Ein generisches Getriebemodell für hybrid-elektrische Antriebe

Citation for published version (APA):

van Harselaar, W., Brouwer, M., & Hofman, T. (2019). Ein generisches Getriebemodell für hybrid-elektrische Antriebe: Die Integration des dynamischen Verhaltens für alle konventionelle, elektrische und hybridische Antriebsmodi in einem Modell. *Forschung im Ingenieurwesen*, 83(1), 1-9. <https://doi.org/10.1007/s10010-018-00295-5>

Document license:
TAVERNE

DOI:
[10.1007/s10010-018-00295-5](https://doi.org/10.1007/s10010-018-00295-5)

Document status and date:
Gepubliceerd: 07/03/2019

Document Version:
Uitgevers PDF, ook bekend als Version of Record

Please check the document version of this publication:

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A generic transmission model for hybrid electric drives

Integration of the dynamic behavior of all conventional-, electric-, and hybrid-transmission mode types in a single model

W. van Harselaar¹ · M. Brouwer¹ · T. Hofman²

Received: 2 May 2018 / Accepted: 24 December 2018 / Published online: 1 February 2019
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Abstract

The design problem of hybrid electric drivetrains has a large design space, especially, when complex dedicated hybrid transmissions that combine an internal combustion engine with multiple electric machines are considered. To evaluate a drivetrain design over a driving cycle, a transmission model is needed. To avoid the need of modeling every drivetrain concept individually, a generic transmission model for hybrid electric drives is developed. The developed model integrates the dynamic behavior of all conventional-, electric-, and hybrid- transmission mode types. Mode types and mode type dependent control variables are defined. The outputs of the model are the torque and rotational speed of all power sources of the drivetrain. The inputs of the model are the torque and speed of the wheels, and the control variables. As a practical example, the model is used to compare the energy efficiency of two complex dedicated hybrid transmissions by optimizing the control for both over a driving cycle.

Ein generisches Getriebemodell für hybrid-elektrische Antriebe

Die Integration des dynamischen Verhaltens für alle konventionelle, elektrische und hybridische Antriebsmodi in einem Modell

Zusammenfassung

Beim Entwurf von Hybrid-Antriebssträngen gibt es viele Freiheitsgrade, insbesondere wenn komplexe Topologien die einen Verbrennungsmotor mit mehreren elektrischen Maschinen kombinieren mitbetrachtet werden. Um eine Antriebsstrangkonfiguration anhand eines Fahrzyklus bewerten zu können, wird ein Modell des Getriebes benötigt. Um nicht jedes Antriebsstrangkonzept einzeln modellieren zu müssen, wird ein generisches Getriebemodell für Hybridantriebe entwickelt. Das entwickelte Modell beschreibt das dynamische Verhalten aller konventionellen, elektrischen und hybridischen Betriebsmodi. Um das Verhalten zu beschreiben werden Betriebsmodi und die von diesen Modi jeweils abhängigen Steuergrößen definiert. Die Eingangsgrößen des Modells sind Drehmoment und Drehzahl am Rad, sowie die Steuergrößen. Die Ausgangsgrößen sind die Drehmomente und Drehzahlen aller Leistungsquellen des Antriebs. Anhand eines praktischen Beispiels wird das vorgestellte Modell angewandt um die Energieeffizienz zweier komplexe Hybridantriebe zu vergleichen indem die Steuervariablen für beide über einen Fahrzyklus optimiert werden.

✉ W. van Harselaar
wilco.van_harselaar@daimler.com

¹ Daimler AG, 70546 Stuttgart, Germany

² Control Systems Technology Group, Mechanical Engineering
Department, Eindhoven University of Technology, 5612
AZ Eindhoven, The Netherlands

1 Introduction

The drivetrain topology defines which power sources a vehicle contains, and how these power sources are connected to the wheels of the vehicle. Distinct topologies enable distinct behavior, which can be represented by a transmission model. In the drivetrain development process, transmission models are used to evaluate drivetrain topologies in the concept phase, while in the design phase models are used to assess the influence of component design on vehicle level [1]. In the case of backward simulation, the inputs of the transmission model are the torque and rotational speed at the wheels of the vehicle together with the control variables. The required torque and speed at the wheels are determined by the driving cycle and vehicle model. The control variables depend on the drivetrain topology, and can determine, e.g., the torque split. The outputs of the transmission model are the torques and rotational speeds of all power sources of the drivetrain. These outputs depend on the drivetrain topology, as the topology defines which power sources are contained. In addition to the dependency of the inputs and outputs on the topology, the model must also represent the dynamic behavior which is strongly dependent on the drivetrain topology. This dynamic behavior can be particularly complex for topologies with multiple power sources. Topologies with multiple power sources may include transmission modes where ratios are not fixed, but electrically variable, so called electric variable transmissions (EVTs). Recently, there has been an increased research interest regarding complex dedicated hybrid transmission (DHT) topologies, which often enable EVT modes [2, 3]. Another trend is the automated generation, or synthesis, of DHT topologies [4, 5]. In [6] a framework is presented for automated synthesis and optimization over different levels of the powertrain system. The generation of many topologies does, however, also require the evaluation of all these topologies, preferably over a driving cycle using an appropriate transmission model. The application of a generic transmission model to this end is described in [5]. In that work results of using the model are presented for four distinct hybrid drivetrain topologies, yet the equations of the used generic transmission model are not presented. In [7] a novel approach is presented to enable the modeling of arbitrary hybrid drivetrain topologies by using an object oriented library and finding a causal flow for the equations that define the topology. A downside of that method is that the descriptive equations can be very different for different transmission modes of the same topology, especially in the case of complex DHT topologies. For such complex topologies it is often not possible to use the same variable(s) as directly controlled variable(s) for all transmission modes.

In this work, a generic transmission model is presented that can be used to represent all possible drivetrain topolo-

gies with up to one internal combustion engine (ICE), and up to two electric machines (EMs). The developed generic transmission model needs parameters to represent a specific topology. Although these parameters are addressed, this work focuses on the model itself. For the interested reader, the automated generation of the parameters is explained in detail in [8].

This paper starts with the definition of drivetrain topologies in Sect. 2. In Sect. 3 the problem of developing a generic transmission model is explained. Subsequently, the solution to the problem is presented by the mode type dependent control variables in Sect. 4 and the developed generic transmission model itself in Sect. 5. Sect. 6 provides a practical application of the explained model, after which this paper ends with a conclusion in Sect. 7.

2 Topology definition and representation

The developed generic transmission model has to be able to represent all drivetrain topologies with a maximum of one ICE and two EMs. Therefore, the possible topologies define the behavior that the model has to include, which underlines the importance of defining topologies before explaining the generic transmission model. In this work, drivetrain topologies are defined on the level of gears, planetary gear sets (PGSs), clutches, and brakes. All components that can be used to build a topology are defined by the library of components listed in Table 1. In the library of components, no distinction is made between a friction clutch and a dog clutch. A clutch pair represents a pair of two clutches of which only one can be closed at a time. A ground is a permanent connection with the transmission housing. The component *wheels* represents all driven wheels without distinguishing between front and rear. An important boundary to the presented generic transmission model is the maximum number of instances for each component, as also listed in Table 1. Stick diagrams are used to visually represent drivetrain topologies that are built from the listed

Table 1 Library of components

Component type	Abbreviation	Max. number of instances
1: Internal combustion engine	ICE	1
2: Electric machine	EM	2
3: Wheels	W	1
4: Gear pair	GP	n.l.
5: Planetary gear set	PGS	n.l.
6: Clutch	C	n.l.
7: Clutch pair	CP	n.l.
8: Brake	B	n.l.
9: Ground	G	n.l.

n.l. not limited

components, as e.g. in Fig. 1. Stick diagrams are widely used to visualize drivetrains, though graph diagrams can be found as an alternative that offers advantages for the mathematical definition of topologies [4].

3 Problem definition

With the method of describing drivetrain topologies explained in the previous section, this section defines the problem of developing a generic transmission model, and introduces definitions which are important throughout the remaining of this publication. An example topology is presented, which is then used to illustrate the definition of transmission mode types. First, let us define transmission modes: distinct transmission modes are defined by distinct clutch combinations of the transmission. Furthermore, every distinct transmission mode describes a unique mechanical connection between the power sources and the wheels of the vehicle. This definition means that drivetrains without clutches and brakes have only one transmission mode. An example of a complex DHT topology with multiple transmission modes is the so-called *two-mode hybrid system* [9, 10], as displayed in Fig. 1. In total, the topology contains six distinct transmission modes that enable propulsion, and three transmission modes that do not enable propulsion, but only enable charging of the battery.

To discuss transmission mode types, control decision variables (CDVs) are defined as mechanical degrees of freedom of the drivetrain for a fixed speed and torque at the wheels. A distinction is made between kinematic and kinetic CDVs. Furthermore, only functional CDVs are considered in this work. To explain the concept and necessity of the definition of transmission mode types, three modes of the two-mode hybrid system are considered:

1. *Mode M1*, in which clutch C1 and brake B2 are closed. As EM2 is directly connected to B2, this EM cannot provide power. Both the ICE and EM1 have a fixed gear ratio to the wheels and can provide power. For a given speed

Fig. 1 Stick diagram of the two-mode hybrid system topology, abbreviations as defined in Table 1

Table 2 Three example modes of the two-mode hybrid system

Mode	M1	M2	M3
Closed clutches/brakes	C1 & B2	C1 & C2	C1
$N_{cdv,\omega}$	0	0	1
$N_{cdv,\tau}$	1	2	1

and torque at the wheels, the speeds of the power sources are fixed, but there is one kinetic CDV as the torque demand at the wheels can be provided by both the ICE and EM1.

2. *Mode M2*, in which clutches C1 and C2 are closed. The ICE, EM1, and EM2 are all connected to the wheels with a fixed gear ratio. For a given speed and torque at the wheels, the speeds of the power sources are fixed, but the torque demand at the wheels can be divided over all three power sources by two kinetic CDVs.
3. *Mode M3*, in which only clutch C1 is closed. All three power sources are mechanically connected with the wheels, but not with a fixed ratio. This is an EVT mode in which the rotational speeds of the power sources are defined by one kinematic CDV, and their torques are determined by one kinetic CDV.

Table 2 lists these three modes with the clutches and brakes that are closed, the number of kinematic CDVs $N_{cdv,\omega}$, and the number of kinetic CDVs $N_{cdv,\tau}$.

The considered example topology shows that depending on the selected transmission mode different quantities, and number thereof, have to be controlled. Even when the same components are mechanically connected to the wheels, as in M2 and M3, the CDVs can differ. This demonstrates the necessity to define transmission mode types for which the quantities that have to be controlled are standardized. This is done by defining mode types according to the existence of mechanical connections between the power sources and the wheels, the number of kinematic CDVs, and the number of kinetic CDVs. A generic transmission model must include the behavior of all possible mode types. In the previous section, the number of ICEs and the number of EMs are limited to a maximum. This boundary limits the number of possible mode types and thereby enables the development of a generic transmission model. The possible mode types can be divided into five general groups:

1. *ICE only*: mode type in which only the ICE can provide power.
2. *EM only*: mode types in which only EMs can provide power.
3. *Hybrid*: mode types in which both the ICE and at least one EM can provide power.

4. *Charge*: mode types in which the battery can be charged with power provided by the ICE, but propulsion of the vehicle is not possible.
5. *Neutral*: mode type in which no propulsion and charging is possible, but the wheels can rotate freely.

Considering the before stated definition of mode types, there are a total of 14 possible mode types for all topologies that are built according to the library of components defined in Table 1. To name the individual mode types, the abbreviations FG (fixed gear) and EVT are used, as are the terms *parallel* and *series*, referring to the respective types of hybrid drives. The 14 possible mode types are listed in Table 3. The problem for a generic transmission model is to integrate the dynamic behavior of all mode types. To represent a specific topology, the list of modes with their corresponding mode type classification, indicated by the symbol Ψ , and their relevant transmission ratios have to be provided to the model. These are considered parameters to the model, and the automated generation of these parameters for arbitrary hybrid and electric topologies is discussed in [8]. The torque and speed at the wheels, which are inputs of the transmission model, are a function of the drive cycle Λ :

$$\Lambda(t) = [v(t) \quad s(t)]^T \quad (1)$$

with velocity v and slope s , both dependent on the time t .

Table 3 All transmission mode types that are possible for topologies built from the library of components defined by Table 1, with for each mode type the quantities that are directly controlled by the continuous control variables

Mode Type, Ψ	$x_{c,2}$	$x_{c,3}$	$x_{c,4}$
ICE only			
1: FG	–	–	–
EM only			
2: FG 1 EM	–	–	–
3: FG 2 EMs	$\tau_{em_1}(x_{c,2})$	–	–
4: EVT 2 EMs	$\omega_{em_1}(x_{c,2})$	–	–
Hybrid			
5: FG parallel 1 EM	$\tau_{ice}(x_{c,2})$	–	–
6: FG parallel 2 EMs	$\tau_{em_1}(x_{c,2})$	$\tau_{ice}(x_{c,3})$	–
7: Series	$P_{ice}(x_{c,2})$	–	–
8: EVT 1 EM	$\omega_{ice}(x_{c,2})$	–	–
9: EVT 2 EMs 2 CDVs	$\omega_{ice}(x_{c,2})$	$\tau_{ice}(x_{c,3})$	–
10: EVT 2 EMs 3 CDVs	$\omega_{ice}(x_{c,2})$	$\tau_{ice}(x_{c,3})$	$\omega_{em_1}(x_{c,4})$
Charge (No propulsion possible)			
11: FG 1 EM	$P_{ice}(x_{c,2})$	–	–
12: FG 2 EMs	$P_{ice}(x_{c,2})$	–	–
13: EVT 2 EMs	$P_{ice}(x_{c,2})$	–	–
Neutral			
14: Neutral	–	–	–

4 Mode type dependent control variables

With the problem definition and boundary considerations for a generic transmission model in the previous sections, this section will explain the first step towards the definition of the mode type dependent control variables of the model. To enable the formulation of a generic model, one generic set of control variables is defined for all possible topologies and mode types. The first control variable, $x_{c,1}$, is defined as a discrete control variable that determines the transmission mode:

$$x_{c,1} \in [1, 2, \dots, N_m], \quad (2)$$

with N_m the number of modes of the topology. For topologies with only one transmission mode control variable $x_{c,1}$ can only be one discrete value. This leads to $x_{c,1}$ not being a variable anymore, but a constant. For the developed generic transmission model $x_{c,1}$ is however retained to enable the generic definition and encompass all possible topologies. Additionally to $x_{c,1}$ continuous control variables are needed. The maximum number of required continuous control variables for topologies with three power sources is three: two kinematic CDVs and one kinetic CDV. Added to the previously defined $x_{c,1}$ this results in a total of 4 control variables for the developed generic model, with the continuous control variables being $x_{c,2}$, $x_{c,3}$, and $x_{c,4}$. As the quantities controlled by the three continuous control variables are dependent on the mode type, the continuous control variables are discretized as scaled variables with a value between zero and one:

$$x_{c,j} \in \left[0, \frac{1}{N_{x_{c,j}} - 1}, \frac{2}{N_{x_{c,j}} - 1}, \dots, \frac{N_{x_{c,j}} - 1}{N_{x_{c,j}} - 1} \right], \quad (3)$$

$$j = 2, 3, 4$$

with the number of discrete values of control variable $x_{c,j}$ being denoted as $N_{x_{c,j}}$. These mode type dependent control variables are defined independently of the drivetrain topology. Which quantity is directly controlled by which CDV for all mode types is listed in Table 3.

One special case for which the number of continuous control variables does not match the actual number of kinematic and kinetic degrees of freedom is the series hybrid mode type ($\Psi = 7$). For a given rotational speed and torque at the wheels, this mode type has two degrees of freedom: the rotational speed of the ICE and the torque of the ICE. Assuming sufficient battery power and charge, both of these variables are however completely decoupled from the demands at the wheels. Let the charging efficiency η_{charge} be

defined as the efficiency of the conversion of chemical energy to electrical energy delivered by the generator:

$$\eta_{\text{charge}} = \frac{E_{\text{gen}}}{E_{\text{ice}}}, \quad (4)$$

where E_{ice} is the chemical energy from the fuel used by the ICE and E_{gen} is the electrical energy delivered by the generator. For each possible power output of the ICE, $P_{\text{ice}} = \tau_{\text{ice}} \cdot \omega_{\text{ice}}$, there is one combination of torque and speed that leads to the highest charging efficiency. To reduce the computational effort for control optimization, series hybrid mode types are implemented with one continuous control variable that determines P_{ice} . The corresponding torque and rotational speed of the ICE and generator are independent of the demands at the wheels and predetermined as a function of P_{ice} .

In addition to the series hybrid mode type, the three charging modes represent a second special case where the number of continuous control variables does not match the actual number of kinematic and kinetic degrees of freedom. For mode type $\Psi = 11$, the charging efficiency is again defined by Eq. 4. For mode types $\Psi = 12$ and $\Psi = 13$ the charging efficiency becomes

$$\eta_{\text{charge}} = \frac{E_{em1} + E_{em2}}{E_{\text{ice}}}, \quad (5)$$

where E_{em1} and E_{em2} are the electrical energies delivered by EM1 and EM2, respectively. As with the series hybrid modes, charging modes have one optimal combination of component torques and speeds for every value of P_{ice} . Again, the computational effort for control optimization is reduced by implementing only one continuous CDV that determines P_{ice} .

5 The generic transmission model

Using the mode type dependent control variables explained in the previous section, this section provides the equations that build the developed generic transmission model. The first contribution is the definition of all required transmission ratios. Subsequently, the component speeds and torques are defined using these ratios.

5.1 Transmission ratios

For fixed gear modes, transmission ratios i_j are defined. These ratios are used to determine rotational speeds and torques in modes of type $\Psi \in \{1, 2, 3, 5, 6\}$ as

$$i_j = \frac{\omega_j}{\omega_w} = \frac{\tau_w}{\tau_j}, \quad (6)$$

where subscript j indicates an arbitrary power source, and subscript w refers to the wheels of the vehicle. In this work, an EVT mode is defined as a mode where at least one component speed is variable for a given rotational speed of the wheels due to the application of a PGS. In EVT mode types $\Psi = 4$ and $\Psi = 8$ there is only one kinematic CDV, and all torques are determined by the torque demand at the wheels. For these mode types, a torque transmission ratio i_j^τ is defined as follows:

$$i_j^\tau = \frac{\tau_w}{\tau_j} \neq \frac{\omega_j}{\omega_w} \quad (7)$$

where superscript τ indicates that this ratio is a torque ratio. In EVT mode types, component speeds and torques may be a function of multiple other speeds and torques. To calculate these component speeds and torques, λ -vectors are defined. These λ -vectors are row vectors with a length of 2 or 3, depending on the mode type. As with i_j^τ , a superscript differentiates between speed vectors λ^ω and torque vectors λ^τ . Subscripts are used to indicate the concerning component. The rotational speed of the first EM for mode types $\Psi \in \{8, 9\}$, for example, is defined as

$$\omega_{em1} = \lambda_{em1}^\omega \cdot [\omega_w \ \omega_{ice}]^T, \quad (8)$$

which means that λ_{em1}^ω is a 1×2 vector, which is defined as

$$\lambda_{em1}^\omega = \omega_{em1} \cdot \begin{bmatrix} \omega_w \\ \omega_{ice} \end{bmatrix}^\dagger, \quad (9)$$

where $A^\dagger$ is the Moore-Penrose pseudoinverse of matrix A [11]. Next to vector λ_{em1}^ω , the following λ -vectors are also defined:

$$\lambda_{em1}^\tau = \tau_{em1} \cdot \begin{bmatrix} \tau_w \\ \tau_{ice} \end{bmatrix}^\dagger \quad (10)$$

$$\lambda_{em2}^\omega = \omega_{em2} \cdot \begin{bmatrix} \omega_w \\ \omega_{ice} \end{bmatrix}^\dagger \quad (11)$$

$$\lambda_{em2}^\tau = \tau_{em2} \cdot \begin{bmatrix} \tau_w \\ \tau_{ice} \end{bmatrix}^\dagger \quad (12)$$

$$\lambda_{em2, \text{ev}}^\omega = \omega_{em2} \cdot \begin{bmatrix} \omega_w \\ \omega_{em1} \end{bmatrix}^\dagger \quad (13)$$

$$\lambda_{em2, 3\text{cdv}}^\omega = \omega_{em2} \cdot \begin{bmatrix} \omega_w \\ \omega_{ice} \\ \omega_{em1} \end{bmatrix}^\dagger \quad (14)$$

Note that the last two λ -vectors differ from the others, and are required to enable the definition of the rotational speed of EM2 in mode types $\Psi = 4$ and $\Psi = 10$, respectively.

5.2 Component speeds and torques

The outputs of the developed generic transmission model are the rotational speeds and torques of all power sources. This section provides the actual equations that define these speeds and torques using the previous explained ratios and definitions. The rotational speed and torque of the ICE, ω_{ice} and τ_{ice} respectively, are defined as

$$\omega_{ice} = \begin{cases} \omega_w(\Lambda) \cdot i_{ice}(x_{c,1}), & \text{if } \Psi(x_{c,1}) \in \{1, 5, 6\} \\ 0, & \text{if } \Psi(x_{c,1}) \in \{2, 3, 4, 14\} \\ \omega_{ice}(P_{ice}(x_{c,2})), & \text{if } \Psi(x_{c,1}) \in \{7, 11, 12, 13\} \\ x_{c,2} \cdot \bar{\omega}_{ice}, & \text{if } \Psi(x_{c,1}) \in \{8, 9, 10\}, \end{cases} \quad (15)$$

and

$$\tau_{ice} = \begin{cases} \frac{\tau_w(\Lambda)}{i_{ice}(x_{c,1})}, & \text{if } \Psi(x_{c,1}) = 1 \\ 0, & \text{if } \Psi(x_{c,1}) \in \{2, 3, 4, 14\} \\ \underline{\tau}_{ice}(\omega_{ice}) + x_{c,2} \cdot (\bar{\tau}_{ice}(\omega_{ice}) - \underline{\tau}_{ice}(\omega_{ice})), & \text{if } \Psi(x_{c,1}) = 5 \\ \underline{\tau}_{ice}(\omega_{ice}) + x_{c,3} \cdot (\bar{\tau}_{ice}(\omega_{ice}) - \underline{\tau}_{ice}(\omega_{ice})), & \text{if } \Psi(x_{c,1}) \in \{6, 9, 10\} \\ \tau_{ice}(P_{ice}(x_{c,2})), & \text{if } \Psi(x_{c,1}) \in \{7, 11, 12, 13\} \\ \frac{\tau_w(\Lambda)}{i_{ice}(x_{c,1})}, & \text{if } \Psi(x_{c,1}) = 8, \end{cases} \quad (16)$$

with maximum ICE speed $\bar{\omega}_{ice}$, minimum ICE torque $\underline{\tau}_{ice}$, and maximum ICE torque $\bar{\tau}_{ice}$. Let Eqs. 15 and 16 be explained step by step. For mode type $\Psi = 1$, the first cases of Eqs. 15 and 16 relate the ICE speed and torque with i_{ice} to τ_w and ω_w , respectively, with τ_w and ω_w both depending on the drive cycle Λ . Both i_{ice} and Ψ are directly dependent on $x_{c,1}$, as this control variable dictates the transmission mode. The second cases of Eqs. 15 and 16 define τ_{ice} and ω_{ice} zero for electric and neutral mode types, in which neither propulsion nor charging is possible using the ICE. The third and fourth cases of Eq. 16 define τ_{ice} with $x_{c,2}$ for $\Psi = 5$, and $x_{c,3}$ for $\Psi \in \{6, 9, 10\}$, according to the definitions in Table 3. Here, τ_{ice} is controlled between $\underline{\tau}_{ice}$ and $\bar{\tau}_{ice}$ with $\underline{\tau}_{ice} < 0$, which enables the modeling of ICE drag torque. For fixed gear modes of types $\Psi = 5$ and $\Psi = 6$ the ICE speed is still defined by the first case of Eq. 15. The third case of Eq. 15 and the fifth case of Eq. 16 state that τ_{ice} and ω_{ice} are a function of P_{ice} for series hybrid and charge modes, with P_{ice} a function of $x_{c,2}$ as explained in Sect. 4. In mode types $\Psi \in \{8, 9, 10\}$ the ICE speed is a CDV

and controlled between zero and $\bar{\omega}_{ice}$ by $x_{c,2}$, as stated by the last case of Eq. 15. For mode type $\Psi = 8$ the last case of Eq. 16 states that τ_{ice} is determined by τ_w and torque transmission ratio i_{ice}^τ .

The rotational speed of EM1 is defined as

$$\omega_{em1} = \begin{cases} 0, & \text{if } \Psi(x_{c,1}) \in \{1, 14\} \\ \omega_w(\Lambda) \cdot i_{em1}(x_{c,1}), & \text{if } \Psi(x_{c,1}) \in \{2, 3, 5, 6\} \\ & \text{or } \Psi(x_{c,1}) = 7 \wedge i_{em1}(x_{c,1}) \neq 0 \\ \underline{\omega}_{em1} + x_{c,2} \cdot (\bar{\omega}_{em1} - \underline{\omega}_{em1}), & \text{if } \Psi(x_{c,1}) = 4 \\ \omega_{em1}(P_{ice}(x_{c,2})), & \text{if } \Psi(x_{c,1}) = 7 \wedge i_{em1}(x_{c,1}) = 0 \\ & \text{or } \Psi(x_{c,1}) \in \{11, 12, 13\} \\ \lambda_{em1}^\omega(x_{c,1}) \cdot [\omega_w(\Lambda) \cdot \omega_{ice}]^T, & \text{if } \Psi(x_{c,1}) \in \{8, 9\} \\ \underline{\omega}_{em1} + x_{c,4} \cdot (\bar{\omega}_{em1} - \underline{\omega}_{em1}), & \text{if } \Psi(x_{c,1}) = 10, \end{cases} \quad (17)$$

with minimum speed of EM1 $\underline{\omega}_{em1}$, and maximum speed of EM1 $\bar{\omega}_{em1}$. Under the assumption of $\underline{\omega}_{em1} = -\bar{\omega}_{em1}$, an uneven discretization of the continuous control variables allows for the control of $\omega_{em1} = 0$ in mode types $\Psi \in \{4, 10\}$. For series hybrid modes, $\Psi = 7$, it is arbitrary which of the EMs is the generator and which is the traction machine. Ratio i_{em1} is defined as zero when there is no mechanical connection between EM1 and the wheels. This property is employed by defining ω_{em1} separately for $i_{em1} = 0$ and $i_{em1} \neq 0$ in a series hybrid mode. Using this definition a differentiation can be made as to whether EM1 is the generator ($i_{em1} = 0$) or the traction motor ($i_{em1} \neq 0$) in a series hybrid mode. In modes of type $\Psi \in \{2, 5, 8\}$ only one EM can provide power for the propulsion of the vehicle. For topologies that contain two EMs, this could be either of the EMs while the other is mechanically not connected to the wheels. As $i_{em1} = 0$ and $\lambda_{em1}^\omega = 0$ when EM1 is not connected to the wheels, ω_{em1} inherently becomes zero

in these cases according to the definitions of the second and fifth cases in Eq. 17. The torque of EM1 is defined as

$$\tau_{em1} = \begin{cases} 0, & \text{if } \Psi(x_{c,1}) \in \{1, 14\} \\ & \text{or } \Psi(x_{c,1}) = \{2, 5\} \wedge i_{em1}(x_{c,1}) = 0 \\ & \text{or } \Psi(x_{c,1}) = \{3, 6\} \wedge x_{c,2} = 0 \\ & \text{or } \Psi(x_{c,1}) = 8 \wedge i_{em1}^{\tau}(x_{c,1}) = 0 \\ & \frac{\tau_w(\Lambda)}{i_{em1}(x_{c,1})}, \\ & \text{if } \Psi(x_{c,1}) \in \{2, 7\} \wedge i_{em1}(x_{c,2}) \neq 0 \\ & \tau_{em1}(\omega_{em1}) + a, \\ & \text{if } \Psi(x_{c,1}) \in \{3, 6\} \wedge x_{c,2} \neq 0 \\ & \frac{\tau_w(\Lambda)}{i_{em1}^{\tau}(x_{c,1})}, \\ & \text{if } \Psi(x_{c,1}) = 4 \\ & \text{or } \Psi(x_{c,1}) = 8 \wedge i_{em1}^{\tau}(x_{c,1}) \neq 0 \\ & \frac{\tau_w(\Lambda) - \tau_{ice} \cdot i_{ice}(x_{c,1})}{i_{em1}(x_{c,1})}, \\ & \text{if } \Psi(x_{c,1}) = 5 \wedge i_{em1}(x_{c,2}) \neq 0 \\ & \tau_{em1}(P_{ice}(x_{c,2})), \\ & \text{if } \Psi(x_{c,1}) = 7 \wedge i_{em1}(x_{c,1}) = 0 \\ & \text{or } \Psi(x_{c,1}) \in \{11, 12, 13\} \\ & \lambda_{em1}^{\tau}(x_{c,1}) \cdot [\tau_w(\Lambda) \quad \tau_{ice}]^T, \\ & \text{if } \Psi(x_{c,1}) \in \{9, 10\}, \end{cases} \quad (18)$$

with

$$a = (x_{c,2} \cdot N_{x_{c,2}} - 1) \frac{\bar{\tau}_{em1}(\omega_{em1}) - \underline{\tau}_{em1}(\omega_{em1})}{N_{x_{c,2}} - 1}, \quad (19)$$

and with minimum and maximum EM1 torques $\underline{\tau}_{em1}$ and $\bar{\tau}_{em1}$, respectively. In general, for EMs it can be stated that $\underline{\tau}_{em}(\omega_{em}) \neq -\bar{\tau}_{em}(\omega_{em})$. To still enable the control $\tau_{em1} = 0$, the value of τ_{em1} is defined separately for $x_{c,2} = 0$ in modes of type $\Psi \in \{3, 6\}$. The definition of a in Eq. 19 ensures that τ_{em1} can still be controlled over its full range by the other discrete values of $x_{c,2}$. In modes of type $\Psi \in \{2, 5, 8\}$ only one EM is mechanically connected to the wheels. For topologies with two EMs this could be either of the two, while the other is not connected to the output. As i_{em1} and i_{em1}^{τ} are zero when EM1 is not connected

Fig. 2 The single EM example (SEE) topology, made up for this exemplary comparison

to the wheels, τ_{em1} is explicitly defined as zero for the cases $\Psi \in \{2, 5\} \wedge i_{em1} = 0$ and $\Psi = 8 \wedge i_{em1}^{\tau} = 0$. A similar complication applies to the series hybrid mode type, $\Psi = 7$, where it is arbitrary which EM provides propulsion and which EM is connected to the ICE. Therefore, in this mode type τ_{em1} is also defined separately for $i_{em1} = 0$ and $i_{em1} \neq 0$.

The rotational speed of EM2, ω_{em2} , is defined as

$$\omega_{em2} = \begin{cases} 0, & \text{if } \Psi(x_{c,1}) \in \{1, 14\} \\ & \omega_w(\Lambda) \cdot i_{em2}(x_{c,1}), \\ & \text{if } \Psi(x_{c,1}) \in \{2, 3, 5, 6\} \\ & \text{or } \Psi(x_{c,1}) = 7 \wedge i_{em2}(x_{c,1}) \neq 0 \\ & \lambda_{em2, \text{ev}}^{\omega}(x_{c,1}) \cdot [\omega_w(\Lambda) \quad \omega_{em1}]^T, \\ & \text{if } \Psi(x_{c,1}) = 4 \\ & \omega_{em2}(P_{ice}(x_{c,2})), \\ & \text{if } \Psi(x_{c,1}) = 7 \wedge i_{em2}(x_{c,1}) = 0 \\ & \text{or } \Psi(x_{c,1}) \in \{11, 12, 13\} \\ & \lambda_{em2}^{\omega}(x_{c,1}) \cdot [\omega_w(\Lambda) \quad \omega_{ice}]^T, \\ & \text{if } \Psi(x_{c,1}) \in \{8, 9\} \\ & \lambda_{em2, 3 \text{cdv}}^{\omega}(x_{c,1}) \cdot [\omega_w(\Lambda) \quad \omega_{ice} \quad \omega_{em1}]^T, \\ & \text{if } \Psi(x_{c,1}) = 10. \end{cases} \quad (20)$$

This definition of ω_{em2} shows similarities with the definition of ω_{em1} . The two definitions differ the most for mode types $\Psi \in \{4, 10\}$, as ω_{em1} is determined by control variables while ω_{em2} is a function of other rotational speeds for these mode types. Finally, the torque of the second EM, τ_{em2} , is defined as

$$\tau_{em2} = \begin{cases} 0, & \text{if } \Psi(x_{c,1}) \in \{1, 14\} \\ & \text{or } \Psi(x_{c,1}) = \{2, 5\} \wedge i_{em2}(x_{c,1}) = 0 \\ & \text{or } \Psi(x_{c,1}) = 8 \wedge i_{em2}^{\tau}(x_{c,1}) = 0 \\ & \frac{\tau_w(\Lambda)}{i_{em2}(x_{c,1})}, \\ & \text{if } \Psi(x_{c,1}) \in \{2, 7\} \wedge i_{em2}(x_{c,2}) \neq 0 \\ & \frac{\tau_w(\Lambda) - \tau_{ice} \cdot i_{ice}(x_{c,1}) - \tau_{em1} \cdot i_{em1}(x_{c,1})}{i_{em2}(x_{c,1})}, \\ & \text{if } \Psi(x_{c,1}) \in \{3, 6\} \\ & \text{or } \Psi(x_{c,1}) = 5 \wedge i_{em2}(x_{c,1}) \neq 0 \\ & \frac{\tau_w(\Lambda)}{i_{em2}^{\tau}(x_{c,1})}, \\ & \text{if } \Psi(x_{c,1}) = 4 \\ & \text{or } \Psi(x_{c,1}) = 8 \wedge i_{em2}(x_{c,1}) \neq 0 \\ & \tau_{em2}(P_{ice}(x_{c,2})), \\ & \text{if } \Psi(x_{c,1}) = 7 \wedge i_{em2}(x_{c,1}) = 0 \\ & \text{or } \Psi(x_{c,1}) \in \{11, 12, 13\} \\ & \lambda_{em2}^{\tau}(x_{c,1}) \cdot [\tau_w(\Lambda) \quad \tau_{ice}]^T, \\ & \text{if } \Psi(x_{c,1}) \in \{9, 10\}. \end{cases} \quad (21)$$

Note that τ_{em2} can be defined by the same statement for electric mode type $\Psi = 3$ and hybrid mode type $\Psi = 6$, as τ_{ice} and ω_{ice} are always zero for $\Psi = 3$.

6 Practical application

Whereas the previous section explains the developed generic transmission model, this section provides a practical application of that model by showing the evaluation of complex drivetrain topologies over a driving cycle. The charge sustaining fuel consumptions of two topologies are compared in an exemplary case study. The goal of the case study is to demonstrate that the presented model can be used to evaluate the fuel consumption potential of arbitrary hybrid drivetrain topologies over a driving cycle.

6.1 Study description and assumptions

The presented study is an exemplary comparison of two distinct topologies. For this study, the authors derived an example topology from the two-mode presented in Sect. 3 by removing one EM and adding an additional clutch to enable the decoupling of the ICE. This single EM example (SEE) topology is displayed in Fig. 2.

The two changes to the two-mode topology result in a completely different set of transmission modes. Table 4 lists all transmission modes enabled by the two-mode topology and Table 5 lists all transmission modes of the SEE topology, with both tables including the corresponding clutch combinations.

In the exemplary study the charge sustaining fuel consumptions of the SEE topology and two-mode topology are compared for a full size luxury car using exemplary power sources. These power sources are a 100kW ICE, a 100kW EM, and a 20kW EM. For the two-mode topology, the 100kW EM is EM2, whereas for the SEE topology the 100kW EM is EM1. The fuel consumption of both topologies is determined by optimizing the control over the worldwide harmonized light vehicles test cycle (WLTC) using dynamic programming [12]. The application of dynamic programming leads to a non-heuristic control strategy that is globally optimal.

The focus of the comparison is to demonstrate the application of the presented model and to show the potential to evaluate arbitrary hybrid drivetrain topologies over a driving cycle, not the results of the comparison. The exemplary comparison is subject to a number of assumptions that influence the results. The assumptions include neglecting drivetrain inertias, appropriate design of the clutches and transmission losses. Also the procedures for transitions between operating modes are not considered. In this exemplary study, mode changes are not penalized. Yet by including the transmission modes as a state to the optimization problem, mode changes could be penalized using the same model. The weights of the distinct drivetrain systems are assumed to be equal. The changes to the two-mode topology might also influence functionalities as e.g.

Table 4 Transmission modes of the two-mode topology

Mode	Ψ : Mode type	C1	C2	B1	B2
1	5: Hybrid FG parallel 1 EM	o	•	•	o
2	6: Hybrid FG parallel 2 EMs	o	•	o	•
3	6: Hybrid FG parallel 2 EMs	•	o	o	•
4	6: Hybrid FG parallel 2 EMs	•	•	o	o
5	9: Hybrid EVT 2 EMs 2 CDVs	o	o	o	•
6	9: Hybrid EVT 2 EMs 2 CDVs	o	•	o	o
7	11: Charge FG 1 EM	o	o	•	o
8	12: Charge FG 2 EMs	•	o	o	o
9	13: Charge EVT 2 EMs	o	o	o	o

o Open, • Closed

Table 5 Transmission modes of the SEE topology

Mode	Ψ : Mode type	C1	C2	C3	B1	B2
1	2: EM only FG 1 EM	o	•	o	•	o
2	2: EM only FG 1 EM	•	o	o	o	•
3	2: EM only FG 1 EM	•	•	o	o	o
4	2: EM only FG 1 EM	•	o	o	•	o
5	5: Hybrid FG parallel 1 EM	o	•	•	•	o
6	5: Hybrid FG parallel 1 EM	•	o	•	•	o
7	5: Hybrid FG parallel 1 EM	•	•	•	o	o
8	5: Hybrid FG parallel 1 EM	•	o	•	o	•
9	8: Hybrid EVT 1 EM	o	o	•	•	o
10	8: Hybrid EVT 1 EM	•	o	•	o	o
11	11: Charge FG 1 EM	o	o	•	o	•
12	11: Charge FG 1 EM	o	•	•	o	o

o Open, • Closed

maximum electric speed, launch performance and behavior in battery charge boundaries, which is also neglected here as only an illustrative comparison is to be shown. Furthermore, fixed component sizes (i.e. maximum power and torque) are used, while they should be optimized for a fair comparison of distinct topologies. All these assumptions lead to the results not being comparable with real life fuel consumption. The generic transmission model can however be used in more detailed simulations and support in the concept phase of drivetrain development.

6.2 Results of the exemplary study

To summarize the results of the exemplary study, a table and a figure are provided. Table 6 shows the relative fuel usage, time the ICE is on, and the average efficiencies of the power sources.

The velocity profile and resulting battery state of charge (SOC) over time are shown by Fig. 3. The results show a 4% lower fuel consumption for the SEE topology, with reduced ICE on time and a clearly increased average efficiency of the 100kW EM. These effects can be explained by the presence of the 4 electric fixed gear modes of type $\Psi = 2$,

Table 6 Results of the topology comparison; relative fuel usage, time the ICE is on, and average efficiencies of the power sources over the driving cycle

Description	Unit	Two-mode	SEE
Fuel usage comparison	%	100	95.9
ICE on time	s	793	547
$\eta_{ice,brake}$	%	33.8	34.3
$\eta_{em,100kW}$	%	82.5	85.2
$\eta_{em,20kW}$	%	84.5	–

Fig. 3 Velocity and battery state of charge (SOC) over time **Fig. 4** Cumulative number of mode changes over time

which allow for more efficient electric driving with the 100kW EM and full electric driving at high speeds. The switching between these four electric transmission modes however leads to a higher number of mode changes, as shown in Fig. 4. The number of mode changes over the cycle should be considered in a topology comparison, as a high number of mode changes could reduce comfort. As mentioned before, mode changes are not penalized in the presented study.

7 Conclusion

A generic transmission model for all possible drivetrain topologies with a maximum of one ICE and two EMs is presented. For the defined library of components, all possi-

ble transmission mode types are identified. The developed model integrates the dynamic behavior of all conventional-, electric-, and hybrid-transmission mode types. A complete set of transmission ratios, including ratio vectors for EVT mode types, is defined to enable the formulation of the model. Equations that define the torques and speeds of all power sources, as a function of the torque and speed demand at the wheels and the control variables, are defined and explained. To demonstrate the application of the presented model to evaluate arbitrary hybrid drivetrain topologies over a driving cycle, the transmission modes and ratios of two complex DHT topologies are derived and used to optimize the control over a driving cycle with dynamic programming.

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