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Citation for published version (APA):

DOI:
10.1109/TPWRD.2002.803748

Document status and date:
Published: 01/01/1998

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
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Download date: 03. Mar. 2021
Power Engineering Letters

A New Application of Graph Theory for Coordination of Protective Relays

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Abstract: The coordination of protective relays of multiloop networks is a tedious and time-consuming process. The complicated part of this process is the determination of a set of relays referred to as the break-point set (BPS), with a minimal size to start the coordination procedure. This letter introduces a new graph-theoretical approach to determine the BPS. This method reduces the complexity of the problem, by exploiting the available sparsity of the dependencies among relays in protection systems.

Introduction: The relays of a protection system must be set in coordination with their primary relays, to ensure selective operation. In multiloop networks each relay may belong to different loops, and its setting must be coordinated simultaneously with the settings of a number of primary relays. This is a complicating process.

To visualize the problem, the coordination of protection system $P_1$ in Figure 1 is taken as an example. In this system, the primary relays of each relay $r_i$ are located at the remote bus of each relay with the exception of its adjacent relay. The adjacent relay of each relay is the one that is located at the opposite end of the same line of each relay. As an example, $r_i$ is the adjacent relay of $r_j$. Thus, $r_i$ and $r_j$ are the primary relays of $r_j$, which are represented by primary-backup pairs $(r_i, r_j)$ and $(r_j, r_i)$ respectively.

In each loop of the network of Figure 1, the settings of each relay depends on the settings of other relay of that loop. As an example, in the loop containing relays $r_i$, $r_j$, and $r_k$, the settings of relay $r_i$ must be made based on the settings of $r_j$, according to the primary-backup pair $(r_j, r_i)$. In the same way, the settings of relay $r_j$ are the same as $r_i$, the settings of relay $r_k$ are the same as $r_i$. Then, if the setting of relay $r_i$ is changed, $r_i$ has to be reset for proper coordination with $r_k$ and another iteration must be done. In a multiloop network, there are many of these loops adjacent to one another, hence, a large number of iterative calculations must be performed to achieve system-wide coordination.

A recently proposed solution is to determine a proper set of relays as a starting point for the coordination procedure [1]. This starting set is called a break point set (BPS), each member of which is called a break point (BP). The main property of a BPS is that if the settings of its relays are known, coordinated setting of the rest of the relays can be determined successively. Starting coordination with a BPS relays ensures

Figure 1. Protection system $P_1$

that each relay’s settings is calculated only once at each iteration. Since
the computation time for a coordination procedure increases with the
size of its corresponding BPS, a BPS with a minimum size is desirable.

The proposed method converts determining the BPS into a graph-
theoretical problem, allowing use of effective tools of graph theory to
reduce the complexity of the problem. The method approaches to a BPS
step by step, choosing an appropriate relay as a BP in each step. This
approach results in a near-to-minimum or a minimum number of BPS.

**Graph-Theoretical Representation of the Problem:** To illustrate
this method, coordinating protection system \( P \), (Figure 1) is consid-
ered. The primary-backup relations of this protection system are dis-
played in Figure 2 by a directed graph, referred to as a dependency
\( D_r \).

In a dependency-diagram each vertex (node) represents a relay, and
each arc (directed branch) indicates the primary-backup relation be-
tween two relays. For example, arc \( v_1 \rightarrow v_3 \) (the arc from vertex 1 to 3) rep-
resents that \( r_1 \) is a primary of \( r_3 \) (or equally, \( r_3 \) is a backup of \( r_1 \)).

If a dependency-diagram has no cycle (directed loop), its vertices
can be ordered in a sequence of sets in which the vertices of each set
have incoming arcs only from the vertices of the previous sets, imply-
ing that the corresponding relays can be coordinated set by set consecu-
tively. Accordingly, a minimum BPS is a minimum set of vertices
which: if they are removed, all cycles of the dependency-diagram will
be opened. Therefore, the determination of a minimum BPS is in fact a
problem in graph theory known as **Feedback Vertex Set** (FVS). The
problem was shown to be an **NP-complete** problem which cannot be car-
ried out in a polynomial time period.

**Reduction Rules for Determination of a Minimum BPS (or FVS):** These rules identify the vertices which have to be members
of every minimum BPS, and also the vertices which need not to be mem-
ers of every minimum BPS.

1. **Rule 1:** If vertex \( v \) of a dependency-diagram \( D \) has no outgoing
   arc, \( v \) is not a BP and can be deleted from \( D \) (Figure 3). In the
   same way, if vertex \( w \) of a dependency-diagram \( D \) has no incom-
ing arc, \( w \) can be deleted from \( D \).

2. **Rule 2:** If vertex \( v \) has only one outgoing arc \( a \) which goes to ver-
tex \( w \), \( v \) is not an essential BP. So, join \( v \) to \( w \) and remove the con-
necting arc \( a \) (see Figure 3). In the same way, if a vertex \( v' \) has
   only one incoming arc \( a' \) which comes from vertex \( w' \), join \( v' \) to
   \( w' \) and remove the connecting arc \( a' \).

3. **Rule 3:** If vertex \( v \) has a self-cycle (having arc from \( v \) to \( v \)), \( v \) is a
   member of every minimum BPS. So, remove \( v \) from the
dependency-diagram and consider it as a new BP.

The proof of these rules can be found in [2].

**New algorithm:** The new algorithm results in a near-to-minimum
or a minimum BPS (or FVS), \( B \), and consists of the following steps:

1. **Step 1:** Create the dependency-diagram \( D \) of the protection sys-
tem. Initiate \( B \) as an empty set: \( B = \{ \} \).

2. **Step 2:** Apply rules 1 to 3 to all vertices of \( D \). Applying Rule 3
   may add a new member to \( B \).

3. **Step 3:** If step 2 changed \( B \), go back to Step 2.

4. **Step 4:** If \( B \) is empty, stop.

5. **Step 5:** Choose an arbitrary vertex \( v \) as a new BP. Remove \( v \) from
   \( D \), add \( v \) to \( B \), and go to Step 2.

**Illustrative Example:** To illustrate the algorithm, determining a
BPS for protection system \( P \), (Figure 1) is considered as an example. The
procedure is as follows, and the dependency-diagram \( D_r \) of this
protection scheme is shown in Figure 2:

1. **Initiate** \( B = \{ \} \). In dependency-diagram \( D_1 \) Rules 1 to 3 can not be
   applied to any of vertices in order to reduce the dependency-
diagram. Choose one of the vertices, say \( v_1 \), as a BP. Remove \( v_1 \) from \( D_1 \).
   Add \( v_1 \) to \( B \), \( B = \{ v_1 \} \).

2. **Join** \( v_3 \) to \( v_1 \) and remove arc \( v_1v_3 \), applying Rule 2. **Join** \( v_4 \) to
   \( v_3 \) and remove arc \( v_3v_4 \), applying Rule 2. **Join** \( v_2 \) to \( v_4 \) and remove
   arc \( v_4v_2 \), applying Rule 2. Figure 4 shows the updated diagram up to this
   step. **Choose** \( v_2 \) as a new BP, remove \( v_2 \) from the dependency-diagram.
   Add \( v_2 \) to \( B \), \( B = \{ v_1, v_2 \} \).

3. **Join** \( v_3 \) to \( v_1 \) and remove arc \( v_1v_3 \), applying Rule 2.
BPS. Owing to the flexibility of this approach, it can be easily applied to special protection schemes (e.g., protection systems with non-directional relays) and network configurations (e.g., radial networks and networks containing multiterminal lines). The method has been applied on a PENTIUM (75 MHz, 8 MB RAM) to a 400-line/200-bus network, and achieves an near-to-minimum BPS within 2.0 seconds.

Acknowledgment: The authors would like to thank Ir. G.L.L.M. Janssen for his contribution, and also Ir. R.W.P. Kerkenaar for his comments and proof reading.

References


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