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Eight-dimensional Polarization-ring-switching Modulation Formats

Bin Chen, Member, IEEE, Chigo Okonkwo, Senior Member, IEEE, Hartmut Hafermann Senior Member, IEEE, and Alex Alvarado, Senior Member, IEEE.

Abstract—We propose two 8-dimensional (8D) modulation formats (8D-2048PRS-T1 and 8D-2048PRS-T2) with a spectral efficiency of 5.5 bit/4D-sym, where the 8 dimensions are obtained from two time slots and two polarizations. Both formats provide a higher tolerance to nonlinearity by selecting symbols with nonidentical states of polarization (SOPs) in two time slots. The performance of these novel 8D modulation formats is assessed in terms of the effective signal-to-noise ratio (SNR) and normalized generalized mutual information. 8D-2048PRS-T1 is more suitable for high SNRs, while 8D-2048PRS-T2 is shown to be more tolerant to nonlinearities. A sensitivity improvement of at least 0.25 dB is demonstrated by maximizing normalized generalized mutual information (NGMI). For a long-haul nonlinear optical fiber transmission system, the benefit of mitigating the nonlinearity is demonstrated and a reach increase of 6.7% (560 km) over time-domain hybrid four-dimensional two-amplitude eight-phase shift keying (TDH-4D-2A8PSK) is observed.

Index Terms—Achievable information rates, fiber nonlinearity, generalized mutual information, multidimensional modulation.

I. INTRODUCTION

Fiber nonlinearities are considered to be one of the limiting factors for achieving higher information rates in coherent optical transmission systems [1]. Advanced modulation formats with geometric and probabilistic shaping have been extensively explored with the aim of increasing achievable information rates (AIRs) [2]–[4]. Meanwhile, signal shaping has also been considered to mitigate the effects of fiber nonlinearities [5]–[9].

Polarization multiplexing (PM) naturally allows modulation on a four-dimensional (4D) space, which has the potential to increase achievable information rates when the modulation is truly designed in 4D. Conventional PM-formats such as PM-QAM, however, are only optimized per two dimensions independently, and thus, do not exploit all the available degrees of freedom. Several power-efficient modulation formats have been proposed using sphere-packing and lattice constructions in 4D and 8D space [2], [3], [10], [11]. These designs, however, aim at optimizing the minimum Euclidean distance (ED) of the constellation, and thus, they are optimum only for asymptotically high SNR, in the linear additive white Gaussian noise (AWGN) channel, and for uncoded metrics such as symbol- and bit-error probability only [12– Sec. IV-A]. Some of these multidimensional (MD) modulation formats were also shown to give high mutual information (MI), but are not well-suited for coded systems based on a bit-wise decoder such as bit-interleaved coded modulation (BICM), i.e., their generalized mutual information (GMI) is quite low [13].

MD constant-modulus modulation formats have been proposed [6], [14], [15] to mitigate the nonlinear interference. One example of this is the 4D 64-ary polarization-ring-switching (4D-64PRS) format we recently proposed in [9]. 4D-64PRS was shown to outperform other modulation formats at SE of 6 bit/4D-sym by jointly optimizing the coordinates and labeling. 8D modulation formats have twice as many degrees of freedom, and thus, can improve the AIRs and nonlinearity tolerance. The 8 dimensions can be obtained by two frequencies [16] or two consecutive time slots [5], [10].

Our work builds upon the polarization-balancing concept, proposed for a spectral efficiency (SE) of 2 bit/4D-sym in [5]. This concept was further investigated in terms of the SE and nonlinearity tolerance trade-off in [7], [17]. All the previous works using this concept only consider PM-QPSK with added constraints, and thus, only 8D formats at SE below 4 bit/4D-sym were considered. Generalizing those formats to higher SEs is nontrivial, especially when both the constellation and its binary labeling are taken into account.

In this paper, we propose an approach to construct two nonlinearity-tolerant modulation formats with a SE of 5.5 bit/4D-sym. The formats are based on set-partitioning 4D-64PRS in two consecutive time slots [5], [10]. This concept was further investigated in terms of the SE and nonlinearity tolerance trade-off in [7], [17]. All the previous works using this concept only consider PM-QPSK with added constraints, and thus, only 8D formats at SE below 4 bit/4D-sym were considered. Generalizing those formats to higher SEs is nontrivial, especially when both the constellation and its binary labeling are taken into account.

In this paper, we propose an approach to construct two nonlinearity-tolerant modulation formats with a SE of 5.5 bit/4D-sym. The formats are based on set-partitioning 4D-64PRS in two consecutive time slots. The first format is suitable for a high code rate coded modulation system. The second is well-suited for lower code rates and also exhibits higher nonlinearity tolerance. Numerical simulations demonstrate increased nonlinearity tolerance and transmission reach increase with respect to other modulation formats.

II. 8D POLARIZATION-RING-SWITCHING FORMATS

In optical transmission systems, the performance of a given modulation format is determined by its tolerance to both nonlinear interference arising from the Kerr effect, and accumulated amplified spontaneous emission noise. Therefore, designing modulation formats which increase the AIRs in the
presence of linear and nonlinear impairments is crucial. In [9], we designed the 4D-64PRS format with SE 6 bit/4D-sym, which has a constant modulus and an optimized binary labeling. 4D-64PRS provides excellent linear gain and nonlinear gain with respect to other modulation formats at the same SE. The structure and binary labeling of 4D-64PRS is shown in Fig. 1. The bits \( b_1, b_2, b_4, b_5 \) determine the two 2D quadrants while \( b_3, b_6 \) determine the actual transmitted symbol.

Let \( S = [S_1, S_2, S_3] \) denote the Stokes vectors with \( S_1 = |X|^2 - |Y|^2, S_2 = 2R\{XY^*\} \), and \( S_3 = 23\{XY^*\} \), and where \( X \) and \( Y \) are complex numbers representing the constellation symbols in \( x \)- and \( y \)-polarization, resp. The symbols 4D-64PRS result in 16 distinct states of polarization (SOPs) and have a constant modulus \( ||S|| = 1 \). This is shown in Fig. 2 (ignoring the colors). If the 4D-64PRS format was to be used in two consecutive time slots (\( T_1 \) and \( T_2 \)), there are \( 2^{12} = 4096 \) 8D symbols as a set \( X \subset \mathbb{R}^8 \), which can be represented by 12 bits \( b_1, b_2, \ldots, b_{12} \). In this paper, we are interested in designing formats with a SE of 5.5 bit/4D-sym (11 bit/8D-sym), and thus, we will use \( b_{12} \) as parity bit to effectively remove 2048 out of the 4096 symbols.

In order to achieve better performance for optical fiber communication system, we designed 8D modulation formats with a better sensitivity and a high nonlinearity tolerance by selecting symbols with larger minimum Euclidean distance and smaller degree of polarization (DOP) in consecutive time slots. The DOP for \( i \)th transmitted 8D symbol is defined as \( p_i = \frac{\|S_{i} + S_{i+2}\|}{\|S_{i} + S_{i+2}\|} \), where \( 0 \leq p_i \leq 1, t_1 \) and \( t_2 \) indicate time slot 1 and time slot 2. It is known that the worst symbols for nonlinearity tolerance are polarization identical (PI) symbols with zero DOP (\( p = 0 \)), which has identical SOPs. Therefore, we first avoid all the strongest cross polarization modulation (XPolM)-inducing PI symbols contained in 4D-64PRS and then jointly consider SOP and Euclidean distance to select 2048 polarization nonidentical symbols (\( p < 1 \)) in 4D-64PRS constellation set for two SNR regimes: high-SNR and medium-SNR. We obtained two types of 8D modulation formats with 5.5 bit/4D-sym. One overhead bit is employed to choose points from the set \( X \) and can be obtained by the following methods:

- Type 1: \( b_{12} \) is a parity bit of single-parity-check code to protecting all information bits, which is an exclusive or (XOR) of all the bits \( b_1, b_2, \cdots, b_{11} \). In this case, the nearest neighboring symbols are removed to maximize minimum ED, which perform better at higher SNR. The parity bit \( b_{12} \) can be obtained as \( b_{12} = b_1 \oplus b_2 \oplus \cdots \oplus b_{11}, \) where \( \oplus \) and \( \ominus \) denote the modulo-2 addition and negation, respectively.
- Type 2: \( b_{12} \) is used to protect only the least significant bits, which are \( b_3, b_6 \) and \( b_9 \). In this case, the modulation will be good for medium SNR. In addition, it has more polarization balanced points in two time slots. The parity bit \( b_{12} \) can be obtained as \( b_{12} = b_3 \oplus b_6 \oplus b_9 \).

Fig. 2 shows the relationship of SOPs for transmitted symbols in two consecutive time slots for two designed 8D modulation formats. The color coding scheme used in Fig. 2 shows the SOP constraint we imposed on the formats. When a blue point is transmitted in the first time slot, only red points are used in the second time slot. No PI symbols with \( p = 0 \) are allowed in both of these two 8D modulation formats.

To inform our intuition on design features that influence linear and nonlinear performance, we list the properties of five modulation formats in Table 1 for comparison. The squared minimum Euclidean distance is denoted as \( d_E^2 \). In addition to constant modulus, we propose two performance metrics for evaluating modulation-dependent nonlinear interference: the maximum DOP and the average DOP, which are calculated for all the possible \( M \) transmitted symbols in two consecutive time slots for a given modulation format. The maximum DOP is defined as \( \alpha = \max_{i=1}^{M} p_i \) and the average DOP is denoted as \( \beta = \frac{1}{M} \sum_{i=1}^{M} p_i \). A larger \( d_E^2 \) should result in better linear sensitivity, while smaller \( \alpha \) and \( \beta \) should in principle result in higher nonlinear noise tolerance. Based on these properties, the two 8D modulation formats should be better than the other three modulation formats for both linear and nonlinear regime, which will be shown in Sec. III.
III. PERFORMANCE EVALUATION

Here we compare the performance of four different modulation formats: PM-8QAM, 5.5b4D-2A8PSK and the two proposed 8D-2048PRS formats. We use the PM-8QAM as baseline to show the nonlinearity-tolerant property of the 8D formats, and choose the 5.5b4D-2A8PSK with the same SE as baseline to show the overall performance improvement. The formats were compared via two performance metrics: normalized GMI (NGMI) and effective SNR (SNR\text{eff}). NGMI is given by NGMI=\text{GMI}/m, where m is the number of bits per 4D of the format and shows the gains for a BICM system with the same soft-decision forward error correction (SD-FEC) overhead. The effective SNR quantifies the gains due to nonlinearity tolerance.

A. Linear Channel Performance

Fig. 3 shows the NGMIs for the linear AWGN channel. 8D-2048PRS-T1 and 8D-2048PRS-T2 are shown to clearly outperform both PM-8QAM and 5.5b4D-2A8PSK for all NGMIs above 0.6 bit. At a NGMI of 0.85 (the state-of-the-art SD-FEC with 25% overhead) 8D-2048PRS-T1 offers gains of 1.15 dB and 0.25 dB with respect to PM-8QAM and 5.5b4D-2A8PSK, resp. These gains increase up to 1.6 dB and 0.7 dB at high SNRs (at NGMI of 0.965 bit).

B. Nonlinear Channel Performance

We consider a dual-polarization multi-span WDM system with 11 co-propagating channels generated at a symbol rate of 45 Gbaud, a WDM spacing of 50 GHz and a root-raised cosine (RRC) filter roll-off factor of 0.1. Each WDM channel carries 2\textsuperscript{16} 4D symbols in two polarizations at the same launch power per channel $P_{ch}$. Each span consists of an 80 km standard single mode fiber (SSMF) through a split-step Fourier solution of the nonlinear Manakov equation with step size 0.1 km and is followed by an erbium-doped fiber amplifier with a noise figure of 5 dB. We also simulate polarization mode dispersion (PMD) with the coarse-step method \cite{19} and fixed-length sections of length 1 km. As for the statistical characterisation of PMD and its effect on fiber transmission, the polarization is uniformly scattered over Poincaré sphere and differential group delays (DGDs) of each section are selected randomly from a Gaussian distribution with standard deviation equal to 20\% of the mean \cite{20}. At the receiver, an ideal receiver is implemented\cite{4} and fiber linear impairments such as the accumulated chromatic dispersion or the polarisation state rotation of the signal are ideally compensated.

First, we consider the propagation without PMD, setting the PMD to zero. We compare the SNR\text{eff} as a function of the transmission distance using $P_{ch} = 0$ dBm (optimum $P_{ch}$ for 100 spans). The results are shown in Fig. 4. The two proposed 8D formats 8D-2048PRS-T1 and 8D-2048PRS-T2 provide a higher SNR\text{eff} than PM-8QAM and 5.5b4D-2A8PSK. Especially, 8D-2048PRS-T2 has higher SNR gains due to its smaller nonlinearity-tolerant property of $\alpha$ and $\beta$ in Table I.

From the results above, we can observe that the proposed 8D-2048PRS formats outperform other modulation formats in both linear and nonlinear channel without PMD. The total nonlinear shaping gain is linear SNR gain (in Fig. 3) plus SNR\text{eff} gain (in Fig. 4). In order to characterise the impact of the fiber PMD parameter, we consider the realistic values of the PMD in the range of 0.01 – 0.2 ps/\sqrt{km} and average the SNR\text{eff} over 50 random realizations of PMD for each data point. In Fig. 5, the average SNR\text{eff} is shown as a function of PMD using 0 dBm launch power per channel over a transmission distance of 8000 km. The SNR\text{eff} without PMD are shown by the dashed lines as a reference. Fig. 5 shows that the PMD has a small positive impact on the SNR\text{eff} for all the modulation formats. This confirmed that random PMD depolarizes signals, averages out nonlinear effects, and thus reduces the nonlinear penalty when PMD itself is fully compensated by DSP at receiver. In addition, the average SNR\text{eff} gain of using 8D formats over PM-8QAM decrease from 0.33 dB to 0.24 dB at high PMD regime because the

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1The constellation 5.5b4D-2A8PSK is generated by using 5b4D-2A8PSK and 6b4D-2A8PSK from \cite{20} with optimized ring ratio in a time-domain hybrid way with a 1:1 ratio.

2The effective SNR (denoted by SNR\text{eff}) represents the SNR after fiber propagation and the receiver digital signal processing (DSP) and is defined as \cite{4} Eq. (16).

3In this paper, we use ideal 8D phase compensation and 8D detection. However, due to the symmetric property and set-partitioned structure of the proposed modulation family, the 8D formats can be equalized and demapped in 4D or even 2D with marginal loss and lower complexity \cite{8, 21}.

4Ideal compensation refers to having at the receiver exact knowledge of the amount of randomly generated angles and DGD values in the fiber simulation.
SNR changes during propagation. The inset of Fig. 5 shows that PM-8QAM has larger SNR variation and 0.36 dB lower SNR in the worst-case scenario w.r.t. 8D2048PRS-T2.

Fig. 6 shows the results without PMD of the NGMI as a function of the transmission distance, using the optimal launch power at each distance. In addition, the recovered 8D-2048PRS-T2 constellation after 20 spans (in Stokes space) is inset. Note that both proposed constellations yield a 26 spans (28.6%) and 7 spans (6.7%) reach increase relative to PM-8QAM and 5.5b4D-2A8PSK at NGMI of 0.85.

IV. CONCLUSIONS
We have designed two new nonlinearity-tolerant 8D modulation formats at spectral efficiency of 5.5 bits/4D-sym and have provided a simple bit-to-symbol mapping by set-partitioning, which shows that these formats can be implemented with slight modifications to 4D-64PRS. Although at a lower SE, the 8D-2048PRS formats outperforms PM-8QAM by significantly improving sensitivity and nonlinearity tolerance. In comparison to modulation formats of the same spectral efficiency such as 5.5b4D-2A8PSK, 6.7% reach increase is observed. The impact of PMD on the proposed modulations were numerically analyzed to show tolerance to nonlinear effects. We believe that the proposed 8D formats are promising candidates for transmission systems with high nonlinearity, and can be extended to higher dimensions, including wavelengths, and mode/core spatial channels. Future work will also address a realistic comparison with probabilistic shaping.

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