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Minimising rotor losses in high-speed high-power permanent magnet synchronous generators with rectifier load

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Indexing terms: Synchronous generators, Rotor losses, Permanent magnet generators, Stator winding configurations

Abstract: In an early stage of the design of a high-speed 1400kW synchronous generator with permanent magnet excitation and loaded by a rectifier, it became apparent that rotor losses are a major problem. The stator currents cause asynchronous components in the air-gap field. Analysis shows that a modified polyphase system reduces the number of these components. An approximate solution for the rotor losses caused by the asynchronous field components has been derived. The formulae show the effects of machine dimensions and harmonics and the effect of a conducting shield in the rotor. The main purpose of the study is to have a tool for making an early choice among several stator winding configurations. A modified nine-phase system, combined with a shield around the permanent magnet rotor, is a prospective option.

List of symbols

\[\begin{align*}
A_{nq} &= \text{surface current density due to the } n\text{th current harmonic and the } q\text{th space harmonic} \\
A_{rg} &= \text{surface current density on the conducting shield with radius } r_g \\
A_3 &= \text{surface current density at the stator bore} \\
B_r, B_\theta &= \text{radial, tangential component of the magnetic induction} \\
B_2 &= \text{radial component of the magnetic induction at radius } r_2 \\
d &= \text{thickness of the conducting shield} \\
d_{skin} &= \text{skin depth} \\
E_2 &= \text{axial component of the electric field strength at radius } r_2 \\
h_m &= \text{height of the permanent magnets} \\
i, j &= \text{integers} \\
I_{dc} &= \text{DC current} \\
I_n &= \text{nth harmonic component in the phase current} \\
J_{rg} &= \text{volume current density in the conducting layer} \\
k &= \text{number of subsystems in the stator} \\
k_1, k_2, k_3 &= \text{integers} \\
l &= \text{active length of the machine} \\
n &= \text{order of the harmonic in the current} \\
N_r &= \text{number of stator slots} \\
p &= \text{pole pair number} \\
q &= \text{absolute order of the space harmonic} \\
P_r &= \text{rotor losses due to one asynchronous component} \\
Q_{s,n,q} &= \text{quality factor for the component with order } n \text{ and } q \\
r &= \text{radial co-ordinate} \\
r_0 &= \text{reference radius} \\
r_g &= \text{radius of the conducting layer in the rotor} \\
r_1 &= \text{radius of the solid rotor iron} \\
r_2 &= \text{radius of the conducting shield} \\
r_3 &= \text{radius of the stator bore} \\
t &= \text{time} \\
Z_q &= \text{winding distribution density of the } q\text{th harmonic} \\
m &= \text{commutation interval} \\
\mu_0 &= \text{permeability of vacuum} \\
\sigma &= \text{specific conductivity} \\
\theta_r &= \text{tangential co-ordinate in rotor co-ordinates} \\
\theta_s &= \text{tangential co-ordinate in stator co-ordinates} \\
\omega_m &= \text{mechanical angular velocity of the rotor} \\
\omega_r &= \text{electric angular frequency in rotor co-ordinates} \\
\omega_s &= \text{fundamental frequency of the stator current}
\end{align*}\]

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The coupling of the turbine with the generator. The important features of such a unit are small volume, low weight and high efficiency.

In the framework of a NOVEM project [1, 2] a design of a 1400kW, 18000rpm synchronous generator has been made. The output voltage with a frequency of 600Hz will be rectified. The generator is of the usual synchronous type with polyphase windings in the stator. The rotor has permanent magnet segments arranged on the surface of a solid iron core. A carbon fibre bandage around the permanent magnets gives the rotor sufficient strength at high speed. A cross-section of the four-pole generator is shown in Fig. 1.

![Fig. 1 Outline of the construction of the high-speed generator](image)

For high power at high speed the power density should be as high as possible. The maximum admissible circumferential speed of the rotor and the possible diameter-to-length ratio set a limit to the dimensions. Consequently, it is difficult to cool the compact construction to a tolerable temperature level. A choice has been made for a liquid cooling system. Therefore, special attention should be paid to the prediction of the losses and it is necessary to find ways to decrease the losses by an adequate construction.

The application of the construction as given in Fig. 1, generally available for a power up to 20kW and a speed of 6000rpm, for a 1400kW generator would cause two problems; the inductance will be too high for commutation of a rectifier load at rated power and the asynchronous components in the air-gap field will induce eddy currents in the solid rotor iron. The eddy current losses can exceed the limit set by cooling capacity and acceptable temperature rise.

A conducting layer around the permanent magnets of the rotor offers a solution to both problems. A good conducting shield will have lower eddy current losses due to asynchronous field components than the solid rotor iron. The idea of a conducting shield in the rotor goes back to at least the times when alternatives with superconducting field windings were considered [3]. The conducting shield had two purposes; damping of mechanical oscillations and screening of an inverse field component. It also decreases the subtransient reactance, which facilitates commutation.

A further reduction of the rotor losses is possible with the use of a modified polyphase system in the stator circuit. The stator coils can be divided in several subsystems with a regular displacement in space. In this way the number of possible asynchronous components in the air-gap field will decrease. This will ultimately result in a loss reduction.

The following Sections deal with an analysis of possible air-gap field components, the losses caused by such components and a comparison of the rotor losses for several configurations of the stator windings with and without rotor shielding.

2 Harmonic field components

2.1 Three-phase winding

In an electrical machine the effects of a nonsinusoidal current and a nonsinusoidal distribution of the windings can be described with Fourier series of the current in time and of the winding distribution in space [4]. First the well known case of a three-phase winding will be treated. In a machine with pole pair number p and a phase current with angular frequency \( \omega_p \), each phase will have a shift in space of \( 2\pi/3p \) and the phase current a shift in time of \( 2\pi/3p_0 \). With \( n \) as the order of the harmonics in the current and \( q \) as the order of the harmonics in space, relative to the pole pair number, the surface current density at the stator bore can be written as

\[
A_{n,q} = I_n Z_q \frac{1}{2} \times \sum_{i=1}^{3} \cos \left[ n \omega_p t - q \varphi_s - (n - q)(i - 1) \frac{2\pi}{3} \right] + \cos \left[ n \omega_p t + q \varphi_s - (n + q)(i - 1) \frac{2\pi}{3} \right]
\]

With \( (n - q) = ... -9, -6, -3, 0, 3, 6, 9, ... \) eqn. 1 becomes

\[
A_{n,q} = \frac{3}{2} I_n Z_q \cos(n \omega_p t - q \varphi_s)
\]

and these components have a positive angular velocity \( n \omega_p/qp \). With \( (n + q) = ... 3, 6, 9, ... \) eqn. 1 becomes

\[
A_{n,q} = \frac{3}{2} I_n Z_q \cos(n \omega_p t + q \varphi_s)
\]

and these components have a negative angular velocity \(-n \omega_p/qp\). Other combinations of \( n \) and \( q \) do not have a resulting current density component.

As most of these field components do not run synchronously with the rotor the elimination of these asynchronous components might improve the behaviour of the machine. The next measures will in a simple way eliminate a lot of these components.

First, make the windings symmetrical in space so that the space distribution of the windings becomes \( Z(p \varphi) \) and only odd harmonics exist. Secondly, make the sum of the stator phase currents

\[
\sum_{i=1}^{3} I_{si} = 0
\]

components with order \( n = 3, 6, 9, ... \) become zero. Thirdly, make the electrical load symmetric with \( I_s(\omega_p t) = -I_s(\omega_p t - \pi) \), so only odd harmonics will exist.

As a result the remaining harmonics have the order \( n = 6k_1 \pm 1 \) and \( q = 2k_2 - 1 \), with \( k_1 = 1, 2, 3, ... \) and
2.2 Transformation to rotor co-ordinates
For the calculation of the losses in the rotor a transformation to rotor co-ordinates is necessary. The transformation is carried out with
\[ \theta_r = \theta_s - \omega_m t \]  
and
\[ \dot{\theta}_m = \omega_s \]  \(4a\) \(4b\)
With \(|n - q| = 0, 6, 12, \ldots\) the transformation of eqn. 2 results in
\[ A_{n,q} = \frac{3}{2} I_n Z_q \cos[(n - q)\omega_s t - q\theta_r] \]  \(5\)
Components with \(|n - q| = 0, 6, 12, \ldots\) cause rotating fields with rotor frequencies \((n - q)\omega_s\) and pole pair number \(pq\), relative to the rotor.
With \(|n + q| = 6, 12, 18, \ldots\) the transformation of eqn. 3 yields
\[ A_{n,q} = \frac{3}{2} I_n Z_q \cos[(n + q)\omega_s t + q\theta_r] \]  \(6\)
These components have a rotor frequency \((n + q)\omega_s\) and pole pair number \(pq\), relative to the rotor.
As a result, the rotor frequencies of the asynchronous field components are multiples of \(+6k\omega_s\).

2.3 Division of the stator windings in subsystems
One phase winding of a three-phase winding occupies \(\pi/3p\) radians in space. A subdivision of each phase of the three-phase system in \(k\) parts, which have a shift in space of \(\pi/3pk\), makes it possible to realise \(k\) three-phase subsystems with a shift in time of \(\pi/3pk\) for the currents. To make the subsystems as symmetrical as possible the subsystems can be connected as given in Fig. 2 for \(k = 3\). Each subsystem is star-connected and the DC sides of the rectifiers are in series, so the current in each subsystem has the same magnitude. The rectifier is 18-pulse for \(k = 3\).

Fig. 2 Generator with three subsystems and 18-pulse rectifier

\(L_c\) = commutation inductivity

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The classic DC machine demonstrates the usefulness of a polyphase system. Each armature coil can be considered as one phase, with a nonsinewave current and winding distribution. Nearly all resulting field components have the same speed relative to the armature (or, at standstill, relative to the brushes).

The stator current density follows from the sum of the \(k\) three-phase subsystems
\[ A_{n,q} = \frac{3}{2} I_n Z_q \]  
\[ \times \sum_{j=1}^{k} \cos \left[ n\omega_s t - q\theta_s - \frac{(n-q)(j-1)\pi}{3k} \right] + \cos \left[ n\omega_s t + q\theta_s - \frac{(n+q)(j-1)\pi}{3k} \right] \]  \(7\)
For each subsystem the first term in eqn. 7 differs from zero for \(|n - q| = 0, 6, 12, \ldots\) and the second term of eqn. 7 differs from zero for \(|n + q| = 6, 12, 18, \ldots\).
For the whole system the result is
\[ A_{n,q} = \frac{3k}{2} I_n Z_q \cos(n\omega_s t - q\theta_s) \]  \(8\)
for \(|n - q| = 0, 6k, 12k, \ldots\)
\[ A_{n,q} = \frac{3k}{2} I_n Z_q \cos(n\omega_s t + q\theta_s) \]  \(9\)
for \(|n + q| = 6k, 12k, 18k, \ldots\).

In this way many combinations of \(n\) and \(q\) do not have a resulting current density. Of course one has to keep in mind that the distribution of the subsystems might be different from the distribution of the undivided three-phase windings.

The transformation to rotor co-ordinates with eqns. 4a and b yields synchronous components with frequency zero and pole pair number \(pq\) for \(|n - q| = 0\), asynchronous components with frequency \((n - q)\omega_s\) and pole pair number \(pq\) for \(|n - q| = 6k, 12k, \ldots\) and asynchronous components with frequency \((n + q)\omega_s\) and pole pair number \(pq\) for \(|n + q| = 6k, 12k, 18k, \ldots\).

The comparison of a regular nine-phase system with a \(3 \times 3\)-system (a three-phase system with \(k = 3\) subsystems) shows that the \(3 \times 3\)-system is a modified nine-phase system. The main property of a \(3 \times 3\)-system (with three separate star connections) is that \(L_c = 0\) for \(n = 3, 6, 9, \ldots\). A nine-phase system is characterised by \(L_c = 0\) for \(n = 9, 18, 27, \ldots\).


3 Rotor losses
3.1 Approximate solution of the field equations
The calculation of the rotor losses can be done in several ways. For instance by treating the conducting parts of the rotor as discrete circuits for each space harmonic and by finding the dissipation from a solution of the equivalent circuit equations. One only needs to find the frequency-dependent circuit parameters from the field equations, but only approximations are possible because of the conducting solid iron core and the end effects. The losses can also be found from an exact solution of the field equations in the two-dimensional case [7, 8] and a calculation of the end effects is
possible for low frequencies and conductivities [9]. The main subject of this study however is the high-frequency and conductivity case. So, it seems easier to start with an approximation that at least includes the effects of material properties, dimensions and harmonics. Besides, the large air gap makes the slot ripple effect [10], i.e. induction variations caused by permeance variations, small.

The field equations for the air gap are
\[ \nabla \cdot \mathbf{B} = 0 \]  
(10)  
and as the volume current density in the air gap equals zero
\[ \nabla \times \mathbf{H} = 0 \]  
(11)  
with the two-dimensional solution in polar co-ordinates [11]
\[ B_\theta = \left[ C_1 \left( \frac{r}{r_0} \right)^{pq-1} + C_2 \left( \frac{r_0}{r} \right)^{pq+1} \right] \cos(pq\theta + \phi) \]  
(12)  
\[ B_r = \left[ C_1 \left( \frac{r}{r_0} \right)^{pq-1} - C_2 \left( \frac{r_0}{r} \right)^{pq+1} \right] \sin(pq\theta + \phi) \]  
(13)

The radius \( r_0 \) which can be freely chosen, will have the value \( r_0 = r_g \).

The integration constants \( C_1 \) and \( C_2 \) can be found when the boundary conditions are known.

At the stator bore
\[ \lim_{r \to r_g} H(\theta) = A_3(\theta) = \hat{A}_3 \cos pq\theta \]  
(14)
for an arbitrarily chosen space harmonic with order \( q \) and with \( A_3 \) as the surface current density at the stator bore. The phase angle \( \phi \) is set to zero as losses do not depend on the phase of the current.

For the rotor, two cases will be treated:
(a) With a conducting shield in the rotor around the permanent magnets at radius \( r_2 \) the rotor currents will flow in a conducting layer at radius \( r_g = r_2 \). The assumption is perfect screening (the limit value for high frequency and/or conductivity) and by consequence the boundary condition is
\[ \lim_{r \to r_g} B_r = 0 \]  
(15)
(b) Without a conducting shield on the rotor the assumption is that the eddy currents in the solid iron will flow in a very thin layer at radius \( r_g = r_1 \). The eddy currents in the permanent magnet segments can be made arbitrarily small by making the segments small. These eddy currents will not shield the solid iron core for the space harmonics of low order because the small segments are not connected. The space harmonics with high-order \( q \) will decrease very fast with decreasing \( r \) and the contribution to the rotor losses will be low for these high-order harmonics. The boundary condition for the solid iron rotor core is
\[ \lim_{r \to r_g} B_r = 0 \]  
(16)

The high rotor speed and the division of the stator windings in subsystems with \( k > 1 \) make the assumption of perfect screening reasonable: high \( \omega_m \), \( \omega_r = pq \omega_m \) and \( \omega_r \) is (a multiple of) \( 6\omega_m \).

The surface current density at radius \( r_g \) can now be found. Combination of eqns. 15 and 13 gives
\[ C_1 = C_3 \left( \frac{r_3}{r_g} \right)^{2pq} \]  
(17)
Combining eqns. 14 and 12 yields
\[ C_1 + C_2 = \hat{A}_3 \mu_0 \]  
(18)
As a result the integration constants become
\[ C_1 = \hat{A}_3 \frac{\mu_0}{1 + \left( \frac{r_3}{r_g} \right)^{2pq}} \]  
(19)
and
\[ C_3 = \hat{A}_3 \frac{\mu_0}{1 + \left( \frac{r_3}{r_g} \right)^{2pq}} \]  
(20)

The field solution at radius \( r_g \) is
\[ B_\theta = \hat{A}_3 \mu_0 \frac{1}{1 + \left( \frac{r_3}{r_g} \right)^{2pq}} \]  
\[ \times \left[ \left( \frac{r_g}{r_3} \right)^{pq-1} + \left( \frac{r_3}{r_g} \right)^{pq} \right] \]  
\[ \times \left( \frac{r_g}{r_3} \right)^{pq+1} \cos pq\theta \]  
(21)
or
\[ \hat{B}_\theta = \hat{A}_3 \mu_0 \frac{2 \left( \frac{r_3}{r_g} \right)^{pq-1}}{1 + \left( \frac{r_3}{r_g} \right)^{2pq}} \]  
(22)

The relation between the current density \( \hat{A}_3 \) at the stator bore and the current density \( \hat{A}_{rg} \) in the conducting layer is
\[ \hat{A}_{rg} = \hat{H}_\theta = \hat{A}_3 \frac{2 \left( \frac{r_3}{r_g} \right)^{pq-1}}{1 + \left( \frac{r_3}{r_g} \right)^{2pq}} \]  
(23)

From this surface current density follows the volume current density
\[ J_{rg} = \frac{A_{rg}}{d_g} \]  
(24)
for a thin layer with thickness $d_g$. For a conducting shield at radius $r_s = r_2$, $d_g$ equals the thickness $d$ of the conducting shield at low frequencies. At high frequencies the skin effect can be taken into account by using

$$d_g = d_{skin} = \sqrt{\frac{2}{\omega \sigma \mu_0}}$$

(25)

3.2 Losses

The conducting losses $P_{rg}$ in the conducting layer at radius $r_s$ and thickness $d_g$ are calculated over the conducting volume and one period of time

$$P_{rg} = \frac{\omega_r}{2\pi} \int_0^{2\pi} \int_0^{r_s} \frac{J_{rg}^2}{\sigma \mu_0} r dr d\theta$$

(26)

As the average of the square of a sine or cosine over one period equals $1/2$, the losses from one component of the eddy currents becomes

$$P_{rg} = \frac{\pi r_s^2}{\sigma d_g} A_{rg}^2$$

(27)

for $d_g << r_s$. The total losses $P_r$ caused by the induced currents in the rotor shield can be obtained by summation of the losses of the relevant components. The square of the total current density should be used in the above integral but as the cross-products of components with different pole pair number, frequency or velocity have an average value equal to zero, the sum of the squares is sufficient.

3.3 Quality factor

The main assumption in the calculation of the rotor losses is the boundary condition (eqn. 15) which depends on the conductivity of the shield. As a test a quality factor can be used.

The currents in the rotor shield are induced by an electric field originating from the asynchronous field components of the stator currents. The electric field strength at radius $r_2$, when the shield is absent, follows from $E = v \times B$

$$P_{rg} = \frac{\pi r_s^2}{\sigma d_g} A_{rg}^2$$

The magnetic induction at radius $r_2$ follows from eqns. 12 and 13 with boundary condition (eqn. 14) at the stator bore and $B_s = 0$ at radius $r_1$ of the solid iron core

$$B_2 = A_3 \mu_0 \left( \frac{r_2}{r_3} \right)^{2q} \left( 1 - \frac{r_1}{r_3} \right)^{2pq}$$

(29)

For a conducting shield at $r_s = r_2$ a quality factor $Q_{s,n,q}$ can be defined as

$$Q_{s,n,q} = \frac{E_2}{B_2}$$

(30)

The current density $J_2$ follows from eqn. 24 with $r_s = r_2$. It should be noted that the approximation $B_s = 0$ at the solid iron surface with radius $r = r_1$ results in too small values of $B_2$, $E_2$ and $Q_{s,n,q}$ at low rotor frequencies.

The analogy with an equivalent short-circuited rotor winding helps with the interpretation. For low values of $Q_{s,n,q}$ (smaller than 1) there is no shielding of the asynchronous field components but mainly a phase shift. $Q_{s,n,q} = 1$ marks the 3dB point for the dissipation in the shield and the field within the shield. For high values of $Q_{s,n,q}$ the shield behaves as an inductive circuit with excellent shielding of the asynchronous field components. At high frequencies however, when the skin depth in the shield becomes important ($d_{skin} << d$), the approximation with one equivalent short-circuited rotor circuit is no longer valid.

When there is no conducting shield around the per­manent magnets the rotor losses in the solid iron core can be estimated using eqn. 27, but a simple check with a quality factor is not possible.

4. Winding distribution and current waveforms

4.1 Surface current density at the stator bore

The usual three-phase stator winding is characterised by $k = 1$: one phase is distributed over three adjacent slots per pole; the winding has a star connection. The $3 \times 3$-phase winding has $k = 3$: one phase per slot per pole; each subsystem has its own star connection.

The order $n$ of the harmonics in the stator current can only be odd because of a balanced load by a full bridge rectifier. The components of the stator current are zero if $n$ equals (a multiple of) 3. Asynchronous components exist if $|n| = q |6k|, 12k, 18k, \ldots$ and the resulting rotor currents have a rotor frequency $o_2 = |n| = q |6k| o_1$ with $o_1$ as the fundamental frequency of the stator currents. The surface current density $A_3$ at the stator bore equals

$$A_3 = \frac{3k}{2} \frac{1}{\pi} \frac{I_n}{f_3}$$

(31)

with $f_3$ the amplitude of the nth harmonic of the stator current and $Z_o$ the winding distribution density for the $q$th space harmonic of a stator phase winding.

4.2 Winding distribution density of the space harmonics

With the use of a Dirac function for a full pitch winding the space harmonics of a three-phase winding can be found. For the $3 \times 3$-phase system with one phase per pole per slot

$$Z_q = 2p \frac{f_3}{\pi r_3}$$

(32)

For the three-phase system with three windings per pole in three adjacent slots and the co-ordinate axis in the middle slot, the distribution density equals

$$Z_q = 2p \left[ 1 + \cos \left( q \frac{\pi}{3} \right) \right]$$

(33)

4.3 Harmonics of the stator currents

If the commutation time of the stator current is neglectable a harmonic with order $n$ has the amplitude

$$I_n = \frac{2\sqrt{2}}{n\pi} I_{dc}$$

(34)

In case the commutation time has to be taken into account, an approximation with linear commutation during an interval $m$ can be used

$$I_n = \frac{2\sqrt{2} \sin \left( \frac{n m}{2} \right)}{n \pi} I_{dc}$$

(35)

5. Numerical calculations

5.1 Calculations

The rotor losses have been calculated for two winding configurations of the synchronous generator; the first
with the usual three-phase winding, the second with a 3 x 3-phase winding. For both winding configurations the calculations are carried out with and without a copper shield around the permanent magnets on the rotor to demonstrate the effect of shielding.

The amplitude of all eddy current components in the rotor shield or at the surface of the solid iron when there is no shield and no shielding by the segmented permanent magnets follows from eqn. 23

$$A_{r_g} = A_3 \left( \frac{r_g}{r_2} \right)^{2n-p} \left( 1 + \frac{2}{r_2} \right)$$

with \( r_g = r_2 \) for a conducting shield or \( r_g = r_1 \) for the solid iron in the absence of a shield.

The calculations have been carried out for a four-pole, 18 000rpm, 1400kW generator. The stator frequency equals 600Hz and the rotor frequency (in the case of a 3 x 3-phase system) is (a multiple of) 10 800Hz.

The calculations have been done for the stator current at full load, for several configurations:
(a) with conducting shield and perfect shielding; only the shield will have eddy current losses by asynchronous components of the stator field
(b) without conducting shield and no influence of the conducting permanent magnet segments; the rotor losses will be concentrated in the solid iron
(c) a three-phase winding with one phase in three adjacent slots (of the 9) per pole and six-pulse rectifier
(d) a 3 x 3-phase winding with one phase per slot (of the 9) per pole and 18-pulse rectifier
(e) a 4 x 3-phase winding with one phase per slot (of the 12) per pole and 24-pulse rectifier.

In most cases the skin depth is less than the thickness of the conducting shield and this value is used for the loss calculation. The commutation time of the stator current has been taken as zero.

A two-dimensional finite-element calculation has been done for an air-gap winding at the stator bore with \( n = 17 \), \( q = 1 \), frequency 10 800Hz and \( I_{dc} = 969 \) A. The shielding effect can be seen in Fig. 4. The finite-element calculation and the approximate solution (eqn. 27) gave the same (rounded) value of 57W for the losses.

![Fig. 4](image_url)  
**Fig. 4.** Flux lines for \( n = 17 \) and \( q = 1 \) in case of a conducting shield around the rotor.
level, e.g. 0.1 – 0.2% of the rated power. Consequently, the cooling of the rotor to an acceptable temperature level is feasible.

An evaluation of the end effects with three-dimensional finite-element simulation and/or measurements is still advisable.

7 Acknowledgments

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8 References

1 KERKENAAR, R.: ‘An electromechanical preliminary design of a 500kW, 25000 rpm generator’. Interim report EMV 93-10 of the NOVEM project, 1993 (in Dutch)