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Collision of dipolar vortices on a $\beta$ plane

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The interaction of two dipoles moving perpendicularly to the gradient of background vorticity is studied both numerically and experimentally. In the numerical computations the vorticity distribution is represented either by four point vortices (point-vortex model) or by thousands of them (vortex-in-cell method). The simplest model is used to study the dynamics and the advection of fluid particles in two kinds of interaction: coaxial couples of equal strength but different size, and equal parallel couples with a nonzero impact parameter (the distance between the dipoles' axis). As a result of the interaction fluid masses are exchanged between the two dipoles and between each dipole and the ambient fluid. In the case of equal coaxial couples the amount of fluid exchanged depends on the gradient of ambient vorticity, with the largest mass exchange occurring always between the eastward traveling dipole and the ambient fluid. The collision of parallel couples with nonzero impact parameter leads to a large mass exchange, either because several interactions may occur or because when two independent couples arise, they have a nonuniform motion. Laboratory experiments in a rotating fluid (with a flat sloping bottom providing the $\beta$ effect), confirm that an elastic interaction is a rare event. The unstable trajectory of the westward traveling dipole, as well as small perturbations unavoidable in the laboratory, invariably lead to collisions of nonaligned dipoles. The gross features of the vortex motion, as well as of the mass exchange, are well modeled using the point-vortex model, whereas the vortex-in-cell method reproduces many details of the vortex motion, the evolution of the vorticity field, and the exchange of mass. © 1995 American Institute of Physics.

I. INTRODUCTION

Since the turn of the last century, when a dipole-vortex solution of the Euler equations was obtained, the dipole has been the subject of numerous studies. In recent years the stability of the dipolar vortex has been investigated numerically by following its evolution after a small perturbation has been imposed on the structure; for instance, by slightly changing the dipole's shape or by the addition of a small random flow in the vicinity of the vortex (see McWilliams et al.$^2$). Another type of perturbation is the presence of a nonuniform external velocity field, like a strain (Kida et al.$^3$) or a shear. These external fields have been thought of as an idealization of the influence of other vortices acting at a large distance compared to the size of the dipole. The character of these external influences changes if the distance between the vortices is comparable to the dipole's diameter, which has led to the study of basic interactions, such as the collision of two dipolar vortices (McWilliams and Zabusky$^4$). Besides the stability issue, a second stimulus for the study of dipolar interactions is the fact that these structures tend to emerge spontaneously in two-dimensional flows. If the dipolar structures are abundant then some collisions must take place. In this context, dipolar interactions might play an important role in the redistribution of vorticity (e.g., Couder and Basdevant$^5$).

In the presence of gradients in the background vorticity steadily translating dipolar vortices only exist if they propagate perpendicularly to those gradients. In the context of geophysical fluid dynamics the gradient in the background vorticity, which points in the northward direction, is generally referred to as a $\beta$ effect, and the dipolar vortices, which translate steadily in the zonal direction (i.e., along the east–west axis), are usually called modons. It has been suggested that collisions of zonally moving dipoles could give rise to transversally propagating ones.$^6$ The collision of dipoles would therefore act as a triggering mechanism of the rich variety of phenomena displayed by a meandering dipole, as has been discussed in, e.g., Velasco Fuentes and van Heijst$^7$ (hereafter referred to as VFvH) and Velasco Fuentes, van Heijst, and Cremer$^8$ (hereafter referred to as VFvHC). The argument, however, leaves unexplained why the processes that generate zonally traveling dipoles (whichever processes are) would not be able to generate transversally propagating ones.

Kroo and Yamagata$^9$ first introduced the modulated point-vortex model for the $\beta$ plane. They studied the dynamics of a single dipole and the head-on collision of dipoles of equal size but different circulations. In all these cases they observed the exchange of partners, a looping motion of the newly formed couples and a second interaction with partner exchange as the couples return to the system's symmetry line. After the second interaction, the two couples recover their initial size, speed, and direction of propagation. For this reason this is sometimes called a soliton like or elastic interaction. Hobson$^{10}$ showed that the motion of the individual point vortices during the interaction of two coaxial couples is integrable. The symmetry present in this problem enabled him to find a Hamiltonian, and, by using this advantageous
formalism, to study the regimes of motion that arise as a function of the system parameters. Makino et al.\textsuperscript{11} studied numerically the interaction of dipoles—with a continuous distribution of vorticity—of different sizes and speeds, and found the same fundamental behavior as predicted by the point-vortex model of Kono and Yamagata.\textsuperscript{9} However, a more detailed study by McWilliams and Zabusky\textsuperscript{14} showed that a soliton-like collision is a rare event among the variety of dipole interactions. An elastic collision arises only if, after the looping motion, the two dipoles reapproach the symmetry line perpendicularly. This is the case for modons of approximately the same size and speed, and for $\beta$ small compared with the dipole’s vorticity.

In addition to the motion of the couples, the advection of fluid during the interaction process has received increasing attention in recent years. This has been mainly motivated by the interest in transport issues in plasmas. Horton\textsuperscript{6} studied numerically meridional transport due to the interaction of two equal modons traveling along parallel lines. He found that the net transport is maximal when the distance between the symmetry lines is of the same order as the modon radius, while it is minimal when the couples have a common symmetry line. Similar results were obtained for transport during the interaction of point-vortex couples by Kono and Horton.\textsuperscript{15} In contrast, Nycander and Isichenko,\textsuperscript{12} using theoretical considerations about the oscillating motion of the resulting couples, found a larger net transport.

We are interested here in the motion of the vortex centers and the advection of passive fluid during the interaction of two zonally moving dipoles. These issues are first discussed using four modulated point vortices (Sec. II). In this model mass transport is approached in the same spirit as the lobe shedding by a dipole on a $\beta$ plane studied by VFvHC, that is to say, the aim is to determine the region of fluid that is detrained or entrained and how much area undergoes such a process. The interaction of dipoles with a continuously distributed vorticity, both in laboratory experiments and in numerical simulations using the vortex-in-cell method, is discussed in Sec. III. It will be shown that the essential features of the interaction of continuous dipoles are also displayed by the point-vortex model.

II. INTERACTION OF MODULATED POINT–VORTEX DIPOLES

It is well known that, for an inviscid homogeneous fluid, there is a dynamical equivalence between (a) a layer of fluid having constant depth and a linearly varying Coriolis parameter (as on a rotating planet), on the one hand, and (b) a layer of fluid in uniform rotation and having a linearly varying depth, on the other hand.\textsuperscript{14} In both cases, the following expression for the conservation of potential vorticity can be obtained:

\[
\frac{D}{Dt} (\omega + \beta y) = 0,
\]

where $D/Dt$ is the material derivative in the plane $(x,y)$ and $\omega$ is the vertical $(z)$ component of the relative vorticity.

The form of the gradient of ambient vorticity $\beta$ in $y$ direction depends on the case being considered: For a rotating planet $\beta = (2\Omega/R)\cos \phi_0$, with $\Omega$ and $R$ the angular velocity and the mean radius of the planet, respectively; and $\phi_0$ a reference latitude. In this approximation, known as the $\beta$-plane model, all the effects of the spherical domain have been neglected, except for the gradient of the Coriolis parameter. The coordinates $x$ and $y$ point eastward and northward, respectively. For the topographic case $\beta = -sf_0/h_0$, where $f_0$ is the uniform Coriolis parameter, and $s$ is the gradient of the fluid depth, which is given by $h(y) = h_0(1 - sy/h_0)$. Now $y$ points toward shallow water and $x$ points in the direction of uniform depth.

The principle of conservation of potential vorticity can be introduced in the point-vortex model by using the assumption that each point vortex represents a small patch of vorticity.\textsuperscript{9,10,15} Therefore the circulation $\kappa_i$ equals the (uniform) vorticity $\vec{\omega}_i$ multiplied by the area of the patch $a$ ($\kappa_i = \vec{\omega}_i a$). Conservation of potential vorticity implies that the relative vorticity $\omega$ of a moving point vortex changes as expressed by (1). This yields the following modulation equation for the circulation $\kappa_i$ at the current position $y_i$:

\[
\kappa_i = \kappa_{i0} + \beta_s (y_{i0} - y_i),
\]

where $\kappa_{i0}$ is the circulation at initial position $y_{i0}$ and $\beta_s = \beta a$.

We consider here only couples that approach each other while moving in the zonal direction, i.e. every point vortex has an approximately constant $y$. It can therefore be assumed that the motion of the vortices up to the collision occurs as if the relative circulation were preserved. After the partner exchange the newly formed couples begin to move transversally to the background-vorticity isolines, with a departure angle that depends solely on the initial configuration. At this stage the $\beta$ effect becomes important, and both $\beta$ and the departure angle determine the subsequent motion of the two couples as well as the mass exchange (as in the single-dipole case discussed by VFvH and VFvHC).

A. Review of the nonmodulated case

The collision of point-vortex couples (with constant circulations) has been studied by several authors since the last century (for a review see Aref\textsuperscript{16}). Here we present briefly the results that will be used in the discussion of interactions on a $\beta$ plane, namely, how the departure angle depends on the initial configuration of the point vortices. For a thorough study of the nonmodulated case the reader is referred to Eckhardt and Aref.\textsuperscript{17}

Among the large variety of possible interactions, we will confine our attention to the case of equal magnitude of the initial circulation $\kappa_0$ (which is assumed to be positive), and with the two couples propagating in opposite directions along parallel lines. These interactions lead to integrable motions of the vortices in the nonmodulated case (Eckhardt and Aref\textsuperscript{17}). Two types of interaction can be distinguished: (a) coaxial couples, when the couples have different sizes ($d-2e_1$ and $d+2e_2$) but a common symmetry line [Fig. 1(a)]; and (b) parallel couples, when the couples have equal size $d$ but their symmetry lines are separated by a distance...
2\varepsilon, which is usually called the impact parameter [Fig. 2(a)]. The values of \varepsilon, \varepsilon_1, and \varepsilon_2 used here lie within the range that generates a partner exchange. Although two coaxial couples are also parallel, we will use these terms as in the definitions above. Note that a head-on collision of two equal couples belongs to both categories, and will receive special attention in the discussion of mass transport.

It is well known that the motion of a system of \( N \) point vortices is given by the system of Hamiltonian equations,

\[
\dot{x}_i = \frac{\partial H}{\partial y_i}, \quad \dot{y}_i = -\frac{\partial H}{\partial x_i},
\]

with

\[
H = -\frac{1}{4\pi} \sum_{i,j=1}^{N} \kappa_i \kappa_j \ln r_{ij},
\]

where \( r_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 \). The Hamiltonian \( H \) does not depend explicitly on time, therefore it is an integral of motion; three more integrals are easily obtained from translational and rotational invariance (see, e.g., Batchelor),

\[
Q = \sum_{i=1}^{N} \kappa_i x_i,
\]

\[
P = \sum_{i=1}^{N} \kappa_i y_i,
\]

\[
I = \sum_{i=1}^{N} \kappa_i (x_i^2 + y_i^2).
\]

Figure 1(b) shows the configuration after the partner exchange of two coaxial couples. The dependence of the angle \( \alpha_0 \) on the parameters \( \varepsilon_1 \) and \( \varepsilon_2 \) can easily be obtained from the conserved quantities (4)–(7); see Eckhardt and Aref. Here \( Q = I = 0 \), whereas the other integrals of motion before (subindex \( I \)) and after (subindex \( F \)) the interaction are given by the following expressions:

\[
P_I = -2\kappa_0 (\varepsilon_1 + \varepsilon_2),
\]

\[
H_I = -\kappa_0^2 \ln[(d-2\varepsilon_2)(d+2\varepsilon_1)],
\]

\[
P_F = -2\kappa_0 d \cos \alpha_0,
\]

where, in the computation of the Hamiltonian, it has been assumed that the distance between the two couples is much larger than the distance between the vortices forming a couple (e.g., \( b \gg d \) in the initial configuration). Conservation of \( P \) and \( H \) during the whole evolution implies that

\[
\varepsilon_1 = \frac{d \varepsilon_2}{d-2\varepsilon_2},
\]

\[
\varepsilon_2 = \frac{d}{2} \left(1 + \cos \alpha_0 - \sqrt{1 + \cos^2 \alpha_0}\right).
\]

In principle, \( \varepsilon_1 \) and \( \varepsilon_2 \) can be chosen arbitrarily. However, the assumption that after the interaction process two new couples of size \( d \) emerge, uniquely determines the value of \( \varepsilon_1 \) once \( \varepsilon_2 \) has been chosen, or vice versa, see (8). The value of, say, \( \varepsilon_2 \) then uniquely determines the departure angle \( \alpha_0 \) in (9). Previous studies have shown the importance of the initial direction of propagation \( \alpha_0 \) in the evolution of a dipole on a \( \beta \) plane (see, e.g., Makino et al., Zabusky and McWilliams, VFvH, and VFvHC). Therefore, we will discuss the interaction process as a function of the departure angle \( \alpha_0 \) instead of the distances \( \varepsilon_1 \) and \( \varepsilon_2 \).

Figure 2(b) shows the configuration after the collision of two parallel couples. In this case we are also interested in how the departure angle \( \alpha_0 \) varies as a function of the impact parameter \( 2\varepsilon \). However, unlike the coaxial case, now the integrals of motion are not sufficient to obtain that relationship. Numerical experiments show that the departure angle is given approximately by

\[
\alpha_0 = \cos^{-1}(2.35\varepsilon).
\]

This empirical relation agrees within 0.1% with the numerical and analytical results of Eckhardt and Aref, for the parameter region studied here. Therefore, in the discussion of parallel interactions on the \( \beta \) plane, the angle \( \alpha_0 \) given by (10) will be called the departure angle. It must be kept in mind, however, that this is not an exact expression.

O. U. Velasco Fuentes and G. J. F. van Heijst
B. Modulated coaxial couples

As illustrated in Fig. 1, in the nonmodulated case the symmetry of the system with respect to the common axis of the couples is preserved during the interaction. On a $\beta$ plane this symmetry is preserved only if the symmetry line is parallel to the west–east axis ($x$). In that event the collision of coaxial couples is equivalent to the evolution of two vortices on the half-plane $y>0$ with a straight free-slip wall at $y=0$. The evolution equations transform into

\[
\frac{dx_i}{dt} = -\frac{1}{2\pi} \frac{\kappa_{3-i}}{y_i} \left( \frac{y_i - y_{3-i}}{r_{1,3-i}^2} - \frac{y_i + y_{3-i}}{r_{r,3-i}^2} \right),
\]

\[
\frac{dy_i}{dt} = -\frac{\kappa_{3-i}}{2\pi} \left( \frac{x_i - x_{3-i}}{r_{1,3-i}^2} - \frac{x_i + x_{3-i}}{r_{r,3-i}^2} \right),
\]

for $i=1,2$, where $r_{1,3-i}^2 = (x_i - x_{3-i})^2 + (y_i + y_{3-i})^2$; and with the circulations being modulated according to (2).

This symmetry has thus reduced the number of equations from eight (the evolution of $N$ point vortices in the infinite plane is described by $2N$ equations) to four.

We describe the evolution in terms of the coordinates ($\eta, \xi$) of the middle point between the vortices in the upper half-plane and the direction of motion $\alpha$ (see Fig. 3), with the distance $\delta$ between these two point vortices being time dependent. These new variables are given by

\[
\eta = \frac{x_1 + x_2}{2},
\]

\[
\xi = \frac{y_1 + y_2}{2},
\]

\[
\delta^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2,
\]

\[
\alpha = \arccos \left( \frac{y_1 - y_2}{\delta} \right).
\]

The time derivative of these equations and the use of (11)–(12) give, after cumbersome but straightforward algebra, the following equations of motion:


\[
\frac{d\eta}{dt} = \frac{\kappa_1}{2\xi + \delta \cos \alpha} + \frac{\kappa_2}{2\xi - \delta \cos \alpha} + \frac{(\kappa_1 - \kappa_2) \cos \alpha}{\delta},
\]

\[
\frac{d\xi}{dt} = \delta \sin \alpha \left( \frac{1}{\delta^2 - \frac{\delta}{\sin^2 \alpha} + \frac{\kappa_1}{\kappa_2}} (\kappa_1 - \kappa_2),
\]

\[
\frac{d\delta}{dt} = \frac{\sin \alpha}{\kappa_1 - \kappa_2} \left( \frac{1}{2\xi + \delta \cos \alpha} + \frac{1}{2\xi - \delta \cos \alpha} \right) - \frac{(\kappa_1 + \kappa_2) \cos \alpha}{\delta},
\]

\[
\frac{d\alpha}{dt} = \frac{\kappa_1 + \kappa_2}{\delta^2} + \frac{1}{\delta^2 - \frac{\delta}{\sin^2 \alpha} + \frac{\kappa_1}{\kappa_2}} (\kappa_1 - \kappa_2),
\]

where a factor $1/4\pi$ has been removed from the right-hand side of Eqs. (13)–(16). A further reduction can be achieved by using the conserved quantity (Zabusky and McWilliams)\(^{15}\)

\[
W = \sum_{i=1}^{N} \kappa_i^2.
\]
FIG. 4. Trajectories of individual vortices in the course of the interaction of coaxial point-vortex couples on the $\beta$ plane. The thick lines show the trajectories of the positive vortices and the thin lines show those of the negative vortices. (a) A large eastward traveling dipole (ETD) interacting with a small westward traveling dipole (WTD) produces a sine-like looping of the newly formed couples. (b) A large WTD collides with a small ETD and a closed loop is described by the new couples. (c) A yet larger WTD colliding with a small ETD gives rise to a self-intersecting loop.

This leads to an expression that relates the latitudinal position $\xi$ to the direction of propagation $\alpha$.

$$\xi = \xi_0 + \frac{s}{2} \left[ \cos \alpha - \cos \alpha_0 \right] \left( \frac{\kappa_0 - \beta \sin \alpha \cos \alpha_0}{\beta} \right).$$

Substitution of this result in (13)–(16) leaves a system of equations where $\eta$ and $\xi$ do not appear on the right-hand side of any of them. Therefore two first-order ordinary differential equations for $\delta$ and $\alpha$ govern the motion of the vortices; furthermore, these equations are autonomous, which implies that the motion of two coaxial couples on the $\beta$ plane is integrable (see Hobson$^{10}$).

Before presenting the numerical results we make some operational definitions. We consider the partner exchange to occur when $\delta_{13} = \delta_{12}$ ($\delta_{ij}$ is the distance between vortices $i$ and $j$). The time elapsed between the two events $\delta_{13} = \delta_{12}$ is called the scattering time $T_s$. The loop length $\lambda$ is the distance in zonal direction between the points where the partner exchanges occur, and it is considered to be positive when the second point lies to the east of the first one, and negative otherwise. The amplitude of the looping motion of the couples is the maximum value of $\xi - \xi_0$, and can be obtained by substituting $\alpha = 0$ in (17).

For small $\alpha_0$, i.e., the eastward traveling dipole (ETD) larger than the westward traveling dipole (WTD), the loop has a small amplitude and a sine-like shape, the newly formed couples reencounter eastward of the location of the first interaction [Fig. 4(a)]. The loop length decreases as the departure angle $\alpha_0$ increases, and becomes zero for some critical value $\alpha_0 \approx 2.26$ [Fig. 4(b)]. For larger values of $\alpha_0$ the second partner exchange occurs to the west of the first partner exchange leading to a cycloid-like loop [Fig. 4(c)]. In all cases the northern couple rotates clockwise and the southern couple anticlockwise. The shape of the trajectory depends on the departure angle $\alpha_0$ in the same way as that of a single tilted dipole (e.g., Kono and Yamagata$^9$ and Zabusky and McWilliams$^5$).

The scattering time $T_s$ decreases with $\beta_*$ and increases with $\alpha_0$ [Fig. 5(a)]. The qualitative behavior of the loop's

FIG. 5. (a) Scattering time $T_s$, (b) amplitude $A$, and (c) length $\lambda$ of the loop described by the newly formed couples during a coaxial collision as a function of the departure angle $\alpha_0$ and the gradient of ambient vorticity $\beta_*$. 

FIG. 6. The same as Fig. 4, but now for the interaction of two parallel equal couples. In (a), (b), and (c) a single exchange-scattering interaction occurs, and in (d) two exchange-scattering interactions take place. As the departure angle increases, the resultant state is (a) two new couples propagate in opposite directions; (b) one new couple propagates in the eastward direction while the other has no net propagation; (c) two new couples propagate in the eastward direction; and (d) the two original couples propagate approximately in their original direction, with the corresponding stability properties.

amplitude $A$ is the same as the amplitude of a meandering dipole, namely $A$ increases with $\alpha_0$ and decreases with $\beta_\phi$ [Fig. 5(b)]. The loop length $\lambda$ decreases with increasing $\beta_\phi$, as does the wavelength of a dipolar vortex, but it has a maximum for a particular value of $\alpha_0$ [Fig. 5(a)].

C. Modulated parallel couples

In the case of equal couples translating along parallel directions, a departure angle in the range $0<\alpha_0<\pi/2$ produces an equivalent evolution as a departure angle in the range $\pi>\alpha_0>\pi/2$. The labels “northern” and “southern” couples have to be interchanged as one describes the behavior in one interval or the other.

For small values of $\alpha_0$ the new couple formed by the southern vortices performs small oscillations and propagates in the eastward direction, while the new couple formed by the northern vortices propagates westward with a cycloid-like trajectory [see Fig. 6(a), where $\alpha_0$ has a small negative value]. The distance between the new couples increases continuously and their mutual influence decreases. Therefore, soon after the interaction each couple can already be considered as a single dipole moving in an otherwise quiescent fluid. As the magnitude of $\alpha_0$ increases the net zonal velocity of the southern couple decreases while that of the northern couple increases (becomes less negative). For some value of $\alpha_0$ the northern couple shows no net zonal displacement, as is illustrated in Fig. 6(b), which was computed using a value of $\alpha_0$ close to the critical value. For slightly larger values of $\alpha_0$ the two couples propagate eastward but their velocities are different enough for the couples to behave independently a few oscillations after the partner exchange [Fig. 6(c)].

The departure angle $\alpha_0=\pi$ leads to a head-on collision of two identical couples, as discussed in the previous section. For small deviations from this value the newly formed couples reencounter almost at the initial latitude, but they are slightly misaligned—their trajectories are not parallel and they have a zonal shift. Owing to the small (but nonzero) impact parameter a new exchange of partners occurs, which reestablishes the original couples; however, now the ETD propagates with a wavy trajectory, while the WTD moves along a cycloid-like path [Fig. 6(d)]. The misalignment of the couples increases with both increasing $\alpha_0$ and decreasing $\beta_\phi$, and as a consequence the wavy trajectories of the reestablished ETD increases in amplitude and decreases in wavelength, while the WTD trajectory has a decreasing amplitude and a less negative wavelength.

For every value of $\beta_\phi \neq 0$ there is a range of $\alpha_0$ values (e.g., $0.35\pi<\alpha_0<0.46\pi$ for $\beta_\phi=0.1$) for which the couples show multiple interactions, with partner exchange (exchange scattering) as well as without partner exchange (direct scattering). Figure 7(a) shows an example of this type of behavior. After the first partner exchange, three interactions without exchange take place; a second exchange occurs later and after a new interaction without partner exchange, the two couples propagate eastward with different zonal speeds. This example shows that it is impractical to study in detail the motion of the vortices in this parameter region. However, one would like to know, for example, if the dipole interactions continue indefinitely or if eventually two independent couples emerge; and if the latter occurs, whether
the couples are formed by the same vortices as initially. As a diagnostic parameter to explore these issues, we define
\[ D = \log(\delta_{13}/\delta_{12}), \]
where \( \delta_{13} \) is the distance between vortices 1 and 3 (pertaining to different couples originally) and \( \delta_{12} \) is the distance between vortices 1 and 2 (forming one of the original couples). From this definition it is clear that a growing positive value of \( D \) indicates that the two original couples have recovered their identity and move away from each other, as illustrated by the (upper) thin line in Fig. 7(b), which gives the evolution of \( D \) in the course of a head-on collision. If, on the contrary, \( D \) has an increasingly negative value then the two newly formed couples are moving apart from each other without interacting again, as is the case in the parallel collision with small departure angle represented by the (lower) thin line in Fig. 7(b). When multiple interactions occur the graph of \( D \) remains "close" to the x axis for a large period of the evolution. An intersection with the axis indicates an exchange scattering, whereas an approach to this axis without crossing it indicates a direct scattering interaction. An example of this is given by the thick line in Fig. 7(b), which corresponds to the interaction shown in Fig. 7(a).

The long-time average of \( D \) (denoted by \( \bar{D} \)) points out the final result of the interaction: (a) A large positive value indicates that in the final state the original couples have recovered their identity and behave almost independently; (b) a large negative value indicates that two new couples propagate independently; and (c) a value close to zero indicates a collective behavior of the four vortices, which are generally arranged as two couples that intermittently interact with each other either with or without partner exchange.

The behavior of \( D \) as a function of the departure angle \( \alpha_0 \) and the gradient of ambient vorticity \( \beta_* \) is shown in Fig. 8(a), and the number of partner exchanges \( (N_E) \) that take place during the evolution is shown in Fig. 8(b). The value of \( \beta_* \) depends evidently on the time interval used to make the average. However, the actual value of \( \bar{D} \) is not as important as its long-term behavior, i.e. if it oscillates indefinitely or if, after a transient period, it grows or decreases monotonically. Numerical integrations using different time intervals have shown that eventually the absolute value of \( \bar{D} \) grows monotonically. This indicates that the final state for all initial conditions (in the parameter region considered here) is that of two independently propagating couples. This result is also confirmed by the observation that the number of partner exchanges \( (N_E) \) does not depend on the time interval if this is large enough.

For every value of \( \beta_* \) the departure angle \( \alpha_0 \) has a

![FIG. 8. (a) The mean value of \( D = \log(\delta_{13}/\delta_{12}) \); and (b) the number of partner exchanges \( (N_E) \) as a function of the departure angle \( \alpha_0 \) and the gradient of ambient vorticity \( \beta_* \).](image-url)
threshold below which $\tilde{D}<0$ and $N_E=1$, that is to say, there is a region where after the first partner exchange the newly formed couples propagate independently [the blue regions in Fig. 8(a)]. Similarly, $\alpha_2$ has a threshold above which $\tilde{D}>0$ and $N_E=2$, which correspond to regions where two partner exchanges take place and the original couples propagate independently [red regions in Fig. 8(a)]. The value of each of these thresholds decreases as $\beta_2$ increases, while the difference between them increases.

In the band of the parameter space located between the regular regions, the outcome of the interaction seems to be sensitive to the initial conditions. In this region multiple interactions, of direct and exchange scattering type, can take place; this is visible in the alternating bands of positive and negative $\tilde{D}$ [Fig. 8(a)]. For low values of $\beta_2$, the bands of positive and negative $\tilde{D}$ correspond to $N_E$ being 1 or 2, but as $\beta_2$ grows the number of partner exchanges increases up to several tens in small regions [Fig. 8(b)].

The time step used here was such that a single couple would require about 630 time steps to advance a distance $a$ (the distance between the point vortices). Results obtained with a halved time step show that $N_E$ is not modified, whereas $\tilde{D}$ changes negligibly (0.001%) in most of the domain and by a small fraction (1%-2%) in the few points where $N_E$ is large (ten or more). Similarly, if the initial distance between the couples is varied moderately around the value used here (8d) the numerical results change marginally, while the structure of the graph as a whole is conserved. However, if this distance becomes much larger the picture changes completely because of the trajectory instability of the west traveling couple.

**D. Mass exchange**

It is well known that two point vortices with circulations of equal magnitude but opposite sign carry with them a fixed area of fluid of almost elliptical shape. The vortices can lose this fluid (or capture some more) only through an external perturbation (such as a prescribed velocity field) or the influence of other vortices or by a change in their circulations ($\text{VFvHC}$). Two point-vortex dipoles on the $\beta$ plane can exchange mass (with one another or with the ambient fluid) both by their mutual interactions and by the modulation of their circulations. In either case, the perturbation changes the shape of the fluid area that can be instantaneously trapped by the vortices, thus producing entrainment or detrainment of fluid. The reader is referred to $\text{VFvHC}$ for a detailed study of mass transport by a single dipole on the $\beta$ plane and to Meleshko et al. for an analysis of mass exchange during the collision of point-vortex couples (with constant circulations).

In the previous section we showed the strong qualitative differences of the vortex motion in the coaxial and the parallel cases. Now we will discuss how these differences affect the exchange of mass. On the one hand, the collision of coaxial couples results in two exchange interactions, which strongly affect the vortex motion. However, this occurs only for a finite time, because the second interaction reestablishes the original couples, which subsequently travel perpendicularly to the gradient of ambient vorticity. The vortices therefore have a constant circulation after the interaction, and neither interior fluid can escape, nor ambient fluid can be entrained (see $\text{VFvHC}$). On the other hand, parallel couples never recover their initial zonal propagation. Both as independent couples and as mutually interacting ones, the couples meander around lines of constant ambient vorticity. The circulations are thus continuously changing and fluid masses can be entrained and detrained ($\text{VFvHC}$).

**1. Coaxial couples**

The exchange of mass during the interaction of two equal coaxial couples is asymmetric (see Fig. 9). A large amount of the fluid initially carried by the ETD is replaced by ambient fluid during the interaction, as can be seen by the thin filaments left by the ETDs halves (Fig. 9). This is because the ETDs positive half moves northward while its negative half moves southward during the looping motion. Hence, the two halves become weaker, see (2), and the area of trapped fluid decreases [Fig. 9(c)], which implies entrainment of fluid. However, the vortices recover their initial circulation as they return to the symmetry line, therefore, they capture ambient fluid [Fig. 9(e)].

In contrast, the WTQ exchanges a negligible amount of fluid (see the little deformation of the thick lines in Fig. 9), because after the first partner exchange its negative half moves northward and its positive half southward. Hence, the two halves acquire a stronger circulation and the area of fluid trapped by each half increases, which implies entrainment of ambient fluid [Fig. 9(e)]. However, most of this fluid is detrained as the temporary couples return to the symmetry line, where the vortices recover their initial circulation and conse-
FIG. 10. Detrainment and entrainment regions of the ETD (left-hand side) for a head-on collision with an equal WTD. The broken lines indicate the fluid initially trapped by the couples. The results are plotted for \( \beta = 0.1 \) (a), 0.2 (b), and 0.3 (c).

Correspondingly the area of trapped fluid recovers its initial size [Fig. 9(e)].

The mass exchanged by the ETD with the ambient fluid was evaluated as a function of \( \beta_+ \). In Fig. 10, the broken lines represent the boundaries of the areas of fluid initially trapped by the original couples, the thick line within the positive vortex of the ETD encloses the area of fluid that will be detrained and the thick line between the two couples indicates the (equal) area of fluid that will be entrained into the positive vortex of the ETD. As \( \beta_+ \) increases the detrainment lobe comprises regions closer to the point vortices, although the lobe areas do not change significantly in the range \( 0.1 < \beta_+ < 0.3 \). The entrainment area has a larger latitudinal span for small \( \beta_+ \) values, which is a consequence of the increasing amplitude of the loop as \( \beta_+ \) decreases. Corresponding areas in the lower half-plane are not drawn in Fig. 10, since they are the mirror images, with respect to the \( x \) axis, of the ones shown here. Moreover, if the figures are mirrored about a vertical line drawn through the middle point between the two couples, one obtains the position of the entrained fluid within the ETD and the tail of detrained fluid after the interaction. These locations are relative to the couples, in absolute space a shift in an eastward direction—the loop length—has to be added. Figure 11 shows the area \( \mu \) of the detrainment lobe (scaled by the area of trapped fluid in the unperturbed case) as a function of the gradient of ambient vorticity \( \beta_+ \). The value of \( \mu \) increases with increasing \( \beta_+ \), up to a maximum of 0.2 at \( \beta_+ = 0.2 \), and beyond that value \( \mu \) decreases to 0.19 for \( \beta_+ = 0.3 \).

When couples of different size collide the exchange of fluid is larger than in the case of two equal couples. This larger exchange is mainly the result of the size differences between the new couples and the original ones; variations of the vortex circulations as the couples move latitudinally play a secondary role here. It is helpful to analyze the mass exchange due to this geometrical mechanism in the nonmodulated case. The amount of mass trapped by each half of a point-vortex dipole is proportional to the square of the distance \( d \) between the vortices; i.e., \( S = k d^2 \), where \( k \approx 1.4248 \). Therefore the fluid areas carried by each vortex before the interaction are \( k(d - \epsilon_1)^2 \) and \( k(d + \epsilon_2)^2 \), for the vortices in the small and large couples, respectively. Note that here \( \epsilon_1 \) and \( \epsilon_2 \) are not independent of each other, they are related by the condition that the new couples have size \( d \), see (8). After the partner exchange all vortices carry the same amount of fluid \( (kd^2) \). Therefore the vortices originating from the small couple entrain an area of fluid \( S_+ \approx 2k d \epsilon_1 \), while an area \( S_d = 2k d \epsilon_2 \) is detrained from the vortices originating from the large couple. In the modulated case the new couples return to the symmetry line of the system and, after a second interaction, the original couples are reestablished. As a consequence, the small couple detrains the fluid captured during the first partner exchange and only retains its original fluid, while the large couple has to compensate the large portion of its original mass lost during the interaction by entraining ambient fluid.

Figure 12(a) illustrates a collision of a large ETD against a small WTD [the trajectory is shown in Fig. 4(a)]. The broken lines represent the fluid trapped by the original couples, the thin lines an arbitrarily chosen streamline close to the separatrices. As the new couple reaches its northernmost position, these patches are advected to the position indicated by the thick lines. The fluid patch carried by the
WTD has suffered little distortion, while the patch carried by the ETD is strongly deformed, and a large fraction has already been detrained. The broken line (hardly visible here) gives the current shape of the separatrix of the newly formed couple. Equivalent features are observed when a large WTD collides with a small ETD (Fig. 12(b), the vortices' trajectories being shown in Fig. 4(b), but now it is the ETD that preserves most of its original fluid, while the WTD loses a large fraction of it and, consequently, entrains a corresponding area of ambient fluid as the couple is reestablished.

In summary, when two equal coaxial couples collide the ETD exchanges fluid with the surroundings while the WTD preserves its mass; and when the couples have a significant difference in size the larger couple exchanges large amounts of fluid, while the smaller couple preserves most of its initial mass.

2. Parallel couples

The exchange of mass caused by the interaction of parallel couples is larger than in the case of the head-on collision of equal couples. Here the process can be divided in different stages. The (first) partner exchange occurs very much like that in the nonmodulated case, and so does the exchange of mass, which implies that only masses of fluid close to the initial separatrix change of region as a result of the interaction (also see Meleshko et al.19). The vortices that have the closest approach exchange the largest amount of fluid. After this partner exchange the evolution strongly depends on the departure angle $\alpha_0$, as has been described in the previous section. When only one exchange of partners takes place, the newly formed couples have an independent behavior, moving periodically around the equilibrium latitude while propagating in the zonal direction. Consequently, in this case mass transport is governed by the periodic perturbation of the vortex strengths, as discussed in VFvHC. On the other hand, when the couples exchange partners twice mass transport during the looping phase is similar to that due to a head-on collision, where the ETD exchanges significant amounts of mass, while the WTD carries with it most of its original fluid (see Sec. II D 1). However, after the second partner exchange each couple has a meandering motion; therefore mass exchange at this stage occurs again as in the single-dipole case (VFvHC).

When multiple interactions take place the evolution of the fluid initially trapped by the couples is more complicated. As a result of the various interactions—especially the direct scattering ones—most fluid initially carried by the couples will be detrained. This is illustrated in Fig. 13, which shows the evolution of fluid patches carried by the point vortices of which the trajectory is shown in Fig. 7(a). The evolution of the fluid patch to the first return to the equilibrium latitude is shown in Figs. 13(a)–13(d). The regions have suffered almost no deformation in frames (a)–(c), but after the direct scattering interaction of the new couples the two negative vortices have exchanged a large fraction of their mass [as is illustrated in Fig. 13(e), which is an enlarged representation of frame (d)]. This large mass exchange is the result of...
the close approach between the two vortices, but more importantly, it is a consequence of their like-signed circulation.

III. INTERACTION OF CONTINUOUS DIPOLES

A. Experimental results

The dynamical equivalence between the bottom topography and the gradient of the Coriolis parameter is used to study the interaction of β-plane dipoles in the laboratory. The experimental setup consists of a rectangular tank (100×150 cm$^2$) mounted on a rotating table, as described in VFvH. The tank is filled with tap water up to a height of about 20 cm, and an inclined plate (with gradient 0.04) placed on the bottom provides the topographic β effect. Two dipoles moving vertically (i.e., along lines of equal depth) were simultaneously generated by dragging two small bottomless cylinders through the fluid while gradually lifting them (see VFvH).

The objective was to produce a head-on collision of two equal dipoles moving perpendicularly to the gradient of ambient vorticity. However, from the knowledge about the behavior of a single dipole it was expected that achieving such an interaction would be difficult. In the first place one has the different stability properties of the dipole trajectory: the point-vortex model and laboratory experiments show that an ETD has a stable trajectory, whereas that of a WTD is unstable (see, e.g., Makino et al.,11 VFvH), i.e., it is easy to produce a dipole traveling straight to the east, but a dipole initially traveling west acquires a latitudinal velocity component and makes a large looping motion (VFvH). A second difference is that, as a consequence of the generation of relative vorticity in the ambient fluid, the ETD broadens and becomes slower, whereas the WTD shrinks and moves faster. These features are not displayed by the point-vortex model, but have been observed in experiments and numerical simulations (VFvH). The two asymmetries make it difficult to achieve a collision of two equal dipoles: in any real situation it is more likely that the collision will occur between unequal dipoles, propagating along nonparallel directions.

Even if two equal couples collide with zero impact parameter, there exists a further hindrance for the realization of a soliton-like collision on a topographic β plane (e.g., a laboratory tank with a sloping bottom). It consists in the following: the "gradient of ambient vorticity" on a topographic β plane depends on $1/h$ (where $h$ is the fluid depth); therefore the newly formed couples encounter different local gradients of ambient vorticity as they move uphill (north) or downhill (south). The northward moving couple experiences a larger β than the couple moving southward, which leads to a northern loop with smaller amplitude and a shorter length than the southern one. The couples return to their equilibrium latitude with a nonzero impact parameter, therefore after being re-established the original couples have a small perturbation with respect to their original direction of propagation. Note that this asymmetry also appears on a rotating planet, where the gradient of ambient vorticity is proportional to the cosine of the latitude. However small, these effects will prevent the realization of a soliton-like interaction, as we have confirmed during the numerous experiments performed, some of which are described below.

1. Visualization experiments

In a first series of experiments, the motion was visualized with dye, and the flow evolution was recorded with photo or video cameras. The positions of the vortex centers were determined from the apparent center of rotation of each vortex. This was done at fixed time intervals (5 s) when the experiment was recorded on video, and at variable times (10 or 15 s) when a photo camera was used.

Figure 14 shows several stages of the interaction between an ETD and a WTD. At time $t=2.25T$ (with time scaled by the rotation period of the table $T=11.1$ s) the couples are approaching each other. At this stage the ETD is slightly larger and better developed, while the WTD is relatively small and its anticyclonic half is not completely developed yet [Fig. 14(a)]. At $t=4.5T$ the distances between the four vortex centers are of similar magnitude, this can thus be considered as the moment of partner exchange [Fig. 14(b)]. The cyclonic halves are slightly closer to each other than the anticyclonic halves, which leads to mass exchange between the positive vortices. This mass exchange is a consequence of the tendency to merge displayed by like-signed vortices; however, this process is inhibited by the presence of the other two vortices. As the exchange process is almost completed (at $t=5.4T$) large portions of dyed fluid are exchanged between the positive halves, which also entrain fluid from the new partner [Fig. 14(c)]. In addition to producing a large exchange of mass during the interaction, the misalignment also makes the northern couple move northwestward and the southern couple move southeastward [Fig. 14(d)].

The northern couple performs therefore a large loop before the asymmetry induced by the β plane is able to pull the vortex back to the equilibrium latitude, while the southern couple moves along a more sine-like trajectory [Fig. 14(e)]. At this stage ($t=12.1T$) two lobes are observed at the rear of the southern couple, in addition to the band of fluid connecting it with the northern couple. The lobe that is being detrained by the cyclonic half contains undyed fluid that was entrained during the interaction. In contrast, the northern couple shows large entrainment of undyed fluid, which is located between the two halves and at the front side; and almost no detrainment. In the last stage shown here (at $t=16.21T$) the vortices are approaching their equilibrium latitude [Fig. 14(f)], but the southern couple has advanced a larger distance in the zonal direction and a second collision is not produced in this case.

Figure 15(d) shows the trajectories of the individual vortices during the interaction discussed above, and frames (a)-(c) show three examples of similar collisions, which lead to different departure angles. The initial position of the vortices is indicated by dots, the thick lines represent the trajectories of the cyclonic vortices, and the thin lines those of the anticyclonic partners. In Fig. 15(a) the negative halves of both the ETD and the WTD are approximately aligned when the dipoles collide. After the partner exchange the northern couple has an almost eastward direction of propagation, therefore it is stable and moves along a sine-like trajectory. The southern couple, on the other hand, has an almost westward direction of propagation, it is therefore unstable and makes a loop with a large latitudinal displacement. Figure
FIG. 14: Sequence of experimental images showing some stages of the collision between two antially moving dipolar vortices on a topographic $R$ plane. The dipoles were generated in the zonal direction. The images were taken at times $t=(a) 2.25T$, (b) 4.5$T$, (c) 5.4$T$, (d) 8.1$T$, (e) 12.1$T$, and (f) 16.2$T$ after withdrawing the cylinders, with $T=11.1$ s the period of the rotating table. Experimental parameters: $f=1.13$ s$^{-1}$, gradient of the fluid depth $0.04$, $h_0=0.16$ m, and $\beta=0.28$ m$^{-1}$ s$^{-1}$.

15(b) shows a similar case but with a smaller impact parameter. The newly formed couples have a larger latitudinal component than in case (a), but it is still possible to observe a sine-like path of the northern couple and a cycloid-like path of the southern couple. Figure 15(c) shows a collision with almost zero impact parameter, although the WTD seemed to be larger than the ETD. The new couples return to their equilibrium latitude and a second collision takes place (this can be best observed in the video film of the experiment). The anticyclonic half on the original WTD was
strongly deformed during this interaction, but a second partner exchange did not take place. This experiment is the best approximation of a soliton-like interaction we have observed.

2. Measurements

Experiments using small particles floating on the free surface of the fluid were done in order to study the evolution of the flow field during the interaction process. The velocity field was measured and properties as vorticity and circulation were derived in the way described in VFvH. In the experiment discussed below the mean depth of the water was $h_0=16$ cm, and the sloping bottom had a gradient $s=0.04$. The rotation period of the table was $T=11.1$ s, therefore the Coriolis parameter has a value of $f=1.13$ s$^{-1}$. The equivalent value of $\beta=sf_0/h_0$ is 0.28 m$^{-1}$ s$^{-1}$.

Figure 16 shows the vorticity distribution at four stages in the interaction process. At time $t=2.5T$ [Fig. 16(a)] the exchange of partners has just taken place. The cyclonic vortices are stronger in both couples, and the vortices originally belonging to the WTD are in turn stronger than those originated from the ETD. The ratio of the total positive circulation to the negative circulation, $\varepsilon=\Gamma^+/\Gamma^-$, within the dipoles are $\varepsilon_n=0.95$ and $\varepsilon_s=1.42$, where the subscripts $n$, $s$ denote the northern and the southern couples, respectively. At time $t=5.7T$ [Fig. 16(b)] the southern couple has passed its southernmost position, and it rotates anticlockwise, as can be expected from the stronger positive vortex ($\varepsilon_n=1.39$). On the other hand, the northern couple is just approaching its extreme latitudinal position, as it rotates clockwise due to the stronger negative vortex (the extreme vorticity values have similar magnitude, but the net circulation is negative: $\varepsilon_n=0.41$). At $t=7T$ [Fig. 16(c)] both couples are approaching the equilibrium latitude. The net circulation in the couples has the same sign as in the previous frame, but its magnitude gets closer to zero, i.e., the ratios $\varepsilon_n=0.6$ and $\varepsilon_s=1.26$ approach unity.

The positive vortices have come very close to one another at time $t=10.27$ [Fig. 16(d)], at this stage the negative halves, especially the one of the southern dipole, seem to have been decoupled from the positive halves, making possible a merger event between the two cyclonic vortices. After the merger event between the cyclonic vortices was completed, the negative vortices merged themselves; although in this final stage the flow was very weak.

B. Numerical simulations using a vortex-in-cell method

For comparison, numerical simulations were made using the vortex-in-cell method described by Christiansen. The model solves the two-dimensional potential vorticity equation by representing the field of relative vorticity by a large amount of point vortices, which have a strength modulated on the basis of conservation of potential vorticity (see VFvH). The model is initialized using two equal Lamb dipoles separated by a distance $d$ (the dipole’s radius). The impact parameter is $0.2d$. The vorticity gradient $\beta$ and the size $d$ and maximum vorticity $\omega$ of the dipole were chosen such that the parameter $\beta d/\omega=0.046$ is of the same order as in the experiment discussed in the previous section (0.03–0.05). In the simulation discussed below time has been scaled, as in the experimental situation, by the rotation period of the table.


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FIG. 15. Experimental trajectories of the vortex centers during dipolar collision on a topographic $\beta$ plane. The period of the rotating table ($T=11.1$ s) and the gradient of the fluid depth (0.04) were the same in all experiments shown; the mean depth $h_0$ (in cm) and the equivalent $\beta$ value (in m$^{-1}$ s$^{-1}$) are, respectively, (a) 18, 0.25; (b) 16, 0.28; (c) 18, 0.25; and (d) 16, 0.28.

FIG. 16. Experimental measurements of vorticity distributions at several stages of the interaction of two dipolar vortices on a topographic $\beta$ plane. Experimental parameters are $T=11.1$ s, $f=1.13$ s$^{-1}$, $h_0=16$ cm, and $\beta=0.28$ m$^{-1}$ s$^{-1}$. Graphs are shown for $t=(a) 2.5T; (b) 5.7T; (c) 7T; and (d) 10.2T.$
observed in the form of thin regions of oppositely signed vorticity at the flanks of the dipoles [Fig. 17(d), \( t = 10.8 \)]. The southern dipole has a larger eastward velocity component than the northern couple [Fig. 17(e)], and as a result the couples return misaligned to the initial latitude [Fig. 17(f)].

One of the main features of the method is that the positions of all points are followed in time (i.e., it is a Lagrangian technique). This enables us to follow the location of species during the flow evolution. We will identify the species with the letters \( E \) or \( W \), for point vortices initially located within the ETD or the WTD, respectively, and the subindexes \( p \) or \( n \) to indicate the initial sign of the vortex’ circulation. For example \( E_p \) denotes the vortices located initially in the cyclonic (positive) half of the ETD.

The initial conditions in this case approximate those of the experiment shown in Fig. 14. The numerical parameters and the initialization of the vorticity field are equal to those in the numerical experiment discussed above, except for the impact parameter, which is 0.4\( d \) is this case. Figure 18(a) shows the distribution of species corresponding with the experimental situation shown in Fig. 14(e). Species \( E_p \) is represented by red dots, \( E_n \) by yellow dots, \( W_p \) by blue dots, and \( W_n \) by green dots. The largest mass exchange occurs between the two cyclonic vortices. A thin ring formed by species \( E_p \) (red) surrounds the core formed by species \( W_p \) (blue), while a similar ring of species \( W_p \) surrounds the core \( E_p \). This is the result of their close approach during the partner exchange. The core \( W_n \) (green) shows a long tail, where one can also find species \( E_p \) and \( W_p \). Similarly, the core \( W_p \) has a tail that also contains species \( E_p \). These long tails, as well as the cusp observed where the two tails encounter each other, are partially the result of the shrinking process undergone by the WTD in the short time elapsed before the interaction. Ambient fluid is found in the interior of both newly formed couples, in the form of thin rings surrounding the vortex cores in the southern couple and as a relatively thicker intrusion of uncolored fluid separating the two halves of the northern couple.

The same experiment was simulated using two parallel point-vortices couples. The parameter \( \rho d \omega = 0.05 \) used here is a typical value in experimental situations, and the impact parameter 0.4\( d \) was chosen as in the vortex-in-cell simulation above. Two fluid contours per point vortex were followed: one is initially located close to the separatrix and the second one is closer to the point vortex. This is done in order to show that the vortices retain an area of fluid that undergoes little deformations, whereas the regions that take part in mass exchange are thin bands of fluid located close to the separatrix. This simple model captures the main features observed in both the experiment and the vortex-in-cell simulation, namely, vortex trajectories and the large exchange of mass between the cyclonic vortices [Fig. 18(b)]. However, in comparison with the experiment, less fluid is entrained and the tail of the anticyclonic half of the northern couple is almost absent in the point-vortex model.

The collision of dipolar vortices on the \( \beta \) plane has been studied using both laboratory experiments and numerical simulations using modulated point vortices. The point-vortex
FIG. 18. Numerical simulations of the dipole collision experiment shown in Fig. 14(e). (a) Vortex-in-cell method: Position of point vortices initially located in the interior of the ETD (red: positive; yellow: negative) and the interior of the WTD (blue: positive; green: negative). Numerical parameters have the same values as in the simulation of Fig. 17, except that the impact parameter is 0.4d. (b) Point-vortex method: deformation of fluid patches initially trapped by the vortices. The color representation is the same as in (a).

model uses only four active particles in the flow and a number of passive contours, whereas a large amount of active particles (up to 250 000) are used in the vortex-in-cell method. The first model renders the qualitative evolution observed in laboratory experiments, whereas the vortex-in-cell model reproduces the flow evolution in more detail, especially the advection of fluid masses.

A discussion of the collision of coaxial point-vortex dipoles, a problem known to be integrable, has been presented. In contrast, the interaction of two equal point-vortex couples with nonzero impact parameter is, in general, not integrable. However, for a large range of the departure angle \( \alpha_0 \) and the gradient of ambient vorticity \( \beta \) the behavior is regular: there is a single exchange scattering with the two newly formed couples having different net zonal speed. A smaller region seems to be sensitive to initial conditions.

In the point-vortex model the head-on collision of two equal couples is not elastic when the trapped fluid is also taken into account. The ETD detrains up to 20% of the original fluid—and consequently entrains the same amount of ambient fluid—while the WTD preserves almost all its original mass. The amount of mass exchanged by the ETD with the ambient fluid increases with \( \beta_0 \) in the range \( 0<\beta_0<0.2 \) and then it slightly decreases. The latitudinal displacement of the detrained fluid, however, decreases with increasing \( \beta_0 \). In contrast, when the sizes of the couples are significantly different, large amounts of fluid are detrained from the larger couple, with the latitudinal displacement of the detrained mass being larger when the WTD is larger. The numerical results suggest that the motion of a passive tracer is regular in the coaxial case.

A large exchange of mass arises during the collision of parallel couples with nonzero impact parameter. The multiple interactions occurring for some initial conditions are very effective in destroying the cores of trapped fluid. In the final state at least one of the couples shows a meandering motion with large amplitude, resulting in large exchange of mass and chaotic particle motion (VFvHC).

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