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RESEARCH ARTICLE

Network-turbo-coding-based cooperation with distributed space-time block codes

Bin Chen and Mark F. Flanagan*

School of Electrical, Electronic and Communications Engineering, University College Dublin, Ireland

ABSTRACT

In this paper, we propose a scheme to improve the performance of turbo-coded two-user cooperative uplink transmission. In the proposed scheme, each user transmits its partner's second parity frame in addition to its own, using a network-coded combination, which is further protected by a space-time block code. The proposed system is simulated in slow and fast fading environments. Numerical simulation results indicate that for slow fading, the proposed scheme can yield up to 20% increase in data throughput compared to the traditional distributed turbo coding, while simultaneously providing a gain of 0.7 dB in bit error rate. This latter gain rises to approximately 2.5 dB when a fast fading environment is considered. Furthermore, an outage probability analysis is conducted, which corroborates the simulated performance results. Copyright © 2014 John Wiley & Sons, Ltd.

*Correspondence

M. F. Flanagan, School of Electrical, Electronic and Communications Engineering, University College Dublin, Ireland.

E-mail: mark.flanagan@ieee.org

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1. INTRODUCTION

Diversity techniques offer an effective countermeasure against multipath fading by providing the receiver with different versions of the data-bearing signal transmitted over channels with independent channel gains [1]. Because of the limited size of mobile devices, user cooperation has been proposed to provide diversity gain; this is called *cooperative diversity* [2, 3]. Data are transmitted multiple times, from the source node and then from the partner acting as relay, thus achieving spatial diversity.

Furthermore, in *coded cooperation* [4], encoding operations for channel coding are distributed among cooperating nodes; this may provide coding gain as well as diversity gain. Based on this concept, distributed turbo codes and distributed low-density parity-check codes were studied in [5–8].

The idea of network coding was first proposed by Ahlswede *et al.* in [9] to enhance the capacity of wired networks. When extended to wireless communication channels, network coding can be used as a further approach to achieve diversity gain.

In wireless networks, channel coding is used to combat channel interference and noise, while network coding

is reserved as a technique to achieve diversity gain. Motivated by these goals, joint channel and network coding has been studied in different communication scenarios, such as the multiple-access relay channel [10–14], two-way relay channel [15] and the cooperative multiple-access channel (CMAC) [16]. The focus of this paper is on the two-user cooperative multiple-access channel, whereby two sources wish to transmit their own messages to the same destination. In contrast to the work of Wu *et al.* [16], which does not consider channel coding, the proposed scheme studies the combination of network coding and turbo-coded cooperation. As the network-coded cooperative transmission (used to transmit the second parity frames of both users) is the same for both users, a distributed space-time block code can be used to achieve transmit diversity for the third transmission phase. Note that a distributed STBC (DSTBC) was proposed in previous work as an effective way to protect cooperative parity transmission in the context of the relay channel [17].

The proposed scheme has a three-phase transmission protocol. During the first phase, the first user broadcasts its message and first parity frame; the signals are received by the second user as well as by the destination. In the second phase, the second user transmits likewise its message and first parity frame. In the third phase, each user decodes its partner's message and forms the corresponding second parity frame, which it then combines with its own second parity

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ity frame using a simple network code (XOR); both users then transmit this same network-coded packet using a distributed STBC. The STBC and network-coded structures efficiently protect the second parity transmission for use with iterative decoding at the destination. In our proposed decoding scheme, different network-channel decoders are used at the destination depending on the different cases regarding initial decoding results. Numerical simulation results show that the proposed scheme yields significant performance improvement when compared with distributed turbo coding (DTC) and non-cooperative schemes. Also, we provide an outage probability analysis that lies in close alignment with the observed simulation results for both slow and fast fading channels.

2. SYSTEM MODEL

We consider the cooperative communication model shown in Figure 1, where the two users S_1 and S_2 wish to transmit their own (independent) information to the same destination D .

We assume that all links in the systems suffer from quasi-static Rayleigh fading, where the fading coefficient on the link between transmitting node $i \in \{S_1, S_2\}$ and receiving node $j \in \{S_1, S_2, D\}$ (where $i \neq j$) during phase $k \in \{1, 2, 3\}$ is denoted by h_k^{ij} . This coefficient is circularly symmetric complex Gaussian with zero mean and variance σ_{ij}^2 , that is, $h_k^{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2)$. Also, the complex Gaussian random vector $\mathbf{n}_k^j$, with statistically independent entries $n_k^j[n] \sim \mathcal{CN}(0, N_0)$ models the additive white Gaussian noise with power spectral density N_0 at receiving node j during phase k .

Based on the assumption that all sources are half-duplex, a three-phase protocol is considered for the proposed user cooperation scheme. In the first phase, S_1 transmits its own $2N$ bits (N information bits $\mathbf{x}_1$ and N first parity bits $\mathbf{p}_1$) to S_2 and destination D . In the second phase, S_2 makes an analogous transmission, transmitting the $2N$ bits $\mathbf{x}_2$ and $\mathbf{p}_2$ to both S_1 and D . S_1 and S_2 each decode

Figure 1. Transmission protocol of the proposed cooperative scheme.

their partner's information bits using a Viterbi or BJCR decoder, after which each computes its partner's second parity frame and combines with its own second parity frame through network coding. In the third transmission phase, the two users share their antennas to create a virtual transmit array and use distributed space-time coding to transmit the network-coded second parity frame.

3. THE PROPOSED STRUCTURE AND DECODING SCHEME

3.1. Transmission structure

In this section, we provide details of the network-turbo-coded cooperative transmission scheme shown in Figure 2.

3.1.1. Network-turbo code encoding.

A distributed encoding scheme is employed at both users. The information vectors associated with S_1 and S_2 are denoted by $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. Recursive systematic convolutional (RSC) encoders with channel encoding operation $\Gamma(\cdot)$ are used, and each user S_i interleaves its information vector $\mathbf{x}_i$ using the interleaver $\Pi(\cdot)$. Thus, the encoding operations may be represented by

$$\mathbf{c}_i = [\mathbf{x}_i, \mathbf{p}_i], \mathbf{p}_i = \Gamma(\mathbf{x}_i), \mathbf{q}_i = \Gamma[\Pi(\mathbf{x}_i)] \quad i = 1, 2 \quad (1)$$

as illustrated in Figure 2. Finally, after receiving the partner's signals and decoding successfully, in the third phase, the two users generate the partner's second parity bits and combine these with its own second parity frame by network coding. A simple XOR operation is used, so that both users form the network-coded parity frame $\mathbf{c}_3 = \mathbf{q}_1 \oplus \mathbf{q}_2$. As we shall see in Section 3.2, the combination of $\mathbf{c}_1$, $\mathbf{c}_2$ and $\mathbf{c}_3$ has the structure of a distributed network-turbo code, which can be used to aid decoding. We assume binary phase-shift keying modulation, so that the modulated symbols are given by

$$\mathbf{u}_k = 2\mathbf{c}_k - 1, \quad k = 1, 2, 3 \quad (2)$$

3.1.2. First and second phase transmission.

In the first two transmission phases, each user transmits its own modulated symbols to partner and destination. Given that the vector of symbols transmitted during phase k is denoted by $\mathbf{u}_k$ and the vector of received signals at node j during phase k is denoted by $\mathbf{y}_k^j$, in the first phase, the received signals are

$$\mathbf{y}_1^{S_2} = h_1^{S_1 S_2} \cdot \mathbf{u}_1 + \mathbf{n}_1^{S_2} \quad ; \quad \mathbf{y}_1^D = h_1^{S_1 D} \cdot \mathbf{u}_1 + \mathbf{n}_1^D \quad (3)$$

and in the second phase, the received signals are

$$\mathbf{y}_2^{S_1} = h_2^{S_2 S_1} \cdot \mathbf{u}_2 + \mathbf{n}_2^{S_1} \quad ; \quad \mathbf{y}_2^D = h_2^{S_2 D} \cdot \mathbf{u}_2 + \mathbf{n}_2^D \quad (4)$$

Figure 2. Structure of the proposed network-turbo-coded cooperative scheme with distributed space-time block codes.

3.1.3. Third phase transmission.

Assuming that each user successfully decodes its partner's transmission, both users form the network-coded second parity frame $\mathbf{c}_3 = \mathbf{q}_1 \oplus \mathbf{q}_2$. Then, assuming that the two users are perfectly synchronised, this network-coded frame is transmitted to the destination using STBC. The antennas available at two users create a virtual two-antenna array. The Alamouti STBC is used, which has coding matrix

$$\mathbf{C} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \quad (5)$$

where $(\cdot)^*$ denotes the complex conjugate. The columns in this matrix represent the different time slots, while the rows represent the transmitted symbols from two users' antennas.

Thus, in the third phase, the signal streams transmitted from users S_1 and S_2 are given respectively by

$$\begin{aligned} \mathbf{v}_1 &= [u_3[1] - u_3^*[2] \quad u_3[3] - u_3^*[4] \quad \cdots]; \\ \mathbf{v}_2 &= [u_3[2] \quad u_3^*[1] \quad u_3[4] \quad u_3^*[3] \quad \cdots] \end{aligned} \quad (6)$$

and the received signal at the destination is

$$\mathbf{y}_3^D = h_3^{S_1D} \cdot \mathbf{v}_1 + h_3^{S_2D} \cdot \mathbf{v}_2 + \mathbf{n}_3^D \quad (7)$$

Because the transmission scheme may be viewed as a network-turbo code (NTC) whose second parity transmission is protected by a distributed STBC, we refer to our scheme as a DSTBC-NTC scheme.

3.2. Receiver structure

The destination receives the signals $\mathbf{y}_1^D$, $\mathbf{y}_2^D$ and $\mathbf{y}_3^D$, indicated in (3), (4) and (7) respectively, in the three phases of transmission. Before decoding, the vectors of *a priori* log-likelihood ratios of the received signals in first two phases, with entries $L_c(\mathbf{c}_k[n]) = \log(\Pr(\mathbf{c}_k[n] = 1 | \mathbf{y}_k^D[n]) / \Pr(\mathbf{c}_k[n] = 0 | \mathbf{y}_k^D[n]))$, can be calculated using

$$L_c(\mathbf{c}_k) = 4\Re\{ \mathbf{y}_k^D (h_k^{S_kD})^* \} / N_0 \quad k = 1, 2 \quad (8)$$

To compute the LLRs for the combined parity frame $\mathbf{c}_3$, we rewrite (7) as

$$\mathbf{y}_3^D[n] = \mathbf{H} \mathbf{u}_3[n] + \mathbf{n}_3[n] \quad (9)$$

for each n , where

$$\mathbf{y}_3^D[n] = \begin{bmatrix} y_3^D[2n-1] \\ (y_3^D)^*[2n] \end{bmatrix}; \quad \mathbf{u}_3[n] = \begin{bmatrix} u_3[2n-1] \\ u_3[2n] \end{bmatrix} \quad (10)$$

and

$$\mathbf{H} = \begin{bmatrix} h_3^{S_1D} & h_3^{S_2D} \\ (h_3^{S_2D})^* & -(h_3^{S_1D})^* \end{bmatrix}; \quad \mathbf{n}_3[n] = \begin{bmatrix} n_3[2n-1] \\ n_3^*[2n] \end{bmatrix} \quad (11)$$

The receiver then computes

$$\begin{aligned} \hat{\mathbf{y}}_3^D[n] &= \mathbf{H}^H \mathbf{y}_3^D[n] = \mathbf{H}^H (\mathbf{H} \mathbf{u}_3[n] + \mathbf{n}_3[n]) \\ &= E_h \mathbf{u}_3[n] + \mathbf{H}^H \mathbf{n}_3[n] \end{aligned} \quad (12)$$

Figure 3. Complete iterative decoder structure at the destination.

Figure 4. Network-turbo decoder.

where $E_h = |h_3^{S_1 D}|^2 + |h_3^{S_2 D}|^2$ and $\mathbf{H}^H \mathbf{n}_3 \sim \mathcal{CN}(\mathbf{0}, E_h N_0 \mathbf{I})$. It is then straightforward to show that

$$L_c \left(\begin{bmatrix} c_3[2n-1] \\ c_3[2n] \end{bmatrix} \right) = \frac{4\Re\{\hat{\mathbf{y}}_3^D[n]\}}{N_0} \quad (13)$$

In Figures 3–5, the symbol L_c indicates this process of *a priori* LLR calculation.

These *a priori* LLRs form the soft inputs to the soft-in soft-out (SISO) decoders and the network-turbo decoder. The complete decoding structure at the destination is shown in Figure 3; for some decoding situations, only a subset of this hardware will be required.

Upon receiving the signals $\mathbf{y}_1^D$ and $\mathbf{y}_2^D$ in the first two phases, the destination will first attempt to decode $\mathbf{x}_1$ and $\mathbf{x}_2$ using the SISO decoder for the RSC code, which is implemented by the log-MAP Algorithm [18].

Meanwhile, the message $\mathbf{x}_1 \oplus \mathbf{x}_2$ can be decoded by the network-turbo decoder. Using the linearity property of channel coding, interleaving and network coding (XOR)[†], we may write [19]

$$\begin{aligned} \Gamma(\mathbf{x}_1 \oplus \mathbf{x}_2) &= \Gamma(\mathbf{x}_1) \oplus \Gamma(\mathbf{x}_2) = \mathbf{p}_1 \oplus \mathbf{p}_2; \\ \Gamma[\Pi(\mathbf{x}_1 \oplus \mathbf{x}_2)] &= \Gamma[\Pi(\mathbf{x}_1)] \oplus \Gamma[\Pi(\mathbf{x}_2)] = \mathbf{q}_1 \oplus \mathbf{q}_2 \end{aligned} \quad (14)$$

Thus, $L(\mathbf{x}_1 \oplus \mathbf{x}_2)$, $L(\mathbf{p}_1 \oplus \mathbf{p}_2)$ and $L(\mathbf{q}_1 \oplus \mathbf{q}_2)$ may be seen to constitute LLRs for decoding of an ordinary turbo code, and they can therefore be used to decode $\mathbf{x}_1 \oplus \mathbf{x}_2$. The result of this decoding may then be used to aid the next iteration of decoding for each user's message.

Figure 4 shows the network-turbo decoding process, $L(\mathbf{q}_1 \oplus \mathbf{q}_2)$ being derived from STBC decoding of the third transmission phase. Also, $L(\mathbf{x}_1 \oplus \mathbf{x}_2)$ and $L(\mathbf{p}_1 \oplus \mathbf{p}_2)$ may be calculated by using log-likelihood algebra, according to

$$\begin{aligned} L(A \oplus B) &= L(A) \boxplus L(B) \\ &= \ln \left[\frac{\exp(L(A)) + \exp(L(B))}{1 + \exp(L(A) + L(B))} \right] \\ &\approx -\text{sign}[L(A)] \cdot \text{sign}[L(B)] \cdot \min(|L(A)|, |L(B)|) \end{aligned} \quad (15)$$

[†]Note that in order to exploit this linearity property, both users must employ the same interleaver $\Pi(\cdot)$.

Table I. Different decoding cases.

	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_1 \oplus \mathbf{x}_2$
Case 1	1	1	X
Case 2	1	0	X
Case 3	0	1	X
Case 4	0	0	0

Initially, the destination makes hard decisions based on $L(\mathbf{x}_1)$, $L(\mathbf{x}_2)$ and $L(\mathbf{x}_1 \oplus \mathbf{x}_2)$, these decoding results being output by the two SISO decoders and the network-turbo decoder; these hard decisions are denoted by $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_1 \oplus \mathbf{x}_2$, respectively. The decoding process may then be divided into several cases as shown in Table I. A cyclic redundancy check is used for all error detection. In Table I, '1' indicates successful decoding (the decoded frame passes the cyclic redundancy check), '0' indicates unsuccessful decoding and 'X' represents either case (1 or 0).

Case 1: When both of the messages $\mathbf{x}_1$ and $\mathbf{x}_2$ can be decoded correctly at the destination, no further operations are necessary. In this case, the additional combined parity and further decoding process are not required.

Case 2: When only one of the messages $\mathbf{x}_1$ and $\mathbf{x}_2$ can be decoded correctly, $\mathbf{y}_3^D$ will be used to drive the iterative turbo decoding process for the partner's message. Without loss of generality, we assume that $\mathbf{x}_1$ is correctly decoded and that $\mathbf{x}_2$ is not. For this case, the decoder structure at the destination is described in Figure 5. The main idea here is to remove the knowledge of the parity frame $\mathbf{q}_1$ from $\mathbf{y}_3^D$. The decision on $\mathbf{x}_1$ is interleaved and then re-encoded by an RSC encoder to generate the second parity frame $\mathbf{q}_1$. *A priori* information on $\mathbf{q}_2$ may then be calculated according to

$$L(\mathbf{q}_2) = (-1)^{\mathbf{q}_1} \cdot L(\mathbf{q}_1 \oplus \mathbf{q}_2) \quad (16)$$

which is a limiting case of (15). The message $\mathbf{x}_2$ may then be decoded by the more powerful turbo decoder.

Case 3: When both of the messages $\mathbf{x}_1$ and $\mathbf{x}_2$ fail to be decoded, but the decoding of $\mathbf{x}_1 \oplus \mathbf{x}_2$ is successful, the iterative decoder structure operates as depicted in Figure 3. Iterative decoding begins at the SISO decoder with the more 'reliable' *a priori* inputs; this is performed

Figure 5. Decoder structure for Case 2.

by choosing the SISO whose LLRs have the larger average absolute value. The two SISO decoders then exchange extrinsic information; the output of each SISO module, which contains LLR-based information about one user's data frame, is passed through a boxplus operation. The boxplus operates according to

$$\begin{aligned} L^p(\mathbf{x}_1) &= (-1)^{\mathbf{x}_1 \oplus \mathbf{x}_2} \cdot L(\mathbf{x}_2); \\ L^p(\mathbf{x}_2) &= (-1)^{\mathbf{x}_1 \oplus \mathbf{x}_2} \cdot L(\mathbf{x}_1) \end{aligned} \quad (17)$$

using the turbo-network decoder output $\mathbf{x}_1 \oplus \mathbf{x}_2$. This process converts each output LLR vector into information on the other user's data, which is then fed as *a priori* information (denoted by $L^p(\mathbf{x}_1)$ and $L^p(\mathbf{x}_2)$ in Figure 3) into the other SISO module.

Case 4: When all three messages $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_1 \oplus \mathbf{x}_2$ fail to be decoded, the decoding structure of Figure 3 will again be used. However, decoding proceeds differently to Case 3 in that the LLRs for $\mathbf{x}_1 \oplus \mathbf{x}_2$ are used instead of the corresponding hard decisions. In this case, all three transmission phases meet with deep fading. Note that poor quality of soft information from the network-turbo decoder (XOR message) can inhibit decoding convergence for this case.

3.3. Different interuser channel situation

The aforementioned description is based on the assumption of a perfect channel between users, but in practice, this assumption will not hold. According to the interuser channel conditions, we consider four possible situations for transmission in the third phase. In Situation 1, both users decode each other's messages correctly. In Situation 2, both users fail to decode their partner's message. In Situation 3, only user S_2 decodes its partner's message successfully, and the converse of this comprises Situation 4. For each of these situations, a different transmission scheme is adopted for each user as shown in Table II (a similar comparison of conditional transmission strategies was also considered in [16]). Note that in the proposed DSTBC-

NTC scheme, the Alamouti STBC is used whenever the two users transmit the same message in the third phase (Situations 1, 3 and 4); otherwise, the two users sequentially transmit their punctured second parity frames (Situation 2). This ensures usage of the same number of time slots for transmission, irrespective of the situation which arises.

4. OUTAGE PROBABILITY ANALYSIS

In this section, we provide an outage probability analysis for the proposed cooperation communication scheme. For a single fading link, the capacity can be expressed using Shannon's formula $C(\gamma) = \log_2(1 + \gamma)$ bits/s/Hz, where γ denotes the link signal-to-noise ratio (SNR). An outage event occurs when the capacity falls below the desired rate R . The corresponding *outage event* can be expressed as $\{C(\gamma) < R\}$. Then the *outage probability* is

$$P_o = \Pr(\gamma < g(R)) = \int_0^{g(R)} p_\gamma(\gamma) d\gamma \quad (18)$$

where $g(R) = 2^R - 1$ and $p_\gamma(\gamma)$ denote the probability density function of the random variable γ .

For the case of Rayleigh fading, γ has an exponential probability density function with the parameter $1/\Gamma$, where Γ indicates the mean value of the SNR. The outage probability for the Rayleigh fading channel can therefore be evaluated as

$$P_o = \int_0^{g(R)} \frac{1}{\Gamma} \exp\left(-\frac{\gamma}{\Gamma}\right) d\gamma = 1 - \exp\left(-\frac{2^R - 1}{\Gamma}\right) \quad (19)$$

In this paper, we consider a *per-user outage event*. An outage event for User 1 occurs when the message of User 1 cannot be decoded at the destination. Because of the symmetrical nature of our transmission system, we may derive the outage probability expression for User 1 only, as the expression for User 2 is identical.

Recall that we assume all links in the system suffer from Rayleigh fading and also that the fading coefficient on the link between transmitting node $i \in \{S_1, S_2\}$ and receiving node $j \in \{S_1, S_2, D\}$ (where $i \neq j$) during phase $k \in \{1, 2, 3\}$ is denoted by h_k^{ij} . Also, we consider binary phase-shift keying transmission over both slow and fast fading channel models. In the slow fading model, h_k^{ij} is constant for one frame, then it varies independently from one frame (consisting of the three transmission phases) to the next; therefore, $\gamma_1^{S_1D} = \gamma_3^{S_1D}$ and $\gamma_2^{S_2D} = \gamma_3^{S_2D}$. In the fast fading model, h_k^{ij} varies from one transmission phase to the next, and the four different variables $\gamma_1^{S_1D}, \gamma_2^{S_2D}, \gamma_3^{S_1D}, \gamma_3^{S_2D}$ will therefore also be statistically independent.

According to the normal Turbo encoding method, an overall rate $R = 1/3$ is used in the system. Thus, there is a total of $3N$ total coded bits per frame. In the first and second phases, each user transmits a rate $R_1 = 1/2$

Table II. The different transmission strategies in phase 3.

Situation	1	2	3	4
DTC				
S1	$\mathbf{q}_2$	$\mathbf{q}_1$	$\mathbf{q}_1$	$\mathbf{q}_2$
S2	$\mathbf{q}_1$	$\mathbf{q}_2$	$\mathbf{q}_1$	$\mathbf{q}_2$
DSTBC-NTC				
S1	$\mathbf{q}_1 \oplus \mathbf{q}_2$	$\mathbf{q}_1(1:2:N)$	$\mathbf{q}_1$	$\mathbf{q}_2$
S2	$\mathbf{q}_1 \oplus \mathbf{q}_2$	$\mathbf{q}_2(1:2:N)$	$\mathbf{q}_1$	$\mathbf{q}_2$

DTC, distributed turbo coding; DSTBC, distributed space-time block codes; NTC, network-turbo code.

The notation $\mathbf{q}_i(1:2:N)$ represents the parity frame $\mathbf{q}_i$ with every second bit punctured. Note that in the proposed DSTBC-NTC scheme, the STBC is only implemented in Situations 1, 3 and 4.

$$\begin{aligned}
A &\equiv \left\{ (\gamma^{S_1 D}, \gamma^{S_2 D}) : (1 + \gamma^{S_1 D})^\alpha (1 + \gamma^{S_1 D} + \gamma^{S_2 D})^{1-\alpha} < 2^R \cap (1 + \gamma^{S_2 D})^\alpha > 2^R \right\} \\
B &\equiv \left\{ (\gamma^{S_1 D}, \gamma^{S_2 D}) : (1 + \gamma^{S_1 D})^\alpha (1 + \gamma^{S_1 D} + \gamma^{S_2 D})^{1-\alpha} > 2^R \cap (1 + \gamma^{S_2 D})^\alpha (1 + \gamma^{S_1 D} + \gamma^{S_2 D})^{1-\alpha} \right. \\
&\quad \left. > 2^R \cap (1 + \gamma^{S_1 D} + \gamma^{S_2 D})^\alpha < 2^R \right\}
\end{aligned} \tag{23}$$

$$\begin{aligned}
C &\equiv \left\{ (\gamma^{S_1 D}, \gamma^{S_2 D}) : (1 + \gamma^{S_1 D})^\alpha (1 + \gamma^{S_1 D} + \gamma^{S_2 D})^{1-\alpha} < 2^R \cup (1 + \gamma^{S_2 D})^\alpha (1 + \gamma^{S_1 D} + \gamma^{S_2 D})^{1-\alpha} \right. \\
&\quad \left. < 2^R \cap (1 + \gamma^{S_1 D})^\alpha < 2^R \cap (1 + \gamma^{S_2 D})^\alpha < 2^R \right\} \\
\Phi(\gamma^{S_1 D}, \gamma^{S_2 D}) &= \frac{1}{\Gamma^{S_1 D}} \exp\left(-\frac{\gamma^{S_1 D}}{\Gamma^{S_1 D}}\right) \frac{1}{\Gamma^{S_2 D}} \exp\left(-\frac{\gamma^{S_2 D}}{\Gamma^{S_2 D}}\right)
\end{aligned} \tag{24}$$

codeword with $2N$ bits. N additional parity bits that are encoded by network coding will be transmitted in the third phase. If we define

$$\alpha = 2N/3N = R/R_1 = 2/3 \tag{20}$$

then $1 - \alpha$ indicates the fraction of each user's bits, which is allocated for the third phase (we call this the *level of cooperation*).

In the first two phases, S_1 and S_2 transmit a rate- $1/2$ codeword to its partner and destination. The $S_i - D$ link outage probability ($i \in \{1, 2\}$) is given by

$$\begin{aligned}
P_o^{S_i D} &= Pr(C^{S_i D} < R/\alpha) \\
&= Pr[\log_2(1 + \gamma^{S_i D}) < R/\alpha]
\end{aligned} \tag{21}$$

In the third phase, after decoding its partner's message successfully and encoding by using joint channel-network coding, S_1 and S_2 will transmit the combined additional parity frame using the Alamouti STBC. The channel capacity for the Alamouti STBC transmission is described by [20]:

$$C^{STBC} = \log_2(1 + \gamma_3^{S_1 D} + \gamma_3^{S_2 D}) \tag{22}$$

4.1. Slow fading

Under the slow fading channel model, h_k^{ij} changes from one block to next block independently, thus $\gamma_1^{S_1 D} = \gamma_3^{S_1 D}$, $\gamma_2^{S_2 D} = \gamma_3^{S_2 D}$. In order to simplify the calculation, the subscript k , which indicates that the transmission phase has been omitted in this section.

Before we analyse the overall outage probability for the case of slow Rayleigh fading, we present an approximate analysis, which is valid in the range of medium-to-high SNR. This simplified analysis will serve to facilitate a prediction of the diversity order of the proposed scheme and its competitors.

For the slow Rayleigh fading channel, the outage probability for the link connecting nodes S_i ($i \in \{1, 2\}$) and D is given by

$$P_e = Pr\left\{\left|h_k^{S_i D}\right| < g\right\} \approx g \tag{25}$$

where $g = \frac{2^{R/\alpha}-1}{\Gamma}$ and Γ indicate the mean value of the link SNR. We also assume that the interuser channels have a very high SNR, so that neither is in outage. The destination receives the three packets $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$. Without loss of generality, we consider the performance of User 1. The frame $\mathbf{x}_1$ cannot be decoded by destination precisely when the direct transmission of $\mathbf{u}_1$, and at least one of the packets $\mathbf{u}_2$ and $\mathbf{u}_3$, cannot be recovered by the destination. In this case, the outage probability for User 1 is

$$P_o \approx P_e(P_e + P_t) \approx P_e^2 \tag{26}$$

where P_t is the probability that the destination correctly decodes the network-coded (XOR) message. Because the packet $\mathbf{u}_3$ is transmitted using STBC, $P_t \approx 0.5P_e^2$.

A more precise analysis is given in the following text. In the previous section, we considered four cases based on the decoding conditions as shown in Table I. The corresponding conditional capacities and outage events for each case may be analysed as follows:

Case 1: When the $S_1 D$ link is not in outage, the destination has no need of the additional parity bits following the first two phases. In this case, the outage probability is $P(1) = 0$ for User 1.

Case 2: When exactly one of the links $S_i D$ is in outage, the extra parity from the STBC decoder can be combined with the codeword from the link $S_j D$ that is not in outage.

Figure 6. Integral region for slow fading.

In this case, given that S_1D link is in outage and S_2D is not in outage, this corresponds to the events

$$\begin{aligned} C^{S_1D} &= \alpha \log_2(1 + \gamma^{S_1D}) < R \\ C^{S_2D} &= \alpha \log_2(1 + \gamma^{S_2D}) > R \end{aligned} \quad (27)$$

Then, we can write the outage event for User 1 as

$$\begin{aligned} C(\gamma^{S_1D}, \gamma^{S_2D} | \text{Case 2}) &= \alpha \log_2(1 + \gamma^{S_1D}) \\ &+ (1 - \alpha) \log_2(1 + \gamma^{S_1D} + \gamma^{S_2D}) < R \end{aligned} \quad (28)$$

The outage probability under Case 2 is then

$$P(2) = \iint_A \Phi(\gamma^{S_1D}, \gamma^{S_2D}) d\gamma^{S_1D} d\gamma^{S_2D} \quad (29)$$

where A is the integral region given in (23).

Note that the integration region A contains the region for which the S_1D link is in outage; thus in specifying A , we may omit the constraint $(1 + \gamma^{S_1D})^\alpha < 2^R$. The graphical representation of A is given in Figure 6 (green).

Note on Cases 3 and 4: When both S-D links are in outage, the destination first tries to decode the network-coded (XOR) message. As the turbo code structure can be constructed from the combined codeword's LLR received during the first two phases and the XOR parity received during the third phase, the event of successful decoding of the XOR message may be described by:

$$\alpha \cdot \min(C^{S_1D}, C^{S_2D}) + (1 - \alpha)C^{STBC} \geq R \quad (30)$$

The $\min(\cdot)$ operation requires both S-D links to be good enough (because of the characteristic of XOR message decoding). Similar considerations have been used in [21].

Case 3: When both S-D links are in outage and XOR decoding is successful, the two SISO decoders can exchange extrinsic information directly. This corresponds to the events

$$\begin{aligned} C^{S_1D} &= \alpha \log_2(1 + \gamma^{S_1D}) < R \\ C^{S_2D} &= \alpha \log_2(1 + \gamma^{S_2D}) < R \\ \alpha \cdot \min(C^{S_1D}, C^{S_2D}) + (1 - \alpha)C^{STBC} &\geq R \end{aligned} \quad (31)$$

In this case, the capacity calculation is facilitated by noting that the availability of channel-derived LLRs regarding one of the frames $\mathbf{x}_1, \mathbf{x}_2$ is equivalent to availability of the same quality LLRs on the other frame, once the network-coded combination $\mathbf{x}_1 \oplus \mathbf{x}_2$ is known *exactly*. Thus, the corresponding outage event is

$$\begin{aligned} C(\gamma^{S_1D}, \gamma^{S_2D} | \text{Case 3}) \\ = \alpha \log_2(1 + \gamma^{S_1D} + \gamma^{S_2D}) < R \end{aligned} \quad (32)$$

The outage probability under Case 3 is then

$$P(3) = \iint_B \Phi(\gamma^{S_1D}, \gamma^{S_2D}) d\gamma^{S_1D} d\gamma^{S_2D} \quad (33)$$

where the graphical representation of the overall integration region B for Case 3 is given in Figure 6 (blue).

Case 4: When both S-D links are in outage and XOR decoding is unsuccessful, all three links are in outage. Thus, for this case, we have the events

$$\begin{aligned} C^{S_1D} &= \alpha \log_2(1 + \gamma^{S_1D}) < R \\ C^{S_2D} &= \alpha \log_2(1 + \gamma^{S_2D}) < R \\ \alpha \cdot \max(C^{S_1D}, C^{S_2D}) + (1 - \alpha)C^{STBC} &\leq R \end{aligned} \quad (34)$$

It follows that the cooperation channel will be always in outage for this case. Therefore, the outage probability for User 1 is

$$P(4) = \iint_C \Phi(\gamma^{S_1D}, \gamma^{S_2D}) d\gamma^{S_1D} d\gamma^{S_2D} \quad (35)$$

where the graphical representation of the overall integration region C for Case 4 is given in Figure 6 (red).

Summarising the aforementioned analysis, the overall probability of the proposed scheme is given by

$$P_o = P(1) + P(2) + P(3) + P(4) \quad (36)$$

where each term in the aforementioned text may be calculated using numerical integration.

4.2. Fast fading

In contrast to the previous analysis, for the fast fading channel, we need to consider the four independent integration variables $\{\gamma_1^{S_1D}, \gamma_2^{S_2D}, \gamma_3^{S_1D}, \gamma_3^{S_2D}\}$ in three independent phases. Thus, we need to rewrite the calculation process and integral region in (37) and (38). In (37), the probability density function $\Phi(\gamma_1^{S_1D}, \gamma_2^{S_2D}, \gamma_3^{S_1D}, \gamma_3^{S_2D})$ is the natural four-dimensional analogue of the two-dimensional (24).

$$\begin{aligned}
P(2) &= \iiint\limits_D \Phi(\gamma_1^{S_1D}, \gamma_2^{S_2D}, \gamma_3^{S_1D}, \gamma_3^{S_2D}) d\gamma_1^{S_1D} d\gamma_2^{S_2D} d\gamma_3^{S_1D} d\gamma_3^{S_2D} \\
P(3) &= \iiint\limits_E \Phi(\gamma_1^{S_1D}, \gamma_2^{S_2D}, \gamma_3^{S_1D}, \gamma_3^{S_2D}) d\gamma_1^{S_1D} d\gamma_2^{S_2D} d\gamma_3^{S_1D} d\gamma_3^{S_2D} \\
P(4) &= \iiint\limits_F \Phi(\gamma_1^{S_1D}, \gamma_2^{S_2D}, \gamma_3^{S_1D}, \gamma_3^{S_2D}) d\gamma_1^{S_1D} d\gamma_2^{S_2D} d\gamma_3^{S_1D} d\gamma_3^{S_2D}
\end{aligned} \tag{37}$$

$$\begin{aligned}
D &\equiv \left\{ (\gamma_1^{S_1D}, \gamma_2^{S_2D}, \gamma_3^{S_1D}, \gamma_3^{S_2D}) : (1 + \gamma_1^{S_1D})^\alpha (1 + \gamma_3^{S_1D} + \gamma_3^{S_2D})^{1-\alpha} < 2^R \cap (1 + \gamma_2^{S_2D})^\alpha > 2^R \right\} \\
E &\equiv \left\{ (\gamma_1^{S_1D}, \gamma_2^{S_2D}, \gamma_3^{S_1D}, \gamma_3^{S_2D}) : (1 + \gamma_3^{S_1D} + \gamma_3^{S_2D})^\alpha < 2^R \right\} \cap \left\{ (1 + \gamma_1^{S_1D})^\alpha < 2^R \cap (1 + \gamma_2^{S_2D})^\alpha < 2^R \right. \\
&\quad \left. \cap (1 + \gamma_1^{S_1D})^\alpha (1 + \gamma_3^{S_1D} + \gamma_3^{S_2D})^{1-\alpha} > 2^R \cap (1 + \gamma_2^{S_2D})^\alpha (1 + \gamma_3^{S_1D} + \gamma_3^{S_2D})^{1-\alpha} > 2^R \right\} \\
F &\equiv \left\{ (\gamma_1^{S_1D}, \gamma_2^{S_2D}, \gamma_3^{S_1D}, \gamma_3^{S_2D}) : (1 + \gamma_1^{S_1D})^\alpha < 2^R \cap (1 + \gamma_2^{S_2D})^\alpha < 2^R \right. \\
&\quad \left. \cap (1 + \gamma_1^{S_1D})^\alpha (1 + \gamma_3^{S_1D} + \gamma_3^{S_2D})^{1-\alpha} < 2^R \cup (1 + \gamma_2^{S_2D})^\alpha (1 + \gamma_3^{S_1D} + \gamma_3^{S_2D})^{1-\alpha} < 2^R \right\}
\end{aligned} \tag{38}$$

5. SIMULATIONS AND DISCUSSION

In this section, simulation results are presented for the proposed DSTBC-NTC-based cooperative scheme and are compared with three reference schemes: the direct transmit turbo-coded (DT-TC) scheme, where users do not cooperate but each transmits its own turbo-coded data; and two DTC schemes (one without puncturing and one with every second bit punctured in the second parity frame).

We first present outage probability results for the proposed scheme. Due to the assumption of an equal mean SNR ($\Gamma^{S_1D} = \Gamma^{S_2D}$) for both users and destination (uplink channels), the outage probabilities for both users are equal. All of the results in Figures 7 and 8 clearly show that the cooperative schemes achieve diversity order two, compared with the DT (non-cooperative) scheme which has diversity order one. Moreover, our scheme can achieve a better performance than the conventional DTC scheme in both slow and fast fading channels. Note that as can be seen from Figure 7, the outage probability approximation given by (26) is in good agreement with the actual outage probability at medium-to-high SNR.

Next, the different schemes are compared from the perspective of throughput as well as bit error rate (BER) performance versus E_b/N_0 via Monte Carlo simulations. In these simulations, each original message frame consists of $N = 500$ bits. The turbo encoder for each user consists of two RSC encoders with a constraint length of 3, each with generator matrix $G(D) = [1 \ (1 + D + D^2)/(1 + D^2)]$. A pseudorandom interleaver is used in generating the second parity frame for the turbo codes.

Considering the throughput, we assume signalling occurs at a constant rate in the system, where the transmission time for N bits is T seconds. After channel encoding, the transmission of one RSC-encoded message costs $2T$ seconds and the transmission of one turbo-coded message costs $3T$ seconds. Table III compares the required time for both users' transmission to destination for the different

Figure 7. Slow fading.

Figure 8. Fast fading.

Table III. Transmission time for different schemes.

Tx Time	DT-TC	DTC		DSTBC-NTC
		w/ punc.	w/o punc.	
	6T	5T	6T	5T

DTC, distributed turbo coding; DSTBC, distributed space-time block codes; NTC, network-turbo code; DT-TC, direct transmit turbo-coded.

schemes. In network-coded cooperation mode (Situation 1 described in Section 3.3), the proposed scheme costs only 5T seconds to transmit the users' messages due to the efficiency of using STBC and network coding. It is more efficient than the direct transmit turbo-coded scheme and DTC without puncturing, which both cost 6T seconds for transmission; this yields an increase in throughput of up to 20%.

Figure 9 depicts the comparison of the average throughput for the different schemes. We use throughput to indicate the average number of correct received data packets in five-time slots for one user. It can be seen that the throughput of the proposed scheme is significantly improved.

In the following text, we focus on the BER results for simulated performance over the Rayleigh fading channel. Also, we consider two quasi-static fading scenarios, as follows. First, we consider performance over *slow fading*, that is, where the fading coefficient h_k^{ij} on any link remains constant for the entire duration of each of the three phases $k \in \{1, 2, 3\}$ of transmission (and changes independently from one phase to the next). Figure 10 compares the BER performance of the proposed scheme with that of the other two reference schemes. It can be seen that the two cooperative transmission schemes achieve a great diversity gain than the non-cooperative scheme. A further improvement

Figure 9. Throughput performance comparison of the different schemes. Slow fading is assumed as well as a perfect interuser channel. **Figure 10.** Bit error rate performance comparison of the different schemes. Slow fading is assumed as well as a perfect interuser channel. **Figure 11.** Bit error rate performance comparison for different interuser channel conditions (slow fading). The variable interuser channel means the interuser channel have the same signal-to-noise ratio with uplink channels.

of 0.7 dB can be achieved by the proposed scheme at a BER of 10^{-3} .

Figure 11 depicts the comparison for different interuser channel conditions. It may be seen that the performance of the proposed scheme is improved as the SNR of the interuser channel increases and that it also performs better than the DTC scheme for all simulated interuser channel conditions.

Next, we consider performance over *fast fading*, that is, where the fading coefficient h_k^{ij} on any link changes independently from one transmission phase to the next (remaining constant during each phase). Thus, the network-turbo decoder will receive the signals from four different chan-

nels, and the performance of iterative decoding can be improved. As shown in Figure 12, the proposed scheme yields approximately 2.5 dB gain over DTC at a BER of 10^{-3} , while maintaining the previously mentioned throughput advantage.

Figure 13 depicts the probability of the four different decoding cases mentioned in Section 3.2 (here, a perfect interuser channel is assumed). It may be seen that Cases 1 and 2 are the dominant events when the uplink SNR is higher than 4 dB. Accordingly, Figure 12 shows that the proposed scheme begins to show better performance than DTC at approximately the same uplink SNR. Because all compared schemes can achieve error-free decoding in Case 1, we conclude that the high probability of Case 2 is responsible for the improved performance for the proposed scheme.

Figure 12. Bit error rate performance comparison of the different schemes (fast fading).

Figure 13. The probability of the four different decoding cases (fast fading).

6. CONCLUSION

In this paper, we proposed a DSTBC-NTC scheme in which a distributed NTC is used for transmission using a three-phase protocol and where a distributed STBC is used to protect the combined second parity transmission. We derive expressions for outage probability in Rayleigh fading and two user cooperation model. The results of numerical simulations show that the proposed scheme provides a modest gain over the use of a distributed turbo code while offering a significant throughput advantage. Furthermore, an outage probability analysis has been conducted, producing FER (frame error rate) bounds that strongly support simulated performance results.

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