Optimal basis poles selection for Orthonormal Basis Functions based model
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Published: 01/01/2014

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Citation for published version (APA):
Motivation
Orthonormal Basis Functions (OBFs) have interesting applications in both the LTI and in the Linear Parameter-Varying (LPV) frameworks. In system identification, several known advantages of OBF based methods came from the series expansion nature of the model structure and the orthonormality of the bases. However, to exploit these properties and to obtain a parsimonious parameterization of the model, an adequate set of OBFs need to be selected (tuned) prior to the identification procedure. Hence, solving the basis selection problem has paramount importance for both LTI and LPV identification as well as for the selection of kernel functions in Bayesian identification.

OBF Model Parameterization
For any stable LTI system:

\[ F(x) \approx \sum_{k=1}^{n_o} w_k \phi_k(x), \]

with \( \{ w_k \}_{k=1}^{n_o} \in \mathbb{R} \) being the set of expansion coefficients and the set of OBFs is given as \( \{ \phi_k(x) \}_{k=1}^{n_o} \subset \mathcal{H}_2(\mathbb{D}) \).

Construction:
Characterized by the state readout of all-pass stable transfer functions \( \{ G_i(x) \}_{i=1}^{n_o} \).

![Diagram of OBFs](image)

Types of rational OBFs:
- Pulse bases (FIR, poles at the origin \( G_i(x) = z^{-1} \))
- Laguerre bases (real poles)
- Kautz bases (complex pole pairs)
- Generalized bases (Hambo, Takenaka-Malmquist):

\[ G_i(x) = \pm \prod_{i=1}^{n_o} \frac{1 - \lambda_i^2 z^{-1}}{1 - \lambda_i z^{-1}} = G_i(z), \]

with \( \{ \lambda_i \}_{i=1}^{n_o} \subset \mathbb{D} \) being real and/or complex pole pairs.

Optimal basis poles selection
The rate of convergence of the series expansion (1) is bounded by \( \rho \geq |G_i(z^{-1})| \).

Proposed New Solution
The optimal pole selection problem boils down to minimization of (3), which is none the less than:

\[ \min_{\lambda_1,..,\lambda_{n_o}} \max_{z \in \Omega} \prod_{i=1}^{n_o} \left| \frac{z - \lambda_i}{1 - z \lambda_i} \right|, \]

with \( \Omega \subset \mathbb{C}^n \) is the set of known system poles (possibly with pole uncertainties). This problem is a difficult rational min-max problem over \( \mathbb{D} \).

Problem reformulation
Let

\[ z = x_1 + jy_1 \]
\[ \lambda = x_2 + jy_2 \]

be the cartesian form of complex variable \( z \) and \( \lambda \), where \( x_1, y_1, x_2, y_2 \in \mathbb{R} \). Then we can re-parameterize Problem (4) as:

\[ \begin{align*}
\min_{x_1, y_1, x_2, y_2} & = \rho^2 \\
\text{s.t.} & = \rho^2 \\
\end{align*} \]

The problem can be further recasted as feasibility problem to satisfy \( n_o \) set of inequalities:

\[ \prod_{i=1}^{n_o} ((x_1 - a_i)^2 + (y_1 - b_i)^2) \leq \rho^2 \]
\[ \prod_{i=1}^{n_o} ((x_2 - a_i)^2 + (y_2 - b_i)^2) \leq \rho^2 \]

which can be solved by Bisection Search with (Semidefinite) Sum of Squares programming.

Future works
Reduction of the required set of inequalities of the feasibility problem (7) by vertices-based optimization (polytopic descriptions of the system poles and possibly exploiting results in hyperbolic geometry).

References