After the recent breakthrough to Bose-Einstein condensation (BEC) in dilute ultracold atomic gases of $^7\text{Li}$, $^{23}\text{Na}$, and $^{87}\text{Rb}$ [1–3], it has become a priority to try to understand the finite lifetime of the trapped gas sample, in particular below the transition temperature. Since the mechanism of the decay is not yet completely clarified, it is useful to consider the theoretical predictions for the various possible partial decay-rate constants. In this paper we present the decay-rate constants for the dipolar decay of the hyperfine states involved. For the case of $^7\text{Li}$ that result has been given in a previous paper [4]. The dipolar decay-rate constants for $^{23}\text{Na}$ and $^{87}\text{Rb}$ will be considered here. In Fig. 1 we show the ground-state hyperfine structure for $^{23}\text{Na}$, $^{85}\text{Rb}$, and $^{87}\text{Rb}$ as a function of magnetic field. In the following we refer to each of the hyperfine states in terms of the (lower-case) single-atom quantum numbers $f,m_f$, although $f$ is a good quantum number only for $B=0$. Capital quantum numbers $F,M_F$ are reserved for two-atom states involved in collisions. Instead of the doubly polarized $f,m_f=2,+2$ state in Na that we studied previously [4], we will concentrate here on the $1,−1$ state for which BEC was realized experimentally [3]. As Fig. 1 shows, special significance has to be attributed to the field range up to 316 G in which the $1,−1$ state can be trapped in a static magnetic trap as a low-field seeking state. Also in the case of $^{85}\text{Rb}$ we will concentrate on the dipolar decay of the state involved in the recent BEC experiment [1], the doubly polarized $2,+2$ state. For completeness we will give the analogous result also for the isotope $^{87}\text{Rb}$. The method used has been sketched in Ref. [5]. Decay channels considered are all exothermal two-body decay channels not forbidden by selection rules, i.e., all are available combinations of lower-energy $f_1m_{f_1},f_2m_{f_2}$ combinations with $M_F=m_{f_1}+m_{f_2}$ at

![Diagram](image.png)
most 2 different from the initial total $M_F$ value, taking into account the possible transfer of $2\hbar$ of angular momentum between internal and external atomic degrees of freedom involved in an inelastic collision.

A reservation should be made in connection with the dipolar rate constant values to be presented in this Rapid Communication. For decades it has been known [6] that the magnetic dipole-dipole interaction $V_{\text{dip}}$ among the valence-electron spins of interacting alkali-metal atoms is accompanied by a second type of spin-spin interaction with the same spin-angle structure that arises as a second-order effect in the valence-electron spin-orbit coupling and will be referred to here as $V_{\text{so}}^2$. Both the (totally different) interatomic distance range, where $V_{\text{so}}^2$ is effective, and its magnitude are roughly known. Its strength increases rapidly with $Z$. Mies et al. [7] pointed to its possible role in the spin relaxation of alkali-metal atoms and made a preliminary study in the case of Rb atoms. A later study by Tiesinga et al. [8] concentrated on $^{133}\text{Cs}$. Although a non-negligible $V_{\text{so}}^2$ contribution cannot be excluded in the present case of the Rb isotopes, we believe that a calculation of the purely magnetic dipolar part alone is useful, so that a comparison with the experimental decay rate might form the first experimental indication of a possible significant $V_{\text{so}}^2$ contribution.

In order to calculate the above-mentioned dipolar rate constants, detailed knowledge of the potentials involved is necessary. For Na the triplet and singlet potentials are very well known [9] and for the Rb isotopes the relevant part of the triplet potential has recently been determined by means of an analysis of two photoassociation experiments [10,11]. The influence of the singlet potential on the dipolar decay-rate constant for the fully stretched Rb state is expected to be small, because the dominant decay channels are almost pure triplet states. We use an experimental Rydberg-Klein-Rees potential [12] and obtain an indication of the associated range of uncertainty of each calculated rate constant by introducing a variable radial phase $\phi_S$ in the singlet wave function at an arbitrary point in the radial range where the WKB approximation is valid [9,10]. Upper and lower limits of the rate constants thus obtained by varying $\phi_S$ are indicated in the following figures.

In Fig. 2 we have given the dominant partial decay-rate constants and the total decay-rate constant for $^{87}\text{Rb}$ as a function of the magnetic field in the zero-temperature limit. Along the horizontal scale the quantity $1 + B/B_0$ is plotted logarithmically to combine the advantages of a linear scale at low $B$ and a logarithmic scale at higher fields where the rate constants show a slower variation. The value of $B_0$ chosen, $a_{\text{hf}}/(64 \mu_B)$, with $a_{\text{hf}}$ the hyperfine constant, ensures a favorable separation between the linear and logarithmic parts. The upper horizontal scale indicates the actual $B$ values. For each partial decay-rate constant two lines are presented, indicating upper and lower limits obtained by varying $\phi_S$. Figure 2(a) shows the rate constants leaving the atoms in the highest hyperfine manifold $f = 2$, as well as the total rate constant. Figure 2(b) shows the remaining partial rate constants. A remarkable feature of the rate constants is the oscillatory behavior for small $B$ values. This behavior is analogous to that of atomic H [5] and Na [4] and can be explained in terms of oscillations of the radial wave function in the final channel relative to that in the initial channel, as they occur in the integrand of the radial transition matrix element. The relative phase of these oscillations shifts with $B$.

Note that, in agreement with Ref. [4], zero-temperature decay-rate constants generally do not vanish. They vanish only in the extreme case of zero final phase-space volume, i.e., at $B = 0$ for final channels with both atoms in the upper hyperfine manifold. As a consequence, whereas the total decay-rate constant is dominated by such channels at the higher fields because of the overlap of initial and final radial wave functions, the remaining channels with at least one atom in the lower hyperfine manifold dominate for small $B$. The gradual decrease of the dominant rate constants and thus of the total rate constant at high fields is due to the growing disparity of initial and final kinetic energies with increasing $B$, leading to a decrease of overlap.
Note, furthermore, that the uncertainty in the singlet potential does not change the orders of magnitude of the dominant rate constants and the total rate constant at low B and is even negligible for the higher fields. The uncertainty in the triplet potential, which has not been included in the figures, corresponds to even smaller changes of the rate constants.

In the $^{87}$Rb BEC experiment the magnetic field is about 5 G. For this field strength the predicted two-body dipolar decay-rate constant is $(6 \pm 3) \times 10^{-15}$ cm$^3$ s$^{-1}$ and the dipolar lifetime of the condensate 90–300 s ($n = 2.5 \times 10^{12}$ cm$^{-3}$), including the factor of 2 reduction of the rate constant due to identical-particle effects [14]. The difference between the experimentally observed decay time of 15 s and this two-body dipolar decay time may be due to $V_{2\omega}$, in which case the amplitude due to $V_{\lambda\omega}$ and $V_{2\omega}$ would be comparable in the case of Rb atoms. In a previous paper [13], we have calculated the rate of recombination to Rb$_2$ for spin-polarized $^{87}$Rb leading to a lifetime of $24 \times 10^4$ s, completely negligible with respect to the two-body dipolar decay rate.

In Fig. 3 we have given the dominant partial decay-rate constants and total decay-rate constant for $^{85}$Rb as a function of the magnetic field in the zero-temperature limit. Again the uncertainty in the rate constants due to the singlet potential is indicated. In contrast to the $^{85}$Rb rate constants these rates show no oscillatory behavior. In general the $^{85}$Rb rate constants are about a factor of 5 higher than the $^{87}$Rb rate constants. This is due to the fact that the absolute value of the $^{85}$Rb + $^{85}$Rb scattering length is larger than that for $^{87}$Rb + $^{87}$Rb. In the case of $^{85}$Rb the range of variation of the rate constants associated with the uncertainty in the triplet potential is much larger than for $^{87}$Rb, due to the much larger uncertainty in the $^{85}$Rb scattering length [10,11]. Relative to the curves presented, the variations amount to an upward or downward shift by up to a factor of 3.

Figure 4 shows all three calculated partial dipolar decay-rate constants for $^{23}$Na, as well as the total rate constant. Each of them tends to 0 for $B \rightarrow 0$ due to vanishing phase-space volume, in other words, due to centrifugal potential which is more difficult for the outgoing atoms to penetrate for decreasing kinetic energies. The pronounced minima as a function of field are again due to the varying phase relations of ingoing and outgoing radial waves.

The total dipolar decay-rate constant for $^{23}$Na is thus calculated to be about $6 \times 10^{-17}$ cm$^3$ s$^{-1}$ in the relevant field range around 1 G, again excluding the factor of 2 bosonic reduction of the rate constant in the BEC environment. We thus find the dipolar lifetime for the condensate to be $110$ s ($n = 1.5 \times 10^{14}$ cm$^{-3}$), much longer than the experimentally observed 1-s lifetime. It may very well be that in the experiment of Ketterle et al. [4], with its much higher atom density, the condensate lifetime is primarily determined by the mechanism of recombination to Na$_2$ in three-body collisions. Indeed, since pairs of $^{23}$Na atoms in the 1,–1 hyperfine state interact predominantly via the triplet potential, the recombination rate can very well be estimated from the recombination rate for the collision of two spin-polarized atoms [13] leading to a lifetime of 1.3 s, in good agreement with the experimentally observed lifetime.

In conclusion, as a complement to previous studies that dealt with recombination rates or with dipolar rate constants for other states, we have calculated the dipolar rate constants for the hyperfine states of $^{87}$Rb and $^{23}$Na involved in two recent successful BEC experiments. The calculated rate constants are roughly consistent with the magnitude of the experimentally observed condensate lifetimes for $^{87}$Rb, with a possible discrepancy due to $V_{2\omega}$. In the case of $^{23}$Na, taking into account the high atomic density, the main decay mechanism of the condensate appears to be recombination to Na$_2$.


