Tiling the surface of a cube by 12 identical pentominoes

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by

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TILING THE SURFACE OF A CUBE BY 12 IDENTICAL PENTOMINOES

C.J. Bouwkamp

Abstract

This report deals with tiling the surface of a cube by twelve congruent copies of a planar pentomino, folded over corners, edges and faces of the cube. Only so-called nice tilings are considered, such that the pentominoes are undeformed and easy to recognize. There are no such tilings for pentominoes U and W. The other pentominoes lead to 1054 tilings distinct modulo rotation and reflection. Most of them are asymmetric and of no particular interest. Only 164 of them have some degree of symmetry, as specified in

\[ X(1), T(1), Z(11), V(2), I(2), F(23), N(3), Y(10), L(30), P(81), \]

with, in parentheses, the number of tilings for the corresponding pentomino. Every symmetric tiling is shown in terms of an unfold of the cube surface onto the plane. Numerical code and details of symmetry are added.

1. Introduction

The problem of tiling the surface of a cube, by a complete set of the 12 different pentominoes, was solved in a previous report [1]. Let me recall that the total number of solutions was found to be 26,358,584 of which 284,402 are nice. Compared to these large numbers, the 1054 and 164 of the new problem is a mere trifle, and so is the corresponding computing time.

In the old problem, pentomino X played a special role to obtain all solutions different modulo rotation of the cube. In the new problem X can be left alone, because, as is easy to see, X leads to one solution only, and this solution is rotational-invariant.

The cube to be covered is shown in Figure 1 (reflections are eliminated). The 60 squares (cells) are numbered as shown in Figure 2.

For the computer a pentomino placed on the cube is an integer array of dimension five, the elements of which are the five ordinal numbers of cells covered by the pentomino and ordered to increasing value.

Apart from X, the pentominoes are ordered pentomino-wise in the order


The integer arrays for each pentomino are ordered to increasing lexicographical value. The corresponding matrix of 5 columns and 3576 rows is available from the old problem. In the new problem, backtrack is over small subsets of these rows as correspond to the pentomino under consideration.
Figure 1. The angle indicated equals $\arctan(1/3)$.

Figure 2. Cell numbers on the cube's layout.
2. Cubic symmetry

The solid cube has many axes of symmetry. First, the three axes from face to face X, Y, Z. Second, the four diagonals from vertex to vertex D1, D2, D3, D4. Third, the six axes from edge to edge T1, T2, T3, T4, T5, T6. See Figure 3, where the corresponding symbols are encircled, at one of two places where the axes cut the cube surface.

![Axes of symmetry within circles.](image)

Now assume that the computer has found a tiling, coded by an ordered set of twelve numbers from 1 to 3576, for a cube fixed in space. If you rotate the cube with its tiling attached, you can describe the tiling with new code numbers, based on the fixed cube. These new numbers can be found by permutation of cell numbers, in combination with sorting procedures, both for numbers and strings.
In the following table these permutations are given explicitly. They were obtained manually, by use of two identical cubes with their cells numbered, one cube held fixed, the other rotated about the axis considered, then rotating the two cubes identically and noting, for 1 to 60 of the first cube, the corresponding cell numbers of the second cube. The lines of IDENTITY are added. With them you can follow permutations more easily.

X stands for rotation over 90 degrees about the x-axis; ROT-1 is the file where the 60 numbers will be stored on disk. X*X stands for rotation over 180 degrees about the x-axis; ROT-2 is the corresponding file of storage. And so forth and so on.

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3. Tilings different modulo rotation

Coping with the symmetry of the tilings is more difficult than in the old problem. First, a tiling will be identified by a string of length 48 derived from a concatenation of its twelve code numbers as computed by the backtrack program. Second, it should be clear that successive strings increase in lexicographical value in the course of backtrack.

If a tiling is not symmetric, it has 24 different codes by rotation. The first of them, that is the smaller, could be stored, and the 23 others ignored. However, by choice we ignore all asymmetric tilings.

If a tiling is symmetric, the first string is not greater than the 23 others, and at least one of the latter is equal to the first.

Assume that the computer finds a tiling, TILING say, with string A$. Then we compute 23 strings by permutations corresponding to rotating the cube, and order them to increasing or non-decreasing lexicographical value. Let then B$ be the very first of them. Only if A$=B$ is TILING symmetric, and A$ is stored. For details, the reader should consider, and study, the actual program in GWbasic.

4. Computer program

0 CLS: REM *** This is program 12SELECT.bas/exe ****************************
1 REM *** It is about covering a cube with 12 identical pentominoes ************
2 REM *** It computes all codes that are distinct modulo rotation ***********
3 REM *** and STORES them in file "NICE"+P$ *******************************
100 DEFINT A-Z
105 DIM PIS1(3576),PIS2(3576),PIS3(3576),PIS4(3576),PIS5(3576),TAL(12,5)
110 DIM PNUM(3576),B(12),COD(12),SOL(24,12),A$(23),AR(60),BB$(24),A(24),B$(24)
120 DIM BODY(60),COPIS(12),COHOL(12),HOEK(576),COP(12),GETAL(12,5),X(12)
140 FOR 1=1 TO 23:READ A$(I):NEXT
200 FOR I= 1 TO 216:PNUM(I)= 2:NEXT
220 FOR I= 457 TO 696:PNUM(I)= 4:NEXT
230 FOR I= 697 TO 936:PNUM(I)= 5:NEXT
240 FOR I= 937 TO 1176:PNUM(I)= 6:NEXT
250 FOR I=1177 TO 1296:PNUM(I)= 7:NEXT
260 FOR I=1297 TO 1752:PNUM(I)= 8:NEXT
270 FOR I=1753 TO 2232:PNUM(I)= 9:NEXT
280 FOR I=2233 TO 2688:PNUM(I)=10:NEXT
290 FOR I=2689 TO 3168:PNUM(I)=11:NEXT
300 FOR I=3169 TO 3576:PNUM(I)=12:NEXT
400 OPEN "codel" FOR INPUT AS #1:FOR 1=1 TO 3576:INPUT #1, PIS1(I) :NEXT:CLOSE
410 OPEN "code2/1 FOR INPUT AS #1:FOR 1=1 TO 3576:INPUT #1, PIS2(I):NEXT:CLOSE
420 OPEN "code3" FOR INPUT AS #1:FOR 1=1 TO 3576:INPUT #1, PIS3(I):NEXT:CLOSE
430 OPEN "code4" FOR INPUT AS #1:FOR 1=1 TO 3576:INPUT #1, PIS4(I):NEXT:CLOSE
440 OPEN "code5" FOR INPUT AS #1:FOR 1=1 TO 3576:INPUT #1, PIS5(I):NEXT:CLOSE
450 OPEN "hoek/l FOR INPUT AS #1:FOR 1=1 TO 676:INPUT #1, HOEK(I) :NEXT:CLOSE
460 OPEN "hoek/2" FOR INPUT AS #1:FOR 1=1 TO 676:INPUT #1, HOEK(I) :NEXT:CLOSE
470 FOR 1=1 TO 576:PIS1(HOEK(I»=O:NEXT: REM *** nice tilings only ***********
480 GOSUB 4000
490 OPEN "NICE"+P$ FOR OUTPUT AS #2
500 CLS:FOR I=1 TO 60:BODY(I)=0:NEXT: REM *** Tiling computed in lines ***
600 J=1:FREHOL=1:
608 BEGIN$=DATE$+" "+TIME$ REM *** begin of filling ***********
610 TRYPI$=LOWER
620 IF FREHOL>60 THEN 900
630 IF BODY(FREHOL)<0 THEN FREHOL=FREHOL+1:GOTO 620
640 IF TRYPI$>UPPER THEN 790
660 IF PIS1(TRYPI$)<FREHOL THEN TRYPI$=TRYPI$+1:GOTO 640
670 IFBODY(PIS2(TRYPI$))=1 THEN TRYPI$=TRYPI$+1:GOTO 640
680 IFBODY(PIS3(TRYPI$))=1 THEN TRYPI$=TRYPI$+1:GOTO 640
690 IFBODY(PIS4(TRYPI$))=1 THEN TRYPI$=TRYPI$+1:GOTO 640
700 IFBODY(PIS5(TRYPI$))=1 THEN TRYPI$=TRYPI$+1:GOTO 640
710 COHOL(J)=FREHOL:COPIS(J)=TRYPI$: REM *** begin of filling ***********
725 IF CM$="x" THEN 1300: REM *** interrupt possible ***********
730 BODY(FREHOL)=1
740 BODY(PIS2(TRYPI$))=1
750 BODY(PIS3(TRYPI$))=1
760 BODY(PIS4(TRYPI$))=1
770 BODY(PIS5(TRYPI$))=1
775 CM$=INKEY$: REM *** interrupt possible ***********
780 J=J+1:FREHOL=FREHOL+1:GOTO 610: REM *** end of filling ***********
790 J=J-1:IF J=0 THEN 1300: REM *** begin of erasing ***********
800 K1=COHOL(J):K2=COPIS(J)
810 COHOL(J)=0:COPIS(J)=0
820 BODY(K1)=0:BODY(PIS2(K2))=0:BODY(PIS3(K2))=0:BODY(PIS4(K2))=0:BODY(PIS5(K2))=0
830 FREHOL=K1:TRYPI$=K2+1:GOTO 640: REM *** end of erasing ***********
900 BBB$="": FOR I=1 TO 12:BBB$=BBB$+STR$(COPIS(I)):NEXT
910 GOTO 2000
1000 REM
1010 IF B$(A(I))<BBB$ THEN 1020 ELSE FOR I=1 TO 12:WRITE #2, COPIS(I):NEXT
GOTO 790
REM
PRINT" Start at " + BEGIN$" + TIME$
REM
END

FOR I = 1 TO 12:COD(I) = COPIS(I):NEXT
SS = 0
FOR JJ = 1 TO 23
OPEN "ROT-" + A$(JJ) FOR INPUT AS 1
FOR I = 1 TO 60:INPUT #1, AR(I):NEXT:CLOSE 1
FOR I = 1 TO 12:COD(I) = COPIS(I):NEXT
FOR I = 1 TO 12
X(1) = AR(PIS1(COP(I))):X(2) = AR(PIS2(COP(I))):X(3) = AR(PIS3(COP(I)))
X(4) = AR(PIS4(COP(I))):X(5) = AR(PIS5(COP(I)))
N = 5: GOSUB 10000
FOR K = 1 TO 5: GETAL(I, K) = X(B(K)): NEXT
NEXT
N = 12
FOR I = 1 TO 12:X(I) = GETAL(I, 1):NEXT
GOSUB 10000
FOR I = 1 TO 12:FOR K = 1 TO 5
TAL(I, K) = GETAL(B(I), K):NEXT
NEXT
FOR I = 1 TO 12
FOR M = LOWER TO UPPER
IF (PIS1(M) = TAL(I, 1) AND PIS2(M) = TAL(I, 2) AND PIS3(M) = TAL(I, 3) AND PIS4(M) = TAL(I, 4) AND PIS5(M) = TAL(I, 5)) THEN COD(I) = M
NEXT M
NEXT I
REM
REM
FOR I = 1 TO 12: SOL(SS, I) = COD(I): NEXT
REM
NEXT JJ
GOTO 20000

INPUT" Pentomino letter (IN CAPITAL!) ": P$
IF P$ = "U" THEN LOWER = 1:UPPER = 216
IF P$ = "T" THEN LOWER = 217:UPPER = 456
IF P$ = "Z" THEN LOWER = 457:UPPER = 696
IF P$ = "V" THEN LOWER = 697:UPPER = 936
IF P$ = "W" THEN LOWER = 937:UPPER = 1176
IF P$ = "I" THEN LOWER = 1177:UPPER = 1296
IF P$ = "F" THEN LOWER = 1297:UPPER = 1752
IF P$ = "N" THEN LOWER = 1753:UPPER = 2232
IF P$ = "Y" THEN LOWER = 2233:UPPER = 2688
IF P$ = "L" THEN LOWER = 2689:UPPER = 3168
5. Conclusion

Two other programs need be mentioned. The first deals with providing details about the character of symmetry, in terms of axes, with input file "NICE"+PS. The second program draws the layout on my matrix printer, by inputing the 12 code numbers. The corresponding information is given at bottom of page, but for pentomino X, where code is absent.

As an ardent puzzler, I made a thin-card-board copy of each layout that has more symmetry than one axis of order 2, after adding a few triangular flaps around the border. Scoring the dashed lines with a blunt knife along a metal straight edge. I can fold the layout into a cube in 3-space and fix it with the flaps inserted between the card-board and 3 squares pasted at the inside. I can unfold the cube at will. Many beautiful geometric objects result!
Acknowledgement

My sincere thanks are due to Herman Willemsen for editing text and drawing the originals of Figures 1 through 3.

References


ROTATIONAL-INARIANT
Four axes order 3 (D1 D2 D3 D4) and three axes order 2 (X Y Z)
Three axes order 2 (Y T1 T4)
One axis order 3 (D3) and three axes order 2 (T1 T3 T6)
One axis order 3 (D3)
One axis order 2 (X)
One axis order 3 (D4) and three axes order 2 (T1 T2 T5)
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One axis order 4 (Z) and four axes order 2 (X Y T2 T3)
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One axis order 2 (X)

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1297 1383 1411 1459 1512 1527 1568 1629 1637 1719 1726 1751
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1297 1389 1411 1417 1464 1518 1521 1573 1628 1640 1697 1716

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One axis order 2 (X)
One axis order 2 (X)
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2235 2346 2353 2363 2371 2431 2482 2526 2535 2545 2646 2650
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One axis order 2 (Y)
One axis order 2 \([\mathbb{Z}]\)
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One axis order 2 [Z]
One axis order 2 \((Z)\)
One axis order 2 (Y)
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Three axes order 2 (X Y Z)
One axis order 2 (X)
One axis order 2 \{X\}
One axis order 2 (Z)
One axis order 2 [Z]

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One axis order 2 (X)
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Three axes order 2: \((X\ Y\ Z)\)
One axis order 2 (Z)

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One axis order 2 (Z)

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One axis order 2 (Z)
One axis order 2 (Y)
One axis order 2 (Y)
One axis order 2 (Y)
One axis order 2 (Y)

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One axis order 2 \( (Z) \)
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One axis order 2 (X)

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One axis order 2 (Y)
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One axis order 2 (X)

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