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Application of computed phase transformation power to control shape memory alloy actuators

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Abstract. This paper presents a control law for shape memory alloy (SMA) wire actuators. Unlike most laws reported in the literature, this control law includes both a feedback and an open-loop part. The open-loop part takes into account the stress–strain–temperature behaviour of SMA wires, as well as the dynamics of the heating and cooling process. A constitutive model for a NiTi wire is developed. This model is a compromise between accurate description of the real behaviour, and simplicity, since it must be used on-line for the determination of the open-loop part of the control law. The method is tested both in simulations and experiments on a system with one degree of freedom. The tracking performance of the controlled system was investigated for a number of trajectory-following tasks. The tracking error obtained with the proposed control method is considerably smaller than with feedback control only.

1. Introduction

For many years research has been done on the relationship between the macroscopic properties of materials and their microstructure. The results are used to improve existing materials and to develop completely new materials, a technology which has become known as ‘material design’. In recent years, attention has been shifting to the development of materials whose mechanical behaviour can be changed during service to meet certain functional or structural requirements. Shape memory alloys (SMAs) form an important class of such materials. Their mechanical properties can be changed by altering their temperature. This behaviour results from a change of the microstructure—more specifically a phase transformation [1–3].

One of the best studied and most used SMAs is the nickel–titanium alloy NiTi. By raising the temperature of an unconstrained, previously deformed NiTi wire, its length can be reduced. Strains of about 8% can be recovered. If the wire is clamped at both ends, a large force can be exerted. Hence NiTi wires can be used as actuators in dynamical systems—for example, to adapt the geometry of flexible structures [4,5], and actively damp vibrations [6,7], and for use in robotic devices [8,9].

Active control of the temperature of NiTi actuators is most easily achieved by means of resistive heating. Most of the control methods reported in the literature apply a feedback (FB) control law: the electric power depends on the difference between the measured and the desired value of certain variables. Application of feedback control for active damping of vibrations is demonstrated in [10–13], and for position control in robotics in [14–17].

Very few attempts have been made to add an open-loop control term, based on a mathematical model of the entire system, to improve performance. In [18, 19] a combination of feedback and open-loop control is applied, where the open-loop part takes into account the hysteretic behaviour of NiTi. However, it is based on steady-state conditions, and neglects the first-order nature of the heating and cooling process.

The purpose of this paper is to demonstrate that application of an open-loop term, based on a mathematical model of the stress–strain–temperature behaviour of NiTi actuators and the dynamics of the heating and cooling process, improves the performance of NiTi actuators.

2. Description and modelling of the SMA behaviour

The behaviour of NiTi is due to a change of the crystal structure from austenitic to martensitic and vice versa. When an austenitic NiTi wire is subjected to a number of tensile test cycles at a relatively high temperature, tensile curves like those in figure 1 are obtained. The deformation of the wire is described with the Green–Lagrange strain $e = (\lambda^2 - 1)/2$, where $\lambda = l/l_{\text{ref}}$: $l$ is the current length of the wire and $l_{\text{ref}}$ the reference length of the austenitic, unloaded wire at the reference temperature $\theta_{\text{ref}}$. The stress in the wire is represented by the second Piola–Kirchhoff
stress \( p = F/(\lambda A_{\text{ref}}) \), where \( F \) is the force in the wire, and \( A_{\text{ref}} \) is the reference cross sectional area. For modelling purposes the curved lines in figure 1 are approximated by straight lines. This is not an essential simplification of the real behaviour, but makes the model less complicated.

The mathematical description of the SMA behaviour in terms of the stress \( p \) and the strain \( e \) is rather complicated. It can be simplified by using alternative variables. One of these variables is the martensite fraction \( m \), which is the relative amount of martensite. The domain of this variable is \([0, 1]\), where \( m = 0 \) if the material is purely austenitic and \( m = 1 \) for a completely martensitic structure. The second alternative variable \( \xi \) is such that the rectangle with vertices 1, 4, 6 and 9 in the \((e, p)\)-plane is mapped onto a unit square in the \((m, \xi)\)-plane (see figure 1).

Starting at point 0 \((m = 0, \xi < 0)\), the austenite deforms elastically. When the ‘martensite start’ transition stress \( p_{\text{ms}} \) is reached \((point 1, m = 0, \xi = 1)\), martensite starts to form. Upon further loading, \( \xi = 0, m > 0 \), and strain increases considerably. The austenite–martensite \((AM)\) transformation ends when the stress equals \( p_{\text{ms}} \) at point 4 \((m = 1, \xi = 0)\). Further loading results in elastic deformation of the martensite \((m = 1, \xi > 0)\).

Unloading from point 5 does not yield the same curve as that followed during loading. The reverse MA transformation does not start until the stress has dropped to \( p_{\text{as}} \) \((point 6, m = 1, \xi = 0)\), which is significantly lower than \( p_{\text{af}} \). During the MA transformation, which ends at point 9 \((m = 0, \xi = 0)\), the martensite fraction decreases from 1 to 0. Continued unloading from point 9 results in elastic deformation of the austenite along the line through points 9 and 0 \((m = 0, \xi < 0)\). Hence, a closed hysteresis loop occurs in such a loading cycle.

If unloading is started before the austenite has been transformed entirely into martensite \((0 < m < 1, \xi = 1)\), e.g. at point 2 or 3, the material is unloaded elastically \((m = 0, \xi < 0)\). If reloading is resumed before the lower MA side \((\xi = 0)\) is reached, the same elastic path is followed \((m = 0, \xi > 0)\) until the upper AM side \((\xi = 1)\) is encountered. Continued loading then involves continued AM transformation. Within the loop \((0 < \xi < 1)\) the martensite fraction cannot change, whereas \( m \) can increase, on the AM side \((\xi = 1)\), and decrease, on the MA side \((\xi = 0)\).

It is noted that the set \( D \) of all possible tuples \((m, \xi)\) consists of the unit square \([0, 1] \times [0, 1] \) plus the line segments \([0] \times (-\infty, 0) \) and \([1] \times (1, \infty) \). For \((m, \xi) \in D \) the relation between the strain \( e \) and the stress \( p \) on the one hand and the alternative variables \( m \) and \( \xi \) on the other is given by

\[
e = e_{\text{af}} + \xi (e_{\text{ms}} - e_{\text{af}}) + m (e_{\text{as}} - e_{\text{af}}) + \xi m (e_{\text{mf}} + e_{\text{af}} - e_{\text{ms}} - e_{\text{as}}) \]

\[
p = p_{\text{af}} + \xi (p_{\text{ms}} - p_{\text{af}}) + m (p_{\text{as}} - p_{\text{af}}) + \xi m (p_{\text{mf}} + p_{\text{af}} - p_{\text{ms}} - p_{\text{as}}). \]

The transition strains \( e_{\text{ms}} \) to \( e_{\text{af}} \) and transition stresses \( p_{\text{ms}} \) to \( p_{\text{af}} \) strongly depend on the temperature \( \theta \). It is assumed that this dependence is linear. Hence if the values of the parameters are known for two different temperatures \( \theta_1 \) and \( \theta_2 \), then their values for any relevant temperature \( \theta \) can be determined. For example, the martensite start stress \( p_{\text{ms}} \) at temperature \( \theta \) follows from

\[
p_{\text{ms}}(\theta) = p_{\text{ms}}(\theta_1) + \frac{\theta - \theta_1}{\theta_2 - \theta_1} [p_{\text{ms}}(\theta_2) - p_{\text{ms}}(\theta_1)]. \]

Analogous relations hold for the other transition parameters. Experimentally obtained values are listed in table 1.

Figure 2 illustrates the constitutive model for a NiTi wire in the \((e, p, \theta)\)-space (left), and in the \((m, \xi, \theta)\)-space (right). Changes of state within the hysteresis loop are restricted to planes of constant martensite fraction. The shaded planes in figure 2 represent two planes of this kind.

The rates \( \dot{e} \) and \( \dot{p} \) of change of the strain \( e \) and stress \( p \), respectively, are related to the rates \( \dot{m} \) and \( \dot{\xi} \) through equations (1)–(3):

\[
\dot{e} = e_m (\xi, \theta) \dot{m} + e_{\xi m} (m, \theta) \dot{\xi} + e_{m \theta} (m, \xi) \dot{\theta} \]

\[
\dot{p} = p_{\text{ms}} (\xi, \theta) \dot{m} + p_{\xi m} (m, \theta) \dot{\xi} + p_{m \theta} (m, \xi) \dot{\theta} \]

where \( e_{m \theta} \) stands for the partial derivative of \( e \) with respect to \( \theta \), etc. The variables \( m \) and \( \xi \) are defined such that they
Table 1. SMA parameters.

<table>
<thead>
<tr>
<th>$\theta_1$ (°C)</th>
<th>$\theta_2$ (%)</th>
<th>$\theta_{m}$ (%)</th>
<th>$\theta_{e}$ (%)</th>
<th>$\rho_{ms}$ (MPa)</th>
<th>$\rho_{p}$ (MPa)</th>
<th>$\rho_{p}$ (MPa)</th>
<th>$\rho_{af}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.08</td>
<td>5.43</td>
<td>4.33</td>
<td>0.39</td>
<td>234</td>
<td>369</td>
<td>218</td>
</tr>
<tr>
<td>70</td>
<td>1.62</td>
<td>6.45</td>
<td>4.70</td>
<td>0.09</td>
<td>369</td>
<td>561</td>
<td>369</td>
</tr>
</tbody>
</table>

Figure 2. A three-dimensional representation of all possible states; on the left in $(e, \rho, \theta)$ coordinates, and on the right in $(m, \xi, \theta)$ coordinates.

never change simultaneously, i.e. that

$$m(t) \dot{\xi}(t) = 0 \quad \forall t.$$  \hspace{1cm} (6)

In fact, $m \neq 0$ can occur only on the AM side ($\xi = 1$) and on the MA side ($\xi = 0$) of the hysteresis loop. On the AM side two situations may arise: either $m > 0$ and $\dot{\xi} = 0$ (continued AM transformation), or $m = 0$ and $\dot{\xi} < 0$ (elastic unloading). Similar remarks hold for the MA side. Then either $m < 0$ and $\dot{\xi} = 0$ (continued MA transformation), or $m = 0$ and $\dot{\xi} > 0$ (elastic loading).

3. The system description

The system considered consists of a rigid body with weight $Mg$ suspended by a NiTi wire (see figure 3, left).

At the initial time $t = 0$ the body is at rest, and the wire is not heated. The length of the wire is $l_0$. The initial elongation factor $\lambda_0$, initial strain $e_0$ and initial stress $p_0$ are given by

$$\lambda_0 = \frac{l_0}{l_{ref}}, \quad e_0 = \frac{1}{2}(\lambda_0^2 - 1), \quad p_0 = \frac{Mg}{A_{ref}\lambda_0}.$$ \hspace{1cm} (7)

The initial values of $m_0$ and $\xi_0$ depend on the deformation history of the wire. The weight of the body is such that MA transformation and shortening of the wire occur upon heating, whereas AM transformation and elongation of the wire take place during cooling. By ensuring that AM transformation (cooling) was the last process before $t = 0$, the state is known to be either on the AM side of the hysteresis loop ($\xi_0 = 1$), or beyond the ‘martensite finish’ point ($m_0 = 1$). The remaining two unknowns, $\theta_0$ and $m_0$ or $\xi_0$, can be determined using the constitutive equations (1)–(3).

Temperature changes in the wire affect its length, and therefore the vertical position of the body. The position of the body is denoted by $q$, which is equal to the length of the wire $l$ (see figure 3, right). Because of the fairly large time constant (several seconds) of the cooling process of a NiTi wire, the length of the wire and therefore the position of the body can only change rather slowly. Therefore inertia effects are neglected. Then the internal force $F_{sma}$ in the wire is equal to the weight $Mg$ of the body. The strain $e(t)$ and the stress $p(t)$ corresponding to a position $q(t)$ follow from

$$e(t) = \frac{1}{2}\left(\frac{q(t)}{l_{ref}}\right)^2 - 1 \quad \text{and} \quad p(t) = \frac{Mg}{A_{ref}q(t)}.$$ \hspace{1cm} (8)

Therefore, if $q(t)$ is given for $t \geq 0$, then $e(t)$ and $p(t)$ can be determined from these relations.

The model adopted for the thermal processes in the wire is a linear one. It is assumed that the temperature satisfies [20]:

$$\tau \theta + (\theta - \theta_\infty) = cP \quad \Rightarrow \quad P = I^2R$$ \hspace{1cm} (9)

where $\tau$ is the time constant of the thermal process, $c$ depends on the geometry of the wire and environmental circumstances, $\theta_\infty$ is the temperature of the surrounding
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For the system in figure 3 the electric current $I$ is the input of the system, whereas the position of the body $q$ is the output. The control objective is to force the position $q = q(t)$ to follow a specified, a priori known, desired trajectory $q_d = q_d(t)$. This can only be achieved by changing the temperature of the wire, either actively or passively. In the first case $I \neq 0$ and, apart from current limitations, any temperature rate $\dot{\theta}$ greater than or equal to $- (\theta - \theta_\infty)/\tau$ can be realized. Passive control corresponds to $I = 0$ and thus $\tau \dot{\theta} = - (\theta - \theta_\infty)$. It is impossible to realize any $\dot{\theta}$ smaller than $- (\theta - \theta_\infty)/\tau$.

The position $q$ is measured by a linear variable differential transformer (LVDT), and converted to a digital value by a 12-bit AD converter, with a resolution of 6 $\mu$m. An AT 386 PC equipped with a PCL718 data acquisition card calculates the input, i.e. the electric current $I$. The value obtained for $I$ is converted to an analogue signal, and transmitted to a servo-amplifier. The resolution for the current is 0.5 mA. The maximum current allowed, $I_{\max}$, is 1.0 A.

The temperature of the wire is not measured but nevertheless plays an important role in the determination of the input $I$. One of the objectives of the research reported here is to investigate whether it is possible to accurately control the system of figure 3 without a temperature sensor.

4. The control law

As mentioned before, the control objective is to force the position $q = q(t)$ of the body to track a prescribed path $q_d = q_d(t)$. This must be achieved by prescribing the electric current through the wire or alternatively the electric power $P$. For the system considered a model is available, consisting of equation (9) for the temperature $\theta$, relation (8) between the position $q$ on one hand and the strain $e$ and stress $p$ on the other, and the constitutive equations (1) to (3) given earlier.

Using this model, the power $P_{ol}(t)$ required to track the desired trajectory $q_d(t)$ can be calculated in advance. This power can be seen as the model-based, open-loop part of the input of the system. Due to modelling errors and unmodelled external disturbances, the open-loop part $P_{ol}$ alone will not result in perfect tracking. Therefore a feedback term, $P_{fb}$, that depends on (time derivatives) of the tracking error $e_q = q - q_d$ is added, resulting in the following relation for the input $P$ of the system:

$$P = \begin{cases} P_{ol} + P_{fb} & \text{if } P_{ol} + P_{fb} > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (10)$$

Figure 4 is a block diagram of the controlled system. In the following section first the open-loop part is considered. Next some remarks with respect to the feedback part are made.

4.1. The open-loop part

This section presents the derivation of the open-loop input. Figure 5 is a flow chart of this computation. At the initial
time \( t = 0 \) the state of the material \((m_0, \xi_0, \theta_0)\) is known. Therefore the strain \( e_0 \) and the stress \( p_0 \) are also known. The desired strain \( e_d(t) \) and the desired stress \( p_d(t) \) for \( t \geq 0 \) can be determined from (8) by replacing \( q(t) \) in these equations by \( q_d(t) \). For the rates \( \dot{e}_d \) and \( \dot{p}_d \) this implies that
\[
\dot{e}_d = \frac{q_d}{I_{ref}} \quad \dot{p}_d = -\frac{p_d}{q_d} \quad (11)
\]
Substitution of these relations in (4) and (5) results in two equations for the three unknown rates \( \dot{m}_d, \dot{\xi}_d, \text{and} \dot{\theta}_d \), with the condition \( \dot{m}_d(t) \dot{\xi}_d(t) = 0 \) for all \( t \geq 0 \). The desired state \((m_d(t), \xi_d(t), \theta_d(t))\) for \( t \geq 0 \) can be determined by (numerical) integration. The integration algorithm adopted is based on the fact that \( \dot{m}_d \) can be unequal to 0 only if \( 0 \leq m_d < 1, \xi_d = 1, \text{or} \ 0 < m_d \leq 1, \xi_d = 0 \). Hence in each step of the integration algorithm it is assumed first that \( \dot{m}_d = 0 \). Then \( \dot{\xi}_d \) and \( \dot{\theta}_d \) are solved using (1)–(3). Next it is checked whether or not the assumption that \( \dot{m}_d = 0 \) is justified; the assumption has to be rejected if \( \xi_d \notin [0,1] \) this implies that the AM or MA boundary is encountered, and \( \xi_d = 0 \) or \( \xi_d = 1; \) \( m_d \) and \( \dot{\theta}_d \) then follow from (1)–(3).

The temperature rate \( \dot{\theta}_d \) is determined in a similar way. Assuming \( \dot{m}_d = 0 \), equations (4) and (5) yield \( \dot{\theta}_d \):
\[
\dot{\theta}_d = \frac{p, \dot{e}_d - e, \dot{p}_d}{p, \dot{e}_d - e, \dot{p}_d} \quad (12)
\]
If either \( 0 \leq m_d < 1, \xi_d = 1 \text{ and} \dot{\xi}_d > 0 \), or \( 0 \leq m_d \leq 1, \xi_d = 0 \text{ and} \dot{\xi}_d < 0 \), AM and MA transformation respectively takes place. Then \( \dot{\xi}_d = 0 \), and \( \dot{\theta}_d \) reads
\[
\dot{\theta}_d = \frac{p, m \dot{e}_d - e, m \dot{p}_d}{p, m \dot{e}_d - e, m \dot{p}_d} \quad (13)
\]
With this algorithm, which resembles very much the well established integration algorithms for elastic–plastic material behaviour [21], the desired temperature \( \theta_d(t) \) and desired temperature rate \( \dot{\theta}_d(t) \) can be determined for all \( t \geq 0 \). Finally, the open-loop part of the input \( P_{ol} \) follows from the thermal equation (9):
\[
P_{ol} = \frac{1}{c} (\tau \dot{\theta}_d + \theta_d - \theta_\infty) \quad (14)
\]
Figure 6 is an illustration of this procedure. A sawtooth trajectory is chosen for \( q_d(t) \) (top left). The desired temperature \( \theta_d(t) \) (middle), the desired temperature rate \( \dot{\theta}_d(t) \) (bottom left), and the open-loop input (bottom right) are shown.

The desired temperature rate has a small value during AM or MA transformation (equation (13)), e.g. between points 2 and 3, while the opposite is true during elastic deformation within the hysteresis loop (equation (12)), between points 1 and 2, and 3 and 4 respectively (dashed lines). Upon switching from (13) to (12) and vice versa, a large discontinuous step is encountered. As a result the
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Figure 6. The desired trajectory (top left); desired temperature (middle); desired temperature rate (bottom left); with (solid), and without (dashed) modification; and open-loop input (bottom right): without (chain), and with the unmodified (dashed) and modified (solid) temperature rate.

Table 2. System and control parameters.

<table>
<thead>
<tr>
<th>$l_{ref}$ (mm)</th>
<th>$A_{ref}$ (mm²)</th>
<th>$M$ (kg)</th>
<th>$\tau$ (s)</th>
<th>$c$ (°C W⁻¹)</th>
<th>$\theta_\infty$ (°C)</th>
<th>$R$ (Ω)</th>
<th>$I_{max}$</th>
<th>$f_s$ (Hz)</th>
<th>$K_p$ (W m⁻¹)</th>
<th>$K_i$ (W m⁻¹ s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>178.5</td>
<td>0.06</td>
<td>1.5</td>
<td>4</td>
<td>50</td>
<td>22</td>
<td>5</td>
<td>1.0</td>
<td>20</td>
<td>4000</td>
<td>4000</td>
</tr>
</tbody>
</table>

desired electric power changes discontinuously as well at these moments. This is a useful effect at the start of elastic deformation, i.e. at points 1 and 3, because the hysteresis loop is crossed more quickly this way, and a higher response speed can be gained. However, when the opposite side of the hysteresis loop is reached and AM or MA transformation starts, i.e. at points 2 and 4, a similar, opposite step in the electric power occurs. Such a sharp change is undesirable at these points because in reality the transition from an elastic to a transformational response is rather smooth, demanding a smoothly changing power. To obtain a more gradually varying power at the start of AM and MA transformation, the expression for the desired temperature rate within the hysteresis loop given by (12) is modified by means of linear interpolation, such that at the start of elastic deformation equation (12) is valid, and at the end equation (13):

$$\dot{\theta}_d = a \frac{p_{\xi} \dot{\xi}_d - e_{\xi} \dot{p}_d}{p_{\xi} \dot{e}_{\xi} - e_{\xi} \dot{p}_{\phi}} + (1 - a) \frac{p_m \dot{e}_d - e_{m} \dot{p}_d}{p_m \dot{e}_{\phi} - e_{m} \dot{p}_{\phi}}$$

(15)

with

$$a = \begin{cases} \xi & \text{if } \dot{\xi} < 0 \\ 1 - \xi & \text{if } \dot{\xi} > 0. \end{cases}$$

(16)

The solid line in figure 6 (bottom left) shows the desired temperature rate calculated using equation (15) instead of (12). The solid line in figure 6 (bottom right) represents the
corresponding electric power. The sharp changes at points 2 and 4 have disappeared, while at points 1 and 3 they are still present. The peaks completely disappear when the temperature rate is very small, i.e. in a quasi-static situation; see the chain line.

In practice the controlled system is a sampled data system. Let \( f_s = 1/\Delta t \) be the sample frequency. It is assumed that the sample interval length \( \Delta t \) is small compared to the time constant \( \tau \) of the thermal process and that the input is constant per sample interval (zero-order hold). In each sample interval, say the interval from \( t = t_i \) to \( t = t_{i+1} = t_i + \Delta t \), the desired temperature \( \theta_d(t_{i+1}) \), the desired values of \( m_d(t_{i+1}) \) and \( \xi_d(t_{i+1}) \), and their rates \( \dot{\theta}_d(t_{i+1}) \), \( \dot{m}_d(t_{i+1}) \) and \( \dot{\xi}_d(t_{i+1}) \) at the end of the interval are calculated, using the integration algorithm outlined.

4.2. The feedback part

To achieve a small tracking error in the presence of modelling errors and external disturbances, a feedback term has to be applied as well. A proportional-integral control law with respect to the tracking error appeared to be satisfactory:

\[
P_{\text{fb}} = K_p e_q + K_i \int_0^t e_q \, dt.
\]  

To prevent the integral action from excessive build-up, an anti-wind-up mechanism [22] is introduced: the integral part is not allowed to change when the electric power is very large. The values of the control gains \( K_p \) and \( K_i \) are listed in table 2.

Figure 7. Simulation results for sine-shaped trajectories.
5. Results

The system in figure 3 has been investigated both numerically and experimentally. Simulations have been performed with MATLAB. For the experiment, MATLAB is used for the off-line computations during pre- and postprocessing, while the actual controller is implemented in a C-code. The set up of the programs was the same for both cases. The values of the parameters, used both in the simulations and the experiments, are listed in table 2.

5.1. Simulations

The control task that is taken as an example is that of forcing the mass to track a sinusoidal trajectory with a period of 20 s and an amplitude of 2 mm, during a total time of 40 s. The simulation results are shown in figure 7.

The mass of 1.5 kg is such that the wire consists entirely of martensite at this temperature; the strain equals 4.5% and $\xi$ equals 2.5. The peak in the electric power during the first second is entirely due to the proportional action of the feedback controller. This causes the temperature of the wire to rise sharply. As soon as $\xi$ has become zero, the MA transformation starts, causing the wire to shrink. After one second the tracking error has vanished and the total power supplied to the wire corresponds to the calculated desired power. Meanwhile the martensite fraction decreases to 0.2 at $t = 10$ s. At that moment the temperature must drop considerably, because then $m = 0$ and $\dot{\xi} > 0$, i.e. the hysteresis loop has to be crossed. Because the necessary temperature fall cannot be accomplished, the wire elongates more slowly than it should. Between $t = 10$ s and $t = 13$ s, $\xi$ increases from zero to one, until at $t = 13$ s the opposite side of the hysteresis loop is reached, and the realized trajectory catches up with the desired one. The second cycle passes off in the same way.

5.2. Experiments

After the simulations, the proposed control method was tested on the experimental system. The results of experiments with three different trajectories are presented: a sawtooth, a sinusoid, and a block wave, each with an amplitude of 1.5 mm.

Periods of 5, 10 and 15 s were selected for each trajectory. Signals with a period smaller than 5 s cannot be followed, because of the limited cooling rate of the wire. On the other hand, the performance of the controlled system did not benefit from the open-loop control part for periods exceeding 15 s; application of the PI feedback controller alone yielded similar results. The performance of three controller settings was investigated. In all of the cases the PI feedback action was present, but the open-loop component differed. The first controller consisted of the PI...
feedback part only. In the second case the open-loop term was based on steady-state conditions, i.e. the temperature rate was not used for the determination of the open-loop term. The open-loop component of the third controller was determined with (14), where the temperature rate was included.

Some typical results are presented in figure 8. The tracking errors obtained with the three controller settings during the third and fourth cycles of the trajectories with periods of 5 s are shown. The results show that open-loop control is an effective means of reducing the tracking error. However, if the desired temperature rate is not used to determine the open-loop part of the input (controller 2), a small improvement with respect to the controller with feedback only can be observed. The best results are obtained with the full open-loop controller (3). The peak values in the tracking error that originates during hysteresis crossings are diminished considerably.

To be able to compare the performance of the controllers, the root mean square values of the tracking errors during a number of cycles have been determined, and are shown in figure 9. These values show that the tracking error can be reduced by a factor of 2 to 4 for the sawtooth and sinusoid trajectories. The relatively small improvement for the block wave is not unexpected. The instantaneous changes in \( q_d \) yield a large tracking error and proportional feedback action, that exceeds the bounds of the electric power. Therefore an open-loop term can hardly increase the reaction speed in this case.

6. Conclusions

A control law for the input of NiTi wire actuators is presented. The input consists of a feedback and a model-based open-loop part. The open-loop part takes into account the stress–strain–temperature behaviour of a NiTi wire, as well as the first-order nature of the heating and cooling process. For this purpose a constitutive model is developed that matches the experimentally observed behaviour adequately, yet requires little computational effort to determine on-line the open-loop part of the control law. The method is tested both numerically and experimentally on a system with one degree of freedom.

Experiments with a number of test trajectories indicate that adding open-loop control diminishes the tracking error, in some cases by a factor of four in comparison with that for feedback control only.

References