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Inter-Frame Polar Coding With Dynamic Frozen Bits
Haotian Zheng, Seyyed Ali Hashemi, Bin Chen, Zizheng Cao, and A. M. J. Koonen

Abstract—A new inter-frame correlated polar coding scheme is proposed, where two consecutive frames are correlated-encoded and assist each other during decoding. The correlation is achieved by dynamic configuration of the frozen bits. The frozen bits of the second frame partially depend on the unfrozen bits of the first frame in encoding and the number of bits that are viewed as frozen by decoder is alterable in different decoding modes. Using this new encoding/decoding scheme, a failed decoded frame can be decoded again with extra information which corrects the errors in its highly unreliable unfrozen bits. Thus, the probability of successful decoding is improved. Simulation results show that the performance of the proposed polar codes outperforms that of the classical counterpart significantly with negligible memory and complexity increment.

Index Terms—Polar codes, inter-frame correlated, dynamic frozen bits.

I. INTRODUCTION

POLAR codes were proposed by Arkan in 2008 and have proven to be capacity-achieving for binary-input discrete memoryless channels under successive cancellation (SC) decoding [1]. However, the finite-length performance under SC decoding is not satisfactory. Recent research work has been focused on improving the error-correction performance of polar codes at moderate and short code length.

SC list (SCL) decoding expands the search range of decoding, improving the error-correction performance significantly [2]. In cyclic redundancy check (CRC) aided SCL (CA-SCL) decoding, the source information contains a built-in CRC, which reduces the number of codewords with the minimum Hamming weight, further providing error-correction performance gain [3]. The optimal strategy of CRC design is studied in [4]. SC stack decoding [5] and SC flip decoding [6], [7] bring substantial complexity reduction with respect to SCL decoding but the error-correction performance does not enhance compared with that of SCL decoding. Belief propagation (BP) decoding is an iterative decoding scheme with lower decoding latency [8]. However, there is a gap between its error-correction performance and that of CA-SCL decoding. Polar subcodes with dynamic frozen symbols are proposed in [9], where frozen bits values are determined by some information bits before them in the same frame. This kind of polar coding scheme has large minimum distance, but it only shows advantage over CA-SCL decoding when the list size is sufficiently large (above 32). Star polar subcodes further introduce the concept of star trellis, which enables a parallel decoding and obtains a slight error-correction performance gain with respect to polar subcodes in specific cases [10]. Research on SC flip decoding shows that in most failed decoding cases, the first error occurs in unfrozen bits with low reliabilities, i.e., the ones with small average log-likelihood ratio (LLR) magnitudes. Once it is corrected, the probability of successful decoding increases significantly.

Inspired by this observation, we propose an inter-frame polar coding scheme with dynamic frozen bits. The values of frozen bits are set dynamically in order to make two consecutive frames correlated. The correlation is leveraged to correct the errors in the highly unreliable unfrozen bits of a failed decoded frame. Our results for a polar code of length 1024 and rate 1/2 show a coding gain of 0.28 dB over classical polar coding scheme at the block error rate (BLER) of $10^{-4}$.

II. POLAR CODING AND CA-SCL DECODING

A polar code of length $N$ is constructed with a vector of relative reliabilities of bit indices $v = \{v_0, \ldots, v_{N-1}\}$, where bit index $v_i$ is less reliable than bit index $v_j$ if $i < j$. The classical polar coding scheme divides all $N$ source bits into two sets according to $v$ and the number of unfrozen bits $K$. Unfrozen bits set $A = \{v_{N-K}, \ldots, v_{N-1}\}$ contains the indices of $K$ bits with higher reliabilities, and those of the remaining $N - K$ bits form frozen bits set $A_c = \{v_0, \ldots, v_{N-K-1}\}$. In order to achieve a reasonable error-correction performance for polar codes with finite length, a CRC of length $r$ is concatenated with polar codes and CA-SCL decoding is used. The $r$ CRC bits and the $K - r$ information bits are assigned to the bits with indices in $A$. The frozen bits with indices in $A_c$ are fixed to predefined values known to the decoder.

CA-SCL decoding is a bit-by-bit sequential decoding algorithm that follows the SC decoding schedule. Unlike SC decoding that estimates each unfrozen bit based on its calculated LLR value, CA-SCL decoding estimates each unfrozen bit as either 0 or 1. In order to limit the exponential growth in the complexity of the decoder, at each bit estimation step, $L$ candidate paths are allowed to survive using a path metric. In this letter, we use the LLR-based formulation of the path metric in [11]. After the estimation of the last bit in CA-SCL decoding, CRC is performed on the final $L$ candidate paths. For the paths that pass the CRC, the one with the best path metric is output as the decoding result. If there is no path
The proposed inter-frame assisted (IFA) SCL (IFA-SCL) decoding scheme is summarized in Algorithm 2. It uses an indicator $f$ to indicate whether the decoding succeeded ($f = 0$) or failed ($f = 1$), an indicator $g$ that shows whether consecutive decoding failures occurred ($g = 1$) or not ($g = 0$) in the decoding of the two latest frames, and is composed of four decoding modes as follows:

**M0** Perform CA-SCL decoding on frame $i$ with the frozen bits defined as
\[
\hat{C}^v_{i} = 0, \quad 0 \leq k < N - K.
\]

Set $f$ in accordance with the decoding result.

**M1** Perform CA-SCL decoding on frame $i$ with the frozen bits defined as
\[
\begin{align*}
\hat{C}^v_{i} &= 0, \quad 0 \leq k < N - K - m, \\
\hat{C}^v_{i} &= \hat{C}^v_{i-1} + m, \quad N - K - m \leq k < N - K.
\end{align*}
\]

Set $f$ in accordance with the decoding result.

**M2** Perform CA-SCL decoding on frame $i$ with the frozen bits defined as
\[
\hat{C}^v_{i} = 0, \quad 0 \leq k < N - K - m.
\]

Set $f$ in accordance with the decoding result.

**M3** Perform CA-SCL decoding on frame $i - 1$ with $\alpha_i - 1$ and partial frozen bits $\hat{C}^v_{i-1}, N - K - m \leq k < N - K$, fetched from memory, and define other frozen bits as
\[
\begin{align*}
\hat{C}^v_{i-1} &= 0, \quad 0 \leq k < N - K - m, \\
\hat{C}^v_{i-1} &= \hat{C}^v_{i-1} + m, \quad N - K - m \leq k < N - K + m.
\end{align*}
\]

In order to determine whether the decoding is successful or not, we utilize the small undetected error probability of CRC [12] and define a successful decoding as when the decoding output passes the CRC. Algorithm 2 shows that if the decoding of frame $i - 1$ succeeds, the decoding of frame $i$ is executed in mode **M1**. The decoder uses the values of MUUBs of frame $i - 1$ as those of MRFBs of frame $i$. On the other hand, if the decoding of frame $i - 1$ fails, the decoding of frame $i$ is executed in mode **M2**. The decoder regards the MRFBs as unfrozen bits since correct values of these bits cannot be obtained from frame $i - 1$. The indicator $g$ ensures that frame $i - 1$ undergoes a secondary decoding operation in mode **M3**, only if both frames $i - 2$ and $i$ are decoded correctly. In this case, the MRFB values of frame $i - 1$ which are taken from frame $i - 2$, and the MUUB values of frame $i - 1$ which are taken from frame $i$, are correct. This guarantees that the frames help each other in order to improve error-correction performance.

In the proposed inter-frame polar coding scheme, the frozen bits are dynamic in two ways: first, their values are not all fixed to predefined values that are known to the decoder, and second, the number of them changes in different decoding modes. It should be noted that the proposed scheme is different from the information re-transmission scenario in which all or part
of a frame is re-transmitted until a successful data delivery is acknowledged. In the proposed scheme, the MUUB values of frame \(i-1\) are assigned to the MRFBs of frame \(i\). Therefore, the code rate remains unchanged. However, the information re-transmission scheme inevitably reduces the code rate in order to re-transmit the same frame.

C. Complexity Analysis

The proposed IFA-SCL decoding scheme does not modify the underlying CA-SCL decoding processes. Therefore, different decoding modes use the same CA-SCL decoder. Considering \(M\) quantization bits for channel and internal LLR values, the memory requirement of the CA-SCL decoder with list size \(L\) can be approximated\(^1\) as [13]

\[
M (N + L(N - 1)) + L(2N - 1) \text{ [bits].} \tag{5}
\]

The IFA-SCL decoder requires additional \(MN + 3m\) bits to store \(N\) channel LLR values of a failed decoded frame, \(m\) bit estimations of a failed decoded frame and at most \(2m\) bit estimations of a successfully decoded frame according to Algorithm 2. Thus, the memory required by the proposed IFA-SCL decoding scheme is

\[
M(2N + L(N - 1)) + L(2N - 1) + 3m \text{ [bits].} \tag{6}
\]

It should be noted that only a secondary decoding attempt in mode M3 leads to extra computational complexity in the proposed IFA-SCL decoding scheme which makes the total computational complexity of decoding a frame almost twice as that of the classical scheme. However, as will be shown in Section IV, the decoding in mode M3 occurs with a small probability for good channel conditions or large list sizes. This results in a small increment in the average computational complexity for the proposed IFA-SCL decoding scheme, in comparison with that of the classical CA-SCL decoding.

IV. Simulation Results

To verify the effectiveness of the proposed strategy, the performance of the inter-frame polar coding scheme is evaluated. Additive white Gaussian noise (AWGN) channel model is assumed and binary phase-shift keying (BPSK) modulation is adopted. For the simulation results in this section, the relative reliability sequence in the 5G standard [14] is used for the polar code of length \(N = 1024\), and for the polar code of length \(N = 2048\), the relative reliability vector is generated by the method in [15], using a design signal-to-noise ratio of 2 dB. A CRC of length \(r = 16\) in the 5G standard with generator polynomial \(D^{16} + D^{12} + D^{5} + 1\) is used for CA-SCL decoding and the rate of the code is defined as \(R = (K-r)/N\).

Fig. 2 shows the BLER and bit error rate (BER) of polar codes of different code parameters with respect to energy per bit to noise power spectral density ratio (\(E_{b}/N_{0}\)). It can be seen that the proposed inter-frame polar coding scheme with IFA-SCL decoding algorithm brings a performance gain of 0.28 dB at a BLER of \(10^{-4}\) in comparison with the classical polar coding scheme with CA-SCL decoding when \(N = 1024\), \(R = 1/2\), \(L = 16\), and \(m = 40\). It is worth mentioning that the BLER performance of the proposed inter-frame polar coding scheme with \(N = 1024\), \(R = 1/2\), \(L = 16\), and \(m = 40\) is 0.15 dB better than that of the classical polar coding scheme of the same code length and rate with \(L = 32\), and it is almost similar to that of the classical polar coding scheme of the same code rate and list size with \(N = 2048\), at a target BLER.

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\(^1\)A small memory is required to store path metrics which is neglected here.
while it is 268064 bits for the classical polar coding scheme. As one code length of 1024 and list size of 6.

Table I reports the average and maximum computational complexity gap between the proposed and the classical decoding schemes reduces as the number of arithmetic operations. The number of arithmetic operations per-frame of a polar code of length 1024 and rate 1/2, the proposed scheme can provide an error-correction performance gain of 0.28 dB over the classical scheme at the block error rate of $10^{-4}$. Moreover, we showed that this error-correction performance advantage is achieved with negligible increment in memory requirements and average computational complexity.

**V. CONCLUSION**

An inter-frame polar coding scheme with dynamic frozen bits is proposed to improve the error-correction performance of polar codes. In the proposed decoding scheme, consecutive frames share information to help each other when a decoding operation fails. We showed that for a polar code of length 1024 and rate 1/2, the proposed inter-frame polar coding scheme can provide an error-correction performance gain of 0.28 dB over the classical scheme at the block error rate of $10^{-4}$. Moreover, we showed that this error-correction performance advantage is achieved with negligible increment in memory requirements and average computational complexity.

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