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Disturbance feedforward control for active vibration isolation systems with internal isolator dynamics

Michiel A. Beijen^a, Marcel F. Heertjes^{a,c}, Hans Butler^{b,c}, Maarten Steinbuch^a

^a*Eindhoven University of Technology, Department of Mechanical Engineering, Control Systems Technology group, PO Box 513, 5600 MB Eindhoven, The Netherlands*

^b*Eindhoven University of Technology, Department of Electrical Engineering, Control Systems group, Eindhoven, The Netherlands*

^c*ASML Mechatronic Systems Development, 5504 DR Veldhoven, The Netherlands*

Abstract

This paper presents a disturbance feedforward control strategy for active vibration isolation systems that have internal isolator dynamics. The structure of the feedforward controller follows from modeling, but to achieve robust performance the tuning of the controller parameters will be based on measurement data. More specifically, the controller poles, which correspond to internal isolator dynamics, are obtained offline from system identification experiments, while the controller zeros are obtained from online self-tuning using measured sampled-data systems. Self-tuning is used to reduce parameter uncertainty when having too limited identification possibilities, and to account automatically for the effect of noise amplification. Measurement results obtained from an industrial vibration isolation system show that residual vibrations are largely reduced up to the level of output noise limitations.

Keywords: active vibration isolation, vibration control, air mounts, high-precision mechatronics.

1. Introduction

Vibration isolation systems are widely used in high-precision machines to achieve the often extreme demands on accuracy [1, 2, 3, 4]. The basic idea is to create a low suspension stiffness between an isolated payload and a corresponding vibrating base. As a result, passive vibration isolation is provided beyond the low suspension frequency. However, systems with low suspension frequencies generally have internal isolator dynamics at relatively low frequencies that may compromise isolation performance. An example is given by an air mount system [5, 6, 7], which achieves a low stiffness using large air tank volumes. This generally leads to low internal resonance frequencies within the air tank as a result of air acoustics [8]. Another example is found in isolators using metal springs such as coil springs [9] or leaf springs [10]. For such springs, the internal resonance frequency corresponding to the first structural eigenmode is generally proportional to the suspension frequency [11, 12].

Several methods have been proposed to improve performance of vibration isolators with internal isolator dynamics. In [13], dynamic vibration absorbers (DVAs) are proposed to counteract internal resonances, but these absorbers require hardware modifications to the isolator. Alternatively, feedback control strategies for active vibration isolation are found in [14, 15]. Feedback control, however, may associate with a poor signal-to-noise ratio (SNR), because the sensor is placed on the payload which by itself is passively isolated. Moreover, a high bandwidth is required to control internal isolator dynamics which can result in stability problems. A feasible solution is then given by disturbance feedforward control that uses measured base vibrations as controller input instead. This renders the controller inherently stable and with a much better input SNR. However, to provide an effective feedforward control force an accurate model of the vibration

Email address: m.a.beijen@tue.nl (Michiel A. Beijen)

Figure 1: industrial vibration isolation system consisting of an isolated metrology frame (MF), base frame (BF), and three isolation modules (IMs) with external air tanks. The MF dimensions are $1.50 \times 1.40 \times 0.26$ m, and has a mass of 1300 kg.

isolator is needed. In [16, 17] feedforward controllers are presented which rely on a spring-damper model for the isolator. This model may be suitable for isolators with a relatively high suspension frequency and as such having internal isolator dynamics occurring well beyond the frequency range of interest. For isolators with a low suspension frequency, however, internal isolator dynamics should be included in the feedforward controller model, because these usually impact performance within the frequency range of interest.

To obtain accurate models for mechanical systems such as vibration isolators, one generally resorts to system identification tools [18]. For heavy-weight industrial vibration isolation systems, however, system identification may give unsatisfactory results, because it is difficult (if not impossible) to sufficiently excite the base of the system [19]. To avoid these problems, self-tuning feedforward controllers then provide an efficient solution to estimate the controller parameters online while keeping the model-based controller structure intact. In earlier work [20], the authors presented a self-tuning feedforward control structure to compensate the stiffness and damping properties induced by air mounts, but this approach did not include the internal isolator dynamics. Differently, [21, 22, 23] have considered adaptive feedforward control using FIR filters, with the disadvantage of requiring many adaptive parameters to accurately estimate low-frequency poles and lightly damped resonances. In [24] an adaptive IIR filter is proposed that generally needs much less parameters compared to FIR filters, but the IIR filter can become unstable if the filter poles are shifted outside the unit circle during adaptation. The solution that is pursued in this paper considers an IIR filter with fixed poles [25], also called a generalized FIR filter [26].

This paper has two main contributions. The first contribution is an extension of the Multi-Input Multi-Output (MIMO) feedforward controller design in [17]. The extension involves the inclusion of an arbitrary number of pre-selected poles from FRF measurements, while the zeros are obtained from data-based self-tuning. The second contribution is the application of the MIMO feedforward control strategy to an industrial multi-axis vibration isolation system that contains air mounts having performance-limiting internal isolator dynamics. By measurements obtained from this system, significant improvements in vibration isolation performance are obtained with respect to systems that only use feedback control and feedforward control consisting of spring-damper compensation.

The remainder of this paper is organized as follows. The industrial vibration isolation system is discussed in Section 2. The control problem is formulated in Section 3, and the self-tuning algorithm is presented in Section 4. Experimental validations at the industrial vibration isolation system are given in Section 5, and the main conclusions are summarized in Section 6.

Figure 2: (a) simplified model; $a_{0,j}$ and $a_{1,j}$ represent respectively base frame and metrology frame accelerations measured at the j^{th} isolator location; each isolation module (IM) is regarded as a dual-chamber air mount system with air chambers V_1 and V_2 ; the actuator force u_j acts directly on the metrology frame; (b) Bode plots of the analytical model in (2).

2. Vibration isolation system

The industrial vibration isolation system considered in this paper is shown in Figure 1 for which a schematic representation is depicted in Figure 2. This system basically consists of: (a) 6 degree-of-freedom movable metrology frame (MF), (b) base frame (BF), and (c) three isolation modules (IMs) that are installed between the base frame and the metrology frame to isolate the latter from base frame vibrations. Each isolation module contains a pneumatically controlled dual-chamber air mount system that provides gravity compensation, damping, and is characterized by a low suspension stiffness. To enable active vibration isolation, each isolation module is equipped with two Lorentz motors: one to apply horizontal forces and one to apply vertical forces. Furthermore, the base frame and the metrology frame are equipped with accelerometers to measure the vertical accelerations of at the isolation module locations. The accelerometers contain analog band-pass filters with a passband from 0.1 to 450 Hz. The considered frequency range of interest is from 1 to 100 Hz, which falls inside the passband of the filters and which is representative for high-precision applications such as the wafer scanning industry [27].

The control strategies proposed in this paper will be applied in three directions, i.e. the coordinates (z ; θ_x ; θ_y) which are defined with respect to the center of mass of the metrology frame, see Figure 2a. Using these coordinates, base frame accelerations are denoted as $\mathbf{a}_0 = [\ddot{z}_0, \ddot{\theta}_{x,0}, \ddot{\theta}_{y,0}]^T$, metrology frame accelerations as $\mathbf{a}_1 = [\ddot{z}_1, \ddot{\theta}_{x,1}, \ddot{\theta}_{y,1}]^T$, and actuator forces as $\mathbf{u} = [F_z, M_x, M_y]^T$. These coordinates are chosen because they are related to vertical actuator forces and vertical measurements performed at the isolation modules, i.e.

$$\mathbf{u}(t) = \mathbf{B} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}, \quad \mathbf{a}_0(t) = \mathbf{R}_0 \begin{bmatrix} a_{0,1}(t) \\ a_{0,2}(t) \\ a_{0,3}(t) \end{bmatrix}, \quad \mathbf{a}_1(t) = \mathbf{R}_1 \begin{bmatrix} a_{1,1}(t) \\ a_{1,2}(t) \\ a_{1,3}(t) \end{bmatrix}. \quad (1)$$

In (1), local base frame and metrology frame accelerations are given by respectively $a_{0,j}$ and $a_{1,j}$, where the subscript j refers to the j^{th} isolation module, local actuator forces are denoted by u_j , and $\mathbf{B}, \mathbf{R}_0, \mathbf{R}_1 \in \mathbb{R}^{3 \times 3}$ represent static transformation matrices. The principal coordinates (x ; y ; θ_z) are not considered for practical reasons. The coordinate frames of $\mathbf{a}_0$ and $\mathbf{a}_1$ coincide at the center of mass of the metrology frame, see Figure 2a. By doing so, the frequency response matrix of the transmissibility that will be defined in (5) reduces to an identity matrix for frequencies $f \rightarrow 0$.

In this paper, it is assumed that (a) the sensors measuring $a_{0,1}, a_{0,2}, a_{0,3}$ are perfectly collocated with the attachment points of the air mounts on the base frame, and (b) the three air mounts are the only transmission paths from the base frame to the metrology frame. As such, the vector $\mathbf{a}_0$ in (1) contains all possible contributions from base frame vibrations to metrology frame vibrations. All other metrology frame vibrations are considered as output disturbances and independent of base frame disturbances.

Figure 3: power spectral density plots of the measured base frame accelerations at all three sensor locations (z_{b1} , z_{b2} , z_{b3}); the blue lines show a measurement of the standard base frame spectrum without shaker excitation, while the red lines show a measurement of the increased disturbance spectrum for frequencies between 60 and 90 Hz by using the shakers. The parametric fitting function $\Phi_{a0}(j(2\pi f))$ corresponds to (3) and is used for experimental validation in Section 5.

2.1. Internal isolator dynamics

Each isolation module is considered as a dual-chamber air mount system that acts as a single-axis vibration isolator in vertical direction, see Figure 2a. The lower chamber V_1 corresponds to an air tank as shown in Figure 1, while the upper chamber V_2 corresponds to the air volume inside an isolation module. V_2 is sealed with a freely moving piston that supports the metrology frame. The lower and upper chamber are connected via a small air neck. To show the effect of air acoustics, consider an example in which $A_0(s)$, $A_1(s)$, and $U(s)$ are defined as the Laplace transforms of $a_{0,j}$, $a_{1,j}$, and u_j , respectively. Using A_0 , U as inputs and A_1 as output, it is shown in [8] that the input-output relations are given by

$$P_1(s) := \frac{A_1(s)}{A_0(s)} \approx \underbrace{\frac{ds + k}{ms^2 + ds + k}}_{\text{suspension dynamics}} \underbrace{\frac{s^2 + \zeta_{ar}\omega_{ar}s + \omega_{ar}^2}{s^2 + \zeta_r\omega_r s + \omega_r^2} \frac{\omega_r^2}{\omega_{ar}^2}}_{\text{internal isolator dynamics}}, \quad P_2(s) := \frac{A_1(s)}{U(s)} \approx \frac{s^2}{ms^2 + ds + k}, \quad (2)$$

with $m = 1200$ kg, $d = 500$ Ns/m, $k = 50 \cdot 10^3$ N/m, $\omega_{ar} = 2\pi \cdot 44$ rad/s, $\zeta_{ar} = 0.01$, $\omega_r = 2\pi \cdot 99$ rad/s, $\zeta_r = 0.01$. Bode plots for P_1, P_2 are given in Figure 2b and clearly show the suspension mode at 1 Hz in both P_1 and P_2 . Moreover, the plot of P_1 shows the presence of internal isolator dynamics, i.e. the effect of air tank acoustics due to a Helmholtz resonator effect [8]. This follows from the anti-resonance at 44 Hz and the resonance at 99 Hz. In P_2 , the effect of air acoustics is not visible. Physically this makes sense, because the actuator force is applied in parallel to the tanks and the metrology frame mass is much larger than the total (resonating) air mass inside the tanks. Different from input U , input A_0 is transmitted to the metrology frame via the air tanks, hence a series connection. As such, the effect of air acoustics is clearly visible in P_1 .

2.2. Base frame disturbance spectrum

Vibration isolation performance should be evaluated for base frame disturbance spectra corresponding to industrial environments, but the experimental setup has been placed on a quiet lab floor. To imitate an

Figure 4: block diagram of a vibration isolation system with disturbance feedforward control.

industrial environment, shakers are installed between the lab floor and the base frame to artificially increase the power of base frame disturbances up to the level of industrial floor specifications. For this setup, the ASML floor specification is considered [27]. This specification lies in-between two other widely used specification curves, i.e. the BBN-C and BBN-D (or VC-C and VC-D) curves [28]. Figure 3 compares these three specifications to the measured base frame disturbance spectrum in terms of power spectral density (PSD) plots. It is observed that, without shakers, the measured PSD is generally flat with a magnitude of $3 \cdot 10^{-9} \text{ (m/s}^2\text{)}^2/\text{Hz}$ between 5 and 100 Hz which is far below the specification. Using shakers, the excitation is increased up to the specifications for frequencies between 60 and 90 Hz, which is also the frequency range where internal isolator dynamics are expected to occur and therefore relevant for this paper. For other frequencies, the power is not increased due to limitations in the power that can be provided by the shakers.

A parametric fit of the measured PSD with increased power is given by the main diagonal entries of transfer function matrix $|\Phi_{\mathbf{a}_0}(s)|^2$ when substituting $s = j(2\pi f)$ with frequency f . Here,

$$\Phi_{\mathbf{a}_0}(s) = 13 \cdot \frac{s^2 + 2\zeta_z(2\pi f_z)s + f_z^2}{(s^2 + 2\zeta_1(2\pi f_1)s + f_1^2) \cdot (s^2 + 2\zeta_2(2\pi f_2)s + f_2^2)} \mathbf{I}_3, \quad (3)$$

with 3×3 identity matrix $\mathbf{I}_3$. Each diagonal entry of $\Phi_{\mathbf{a}_0}$ has a complex pair of zeros at $f_z = 73.5$ Hz with damping ratio $\zeta_z = 0.1$, and two complex pairs of poles at $f_1 = 70.7$ Hz and $f_2 = 77$ Hz with damping ratios $\zeta_1 = \zeta_2 = 0.03$. Figure 3 shows that the main diagonal entries of $|\Phi_{\mathbf{a}_0}(j(2\pi f))|^2$ lead to a reasonable fit of the measured spectrum beyond 5 Hz. This fit will be used later on in this paper.

3. Problem formulation

In this paper, only feedforward control is considered for which the general MIMO block diagram is shown in Figure 4. The base frame acceleration $\mathbf{a}_0(t)$ is considered as input disturbance while the metrology frame acceleration $\mathbf{a}_1 = \mathbf{y}_1 + \mathbf{y}_2 + \mathbf{n}_p$ is considered as the signal to be minimized in some norm-based sense. The latter is formed by three signals: $\mathbf{y}_1(t) \in \mathbb{R}^3$ is caused by $\mathbf{a}_0$ via the primary path $\mathbf{P}_1$, $\mathbf{y}_2(t) \in \mathbb{R}^3$ represents the impact of the control signal $\mathbf{u}(t)$ transferred via the secondary path $\mathbf{P}_2$, and $\mathbf{n}_p(t) \in \mathbb{R}^3$ represents metrology frame accelerations due to the process noise caused by direct disturbance forces, actuator noise, et cetera. The feedforward controller $\mathbf{C}_0$ provides disturbance feedforward control based on the measured signal $\tilde{\mathbf{a}}_0 = \mathbf{a}_0 + \mathbf{n}_0$, representing the base frame acceleration with additive measurement noise represented by $\mathbf{n}_0(t) \in \mathbb{R}^3$. In the Laplace domain, the signal $\mathbf{a}_1$ can be written as

$$\mathbf{A}_1(s) = \mathcal{T}(s)\mathbf{A}_0(s) + \mathcal{S}_0(s)\mathbf{N}_0(s) + \mathbf{N}_p(s), \quad (4)$$

with transmissibility matrix $\mathcal{T}$ and noise sensitivity matrix $\mathcal{S}_0$ defined as

$$\mathcal{T}(s) = \mathbf{P}_1(s) + \mathbf{P}_2(s)\mathbf{C}_0(s), \quad \mathcal{S}_0(s) = \mathbf{P}_2(s)\mathbf{C}_0(s). \quad (5)$$

The transmissibility matrix $\mathcal{T}$ is a measure for vibration isolation performance, whereas the noise sensitivity matrix $\mathcal{S}_0$ is a measure for noise amplification.

The control problem can now be formulated as follows. Given a set of (possibly correlated and colored) disturbances $\mathbf{A}_0$, find a feedforward controller $\mathbf{C}_0$ such that the power of $\mathbf{A}_1$ is minimized in the frequency

range of interest, i.e. in the frequency interval between 1–100 Hz. Minimization will be done for a vibration isolation system that has internal isolator dynamics. Under the assumptions that $\mathbf{P}_2$ is a square system, $\mathbf{a}_0$ and $\mathbf{n}_p$ are uncorrelated, and in the absence of $\mathbf{n}_0$, perfect cancellation of base frame vibrations is obtained if $\mathcal{T} = \mathbf{0}$ in (5) which requires

$$\mathbf{C}_0(s) = -\mathbf{P}_2^{-1}(s)\mathbf{P}_1(s), \quad (6)$$

This controller follows from substitution of (6) in (5) and is known as the Wiener solution [29]. However, a discrete-time implementation of the design in (6) generally results in a non-causal and unstable feedforward controller due to time delays and non-minimum phase zeros in $\mathbf{P}_2$. In the context of disturbance feedforward control such unfavorable properties should be avoided. In practice, the aim is therefore to obtain the best causal and stable approximation of (6) [30]. Such an approximation is generally unable to improve performance at all frequencies [31]. Therefore, frequency weighting by adding a transfer function matrix Φ is considered to optimize for a specific disturbance spectrum and frequency range of interest. Using the z -transform, an approximated solution is found in [29] and is given by

$$\mathbf{C}_0(z) = -\mathbf{P}_{2,o}^{-1}(z) \{ \mathbf{P}_{2,i}^*(z) \mathbf{P}_1(z) \Phi_{co}(z) \}_+ \Phi_{co}^{-1}(z), \quad (7)$$

with $z \in \mathbb{C}$. In (7), inner-outer factorization is applied such that $\mathbf{P}_2(z) = \mathbf{P}_{2,i}(z)\mathbf{P}_{2,o}(z)$. The outer factor $\mathbf{P}_{2,o}$ contains all the dynamics of $\mathbf{P}_2$ that can be inverted in a stable and causal manner. All remaining dynamics and time delays are included in the inner factor $\mathbf{P}_{2,i}$. Note that $\mathbf{P}_{2,i}^*(z) = \mathbf{P}_{2,i}^T(z^{-1})$ represents the adjoint which is the transposed and time-reversed system [32] which here equals the inverse since $\mathbf{P}_{2,i}$ is all-pass and square [33]. Similarly, co-inner-outer factorization is applied to Φ such that $\Phi(z) = \Phi_{co}(z)\Phi_{ci}(z)$ with co-outer-factor Φ_{co} and co-inner-factor Φ_{ci} . In this paper, the design of $\Phi(z)$ is based on the discrete-time equivalent of $\Phi_{a0}(s)$ in (3) which can be obtained from zero-order-hold discretization. The causality operator $\{\cdot\}_+$ is used in (7) to find the best causal and stable approximation of the non-invertible part. For more information regarding (co-)inner-outer factorization and the causality operator, the reader is referred to [32, 33].

Although (7) shows a model-based solution to the feedforward control problem, two practical aspects limit a straightforward implementation of (7). First, (7) needs parametric models for $\mathbf{P}_1$ and $\mathbf{P}_2$. Model estimates of $\mathbf{P}_1$ are only available with limited accuracy [19], because in identification experiments it is often difficult (if not impossible) to sufficiently excite the base frame. Although one might still be able to detect resonance peaks (poles) of $\mathbf{P}_1$ under this limiting condition, it generally impedes accurate estimations of the zeros and the mode shapes of $\mathbf{P}_1$ and therefore an accurate model-based design of $\mathbf{C}_0$. Second, (7) does not consider the noise contribution $\mathcal{S}_0\mathbf{N}_0$ while minimization of $\mathbf{A}_1$ in (4) implies finding a balanced tradeoff between minimizing $\mathcal{T}\mathbf{A}_0$ and $\mathcal{S}_0\mathbf{N}_0$. To deal with these two practical aspects, self-tuning is proposed as a solution. Namely, through self-tuning the necessity of requiring accurate estimations of the mode shapes and zeros of $\mathbf{P}_1$ is avoided, whereas the tradeoff between $\mathcal{T}\mathbf{A}_0$ and $\mathcal{S}_0\mathbf{N}_0$ is automatically considered depending on the significance of each contribution in the data. Also, a stable and causal $\mathbf{C}_0$ is guaranteed by fixing the controller poles a priori in so-called basis functions. According to (7), the controller poles are given by the transmission zeros of $\mathbf{P}_{2,o}$ and Φ_{co} together with the poles of $\{\mathbf{P}_{2,i}^*(z)\mathbf{P}_1(z)\Phi_{co}(z)\}_+$ which are identical to the poles of $\mathbf{P}_1(z)\Phi_{co}(z)$ [29, section C.3]. These pole locations are assumed to be measurable a priori such that the basis functions for self-tuning can be constructed. The controller zeros, depending on the unmeasurable zeros of $\mathbf{P}_1$, are obtained online via self-tuning which will be discussed in Section 4.

3.1. Example: simplified SISO air mount control

Reconsider the simplified model for the single-axis air mount system given in (2). A perfect feedforward controller for such an air mount system is given by substitution of (2) in (6), giving

$$C_0(s) = \underbrace{\frac{ds + k}{s^2}}_{\text{spring-damper compensation}} \cdot \underbrace{\frac{s^2 + \zeta_{ar}\omega_{ar}s + \omega_{ar}^2}{s^2 + \zeta_r\omega_r s + \omega_r^2} \frac{\omega_r^2}{\omega_{ar}^2}}_{\text{internal isolator dynamics compensation}}. \quad (8)$$

Figure 5: layout of the self-tuning controller using the FeLMS algorithm with residual noise shaping.

The first rational transfer function in (8) describes the compensation for the fundamental stiffness and damping properties of the air mount system, whereas the second function can be seen as the compensation for internal isolator dynamics.

Remark 1. As shown in (8), the disturbance feedforward controller requires two poles at $s = 0$, hence it has two pure integrators. However, since the input to $\mathbf{C}_0$ is a measured acceleration signal, pure integrators are generally undesired, because they may induce the amplification of low-frequency sensor noise and signal drift. To circumvent this problem, the pure integrators will be replaced by n^{th} -order weak integrators $H_{(\alpha,n)}$ as proposed in [17], or

$$H_{(\alpha,n)} = \frac{1 - \left(\frac{\alpha}{s+\alpha}\right)^n}{s}, \quad (9)$$

with cut-off frequency $\alpha \in \mathbb{R}$ and order $n \geq 1$. Large n implies large roll-off obtained in $\mathcal{T}$ for frequencies beyond α , but more amplification of low-frequency sensor noise. In this paper, $n = 3$ and $\alpha = 2\pi \cdot 0.5 \text{ rad/s}$ (0.5 Hz).

4. Self-tuning disturbance feedforward control

In this section, the self-tuning algorithm is presented to find the zeros of feedforward controller $\mathbf{C}_0$. First, an existing approach for self-tuning control to realize spring-damper compensation is summarized in Section 4.1. Afterwards, this approach is extended with basis functions to account for internal isolator dynamics as well in Section 4.2.

4.1. Spring-damper compensation

This section summarizes a MIMO self-tuning control approach presented in [17] that was developed for spring-damper compensation. The block diagram corresponding to this approach is shown in Figure 5. In this figure, $\mathbf{C}_0$ represents the self-tuning controller, the *update* block contains the update law, and $\mathbf{N}$ represents a residual noise shaping filter. These three elements are summarized in the following.

Defining q representing the backward shift operator, and $k \in \mathbb{N}$ referring to time samples $t_k = kT_s$ with sampling time $T_s = 1/f_s$, a self-tuned feedforward control signal providing spring-damper compensation is defined as

$$\mathbf{u}_{\mathbf{FF}}(k) = H_{(\alpha,n)}(q) \underbrace{\begin{bmatrix} -\mathbf{D}(k) & -\mathbf{K}(k) \end{bmatrix}}_{\mathbf{W}(k)} \underbrace{\begin{bmatrix} \tilde{\mathbf{a}}_0(k) \\ H_{(\alpha,n)}(q)\tilde{\mathbf{a}}_0(k) \end{bmatrix}}_{\tilde{\boldsymbol{\psi}}(k)}. \quad (10)$$

The matrix $\mathbf{W}(k) \in \mathbb{R}^{3 \times 6}$ consists of the self-tuning damping matrix $\mathbf{D}(k)$ and stiffness matrix $\mathbf{K}(k)$, and $\tilde{\boldsymbol{\psi}}(k) \in \mathbb{R}^6$ represents the so-called regression vector. For a practical implementation, (10) is rewritten such

that the parameters from $\mathbf{W}(k)$ are stored in a single column vector $\mathbf{w}(k)$, or

$$\mathbf{u}_{\mathbf{F}\mathbf{F}}(k) = H_{(\alpha,n)}(q) \cdot \underbrace{\begin{bmatrix} \tilde{\boldsymbol{\psi}}^T(k) & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & \tilde{\boldsymbol{\psi}}^T(k) \end{bmatrix}}_{\tilde{\boldsymbol{\Psi}}(k)} \underbrace{\begin{bmatrix} (\mathbf{W}_{(1,:)}(k))^T \\ \vdots \\ (\mathbf{W}_{(6,:)}(k))^T \end{bmatrix}}_{\mathbf{w}(k)}, \quad (11)$$

with regression matrix $\tilde{\boldsymbol{\Psi}}(k) \in \mathbb{R}^{3 \times 18}$, parameter vector $\mathbf{w}(k) \in \mathbb{R}^{18}$ containing the model-based controller parameters, and $(\mathbf{W}_{(i,:)}(k))^T$ denoting the transposed i^{th} row of $\mathbf{W}(k)$.

Given input $\tilde{\mathbf{a}}_0$, the goal is to find the optimal parameter vector $\mathbf{w}(k) = \mathbf{w}^*$ such that $\mathbf{a}_1$ is minimized in a frequency range of interest. This is done using the Filtered-error Least Mean Squares (FeLMS) algorithm [34] with residual noise shaping [35]. In particular, the instantaneous squared filtered-error $\mathbf{e}$ is minimized, where $\mathbf{e}$ is given by

$$\mathbf{e}(k) = H_{(\alpha,n)}^{-1}(q) \mathbf{N}(q) \hat{\mathbf{P}}_2^{-1}(q) \mathbf{a}_1(k). \quad (12)$$

The filtering operations to obtain $\mathbf{e}(k)$ from the measured signal $\mathbf{a}_1(k)$ can be explained as follows. From FeLMS [21], it is known that $\mathbf{a}_1$ must be filtered by the model inverse of the secondary path denoted by $\hat{\mathbf{P}}_2^{-1}$, see Appendix A, and by $H_{(\alpha,n)}^{-1}$ to compensate for the weak integrators used as pre-multiplication filter in (11). Residual noise shaping [35] requires an additional filter $\mathbf{N}$ to specify the frequency range of interest for vibration isolation. Note that, although $\hat{\mathbf{P}}_2^{-1}$ might become non-causal, it is sufficient to render the product $H_{(\alpha,n)}^{-1} \mathbf{N} \hat{\mathbf{P}}_2^{-1}$ stable and causal by designing $\mathbf{N}$ with sufficiently large relative degree, i.e. the number of poles minus the number of zeros is non-negative. In this paper, a multi-loop SISO design for $\mathbf{N}$ is proposed:

$$\mathbf{N}(q) = n_{\text{flat}}(q) n_{\text{perf}}(q) \mathbf{I}_3. \quad (13)$$

The flattening filter n_{flat} is used to make the frequency spectrum of $H_{(\alpha,n)}^{-1} \mathbf{N} \hat{\mathbf{P}}_2^{-1}$ as flat as possible, at least in the frequency range of interest. By doing so, frequency characteristics of the original error signal $\mathbf{a}_1$ are preserved in the filtered error signal $\mathbf{e}$, hence minimization of $\mathbf{e}$ implies minimization of $\mathbf{a}_1$. In this paper, n_{flat} contains a low-pass filter with a pole at $s = \alpha$ to create a slope of -20 dB/decade that compensates the slope of +20 dB/decade induced by $H_{(\alpha,n)}^{-1}$. The performance filter n_{perf} is used to specify the frequency range where vibration isolation is required. According to the frequency range of interest in this paper, n_{perf} is designed as a fifth-order band-pass filter from 1 to 100 Hz. A side effect of the large roll-off in n_{perf} is that high-frequency content is strongly filtered, which prevents convergence problems of $\mathbf{w}(k)$ under the presence of non-modeled plant dynamics at high frequencies [17].

Given the minimization problem, the parameters can be computed online using the update law

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu(k) \left[\mathbf{N}(q) \tilde{\boldsymbol{\Psi}}(k) \right]^T \mathbf{e}(k), \quad (14)$$

with time-varying adaptation rate $\mu(k)$ that is defined by

$$\mu(k) = \frac{\bar{\mu}}{\epsilon + \|\mathbf{N}(q) \tilde{\boldsymbol{\Psi}}(k)\|_2^2}, \quad (15)$$

see [21], with $\epsilon > 0$ a small positive regularization constant to prevent division by zero, and the normalized adaptation rate $0 < \bar{\mu} < 2$. In Section 5, $\bar{\mu} = 0.0001$ is chosen to provide a good tradeoff between convergence speed on the one hand and a sufficient low steady-state variance in the parameters on the other hand, and ϵ is 0.1% of the RMS power of the signals in $\mathbf{N}(q) \tilde{\boldsymbol{\Psi}}(k)$.

4.2. Internal isolator dynamics

This section extends the spring-damper compensation algorithm such that internal isolator dynamics can be compensated as well. First the extended controller structure is presented, and afterwards a design procedure for the basis functions is discussed.

Figure 6: general structure of a self-tuning controller, computing the j^{th} controller output $u_j, j \in \{1, 2, 3\}$; the controller poles are fixed in the basis functions $\mathcal{B}^{[i]}, i \in \{1, \dots, n_p + 2\}$, while the controller zeros are determined by the self-tuning parameter vectors $\mathbf{w}_j^{[i]}$.

4.2.1. Self-tuning controller structure

Consider Figure 6 that shows the proposed structure in the form of a generalized FIR filter for the j^{th} controller signal u_j , with $j \in \{1, 2, 3\}$. The input consists of the measured base frame accelerations $\tilde{\mathbf{a}}_0(k) \in \mathbb{R}^3$, with $k \in \mathbb{N}$ referring to time samples $t_k = kT_s$ with sampling time T_s . The poles are stored in basis functions $\mathcal{B}^{[i]}$, with $i \in \{1, \dots, n_p + 2\}$. The self-tuning parameter vectors are given by $\mathbf{w}_j^{[i]}(k) \in \mathbb{R}^3$. Figure 6 basically consists of two blocks. The left block is used for compensating internal isolator dynamics and represents an extension with respect to Section 4.1 whereas the right block represents spring-damper compensation. Without compensation for internal isolator dynamics ($n_p = 0$), the structure reduces to a spring-damper compensation controller similar to (10) with $\mathcal{B}^{[n_p+1]} = 1$, $\mathcal{B}^{[n_p+2]} = H_{(\alpha, n)}$, and $\mathbf{w}_j^{[n_p+1]}$ and $\mathbf{w}_j^{[n_p+2]}$ corresponding to the estimated damping and stiffness parameters.

The control signals are calculated as follows. First, the measured disturbances $\tilde{\mathbf{a}}_0$ are filtered making use of the basis functions $\mathcal{B}^{[i]}(q)$, with q representing the backward time-shift operator. Hence, the filtered disturbance vectors are given by $\tilde{\psi}^{[i]}(k) = \mathcal{B}^{[i]}(q)\tilde{\mathbf{a}}_0(k)$. Second, the i^{th} contribution to the j^{th} control signal follows from the inner product $u_j^{[i]}(k) = (\mathbf{w}_j^{[i]}(k))^T \tilde{\psi}^{[i]}(k)$. Third, all contributions for $i = 1, \dots, n_p + 2$ are summed and integrated by an n^{th} -order weak integrator $H_{(\alpha, n)}(q)$, resulting in the j^{th} control output $u_j(k) \in \mathbb{R}$, or

$$u_j(k) = H_{(\alpha, n)}(q) \sum_{i=1}^{N_p+2} (\mathbf{w}_j^{[i]}(k))^T \tilde{\psi}^{[i]}(k). \quad (16)$$

Fourth, each u_j given by (16) is stacked in the control vector $\mathbf{u}(k) = [u_1(k), u_2(k), u_3(k)]^T$, leading to

$$\mathbf{u}(k) = H_{(\alpha, n)}(q) \underbrace{[\mathbf{W}^{[1]}(k) \dots \mathbf{W}^{[n_p+2]}(k)]}_{\mathbf{W}(k)} \underbrace{\begin{bmatrix} \tilde{\psi}^{[1]}(k) \\ \vdots \\ \tilde{\psi}^{[n_p+2]}(k) \end{bmatrix}}_{\tilde{\psi}(k)}, \quad \text{with } \mathbf{W}^{[i]}(k) = \begin{bmatrix} (\mathbf{w}_1^{[i]}(k))^T \\ \vdots \\ (\mathbf{w}_3^{[i]}(k))^T \end{bmatrix}. \quad (17)$$

Similar to (10), $\tilde{\psi}(k) \in \mathbb{R}^{3(n_p+2)}$ in (17) represents the regression vector whereas $\mathbf{W}(k) \in \mathbb{R}^{3 \times 3(n_p+2)}$ contains the self-tuning parameters. Next, (17) can be rewritten to a form similar to (11) such that expressions for the new regression matrix $\tilde{\Psi} \in \mathbb{R}^{3 \times 9(n_p+2)}$ and parameter vector $\mathbf{w} \in \mathbb{R}^{9(n_p+2)}$ are obtained and the update law from (18) can be applied again:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu(k) \left[\mathbf{N}(q) \tilde{\Psi}(k) \right]^T \mathbf{e}(k). \quad (18)$$

4.2.2. Basis functions

Two important aspects play a role when designing basis functions. First, the linear combination of basis functions must include all desired controller poles. This requirement can be fulfilled by using rational basis functions [36]. Second, the basis functions must be designed such that all parameters in $\mathbf{w}(k)$ have a fast and uniform convergence speed when updated according to (18). To obtain this, it is shown in [17] that all eigenvalues of $\mathbb{E} [\tilde{\Psi}^T(k) \mathbf{N}^T(q) \mathbf{N}(q) \tilde{\Psi}(k)]$ must be identical. This requirement implies careful design of both the filter $\mathbf{N}$ and the basis functions (which determine $\tilde{\Psi}$). In general, it is not straightforward to design both $\mathbf{N}$ and the basis functions in such a way that all eigenvalues become identical. However, in this paper a solution is found under the following assumption.

Assumption 2. *The system satisfies the following conditions:*

- (I) *The noise shaping filter is multi-loop SISO, i.e. $\mathbf{N}(q) = n(q) \mathbf{I}_3$, with SISO filter $n(q)$ and 3×3 identity matrix $\mathbf{I}_3$;*
- (II) *The set of filtered input disturbance signals $n(q) \tilde{\mathbf{a}}_0(k)$ has a power spectral density matrix corresponding to a set of (possibly correlated) Gaussian disturbances with a flat frequency spectrum.*

Condition (I) in Assumption 2 is easily satisfied since $\mathbf{N}$ is user-defined. Condition (II) in Assumption 2 is often reasonable, because the controller aims at broadband disturbance rejection. This means that the spectral power of base frame vibrations should be sufficiently distributed over a broad frequency range. Note that a power spectrum that not exactly satisfies Condition (II) still leads to convergence, although some of the parameters might suffer from reduced convergence speeds. For more information regarding implementation aspects of basis functions for non-Gaussian disturbances, the reader is referred to [37].

Under Assumption 2, it is derived in Appendix B that fast and uniform convergence of the parameters is obtained if orthonormal basis functions are designed [25, 36], satisfying

$$\langle \mathcal{B}^{[m]}, \mathcal{B}^{[n]} \rangle = 0, \quad \langle \mathcal{B}^{[m]}, \mathcal{B}^{[m]} \rangle = \langle \mathcal{B}^{[n]}, \mathcal{B}^{[n]} \rangle = 1, \quad (19)$$

with basis functions $\mathcal{B}^{[m]}, \mathcal{B}^{[n]}$, where $m, n \in \{1, \dots, n_p + 2\}$, $m \neq n$, and the inner product defined as

$$\langle \mathcal{B}^{[m]}, \mathcal{B}^{[n]} \rangle := \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathcal{B}^{[m]}(e^{j\omega}) \mathcal{B}^{[n]H}(e^{j\omega}) d\omega. \quad (20)$$

For the left block in Figure 6, which compensates internal isolator dynamics, a set of orthonormal basis functions is derived based on Kautz functions [38], which are defined as

$$\begin{aligned} \mathcal{B}^{[2i-1]}(z) &= \frac{\sqrt{1-c_i^2}(z-b_i)}{z^2+b_i(c_i-1)z-c_i} \prod_{k=1}^{i-1} \left(\frac{-c_k z^2 + b_k(c_k-1)z+1}{z^2+b_k(c_k-1)z-c_k} \right), \\ \mathcal{B}^{[2i]}(z) &= \frac{\sqrt{1-c_i^2}\sqrt{1-b_i^2}}{z^2+b_i(c_i-1)z-c_i} \prod_{k=1}^{i-1} \left(\frac{-c_k z^2 + b_k(c_k-1)z+1}{z^2+b_k(c_k-1)z-c_k} \right), \quad i \in \{1, \dots, \frac{n_p}{2}\}. \end{aligned} \quad (21)$$

In (21), the coefficients b_i, c_i in the i^{th} pair of Kautz functions depend on the resonance frequency and damping ratio of the corresponding poles and will be determined from measurements, see Section 5.1. Note that this paper only considers complex pairs of poles in the basis using Kautz functions. If one also wants to include real poles, Laguerre functions or Takenaka-Malmquist functions could be added as well [25, 38].

The basis functions for the spring-damper compensation block in Figure 6 are chosen similar to [17],

$$\mathcal{B}^{[n_p+1]} = 1, \quad \mathcal{B}^{[n_p+2]}(z) = \prod_{k=1}^{n_p} \left(\frac{-c_k z^2 + b_k(c_k-1)z+1}{z^2+b_k(c_k-1)z-c_k} \right) \beta H_{(\alpha, n)}(z). \quad (22)$$

Orthonormality of the basis functions in (21) and (22) can be proven by showing the relevant orthogonality and normalization properties. Orthogonality is shown in three steps. First, let the weak integrator in $\mathcal{B}^{[n_p+2]}$

Table 1: Frequencies and relative damping ratios of the poles estimated from Figure 7.

Char. locus	Frequency (Hz)	Damping ratio
yellow	57.1	0.01
yellow	70.7	0.01
red	57.8	0.01
red	77.0	0.01
blue	76.7	0.01

Figure 7: Measured characteristic loci plots of $\mathbf{P}_2^{-1}\mathbf{P}_1$; the peak at 50 Hz is not caused by a pole, but related to the electrical power grid.

be obtained from ZOH discretization of (9) such that $\mathcal{B}^{[n_p+2]}$ has a relative degree of one, rendering it orthogonal to $\mathcal{B}^{[n_p+1]}$ which can be verified from (20). Second, following the same line of reasoning, it is concluded that the Kautz functions in (21) are orthogonal to $\mathcal{B}^{[n_p+1]}$. Third, the all-pass filter included in $\mathcal{B}^{[n_p+2]}$ renders the function orthogonal to the Kautz functions in (21), see Appendix C. To prove orthonormality, observe that the Kautz functions in (21) are orthonormal from themselves, that $\langle \mathcal{B}^{[n_p+1]}, \mathcal{B}^{[n_p+1]} \rangle = 1$, and let the normalization constant $\beta \in \mathbb{R}$ be determined such that $\langle \mathcal{B}^{[n_p+2]}, \mathcal{B}^{[n_p+2]} \rangle = 1$.

5. Experimental validation

This section presents an experimental validation of the self-tuning control strategy to improve vibration isolation performance of the industrial system presented in Section 2. First, measurements are conducted to determine the poles for the basis functions, see Section 5.1. Second, self-tuning is applied and performance measurements are conducted, see Section 5.2.

5.1. Measurements for the basis functions

To find the desired controller via self-tuning, basis functions must be constructed which needs a priori information about the relevant controller poles. Recall from Section 3 and (7) that these poles are related to the poles of $\mathbf{P}_1$, the transmission zeros of $\mathbf{P}_2$, and the poles and transmission zeros of the spectrum $\Phi_{\mathbf{a}_0}$.

Frequency response function (FRF) measurements of $\mathbf{P}_1$ and $\mathbf{P}_2$ are performed, see Appendix A. Note that the measured $\mathbf{P}_1$ presented in Appendix A is relatively noisy due to limited excitation capabilities, but this is not considered to be a problem because it is only used to detect resonance peaks for selecting the controller poles. Based on the measurements of $\mathbf{P}_1$ and $\mathbf{P}_2$, an estimated FRF matrix of $\mathbf{P}_2^{-1}\mathbf{P}_1$ is obtained for which Figure 7 shows the characteristic loci [39] (also called complex mode indicator functions [40]). These loci represent the eigenvalues of the FRF matrix along the frequency axis and are used to estimate the required poles. An advantage of using characteristic loci is that closely spaced poles can be separated from each other. A set of poles is determined using peak picking, the result of which is listed in Table 1. Note that for all poles the corresponding damping ratio is estimated as 0.01, but this value might be inaccurate due to averaging and a finite frequency grid [41]. To enable better estimations of the true poles one can use more elaborate system identification methods such as PolyMAX [42] or Local Polynomial Methods [43]. An alternative method is overfitting which will be used in this paper. Using overfitting, five different pairs of basis functions are implemented for every complex pair of poles, each using the same frequency but with different damping ratios $\zeta \in \{0.1, 0.03, 0.01, 0.003, 0.001\}$. The characteristic loci are

Figure 8: Measured (a) power spectral density (PSD), and (b) cumulative power spectral density (PSD); Using full-order (FO) control, the (cumulative) PSD is largely reduced.

not plotted for frequencies below 10 Hz, because internal isolator dynamics are not expected to occur below 10 Hz. At low frequencies, the characteristic loci have a slope of -40 dB/decade caused by two poles at $s = 0$. These correspond to the poles for spring-damper compensation, similar to the example in (8).

To account for the disturbance spectral factor Φ_{a_0} , reconsider the parametric fit given in (3). According to this fit, basis functions should be included to represent the zeros and the poles at $f_1 = 70.7$ Hz and $f_2 = 77$ Hz. However, since these contributions are assumed to be sufficiently included already by the overfitting method to account for $\mathbf{P}_1$ and $\mathbf{P}_2$, no additional basis functions are included to account for Φ_{a_0} .

5.2. Vibration isolation performance

Two controllers are implemented, namely a controller of (a) reduced-order (RO), and (b) full-order (FO). For all three considered directions (z, θ_x, θ_y), the RO controller only provides spring-damper compensation using the two basis functions in (22) and serves as a performance reference. The FO controller also includes 25 pairs of basis functions, see Section 5.1, to compensate for internal isolator dynamics and the characteristics of the base frame disturbance spectrum. The self-tuning algorithm for both the RO and FO controller is turned on by setting $\bar{\mu} = 0.0001$, see (15), such that the optimum is reached in less than ten minutes. Afterwards, it is turned off ($\bar{\mu} = 0$) such that the parameter vector $\mathbf{w}$ becomes fixed with the result of the feedforward controller becoming linear time-invariant (LTI). Using the LTI controllers, a second measurement of 10 minutes is conducted to serve as an assessment of performance. Each measurement is split into $M = 60$ subrecords of 10 seconds, and each subrecord is filtered with a Hanning window to reduce the effects of leakage. To compensate for data loss due to windowing, additional subrecords are created by using an overlap factor of 50%. Then, all subrecords are averaged which leads to the averaged (cross) power spectral density (PSD) matrices $\hat{S}_{a_0, a_0}(f_k)$, $\hat{S}_{a_1, a_1}(f_k)$, and $\hat{S}_{a_1, a_0}(f_k)$ calculated at discrete points $f_k = kf_0$, $k = 1, 2, \dots$, $f_0 = 0.1$ Hz. Here, $\hat{S}_{a_1, a_1}$ is a measure for the power spectral density (PSD) of residual vibrations at the isolated metrology frame, whereas $\mathcal{T}(f_k) = \hat{S}_{a_1, a_0}(f_k) \hat{S}_{a_0, a_0}^{-1}(f_k)$ gives a measure for the transmissibility matrix. Additionally, the output noise covariance matrix $\hat{C}_{a_1}$ is estimated as [19, 43]

$$\hat{C}_{a_1}(f_k) = \frac{M}{M-3} \left(\hat{S}_{a_1, a_1}(f_k) - \hat{S}_{a_1, a_0}(f_k) \hat{S}_{a_0, a_0}^{-1}(f_k) \hat{S}_{a_1, a_0}^H(f_k) \right). \quad (23)$$

Figure 9: Bode magnitude plots of the transmissibility functions $\mathcal{T}$ for the systems with feedback and feedforward control; compared to the system with only feedback control, both reduced-order (RO) and full-order (FO) feedforward control lead to a significant reduction in $\mathcal{T}$ at the lower frequencies; at the higher frequencies, FO-control realizes an additional reduction to account for internal isolator dynamics.

Under the assumption that the signals in $\mathbf{n}_p$ and $\mathbf{n}_1$ are uncorrelated with $\mathbf{a}_0$, recall Figure 4, $\hat{\mathbf{C}}_{\mathbf{a}_1}$ gives a lower bound for $\hat{\mathbf{S}}_{\mathbf{a}_1, \mathbf{a}_1}$, i.e. the best performance that can be obtained using disturbance feedforward control.

In Figure 8a, the measured $\hat{\mathbf{S}}_{\mathbf{a}_1, \mathbf{a}_1}$ for feedback and feedforward control systems are compared with $\hat{\mathbf{C}}_{\mathbf{a}_1}$. For the system with only feedback control, it is observed that $\hat{\mathbf{S}}_{\mathbf{a}_1, \mathbf{a}_1}$ is significantly higher than $\hat{\mathbf{C}}_{\mathbf{a}_1}$. When applying the RO feedforward controller, it is observed that the power at low frequencies reduces, but not at high frequencies where internal isolator dynamics occur because the controller structure is insufficient to fit these dynamics. Contrarily, when using the FO controller, the power is significantly reduced at both low and high frequencies to (almost) $\hat{\mathbf{C}}_{\mathbf{a}_1}$, indicating large performance improvements obtained with FO feedforward control. Figure 8b shows plots of the cumulative power spectral density (PSD) which is calculated as

$$\text{cumPSD}(f_N) = \sum_{k=1}^N \hat{\mathbf{S}}_{\mathbf{a}_1, \mathbf{a}_1}(f_k) \Delta f, \quad N = 1, 2, \dots, \frac{f_s}{2f_0}, \quad (24)$$

with sampling frequency f_s . From the cumulative PSD plots, it follows that FO feedforward control outperforms the other controllers. Compared to feedback control, FO control reduces the cumulative power with approximately 79% in Z -direction, 67% in θ_x -direction, and 89% in θ_y -direction. This clearly shows that there is a benefit of using FO control.

Figure 9 shows Bode magnitude plots of the transmissibility functions for the systems with feedback and feedforward control. Below 2 Hz, the measurements have a significant uncertainty due to an insufficient signal-to-noise ratio. In the z -direction, application of both RO- and FO-feedforward control leads to a significant reduction of the transmissibilities at frequencies below 15 Hz. However, this reduction comes at the cost of performance deterioration at the mid-frequencies (between 15 and 50 Hz) due to a waterbed effect

[31]. In the θ_x - and θ_y -directions, this shift in performance is less visible because the signal-to-noise ratio at low frequencies is much smaller, recall Figure 8a. For frequencies between 60 and 80 Hz, the transmissibility is significantly lower when using FO-control. This is beneficial because here the base frame excitations are the strongest, recall Figure 3. This benefit cannot be obtained by RO control because it cannot fit the internal isolator dynamics properly into the feedforward controller structure.

6. Conclusions

This paper presented a disturbance feedforward control strategy for active vibration isolation systems with internal air mount dynamics. The controller is implemented as a self-tuning controller for which the poles are fixed in rational basis functions and the zeros are obtained online via a self-tuning approach. The poles are obtained from measured characteristic loci plots of the system using spectral analysis. Implementation of the self-tuning controller shows a reduction up to 89% under a given set of base frame vibrations representative for industrial environments. It is shown that the proposed full-order self-tuning controller clearly outperforms a feedforward controller which is only based on spring-damper compensation.

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Appendix A. System identification

This section presents system identification results for the industrial system presented in Section 2. These results are used to implement the self-tuning controller in Section 5.

The primary path $\mathbf{P}_1$ represents the transmissibility matrix of the uncontrolled system, and is measured using the method in [19]. In this method, a combination of environmental floor excitations at low frequencies and shaker excitations at high frequencies is used. The shakers are placed in parallel to the base frame support feet, see Figure 2a. Both floor and shaker excitations are considered as random signals, and a measurement record of 10 minutes is recorded at a fixed sampling frequency of 4 kHz. Spectral analysis is used to estimate the FRF and the 95% confidence regions of the FRF, which are both shown in Figure A.10.

The FRF of the secondary path $\mathbf{P}_2$ is also obtained using spectral analysis. In this measurement, a signal generator is used to generate three random excitation signals which are sent simultaneously to all three Lorentz actuators. These signals are recorded and scaled with the motor constants of the actuators to obtain the estimated input forces and moments exerted to the system. The corresponding response at the accelerometers is measured as the output of the system. A data record of three minutes is recorded, which is split in subrecords of 10 seconds and filtered with a Hanning window with 50% overlap. Figure A.11 shows the resulting FRF measurements of $\mathbf{P}_2$, and a fitted parametric model which is given by

$$\hat{\mathbf{P}}_2(s) = \underbrace{\begin{bmatrix} \frac{s^2}{ms^2+d_zs+k_z} & 0 & 0 \\ 0 & \frac{s^2}{J_{xx}s^2+d_xs+k_x} & 0 \\ 0 & 0 & \frac{s^2}{J_{yy}s^2+d_ys+k_y} \end{bmatrix}}_{\hat{\mathbf{P}}_{2,m}(s)} \underbrace{\frac{\omega_n^2}{s^2 + 2\zeta_n\omega_n s + \omega_n^2} e^{\tau s} \mathbf{I}_3}_{\hat{\mathbf{P}}_{2,a}(s)}. \quad (\text{A.1})$$

The parametric model includes the suspended rigid-body behavior of the metrology frame in $\hat{\mathbf{P}}_{2,m}$, and actuator dynamics described by a second-order low-pass filter at $\omega_n = 2\pi \cdot 160$ rad/s with relative damping $\zeta_n = 0.7$ and a time delay of $\tau = 0.001$ s in $\hat{\mathbf{P}}_{2,a}$. The modeled masses and inertias are $m = 1100$ kg, $J_{xx} = 250$ kg m², $J_{yy} = 200$ kg m², the air mount damping values are $d_z = 6000$ Ns/m, $d_{Rx} = 1500$ Ns/m, $d_{Ry} = 1500$ Ns/m, and the air mount stiffness values are $k_z = 50$ kN/m, $k_{Rx} = 5$ kN/m, $k_{Ry} = 5$ kN/m.

Figure A.10: FRF Measurement of the primary path $\mathbf{P}_1$, representing the transmissibility function matrix of the vibration isolation system without feedforward control. The shaded area represents the 95% confidence interval of the FRF measurement. It is observed that the suspension frequencies are around 1 Hz, after which the transmissibility starts to roll off from the 0 dB line. Note that the suspension modes show no resonance peaks because skyhook damping is applied using acceleration feedback.

Appendix B. Proof for uniform convergence speed

This appendix proves uniform convergence speed of all self-tuning parameters. It is shown in [17] that all eigenvalues of $\mathbb{E} [\tilde{\Psi}^T(k) \mathbf{N}^T(q) \mathbf{N}(q) \tilde{\Psi}(k)]$ must be identical. In this paper, $\mathbf{N}(q) = n(q) \mathbf{I}_{3(N_p+2)}$ is used, see Assumption 2. Then, fast and uniform convergence of the parameters is obtained if all eigenvalues of $\mathbb{E} [\tilde{\psi}(k) n^2(q) \tilde{\psi}^T(k)]$ are identical [17]. Define

$$n(q) \tilde{\psi}(k) = \mathcal{B}(q) \otimes (n(q) \tilde{\mathbf{a}}_0(k)), \quad \mathcal{B}(q) = \begin{bmatrix} \mathcal{B}^{[1]}(q) \\ \vdots \\ \mathcal{B}^{[N_p+2]}(q) \end{bmatrix}. \quad (\text{B.1})$$

Then, using Parseval's rule, it is obtained that

$$\mathbb{E} \left[\left(n(q) \tilde{\psi}(k) \right) \left(n(q) \tilde{\psi}(k) \right)^T \right] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(n(e^{j\omega}) \tilde{\psi}(e^{j\omega}) \right) \left(n(e^{j\omega}) \tilde{\psi}(e^{j\omega}) \right)^H d\omega \quad (\text{B.2})$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} (\mathcal{B}(e^{j\omega}) \otimes (n(e^{j\omega}) \tilde{\mathbf{a}}_0(e^{j\omega}))) (\mathcal{B}(e^{j\omega}) \otimes (n(e^{j\omega}) \tilde{\mathbf{a}}_0(e^{j\omega})))^H d\omega \quad (\text{B.3})$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} (\mathcal{B}(e^{j\omega}) \otimes (n(e^{j\omega}) \tilde{\mathbf{a}}_0(e^{j\omega}))) (\mathcal{B}^H(e^{j\omega}) \otimes (n(e^{j\omega}) \tilde{\mathbf{a}}_0(e^{j\omega}))^H) d\omega \quad (\text{B.4})$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} (\mathcal{B}(e^{j\omega}) \mathcal{B}^H(e^{j\omega})) \otimes \mathbf{S}_{\mathbf{a}_0, \mathbf{N} \mathbf{a}_0, \mathbf{N}}(e^{j\omega}) d\omega, \quad (\text{B.5})$$

Figure A.11: Bode magnitude plots of the measured transfer functions $\mathbf{P}_2$ (solid light blue lines) compared to the parametric model $\hat{\mathbf{P}}_2$ (dashed red lines); the off-diagonal magnitudes are low compared to the main-diagonal magnitudes between 1 and 100 Hz, showing good system decoupling and validating the decentralized structure of the parametric model.

where the power spectral density matrix of the filtered disturbances $n(q)\tilde{\mathbf{a}}_0$ is given by

$$\mathbf{S}_{\mathbf{a}_0, \mathbf{N}\mathbf{a}_0, \mathbf{N}}(e^{j\omega}) = (n(e^{j\omega})\tilde{\mathbf{a}}_0(e^{j\omega}))(n(e^{j\omega})\tilde{\mathbf{a}}_0(e^{j\omega}))^H. \quad (\text{B.6})$$

In (B.4) it is used that $(A \otimes B)^H = A^H \otimes B^H$, and in (B.5) it is used that $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$. Now, the problem has been reformulated to a frequency-domain design problem of the basis functions in $\mathcal{B}$ such that the matrix resulting from the integral in (B.5) has all identical eigenvalues. There exists no general solution to this design problem for any arbitrary $\mathbf{S}_{\mathbf{a}_0, \mathbf{N}\mathbf{a}_0, \mathbf{N}}(e^{j\omega})$. However, under the assumption

$$\mathbf{S}_{\mathbf{a}_0, \mathbf{N}\mathbf{a}_0, \mathbf{N}}(e^{j\omega}) = \sigma_0^2 \mathbf{U}, \quad (\text{B.7})$$

with unitary matrix $\mathbf{U}$, the disturbances represent (possibly correlated) Gaussian disturbances with zero mean and variance σ_0^2 , and (B.5) reduces to

$$\mathbb{E} \left[\left(n(q)\tilde{\boldsymbol{\psi}}(k) \right) \left(n(q)\tilde{\boldsymbol{\psi}}(k) \right)^T \right] = \underbrace{\frac{\sigma_0^2}{2\pi} \int_{-\pi}^{\pi} \mathcal{B}(e^{j\omega}) \mathcal{B}^H(e^{j\omega}) d\omega}_{\mathbf{A}} \otimes \mathbf{U}. \quad (\text{B.8})$$

In general, the singular values of $(\mathbf{A} \otimes \mathbf{U})$ are given by $\sigma_{\mathbf{A},i} \sigma_{\mathbf{U},j}$, where $\sigma_{\mathbf{A},i}, i \in \{1, \dots, N_p + 2\}$ and $\sigma_{\mathbf{U},j}, j \in \{1, 2, 3\}$ represent the singular values of $\mathbf{A}$ and $\mathbf{U}$, respectively. Since $\mathbf{U}$ is unitary, it follows that $\sigma_{\mathbf{U},j} = 1$ for all j . Then, all singular values of (B.6) are identical, i.e. a solution of the basis functions design problem is found, when using orthonormal basis functions [25, 36] as defined in (19). Using this design of orthonormal basis functions, the matrix $\mathbf{A}$ in (B.6) reduces to a scaled identity matrix which has eigenvalues $\sigma_{\mathbf{A},i} = \sigma_0^2$ for all i , and which completes the proof.

Appendix C. Sketch of proof for orthonormality of an n^{th} -order weak integrator

This appendix gives a sketch of proof for the orthonormality of a basis function describing the n^{th} -order weak integrator. To this end, consider a Takenaka-Malmquist basis function given by

$$\mathcal{B}^{[1]} = \frac{(1 - \xi_1^* z)}{(z - \xi_1)} \frac{1}{(z - \xi_2)}, \quad (\text{C.1})$$

And two additional basis functions

$$\mathcal{B}^{[2]} = \frac{(1 - \xi_1^* z)}{(z - \xi_1)} \frac{(1 - \xi_2^* z)}{(z - \xi_2)} \cdot \frac{1}{(z - \alpha)}, \quad (\text{C.2})$$

$$\mathcal{B}^{[3]} = \frac{(1 - \xi_1^* z)}{(z - \xi_1)} \frac{(1 - \xi_2^* z)}{(z - \xi_2)} \cdot \frac{(1 - \alpha z)}{(z - \alpha)} \frac{1}{(z - \alpha)}. \quad (\text{C.3})$$

Two basis functions are said to be orthogonal if

$$\langle \mathcal{B}^{[m]}, \mathcal{B}^{[n]} \rangle := \frac{1}{2\pi i} \oint_{\mathbb{T}} \mathcal{B}^{[m]}(z) \mathcal{B}^{[n]H} \left(\frac{1}{z^H} \right) \frac{1}{z} dz = 0. \quad (\text{C.4})$$

It follows straightforwardly from substitution of (C.1)–(C.3) in (C.4) that $\langle \mathcal{B}^{[1]}, \mathcal{B}^{[2]} \rangle = 0$, $\langle \mathcal{B}^{[1]}, \mathcal{B}^{[3]} \rangle = 0$, and $\langle \mathcal{B}^{[2]}, \mathcal{B}^{[3]} \rangle = 0$. Then, it also holds that $\langle \mathcal{B}^{[1]}, (c_2 \mathcal{B}^{[2]} + c_3 \mathcal{B}^{[3]}) \rangle = 0$ with $c_2, c_3 \in \mathbb{R}$. The latter linear combination can be written as

$$\begin{aligned} c_2 \mathcal{B}^{[2]} + c_3 \mathcal{B}^{[3]} &= \frac{(1 - \xi_1^* z)}{(z - \xi_1)} \frac{(1 - \xi_2^* z)}{(z - \xi_2)} \left[\frac{c_2}{z - \alpha} + \frac{c_3(1 - \alpha z)}{(z - \alpha)^2} \right] \\ &= \frac{(1 - \xi_1^* z)}{(z - \xi_1)} \frac{(1 - \xi_2^* z)}{(z - \xi_2)} \left[\frac{(c_3 - c_4 \alpha)z + (c_4 - c_3 \alpha)}{(z - \alpha)^2} \right] \\ &= \frac{(1 - \xi_1^* z)}{(z - \xi_1)} \frac{(1 - \xi_2^* z)}{(z - \xi_2)} \left[\frac{d_1 z + d_0}{(z - \alpha)^2} \right] \end{aligned} \quad (\text{C.5})$$

with the two variables $d_1 = (c_3 - c_4 \alpha)$, $d_0 = (c_4 - c_3 \alpha)$ which are free to choose since $\langle \mathcal{B}^{[1]}, (c_2 \mathcal{B}^{[2]} + c_3 \mathcal{B}^{[3]}) \rangle = 0$ for any $c_2, c_3 \in \mathbb{R}$. Next, if d_1, d_0 are chosen such that $\left[\frac{d_1 z + d_0}{(z - \alpha)^2} \right]$ describes a weak integrator $H_{(\alpha, n)}(z)$ having order $n = 2$, cut-off frequency α , and a relative degree of one, the linear combination of basis functions in (C.5) can be regarded as a single basis function similar to $\mathcal{B}^{[n_p+2]}$ in (22) and orthogonal to the Takenaka-Malmquist basis function in (C.1). This idea can be generalized to any Takenaka-Malmquist basis function, and any arbitrary higher-order weak integrator $H_{(\alpha, n)}(z)$, by extending (C.1)–(C.3) up to n basis functions.

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