A Model Reference & Sensitivity Model-based Self-learning Fuzzy Logic Controller as a Solution for Control of Nonlinear Servo Systems

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Abstract: In this paper, design, simulation and experimental verification of a self-learning fuzzy logic controller (SLFLC) suitable for control of nonlinear servo systems are described. The SLFLC contains a learning algorithm that utilizes a second-order reference model and a sensitivity model related to the fuzzy controller parameters. The effectiveness of the proposed controller has been tested in the position control loops of two chopper-fed dc servo systems, first by simulation in the presence of a backlash nonlinearity, then by experiment in the presence of a gravity-dependent shaft load and fairly high static friction. The simulation and experimental results have proved that the SLFLC provides desired closed-loop behavior and eliminates a steady-state position error.

Keywords: Fuzzy Systems, Fuzzy Control, Sensitivity Model, Self-Learning Fuzzy Logic Controller, Nonlinear Servo System

I. INTRODUCTION

Self-organizing fuzzy logic controllers have been successfully applied to various control processes such as pH-neutralization, inverted pendulum, robot drives, induction motor and many others [1-5]. Since Mamdani [1], many researchers attempted to improve the performance of self-organizing mechanisms and establish a more systematic method for designing and tuning the fuzzy controller. Some of them used neural nets and some of them used a reference model [6-10].

The SLFLC which utilizes a sensitivity model and a 2nd-order reference model has been effectively introduced for control of a nonlinear control system [10]. Sensitivity functions as a measure of influence of a particular variable or parameter on the focused variable are used for planning of intervention in the system. But this presumes a differentiable character of a function describing a dependence of two variables (variables and parameters). There is a class of fuzzy controllers widely used in practical applications, usually called the singleton fuzzy controllers, which can be organized to assume an analytical and differentiable form [10].

The paper presents a design, simulation and experimental verification of an SLFLC that is organized by means of a model reference-based and a sensitivity model-based learning mechanism.

The paper is organized in the following way. First a brief description of the structure of a nonlinear positioning (static) fuzzy control system is given. Then the synthesis of a self-learning fuzzy controller is explained. Further we show simulation results obtained in the position control loop of a chopper-fed dc servo system affected by a backlash nonlinearity. Then we show experimental results obtained in the position control loop of a chopper-fed dc servo system affected by a static friction and dynamic gravitation-dependent load variations.

II. DESCRIPTION OF THE CONTROL PROBLEM

Let us consider control of an unknown time-varying nonlinear static high-order SISO control process described as follows:

\[ y(t) = f(y(t-1), y(t-2), u(t-1), u(t-2), t) \]

where \( y(t) \) denotes a measurement noise.

The fuzzy controller used in the SLFLC scheme is a singleton fuzzy controller. How much will the \( j \)-th fuzzy control rule contribute to the defuzzified control output depends on a degree of contribution described with the fuzzy basis function [10]:

\[ \phi_j[e(k), d_y(k)] = \frac{\mu_i[e(k), d_y(k)]}{\sum_{i=1}^{a} \mu_i[e(k), d_y(k)]}, \quad j = 1, 2, ..., r \]

where:

- \( e(k) = u(k) - y(k) \) - system error,
- \( dy(k) = y(k) - y(k-1) \) - change of system output,
- \( p \) - number of fuzzy subsets of \( e(k) \),
- \( q \) - number of fuzzy subsets of \( d_y(k) \),
- \( r = pq \) - number of rules,
- \( \mu_i \) - membership function of a fuzzy implication.

The SLFLC (Fig. 1) contains a nonintegral (PD) fuzzy controller to be organized and a proportional (P) controller:

\[ u(k) = T[e(k), dy(k), \lambda] \]

where:

- \( A_j \) - centroid of the fuzzy controller output subset activated by the \( j \)-th fuzzy rule, \( \phi_j \) - fuzzy basis function of the \( j \)-th fuzzy rule, \( k_s \) - proportional gain coefficient, \( \lambda \) - fuzzy controller parameter vector.

The controller parameter vector \( \lambda \) contains the following parameters: output subset centroid \( A_k \), input subset centroid of \( c \) and \( d_y \), \( \phi_c^* \) and \( \phi_y^* \), and widths of input subsets of \( c \) and \( d_y \), \( w_c^* \) and \( w_y^* \).

Regarding a determination of desired closed-loop dynamic behavior, a second-order reference model is used:

\[ y_r(k) = a_0 u_c(k-1) + a_1 u_c(k-2) + b_0 u_y(k-1) + b_1 u_y(k-2) \]

The synthesis of the SLFLC has a goal to find a set of control rules which would enforce a control system to have a minimal reference model tracking error.
A sensitivity model of a fuzzy controlled system can be built only if fuzzification and defuzzification operations have a form that allows a fuzzy input-output mapping function to assume an analytical and differentiable form. In this sense, an inference engine will utilize a product operator:

$$y[e(k),dY(k)] = \prod_{j=1}^{r} \prod_{i=1}^{p} \mu_j(dY(k))$$

Following the principle of differentiability, membership functions have a form

$$\mu_j^{e_i}(x) = c_i^j x^{q_j} \quad \mu_j^{d_i}(x) = w_i^j x^{q_j}$$

where $c_i^j$ is a centroid of a membership function $\mu_j^e$, and $w_i^j$ is a width of a membership function $\mu_j^d$.

III. DESIGN OF A SELF-LEARNING FUZZY LOGIC CONTROLLER

A total differential of the system output $y(k)$ w.r.t. small variations of controller parameters ($k_0 = \text{const.}$) is determined by

$$\Delta y(k) = \sum_{i=1}^{N} \frac{\partial y}{\partial \lambda_i} \Delta \lambda_i = \sum_{i=1}^{N} \eta_i \cdot \Delta \lambda_i$$

where $\eta_i$ are sensitivity functions related to parameters of the fuzzy controller:

$$\eta_i(k) = \frac{\partial y}{\partial \lambda_i} = G_i \cdot \frac{\partial \mu_i}{\partial \lambda_i}$$

A sensitivity function of the controller output w.r.t. controller parameter variations has the following form

$$\frac{\partial \mu_i}{\partial \lambda_i} = \frac{\partial \mu_i}{\partial \mu_i} \cdot \frac{\partial \mu_i}{\partial \lambda_i} = G_i \cdot \frac{\partial \mu_i}{\partial \lambda_i}$$

Insertion of (10) into (9) and further in (8) yields:

$$\Delta y(k) = \sum_{i=1}^{N} \frac{\partial y}{\partial \lambda_i} \cdot \frac{\partial \mu_i}{\partial \lambda_i} \cdot \frac{\partial \mu_i}{\partial \lambda_i} = \sum_{i=1}^{N} \eta_i \cdot \Delta \lambda_i$$

Equation (11) can be used for assessment of controller parameter variations that would provide a given change of the system output. The given change of the system output $\Delta y(k)$ coincides in the model reference control concept with a tracking error $e_\text{tr}$. In the proposed concept, parameter system variations will be compensated only by modifying the centroid of the fuzzy output sets $A_i$ (input membership functions remain predefined). Therefore,

$$\frac{\partial y}{\partial \lambda_i} = \frac{\partial y}{\partial \mu_i} \cdot \frac{\partial \mu_i}{\partial \lambda_i} = \frac{\partial [\mu_i(e(k),dY(k),A_i)]}{\partial \lambda_i}$$

From (2) and (3), it follows:

$$\frac{\partial y}{\partial \lambda_i} \cdot \frac{\partial [\mu_i(e(k),dY(k),A_i)]}{\partial \lambda_i} = \sum_{i=1}^{N} \mu_i(e(k),dY(k),A_i)$$

Accordingly, equation (8) assumes a form:

$$\eta_i(k) = G_i \cdot [\mu_i(e(k),dY(k),A_i)]$$

It may be seen that transient behavior of all sensitivity functions depends on the dynamic characteristics of the control process, $G_i = \frac{\partial y}{\partial \mu_i}$. Since the exact model of the control process (1) is unknown, a reference model (4) could be assumed for a reasonable process approximation.

If the reference model (4) is representing the process approximation $G_{\text{ref}}$, then sensitivity functions have a form (for simplicity, $\Phi_i(e(k),dY(k),A_i)$ is replaced by $\Phi_i(k)$):

$$\eta_i(k) = \Phi_i(k) \cdot \eta_i(k_0) \cdot \Phi_i(k_1) \cdot \eta_i(k_2) \cdot \Phi_i(k_3)$$

Let us make the following assumptions: the static part of a control process is inherently stable, the boundary values (limits) of the system output $y(k)$ and the tracking error $e_\text{tr}$ are known, the reference input $y(k)$ is imposed as a sequence of alternating step changes. In this case, a learning algorithm has a form

$$\Delta \lambda_i = f_i \left\{ e_\text{tr}(k), \eta_i(k), \Phi_i(k), \lambda_i \right\}$$

where $\lambda_i$ denotes a current learning iteration (i.e. a current run of the system). The meaning of this is that centroids $A_i$ are changed only once during each run of the system. A new run of the system starts with a new change of $y(k)$. It must be noted, that approximate equality in (11) is intentionally replaced by normal equality in (16).

The sensitivity function equation (15) holds for every sampling interval, while the modification of the centroid $A_i$ is executed only once in the learning iteration. Therefore, it is necessary to pick the moment when influence of a particular controller parameter is the highest, and that is in the maximum of the corresponding sensitivity function:

$$\beta_i = \eta_i^{\text{max}}(k) = \max \left\{ \eta_i^{\text{max}}(k) \right\}$$

On insertion of (17) into (16), the learning law assumes a form

$$\Delta \lambda_i = f_i \left\{ e_\text{tr}(k), \beta_i^{\text{max}}, \lambda_i \right\}$$

The speed of learning depends on dynamic characteristics of the sensitivity functions generated during the system output transient response. During a learning interval, it may happen that some sensitivity function reaches a maximum value lower than a predetermined threshold value $\delta$. This means that the corresponding centroid has a negligible influence on the process behavior, and so remains unchanged.
IV. SIMULATION RESULTS

The proposed SLFLC has been tested by computer simulation in case of controlling the position loop of a laboratory dc servo drive ES 130 (Feedback Inc.) affected by the impact of a backlash nonlinearity. The rated parameter values are: $K_p=175$ rad/Vs (amplifier & motor gain), $T_m=160$ms (electromechanical time constant), $N=28.65$ V/rad (feedback gain), $K_p=0.015$ V/V (P gain). The width of a backlash is set to $\pm 3^\circ$.

The parameters of the 2nd-order reference model are determined according to the selected performance indices: overshoot in response, $\sigma_o=1.52\%$, and a time of maximum $t_{m}=5.53$s, which yields (sampling interval $T_s=0.01$ s):

$$y_d(k)=0.8429y_d(k-1)-0.8521y_d(k-2)+$$
$$+0.0047u_d(k-1)+0.0045u_d(k-2)$$ (19)

The model (19) is also used as a part of the sensitivity model (15). Five linguistic subsets with the corresponding membership functions (7) have been defined for both fuzzy controller inputs (normalized universes of discourse $E$ and $D_Y$): LN $(c=-1, w=0.227)$, MN $(c=-0.4, w=0.048)$, Z $(c=0, w=0.01)$, MP $(c=0.4, w=0.048)$, LP $(c=1, w=0.227)$.

The proposed fuzzy control method has been tested in case of a series of step changes of the reference input equal to $A_0=\pm 30^\circ$. Fig. 2a shows the reference model and the measured system output responses obtained after first three runs of the system. The negative effect of a backlash in the measured system output is clearly seen. Figs. 2b and 2c show the tracking error and the controller output responses, respectively.

In the first run, a dynamic behavior is determined only by the proportional controller and the tracking error exceeds 30% of the imposed change of the reference input.

Figs. 3a, 3b and 3c show the same group of responses obtained after thirteen learning iterations (positive runs) of the system. The system follows the reference model very closely and the steady-state tracking error is kept below 5%, thus proving a convergence of the learning process. In addition, the controller output (Fig. 3c) has a very acceptable oscillatory form. The steady-state error is kept within $\pm 1\%$ which is a significant achievement, since the dead-zone of the backlash would normally cause a static error within $\pm 10\%$.

V. EXPERIMENTAL RESULTS

The proposed SLFLC has been tested by a series of experiments in case of controlling the nonlinear position loop of a chopper-fed dc servo drive (Fig. 4). During experiments, the distribution of membership functions (7) has remained the same.

![Fig. 4. Experimental setup of a dc servo drive.](image-url)
The position loop is affected by the impact of a static friction and nonlinear gravitation-dependent shaft load \( T_L = T_{L_0} \sin \theta \) as shown in Figs. 5 and 6. The block structure of the nonlinear control process is shown in Fig. 7. The rated parameter values are:

- \( k_p = 0.015 \text{ V/V} \) (P gain), \( K_v = 1/45 \, \text{rad} \) (feedback gain), \( K_e = 0.191 \) Nm/A, \( K_r = 0.106 \) V/A (armature gain), \( J = 2.7 \times 10^{-4} \) (no load), \( J = 1.27 \times 10^{-5} \) (max. load), \( N = 4 \) (gear ratio). The length of a load bar is 27 cm, and weight of the weight is 0.22 kg.

The parameters of the 2nd-order reference model are determined according to the selected performance indices: overshoot in response, \( \omega_n = 1.52\% \), and time of maximum \( t_m = 0.6 \) s and 0.4 s. The reference model is also used for description of the sensitivity model \( G_{sw} \). Five linguistic subsets have been defined for both fuzzy controller inputs (universes of discourse \( E \) and \( D_Y \)): LN, MN, Z, MP, LP. A linear distribution of the corresponding membership functions (7) has been selected.

The proposed self-learning fuzzy control method has been experimentally tested for a series of step changes of the reference input equal to \( \Delta \theta = \pm 45^\circ \) in two intentionally selected operating points, \( \theta_0 = 0^\circ \) and \( \theta_{\text{max}} = 90^9 \), which correspond to the extremal magnitudes of a position dependent load torque \( T_L \). Fig. 8a shows the reference model \( t_m = 0.6 \) s and the measured position responses obtained after two runs of the system (one in each direction) under full load weight \( T_{L_{\text{max}}} \) conditions in the operating point \( \theta_0 = 0^\circ \). Effects of friction and nonlinear time-varying load reflect in different dynamics for each direction and in the presence of a steady state error. Figs. 8b and 8c show the tracking error and the controller output responses, respectively. In the first run, a dynamic behavior is determined only by the proportional controller (Fig. 1) and the tracking error exceeds 30\% of the imposed change of the reference input.
Fig. 8. Start of learning (0°-90°): the reference model and the system output responses (a), the tracking error responses (b) and the self-learning fuzzy controller output responses (c).

Fig. 9. End of learning: the reference model and the system output responses (a), the tracking error responses (b) and the self-learning fuzzy controller output responses (c).

Fig. 10. Start of learning (0°-90°): the reference model and the system output responses (a), the tracking error responses (b) and the self-learning fuzzy controller output responses (c).

Fig. 11. End of learning: the reference model and the system output responses (a), the tracking error responses (b) and the self-learning fuzzy controller output responses (c).
Figs. 9a, 9b and 9c show the same group of responses obtained after completion of learning (after twelve positive runs of the system). Now the system follows the reference model very closely and the maximum tracking error value is kept below 5%, thus proving a fast convergence of the learning process. In addition, the controller output (Fig. 9c) has a very acceptable nonoscillatory form and the steady-state system error is kept at zero as required.

Figs. 10a, 10b and 10c show the same group of responses obtained after first two runs of the system under full load weight conditions in the operating point $\alpha=90^\circ$. This is the most difficult starting operating point which results in the highest steady state error (over 150% due to the presence of active load which cannot be compensated only by means of a P controller).

Figs. 11a, 11b and 11c show the same group of responses obtained after completion of learning (slightly longer, i.e. after fourteen iterations). The system follows the reference model very closely and the maximum tracking error value is kept below 5%, while the steady-state system error is kept at zero as required. The controller output (Fig. 11c) has retained a very acceptable nonoscillatory form.

More experiments with changed reference model parameters ($\alpha=0.4s$) indicated that the choice of reference model parameters does not influence much the convergence of the learning process. We also tried to improve performance by adding an integral term in parallel to the P controller and SLFLC to raise the steady state accuracy, but responses obtained in the presence of the integral term did not differ much from those obtained without it. Namely, the integral term was activated conditionally (i.e. only if it was necessary) after major part of the learning process was completed.

VI. CONCLUSIONS

The paper presents a self-learning fuzzy logic control (SLFLC) scheme suitable for control of nonlinear astatic systems. A learning law is based on the use of a reference model and a sensitivity model built w.r.t. to the fuzzy controller parameters. Simulation and experimental results obtained in the nonlinear position control loop of a dc servo motor drive have confirmed a fast convergence of learning and effective control without steady state errors and with very close following of the reference model dynamics.

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VIII. REFERENCES


VIII. BIOGRAPHIES

Zdenko Kovacic (1958) received his Ph.D. in 1993, M.S.E.E. in 1987 and B.S.E.E. in 1981, at University of Zagreb, Croatia. He is currently an assistant professor at The Faculty of Electrical Engineering and Computing, University of Zagreb. His areas of interest are intelligent, adaptive and optimal control systems, control of power converters, electrical drives and robots. During 1990-1991, he spent one year working as a visitor researcher in the Motion Control Laboratory of prof. dr. Kristman Ramus at the Bradley Department of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, USA. Currently, he works on development and implementation of self-learning and adaptive control algorithms applied to the electrical motor drives and robots. He has been an author and coauthor of numerous papers published in journals and presented at the national and international conferences.

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