D.	12	12	12	10	12	Abs. Gap	0.45	0.45	0.18	0.18
	B.Ov.	2.8	2.97	3.25	0.7	3.25					
	B.Ov.inf	–	0.49	0.05	0	0.05	CPU time (s)	43		558	
	R.Ov.inf.	–	0.12	0	0	0					
$P = 36$	Obj. Func.	41.31	88.1	51.50	61.39	51.50	$\bar{v}_{N,M}$	60.70	60.70	48.98	48.98
$I = 4$	W.T.S.	0	0	0	0.025	0	$\hat{v}_{N'}(\hat{x})$	61.39	61.39	51.5	51.5
$Q = 40$	W.T.US.	1.5	1.5	1.5	4.5	1.5	$\hat{\sigma}_{N,M}$	0.16	0.16	0.58	0.58
$D = 2$	N.R.P.	1	1	1	4	1	$\hat{\sigma}_{N'}(\hat{x})$	0.13	0.13	0.60	0.60
$W = 1$	N.S.D.	26	26	26	22	26	Abs. Gap	1.28	1.28	4.93	4.93
	B.Ov.	4.2	4.82	5.21	1.41	5.21					
	B.Ov.inf	–	0.71	0.12	0	0.12	CPU time (s)	214		1038	
	R.Ov.inf.	–	0.17	0	0	0					
$P = 54$	Obj. Func.	53.04	146	79.4	91.63	79.4	$\bar{v}_{N,M}$	89.07	77.81	70.78	56.86
$I = 6$	W.T.S.	1.43	1.43	1.43	1.03	1.43	$\hat{v}_{N'}(\hat{x})$	91.63	91.63	79.4	79.4
$Q = 60$	W.T.US.	0	0	0	6	0	$\hat{\sigma}_{N,M}$	0.58	0.75	0.85	0.49
$D = 3$	N.R.P.	0	0	0	6	0	$\hat{\sigma}_{N'}(\hat{x})$	0.19	0.19	1.34	1.34
$W = 1$	N.S.D.	36	36	36	32	36	Abs. Gap	4.15	15.76	13.02	26.21
	B.Ov.	8.4	8.94	9.64	2.15	9.64					
	B.Ov.inf	–	1.46	0.40	0	0.40	CPU time (s)	4160		4776	
	R.Ov.inf.	–	0.35	0.02	0	0.02					
$P = 72$	Obj. Func.	84.18	192.33	117.21	130.83	117.21	$\bar{v}_{N,M}$	128.53	111.21	110.51	89.56
$I = 8$	W.T.S.	0.97	0.97	0.97	0.57	0.97	$\hat{v}_{N'}(\hat{x})$	130.83	130.83	117.21	117.21
$Q = 80$	W.T.US.	2	2	2	10.33	2	$\hat{\sigma}_{N,M}$	0.45	0.68	1.09	0.57
$D = 4$	N.R.P.	2	2	2	9	2	$\hat{\sigma}_{N'}(\hat{x})$	0.24	0.24	1.62	1.62
$W = 1$	N.S.D.	44	44	44	38	44	Abs. Gap	3.71	21.52	12.13	32.01
	B.Ov.	9.8	10.68	11.48	2.73	11.48					
	B.Ov.inf	–	1.70	0.49	0	0.49	CPU time (s)	4040		4792	
	R.Ov.inf.	–	0.39	0.03	0	0.03					
$P = 90$	Obj. Func.	92.93	232.40	140.01	153.4	140.01	$\bar{v}_{N,M}$	150.68	118.97	131.33	92.61
$I = 10$	W.T.S.	3.52	3.52	3.52	3.67	3.52	$\hat{v}_{N'}(\hat{x})$	153.41	153.41	140.01	140.01
$Q = 100$	W.T.US.	1	1	1	10.33	1	$\hat{\sigma}_{N,M}$	0.36	0.92	1.34	0.62
$D = 5$	N.R.P.	1	1	1	10	1	$\hat{\sigma}_{N'}(\hat{x})$	0.32	0.32	2.18	2.18
$W = 1$	N.S.D.	56	56	56	50	56	Abs. Gap	4.12	37.00	15.75	52.96
	B.Ov.	12.6	13.78	14.75	3.6	14.75					
	B.Ov.inf	–	2.16	0.69	0	0.69	CPU time (s)	4284		4824	
	R.Ov.inf.	–	0.54	0.08	0	0.08

Fig. 2. Number of unscheduled patients in TSS and TSCC test problems: (a) without consideration of emergency patients, (b) with consideration of emergency

Fig. 3. Total number of days all the surgeons come to the hospital to accomplish their surgeries in TSS and TSCC test problems: (a) without consideration of emergency patients, (b) with consideration of emergency patients.

Fig. 4. Total blocks overtime (feasible and infeasible) in TSS and TSCC test problems: (a) without consideration of emergency patients, (b) with consideration of emergency patients.

5.2. Real data experiment

In this subsection, in order to evaluate the performance of the proposed formulation and solution approaches, we apply real data from the Mehregan hospital, located in Qazvin, Iran, derived from Haghi (2014), and compare the real existing situation to the results obtained from the proposed approaches. Mehregan hospital has 5 ORs and provides surgical services of Orthopedic (Ort), Urology (Ur), Ophthalmology (Opt), Female and Maternity (Fem), General (Gen), ENT (Ear, Nose, and Throat) and Neurology (Neu) in an open scheduling strategy. According to Haghi (2014), Table 6 presents a summary of a one-week data of the implemented surgeries in Mehregan hospital.

It can be seen that surgeries have not been properly distributed

throughout the week and we have days with very high traffic and in contrast, there are some days with low workloads. Also, in most cases, the total surgery duration is less than the total available surgery hours per day ($5 \times 8 = 40$), but we can see that the finish time of last surgeries last too late (e.g., till 11 pm) (Haghi, 2014). According to the data, the approximated surgery duration of different specialty groups is presented in Table 7. We suppose that the surgery duration of each specialty group follows in a triangular distribution with the parameters of min, mean and max of surgery duration.

To evaluate the performance of the proposed formulations and solution approaches in generating a better schedule, it is supposed that we have a list of the same number of patients of the specialty groups. We consider a modified block scheduling policy, and we assume that

Fig. 5. Objective function (Big scenario & Big scenario (Best) models) of the TSS and TSCC test problems: (a) without consideration of emergency patients, (b) with consideration of emergency patients.

Fig. 6. The gaps obtained via SAA in solving TSS and TSCC test problems with and without the consideration of emergency patients; (a) absolute gap, (b) relative gap.

Fig. 7. Effect of chance/risk of violation on the objective function, total blocks overtime (feasible) and number of unscheduled patients in SAA approach on a TSCC test problem.

emergency patients and general surgeries could be operated in all ORs. Also, we propose an MSS in Fig. 10 (the way the MSS is obtained is not the concern of this paper). Table 8 summarizes and compares the achieved results.

The results show that the deterministic solution will not be a good feasible solution in a stochastic environment. TSS produces efficient schedules, but TSCC could produce schedules in which all the patients could have been operated. In addition to inpatients, it could produce schedules which support emergency patients, too. According to the solution approaches, SAA, in solving real-sized problems, still has its applicable performance but BD fails in producing satisfactory results. In fact, the required information between the master and sub-problem could not be properly exchanged.

6. Conclusion

In this study, we investigated one of the important concerns in hospitals, assigning dates and operating rooms to surgeries, which is known as advance scheduling problem. Accordingly, a modified block scheduling policy which seems to have the proper flexibility is considered. Surgery durations, in our problem, are reflected as stochastic parameters and the arrival of emergency patients is taken into account. We formulated the problem as a two-stage stochastic program. To

Table 4Summary of the results of the SAA approach based on different chance level of violation on a TSCC test problem ($P = 100$, $I = 0$, $Q = 100$, $D = 5$, $W = 1$).

Variables	Big scenario (Best) ($\alpha^B = \alpha^R$)									
	$\alpha^B = 0$		$\alpha^B = 0.1$		$\alpha^B = 0.2$		$\alpha^B = 0.3$		$\alpha^B = 0.4$	
Obj. Func.	156.04		155.96		154.26		152.02		149.75	
W.T.S.	3.73		3.39		3.91		4.4		3.54	
W.T.US.	11		11.25		10		9.75		9.25	
N.R.P.	10		10		9		9		8	
N.S.D.	54		54		54		54		56	
B.Ov.	3.58		3.58		4.85		4.84		6.09	
B.Ov.inf	0.00		0		0.08		0.03		0.09	
R.Ov.inf.	0.00		0		0.00		0		0	
SAA										
$\bar{v}_{N,M}$	154.93	134.72	151.06	130.76	145.87	124.33	146.83	119	138.45	117.31
$\hat{v}_{N'}(\hat{x})$	156.53	156.53	155.96	155.96	154.26	154.26	152.02	152.02	149.75	149.75
$\hat{\alpha}_{N,M}$	0.28	0.88	0.93	0.71	0.80	0.84	0.82	0.71	1.26	0.77
$\hat{\alpha}_{N'}(\hat{x})$	0.32	0.32	0.32	0.32	0.52	0.52	0.47	0.47	0.65	0.65
Abs. Gap	2.82	24.27	7.46	27.30	11.09	32.71	7.83	35.41	15.22	35.31
CPU time (s)		4735		4789		5031		5159		5254
	$\alpha^B = 0.5$		$\alpha^B = 0.75$		$\alpha^B = 1$					
Obj. Func.	142.67		124.62		110.98					
W.T.S.	3.74		3.74		3.99					
W.T.US.	3.75		3.75		1.5					
N.R.P.	3		3		1					
N.S.D.	58		58		60					
B.Ov.	12.29		12.48		15.10					
B.Ov.inf	0.50		0.16		0.05					
R.Ov.inf.	0.04		0.00		0.05					
SAA										
$\bar{v}_{N,M}$	134.76	115.49	119.38	107.54	106.53	105.68				
$\hat{v}_{N'}(\hat{x})$	142.67	142.67	124.62	124.62	110.96	110.96				
$\hat{\alpha}_{N,M}$	1.50	0.88	1.35	0.79	0.75	0.69				
$\hat{\alpha}_{N'}(\hat{x})$	1.70	1.70	1.30	1.30	1.46	1.46				
Abs. Gap	14.38	32.37	10.61	21.29	8.90	9.61				
CPU time (s)		5371		5324		5164				

capture real-world conditions, we employed the chance-constrained approach and presented a two-stage chance-constrained formulation. We considered different conflicting goals in the objective function consisting of minimizing the patients waiting time, tardiness, cancellation, block overtime, and maximization of the surgeons' satisfaction by minimizing the number of days the surgeons should come to the hospital to accomplish their surgeries in the planning horizon. We applied two solution approaches based on SAA method and Benders decomposition algorithm to solve the proposed formulations. The efficiency of the formulations and the solution approaches are examined through an extended analysis. Finally, we applied the approaches for a real hospital. Our results showed that deterministic solutions do not fit properly in the stochastic environment for this problem. Chance-constrained concept and two-stage stochastic programming approach suitably formulate the problem. High level of violation risk in the chance-constrained approach, basically affects the results and decreases the considerable variation between the deterministic and stochastic

solutions. SAA method and Benders decomposition algorithm could generate efficient solutions in a relatively reasonable computational time in medium-sized problems (Benders decomposition method performed better), but Benders decomposition technique fails in producing satisfactory optimality gaps in large-sized problems.

Finally, in the following, we recommend some directions for future works.

- Modified block scheduling policy seems to be more efficient than open or block scheduling policy. Thus, investigating different features of this policy can be considered as an interesting option.
- Different strategies for admitting emergency patients could be investigated.
- According to Wang et al. (2016), ignoring downstream units of ORs may lead to 18% surgery cancellation. Thus, the consideration of downstream units of ORs seems a useful extension.
- The chance-constrained concept is rarely considered in the literature

Table 5
Summary of the results of the BD approach in TSS test problems.

Problem	Variables	TSS with BD (Inp.)	TSS with BD (Inp. & Emer.)		Inp. Inp. & Emer.	
$P = 20^*$	Obj. Func.	31.7	29.75	Lower Bound	31.70	29.75
$I = 0^*$	W.T.S.	0	0	Upper Bound	31.70	29.75
$P = 18^{**}$	W.T.US.	2	2	Abs. Gap	0	0
$I = 2^{**}$	N.R.P.	2	2			
$Q = 20$	N.S.D.	14	10			
$D = 1$	B.Ov.	0.68	0.68	CPU time (s)	4	5
$W = 1$						
$P = 40^*$	Obj. Func.	63.06	61.36	Lower Bound	63.06	61.36
$I = 0^*$	W.T.S.	1.08	0.25	Upper Bound	63.06	61.36
$P = 36^{**}$	W.T.US.	4.33	4.5	Abs. Gap	0	0
$I = 4^{**}$	N.R.P.	4	4			
$Q = 40$	N.S.D.	24	22			
$D = 2$	B.Ov.	1.41	1.40	CPU time (s)	85	192
$W = 1$						
$P = 60^*$	Obj. Func.	124.3	91.50	Lower Bound	119.67	81.14
$I = 0^*$	W.T.S.	1.5	1.03	Upper Bound	124.28	91.50
$P = 54^{**}$	W.T.US.	9.33	6	Abs. Gap	4.61	10.36
$I = 6^{**}$	N.R.P.	9	6			
$Q = 60$	N.S.D.	32	32			
$D = 3$	B.Ov.	1.86	2.12	CPU time (s)	1000	2000
$W = 1$						
$P = 80^*$	Obj. Func.	178.8	130.7	Lower Bound	167.43	115.11
$I = 0^*$	W.T.S.	1.78	0.57	Upper Bound	178.77	130.7
$P = 72^{**}$	W.T.US.	15.17	10.33	Abs. Gap	11.34	15.59
$I = 8^{**}$	N.R.P.	13	9			
$Q = 80$	N.S.D.	46	38			
$D = 4$	B.Ov.	2.20	2.7	CPU time (s)	1000	2000
$W = 1$						
$P = 100^*$	Obj. Func.	158.8	175.24	Lower Bound	139.35	149.92
$I = 0^*$	W.T.S.	4.60	4.2	Upper Bound	158.76	175.24
$P = 90^{**}$	W.T.US.	11	10.67	Abs. Gap	19.41	25.31
$I = 10^{**}$	N.R.P.	10	10			
$Q = 100$	N.S.D.	58	50			
$D = 5$	B.Ov.	3.54	8.84	CPU time (s)	2000	2000
$W = 1$						

* Test problems considered only inpatients, for example in the first test problem $Q = 20$ means that all 20 patients are inpatients ($P = 20$ and $I = 0$).

** Test problems considered Inpatients and emergency patients, for example in the first test problem $Q = 20$, $P = 18$ and $I = 2$ means that from all 20 patients, 18 patients are inpatients and 2 patients are emergency patients.

Fig. 8. The objective function via SAA and BD in solving TSS test problems with and without the consideration of emergency patients.

of operating room scheduling whereas it basically suits the subject. Of course, the difficulties in solving such formulations should be covered.

- According to the fact that the scenario-based two-stage models, especially in a medium/large number of scenario cases, usually lead to an immense formulation in the extensive form, it is needed to facilitate it with more efficient techniques.
- Standard Benders decomposition (L-shaped decomposition) technique cannot efficiently solve ASP in real-sized problems. Jensen valid inequality is not powerful enough to improve the performance of the algorithm in large size problems. Therefore, it is essential to elevate the performance of such decomposition techniques or to apply tighter valid inequalities or symmetry breaking constraints.
- Finally, big improvements could be obtained by operating room planning and scheduling in a coalition of multiple hospitals in a strategic network, although this is a situation which is uncommon in many countries.

Fig. 9. Absolute gaps and CPU times obtained via SAA and BD in solving TSS test problems with and without the consideration of emergency patients; (a) Absolute gap, (b) CPU time.

Table 6

Summary of a one-week data of the surgeries operated in Mehregan hospital (Haghi, 2014).

Day	Total No. of Surgeries	Total surgery duration (hrs)	Specialty group	Total surgery duration of each specialty group (hrs)	No. of surgeries	No. of surgeons	Start time of the first surgery	Finish time of the last surgery
Saturday	27	16	Ort	0	0	0	9:20	23:00
			Ur	1.5	3	2		
			Opt	5.5	17	2		
			Fem	3.3	4	3		
			Gen	0	0	0		
			ENT	2.2	2	1		
			Neu	3.5	1	1		
Sunday	51	42.8	Ort	4.2	3	2	9:00	23:45
			Ur	8.4	9	2		
			Opt	4.7	14	1		
			Fem	10	14	5		
			Gen	10.2	9	4		
			ENT	5.3	2	1		
			Neu	0	0	0		
Monday	48	44.1	Ort	11.2	6	3	8:45	0:15
			Ur	3.3	6	3		
			Opt	6	17	3		
			Fem	2.8	4	2		
			Gen	9.1	10	3		
			ENT	2.3	1	1		
			Neu	9.4	4	2		
Tuesday	48	34.6	Ort	2.7	2	2	8:40	19:30
			Ur	9.4	13	4		
			Opt	7.4	18	2		
			Fem	2.8	3	3		
			Gen	8	8	5		
			ENT	4.3	4	2		
			Neu	0	0	0		
Wednesday	50	34.9	Ort	1.1	2	2	9:30	23:20
			Ur	2.2	2	1		
			Opt	7.6	25	3		
			Fem	3.8	7	3		
			Gen	7.4	8	4		
			ENT	2.2	1	1		
			Neu	10.6	5	2		
Thursday	23	28.5	Ort	11.5	6	5	8:40	21:45
			Ur	3.6	4	2		
			Opt	0	0	0		
			Fem	8.4	9	3		
			Gen	0	0	0		
			ENT	5	4	1		
			Neu	0	0	0		

Table 7
Surgery durations in different specialty groups.

Specialty group	Min of surgery duration	Mean of surgery duration	Max of surgery duration
Orthopedic (Ort)	0.3	1.4	2.8
Urology (Ur)	0.3	0.8	2.3
Ophthalmology (Opt)	0.2	0.3	1
Female & Maternity (Fem)	0.3	0.7	1.2
General (Gen)	0.3	0.9	2.4
ENT	0.7	1.5	2.8
Neurology (Neu)	1.5	2.5	3.5

	Saturday		Sunday		Monday		Tuesday		Wednesday		Thursday	
	B 1	B 2	B 1	B 2	B 1	B 2	B 1	B 2	B 1	B 2	B 1	B 2
Room 1	Opt	Gen	Opt	Opt	Opt	Gen	Opt	Opt	Opt	Gen	Opt	Opt
Room 2	Ort	Ort	Ort	Ort	Ort	Neu	Ort	Ort	Gen	Ort	Ort	Ort
Room 3	Gen	Neu	Ur	Neu	Ur	Neu	Gen	Neu	Ur	Neu	Gen	Neu
Room 4	Ur	ENT	Ur	ENT	Ur	ENT	Ur	ENT	Ur	ENT	Ur	ENT
Room 5	Fem	Fem	Gen	Fem	Fem	Fem	Fem	Fem	Fem	Fem	Fem	Gen

Fig. 10. The proposed MSS.

Table 8
Summary of the results achieved from the proposed formulations (TSS & TSCC) and solution approaches (SAA & BD) in Mehregan Hospital.

Problem	Variables	Deterministic	Big Scenario		Big Scenario (Best)		SAA				
		TSS/TSCC	TSS	TSCC	TSS	TSCC	TSS		TSCC		
$P = 247$	Obj. Func.	85.6	474.6	439.20	261.66	195.63	$\bar{v}_{N,M}$	194.58	173.8	167.41	157.17
$I = 0$	W.T.S.	13.58	13.58	13.58	13.65	12.14	$\hat{v}_{N'}(\hat{x})$	261.66	261.66	195.63	195.63
$Q = 247$	W.T.US.	0	0	0	1	0	$\hat{\sigma}_{N,M}$	2.55	1.93	1.35	0.70
$D = 6$	N.R.P.	0	0	0	1	0	$\hat{\sigma}_{N'}(\hat{x})$	4.09	4.09	2.98	2.98
$W = 1$	N.S.D.	92	92	92	108	100	Abs. Gap	80.43	99.92	36.89	45.77
	B.Ov.	6.5	29.14	33.63	24.61	25.87					
$\alpha^B = 0.5$	B.Ov.inf	–	8.36	3.30	1.43	0.42	CPU time (s)		4872		5194
	R.Ov.inf.	–	3.07	1.6	0.26	0.18					
$P = 247$	Obj. Func.	85.76	639	282.16	260.87	190.12	$\bar{v}_{N,M}$	195.53	175.44	166.03	157.27
$I = 8$	W.T.S.	12.73	12.73	12.73	13.14	13.47	$\hat{v}_{N'}(\hat{x})$	260.87	260.87	190.12	190.12
$Q = 255$	W.T.US.	0	0	0	2.89	0	$\hat{\sigma}_{N,M}$	3.03	2.24	1.20	0.67
$D = 6$	N.R.P.	0	0	0	3	0	$\hat{\sigma}_{N'}(\hat{x})$	3.62	3.62	2.83	2.83
$W = 1$	N.S.D.	94	94	94	106	100	Abs. Gap	78.77	97.20	32.15	39.81
	B.Ov.	6.5	31.82	29.48	23.49	25.90					
$\alpha^B = 0.5$	B.Ov.inf	–	8.48	1.91	1.24	0.37	CPU time (s)		5311		5221
	R.Ov.inf.	–	0.56	0.18	0.12	0.09					
		TSS with BD (Inp.)	TSS with BD (Inp. & Emer.)				Inp.	Inp. & Emer.			
$P = 247^*$	Obj. Func.	541.97	876.53				Lower Bound	215.83	218.6		
$I = 0^*$	W.T.S.	12.83	18.09				Upper Bound	541.97	876.53		
$P = 237^{**}$	W.T.US.	38.88	73.58				Abs. Gap	326.14	657.94		
$I = 10^{**}$	N.R.P.	390	730								
$Q = 247$	N.S.D.	55	48								
$D = 6$	B.Ov.	45.2	6.84				CPU time (s)		4500		
$W = 1$											

Appendix A. Application of Benders decomposition method

Notations which we additionally applied to formulate the Benders master and sub-problem are as follow.

Indices:

l index showing feasibility cut or optimality cut, respectively ($l = 1, 2$)

k_l index showing different iteration of the BD algorithm ($k_l = 1, \dots, K_l, \forall l = 1, 2$, where K_l is the maximum number of feasibility/optimality cuts added to the master problem)

Parameter:

θ_{LB}

lower bound of the θ , approximation of the Benders sub-problem objective function

Free decision variables:

θ

approximation of the Benders sub-problem objective function

$\pi_{bdw}^{B[k_l]}$

dual variable of the blocks capacity constraint in the Benders sub-problem (S2) in each iteration k_l

$\xi_{bdw}^{B[k_l]}$

dual variable of the blocks capacity constraint in the Benders sub-problem (S3) in each iteration k_l

$\pi_{rdw}^{R[k_l]}$

dual variable of the ORs capacity constraint in the Benders sub-problem (S4) in each iteration k_l

$\xi_{rdw}^{R[k_l]}$

dual variable of the ORs capacity constraint in the Benders sub-problem (S5) in each iteration k_l

The mathematical formulation of the Benders master problem is as follows.

$$Obj_{B^m} = \min \left\{ \begin{aligned} & \left(\sum_q \sum_b \sum_d \sum_w [m_1(D_{dw} - A_q) + m_2(D_{dw} - D_q)^+] x_{qbdw} U_q \right) + \\ & \left(\sum_{q: D_q \leq D \times W} [m_3(D_q - A_q) + m_4((D \times W + 1) - D_q)] (1 - \sum_b \sum_d \sum_w x_{qbdw}) U_q \right) + \\ & m_5(Q - \sum_q \sum_b \sum_d \sum_w x_{qbdw}) + m_6(\sum_s \sum_d \sum_w n_{sdw}) + \theta \end{aligned} \right\} \quad (B1)$$

Subject to: (D2), (D7)–(D11)

$$\theta \geq \theta_{LB} \quad (B2)$$

$$\sum_{\omega} P_{[\omega]} \left(\sum_b \sum_d \sum_w (\pi_{bdw}^{B[k_1]} (\sum_q t_{q[\omega]} x_{qbdw} - \gamma_{bdw} C_{bdw}) + \xi_{bdw}^{B[k_1]} O_b^{\max}) \right) + \sum_{\omega} P_{[\omega]} \left(\sum_r \sum_d \sum_w (\pi_{rdw}^{R[k_1]} (\sum_{b: R_b^B=r} t_{q[\omega]} x_{qbdw} - \gamma_{bdw} C_{bdw}) + \xi_{rdw}^{R[k_1]} O_r^{\max}) \right) \leq 0 \quad \forall k_1 \quad (B3)$$

$$\theta \geq \sum_{\omega} P_{[\omega]} \left(\sum_b \sum_d \sum_w (\pi_{bdw}^{B[k_2]} (\sum_q t_{q[\omega]} x_{qbdw} - \gamma_{bdw} C_{bdw}) + \xi_{bdw}^{B[k_2]} O_b^{\max}) \right) + \sum_{\omega} P_{[\omega]} \left(\sum_r \sum_d \sum_w (\pi_{rdw}^{R[k_2]} (\sum_{b: R_b^B=r} t_{q[\omega]} x_{qbdw} - \gamma_{bdw} C_{bdw}) + \xi_{rdw}^{R[k_2]} O_r^{\max}) \right) \quad \forall k_2 \quad (B4)$$

$$\pi_{bdw}^{B[k_l]}, \xi_{bdw}^{B[k_l]}, \pi_{rdw}^{R[k_l]}, \xi_{rdw}^{R[k_l]}, \theta \geq 0 \quad \forall l \quad (B5)$$

The mathematical formulation of the Benders sub-problem is as follows.

$$Obj_{B^s} = \min \sum_{\omega} P_{[\omega]} \{ m_7 (\sum_b \sum_d \sum_w o_{bdw}[\omega]) \} \quad (B6)$$

Subject to: (S2)–(S6)

It is clear from the formulations that the original two-stage formulation is decomposed to a master problem where the patients and surgeons assigning decisions are made (first stage decision variables), and to a sub-problem corresponding to overtimes in each scenario in the presence of the fixed values of the first stage decision variables (second stage decision variables). According to BD algorithm, in each iteration, the master problem is solved, and the obtained solution is infixed to the sub-problem; then the sub-problem is solved. If it produces infeasible solutions, then, feasibility cuts (B3), which are based on the dual of the sub-problem constraints, are added to the master problem. But, if it produces feasible solutions, optimality cuts (B4) are added to the master problem to progressively recover the approximation of the sub-problem objective function, θ . Therefore in brief, in each iteration of the BD algorithm, one of the feasibility or optimality cuts are attached to the master problem.

It is worth noting that to elevate the performance of the customary and standard BD algorithm and to cover the drawback of poor approximation of θ , (especially early in the algorithm), we take benefit from valid inequalities stemming from Jensen inequality (Jensen, 1906). In this regard, we add set of constraints (D3)–(D6) which are based on the average value of the surgery duration as proposed in Batun et al. (2011) and Xiao et al. (2016) to the master problem.

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