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Investigation on bubble characteristics in a gas fluidized bed

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Abstract—To characterize the bubble behavior in a fluidized-bed reactor experiments were carried out with a capacitance probe. The probe consisted of two measuring tips with a distance of 10 mm. Cross correlation of the two time-averaged signals could not be used to determine the bubble rising velocity because of the log-normal bubble size distribution. Therefore, a statistical method was developed to analyze and fit the experimentally determined time-averaged signals. Quartz sand powders with mean sieve particle sizes of 106, 165, 230 and 587 μm were used as fluidizing powders. The theoretical equivalent bubble diameter calculated with the equation of Darton et al. (1977) was found to be about 2.7 times the determined radial averaged mean pierced bubble length. A stable bubble height \( h^* \) of about 60 cm was determined for the powder with an average particle size of 106 μm. Combined with other data it is argued that \( h^* \) increases with increasing particle size.

KEYWORDS: Bubble behavior; gas fluidized bed.

INTRODUCTION

In a gas fluidized-bed gas is led into a reactor filled with particles that are supported by a distributor plate. If sufficient gas is led into the reactor bubbles will form, which determine in a great extent the behavior of a gas fluidized bed. They maintain the particle movement which gives rise to the excellent heat transfer properties of this type of reactor. A major disadvantage of the bubbles is the fact that they contain most of the gas. Since this gas is not in direct contact with the bed particles, it cannot take part in any reaction between gas and solids. Thus, mass transfer of gas between bubbles and interstitial gas in the dense phase is necessary.

As part in a research project we are interested in the mass transfer from the bubble phase to the dense phase as a function of the particle size (Van Lare et al., 1990, 1991; Van Lare, 1991). An important factor that influences this mass transfer is the gas flow division between the two phases, in which the bubble size is a governing parameter. There have been numerous investigations on the bubble gas flow in a two-dimensional gas fluidized bed (e.g. Pyle and Harrison, 1967; Grace and Harrison, 1969; Geldart, 1967; Geldart and Cranfield, 1977). However, as the bubbles rise in the gas fluidized bed, they grow due to coalescence and split due to instabilities at the bubble boundary. At the stable bubble height \( h^* \) there is an equilibrium between these two processes and the maximum stable bubble diameter is reached. This stable bubble diameter determines the average bubble diameter and hence the average specific surface, which is an important factor that influences the overall mass transfer (e.g. Van Swaaij, 1985).

To measure the bubble characteristics the use of visual methods in three-dimensional beds is limited. Rowe et al. (1979) used a X-ray method with which it was possible to analyze the behavior of one or few bubbles in a three-dimensional bed which was operated in a non-bubbling mode. However, hydrodynamics are different in freely bubbling fluidized beds (Clift and Grace, 1985). For these types of beds several methods have been used, such as observation of the bubble eruption diameter (e.g. Argyriou et al., 1971). There is however not a unique relation between the bubble eruption diameter and the actual bubble diameter when the bubble shape is not known. Also this technique only gives information on the bubbles leaving the bed and not on the bubbles still in the bed. Light probes that were put in the bed have been used by Whitehead and Young (1967) and Glicksman and McAndrews (1985). Halow et al. (1993) used a high-speed three-dimensional capacitance imaging technique to measure voidage distributions within fluidized a bed. From the imaging data quantitative information could be extracted about the bubble properties.

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Another more direct method to measure the bubble characteristics has been used first by Werther (1972) and later by Fan et al. (1983) and is based on electrical capacity differences between the bubble phase and the dense phase. With this method a small capacitance probe is put in the fluidized bed to obtain information on the visible bubble gas flow. (Due to the finite size of the probe not all bubbles are detectable and therefore the term visible is used.)

**EXPERIMENTAL METHOD**

A small capacitive double needle probe (Fig. 1) was used as the basis of the measurement technique. The probe was shaped such that it was possible to put it horizontally in the bed (which is mechanically more stable than vertical) and still have a vertical alignment with a rising bubble (Fig. 1). A somewhat larger probe than the one used by Werther (1972) was designed, because in our investigations larger particles were used.

In the fluid bed the probe responds to porosity variations in the measuring volume as a function of time. Bubbles striking the probe as they rise cause an electric pulse (Fig. 2).

When two probes are used with a known vertical distance between the needle tips, the bubble velocity $u_b$ can be computed from the time difference of the two signals. The duration of the bubble signal gives information on the time the probe has been immersed in the bubble (the so-called bubble contact time $t_b$). Although the bubble can be pierced in an arbitrary place, the average bubble contact time (combined with the bubble velocity) gives a measure for the bubble size when the shape of the bubble is known.

All experiments were performed in a 25 cm diameter stainless-steel bed with a porous plate distributor (flexolith-H). The reactor had two rows of measuring taps. One row for measuring the bed pressure drop and one row for probes. The distance between the taps was 10 cm (Van Lare, 1991). The solids fluidized were quartz sand powders having an average mean sieve particle size of 106, 165, 230 and 587 µm and a particle density of 2650 kg/m³. The experimentally determined minimum fluidization velocities were 1.4, 2.3, 5.1 and 21.3 cm/s, respectively. The powders were fluidized with air ($U/U_{mf} \geq 1.08 - 8.4$) at varying bed heights with a maximum bed height of almost 1 m. Measurements were performed at seven radial positions ($r = 0, 2, 4, 6, 8, 10$ and 12 cm, with $r = 0$ being the center of the bed) (see also Van Lare, 1991).

A schematic diagram of the equipment is shown in Fig. 3. The probe was connected to two capacitance meters (A and B). The signals were transferred to a fast Fourier transform (FFT) analyzer. In order to measure the bubble frequency $k$ one of the signals was also transferred to a comparator and a pulse counter. The comparator generates a block pulse when the incoming signal exceeds a pre-set level. The pulse duration is as long as the signal exceeds this level. The counter counts the number of block pulses $n$ in a given time $T$. The ideal working pre-set level for the comparator was found by using a method developed by Werther (1972): when the level is too low the random noise of the signal generates non-wanted pulses and the measured bubble frequency is too high. When the level is too high none or very few pulses were generated and the measured bubble frequency was too low. In between these two areas there was a region where the frequency was independent of the level. This was the desired pre-set level (Fig. 4). For each experiment this pre-set level was determined.

In a bubbling bed the capacitance probe gave signals as shown in Fig. 2. For determining the bubble velocity the time difference between the signals of the two probes had to be measured. In principle, this can be done by using the cross-correlation function $\varphi_{xy}$.
This function is defined in the following way (e.g. Werther, 1972):

$$\varphi_{xy} = \lim_{T \to \infty} \frac{1}{2 \cdot T} \int_{-T}^{T} x(t)y(t + \tau) \, dt$$  \hspace{1cm} (1)$$

with $T$ being the total measuring time and $\tau$ the real time. The $\tau$ value at which the $\varphi_{xy}$ function shows its maximum is the time difference $\tau_w$ between the two signals $x$ and $y$. The FFT-analyzer had the possibility of measuring this time difference $\tau_w$. A trigger level had to be defined. When a bubble signal passed this level, the total signal was measured during a user defined time (Fig. 5). It was found that $\tau_w$ was strongly dependent on the shape of the signal and that many signals had to be used to obtain an accurate average $\tau_w$.

The FFT-analyzer also had the possibility of time averaging the signal. This means that occasional signal fluctuations could be averaged out. Therefore, a certain trigger level had to be defined. When the signal from the lower probe (channel A) exceeded this level the averaging of the two bubble signals started. As many bubbles as needed were measured to obtain an averaged time signal that was stable (which could be monitored constantly). It was found that 128 or 256 bubble pairs were sufficient (the number of bubbles could only be measured in powers of two).

**STATISTICAL SIGNAL ANALYSIS**

If we consider block pulses it can readily be seen that the time-averaged signal is a cumulative pierced length distribution. All bubbles contribute to the averaged signal of probe A at the trigger time $\tau_{trig}$ (Fig. 6). The bubbles strike the second probe (B) later than the first probe (A). Because the averaging of the two time signals is triggered by the signal of probe A, this means that the time-averaged signal of probe B comprises a bubble size distribution as well as a bubble velocity distribution (Fig. 6). To describe both signals, the following assumptions were made:

1. The bubble size and velocity were both described with a log-normal distribution, defined by (Pollard, 1977)

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ \frac{\ln(x - \theta) - \mu^2}{2 \sigma^2} \right\} \hspace{1cm} (2)$$

where $\mu$ is the average value of log-normal distribution, $\sigma$ the deviation of log-normal distribution, $\theta$ the starting value of the distribution for which $f(x) > 0$ and $\theta = 0$ in many cases, with $f(x)$ being the possibility of finding a value $x$. In this case $\theta = 0$, because it is impossible to find velocities and sizes smaller than zero.

Werther (1972) found a log-normal distribution in rising velocities for the bubbles. The $\tau_w$ values were measured by analyzing signals of many individual bubbles. Figure 7 shows that in our experiments the found probability curve also can be described by a log-normal distribution. The real average $M$ and deviation $S$ can be found from (Pollard, 1977)

$$M = \theta + \exp \left\{ \mu + \frac{1}{2} \sigma^2 \right\} \Rightarrow M = \exp \left\{ \mu + \frac{1}{2} \sigma^2 \right\}, \hspace{1cm} (3)$$

because in this case $\theta = 0$.

$$S^2 = \exp(2\mu)\exp(\sigma^2)[\exp(\sigma^2) - 1]. \hspace{1cm} (4)$$

2. Rising time $\tau_1$ and bubble contact time $\tau_2$ were taken to be stochastically independent. Although bubble diameter and rising time are interdependent, rising time and bubble contact time are not because a bubble is pierced in an arbitrary place.

3. The average bubble time signal of the lower and upper probes were equal. This is, of course, essential, because if this was not the case it would mean that something happened between the two probe points (such as coalescence) and the second signal could not be analyzed using the first signal. For an individual bubble the two signals do not have to be equal because each probe can pierce the bubble at a slightly different place. It was indeed found by observing the probe signal that the two individual signals were not always the same, but that the averaged signals were virtually the same as is shown in Fig. 8. This was
checked for several conditions. This also means that, in
general, the bubbles do not rise side ways but rise
vertically or nearly vertically at the probe tip. Because
if the bubbles consistently rise side ways, the averaged
signals of the two probe tips should have been different.

From the shape of the averaged bubble signal it
also could be concluded that no continuous coalesc-
ing of bubbles took place at the probe tip. If continu-
ous coalescing had taken place this would have result-
ed in a bimodal averaged bubble signal. However, this
bimodal averaged signal never was observed. So, oc-
casional coalescence of bubbles at the probe tip has
been averaged out.

The signal was assumed to be trapezium shaped.
The same description was used as the one given by
Werther (1972) (Fig. 9). He argued that \( \alpha = \frac{l}{u_b} \), where
\( \alpha \) is the trapezium slope and \( l \) is the probe length. That
the signal was indeed trapezium like is shown in
Fig. 5.

The time at which the bubble first reached the lower
probe is called \( t_0 \) (Fig. 9). As the bubble rose the
amplitude of the signal became larger, until a max-
imum value was reached at \( t_2 \). When the probe reach-
ed the ‘bottom’ of the bubble at \( t_3 \), the signal started
to descend, until the bubble had passed the probe
completely at \( t_4 \). The bubble contact time \( t_b \) is the time

---

**Fig. 6. Schematic presentation of how the time-averaged signal is determined. Experimentally determined
curves are shown below.**
the bubble needs to pass the probe tip completely, being $t_3 - t_0$ (or $t_4 - t_0$). The process was described mathematically by the following equations:

\begin{align}
  y(t) &= 0 \quad (t < t_0) \text{ and } (t > t_3) \\
  y(t) &= \frac{(t - t_0)}{(t_0 - t_3)} y_{\text{max}} \quad (t_0 \leq t \leq t_3) \\
  y(t) &= y_{\text{max}} \quad (t_3 < t \leq t_4) \\
  y(t) &= \frac{(t - t_4)}{(t_4 - t_3)} y_{\text{max}} \quad (t_3 \leq t \leq t_4) \text{.} \\
\end{align}

The FFT-analyzer determined the average signal by adding the individual signals and dividing the sum by the total number of signals:

$$
\bar{y}(t) = \frac{1}{n} \sum_{i=1}^{n} y_i(t)
$$

with $n$ being the number of signals and $y_i(t)$ the averaged time signal.

The signals of the lower probe were all triggered at $t_0$, so the first slope is the averaged slope of the bubble signal. All bubbles made contact with the probe within the time $t_0$ giving a maximum $y$ value of $y_{\text{max}}$ at this time value. The probability that a bubble hits the probe at $t_0$ is therefore defined to be equal to one. When the bubble with the longest contact time had passed the probe, the time-averaged signal would of course be equal to zero, meaning that the probability of finding bubbles with an even larger time was also equal to zero. The time-averaged signal is therefore related to a cumulative distribution function of the bubble contact times. This can be shown by

$$
P(t_b \leq t) = F(t) = \frac{\int_0^t f(t) \, dt}{F(t)}
$$

with $P(t_b \leq t)$ being the probability of finding $t_b$ values less than or equal to a given $t$ value, $f(t)$ the probability function defined by eq. (2) and $F(t)$ the cumulative distribution function. If we define the zero point of the curve at $t_0$, the cumulative distribution function is given by the following equation:

$$
F(t) = 1 - \frac{y(t)}{y_{\text{max}}} \text{.}
$$

$F(t)$ could, of course, also be calculated by numerical integration of $f(t)$ [eq. (2)]. Average and deviation of the bubble contact time were found by fitting several points of the averaged curve, using eqs (3) and (4).

The signal of the second probe tip (B) was measured after the rising time $t_{b'}$ at a time $(t_0 + t_3)$ the signal of the second probe started to change. Because there is a probability distribution for the rising velocity, this means that not all bubbles hit the probe at a given constant time, hence there is no point on the averaged curve of the second probe that corresponds to a
'hit-probability' of one. The curve is, however, related to the cumulative distribution function of bubble contact time and rising time, but these distributions could not be determined directly from the curve as was the case for the first probe (A).

This problem was solved by simulating the averaged signal of the second probe with a computer simulation program. The bubble contact time distribution determined from the first probe was used to generate simulated signals describing the second probe. The average and deviation in the rising time were varied and the signal of the second probe was simulated by adding the signals and dividing by the total number of generated signals. A sum of squares technique was used to fit the curves.

The complete scheme for obtaining the average and the standard deviation of the bubble contact time $t_b$ and rising time $t_r$ therefore was:

(i) The measured signals on the FFT-analyzer were described by taking several points of the curves.

(ii) $t_b$, $t_r$, and the slope of the trapezium were determined from the averaged signal of the first probe (A). By extrapolating the first slope to an $y$-value of zero, $t_b$ was determined. Extrapolating to $y_{max}$ (that was measured directly) gave $t_r$.

(iii) For $t > t_r$ the curve was equal to $1 - F(t)$. Six to nine points were taken to describe the curve for $t > t_r$. Using eq. (5), the curve was fitted with a least-squares method thereby determining $t_3$ and $t_4$. Figure 10(a) shows the difference between computed and measured signal.

(iv) The averaged signal of the second probe was simulated by assuming a trapezium-like signal [eq. (5)]. Slope of the left and right side of the trapezium were taken to be equal to the slope determined from the first signal [step (iii)]. A total number of 600 bubbles were 'generated' and an average signal was determined in the same way the FFT analyzer does. The curve was again fitted using a least-squares method. The maximum of the curve was not known on forehand (as was explained earlier). Therefore, bubble signals were generated with an arbitrary height. The maximum of the simulated curve was taken equal to the maximum of the measured curve. All other points of the calculated curve were also corrected with this ratio. In this way measured and simulated curve could be compared. Measured and computed curves are shown in Fig. 10(b).

From the Figs 10(a) and (b) it can be seen that it is possible to describe the time-averaged signals with this method. Discrepancies between measured and calculated curves occur because the actual signal is more gradual and not an exact trapezium with its sudden changes.

Cross correlations could not be used to determine $t_r$ as can be shown mathematically. The time-averaged signal is defined by eq. (6). We use $\rho_{xy}$ for denoting the cross-correlation function of the two time-averaged signals. Hence (the notation \( \lim_{\tau \to -\infty} \) has been left out):

\[
\rho_{xy} = \frac{1}{2T} \int_{-T}^{T} x(\tau) y(\tau + \tau) \, d\tau.
\]  

Here $x(\tau)$ and $y(\tau + \tau)$ denote the time-averaged signals of the two different probes. Substitution of eq. (6)
in eq. (9) leads to:

\[
\bar{\phi}_{xy} = \frac{1}{2 \cdot T n^2} \int_{-T}^{+T} \sum_{i=1}^{n} \sum_{j=1}^{n} x_i(t) x_j(t + \tau) d\tau
\]

\[
- \frac{1}{2 \cdot T n^2} \int_{-T}^{+T} x_1 x_2 + \ldots + x_n d\tau
\]

\[
\times (y_1 + y_2 + \ldots + y_n) d\tau
\]

(10)

(Note: the notations \((t)\) and \((t + T)\) have been left out for simplicity)

\[
- \frac{1}{2 \cdot T n^2} \left\{ \int_{-T}^{+T} x_1 y_1 d\tau + \int_{-T}^{+T} x_1 y_2 d\tau + \ldots + \int_{-T}^{+T} x_n y_n d\tau \right\}
\]

\[
+ \int_{-T}^{+T} x_n y_n d\tau
\]

(11)

\[
= \frac{1}{2 \cdot T n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \int_{-T}^{+T} x_i y_j d\tau
\]

\[
\Rightarrow \bar{\phi}_{xy} = \phi_{11} + \phi_{12} + \phi_{13} + \ldots + \phi_{nn}
\]

(12)

This shows that signals are correlated that are not obtained from the same bubble. If the distributions had been normal distributions it would not make any difference. But the \(t_i\) with the highest probability is not the average \(t_x\), since the probability distribution is log normal. With the average cross-correlation \(\bar{\phi}_{xy}\) there is, in general, a larger probability of finding a time difference that is smaller than the average time difference. A larger time difference is, of course, also possible depending on the skewness of the distribution. But, in general, smaller values will be found. The errors will become larger with higher skewness factors, because then the log-normal distribution differs more from the normal distribution. When the deviation is small enough the difference disappears. The experimentally measured ratio of the rising time \(t\), determined by the statistical method and by the cross-correlation is shown in Fig. 11. This shows that the expected trend indeed was found, though the deviations from the mean values are considerable.

**DETERMINATION OF LOCAL FLUIDIZING STATE**

The equations for calculating all necessary local parameters have all been described extensively by Werther (1972).

The pierced lengths and bubble velocities are stochastic variables. Average values will be used in our calculations.

The pierced length \(l_i\) of one bubble is given by its velocity \(u_{b,i}\) and pierced time \(t_{b,i}\):

\[
l_i = u_{b,i} t_{b,i}.
\]

(15)

Since the pierced length and bubble velocity are stochastically independent, the mean pierced length \(E[I]\) can be calculated by

\[
E[I] = u_b t_b.
\]

(16)

The average bubble velocity \(u_b\) can be calculated from

\[
u_b = \frac{s}{t_b}
\]

(17)

with \(s\) being the distance between the two probe points (for the probes we used \(s = 10\, \text{mm}\)).

The bubble hold up \(\delta\) can be determined from the total time the probe is immersed in bubbles \(n_t\) compared to the total measuring time \(T\):

\[
\delta = \frac{n_t}{T} = k t_b \quad \text{(with } k = n/T)\]

(18)

where \(k\) is the bubble frequency.

The local visible bubble gas flow is given by the measured total amount of gas in the bubbles (with the probe surface \(dA\) as reference) per square cm per

![Fig. 11. \(t_s\) values experimentally determined by statistical method and by cross-correlation as a function of determined deviation \(S_e\) (●) measured data points; (—) curve fit.](image-url)
second:

\[ \dot{V}_k \left( \frac{\text{cm}^3}{\text{cm}^2 \text{s}} \right) = \frac{n \partial A}{T \cdot \partial A} = k \dot{u}_m. \]  

(19)

The average values of these parameters were calculated by the 'cup mixing' principle:

\[ \ddot{v} = \int_0^R \dot{v} \, r \, dr \approx \frac{\sum_j \int_{R_{j-1}}^{R_j} \dot{v} \, r \, dr}{R^2}. \]  

(21)

The dense phase through flow factor \( \varphi \) is defined by Van Lare et al. (1991) and Van Lare (1991):

\[ \varphi = \frac{U - V_s}{U - U_m}. \]  

(20)

This factor gives the extra flow through the dense phase compared to the two-phase theory of Toomey and Johnstone (1952). When \( \varphi = 1 \) the two-phase theory applies. In practice, \( \varphi \) values \( > 1 \) can be found, since \( V_s \) denotes the bubble flow visible for the measuring device (see before).

### RESULTS AND DISCUSSION

It occurred occasionally that a measured time-averaged signal could not be fitted accurately (i.e. errors larger than about 5%). This occurred more often for the larger particle powders (230 and 587 \( \mu m \)) than for the smaller particle powders (106 and 165 \( \mu m \)). This is because: the change from the dense phase to the bubble phase is more gradual for the larger particle powders and the probe-size-particle-size ratio is smaller for the larger particles. The average values

### Table I(a). Results for the 106 \( \mu m \) quartz sand powder (\( U_m = 1.4 \text{ cm/s} \))

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<thead>
<tr>
<th>( H (\text{cm}) )</th>
<th>( U (\text{cm/s}) )</th>
<th>( h (\text{cm}) )</th>
<th>( k (s^{-1}) )</th>
<th>( \dot{E}[\text{W}] )</th>
<th>( u_m (\text{m/s}) )</th>
<th>( \delta )</th>
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### Table I(b). Results for the 106 \( \mu m \) quartz sand powder (\( U_m = 2.3 \text{ cm/s} \))

<table>
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<tr>
<th>( H (\text{cm}) )</th>
<th>( U (\text{cm/s}) )</th>
<th>( h (\text{cm}) )</th>
<th>( k (s^{-1}) )</th>
<th>( \dot{E}[\text{W}] )</th>
<th>( u_m (\text{m/s}) )</th>
<th>( \delta )</th>
<th>Bed expansion ( \varphi )</th>
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<td>~ 0.10 2.87</td>
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<td>~ 0.12 5.46</td>
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<td>6.73</td>
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</table>
were then calculated by extrapolation of the other radial values obtained at one height. Measuring conditions and results obtained for the several powders are given in Tables 1(a)–(d). Here only the radial averaged values at one height are given. Most values were obtained from the seven radial positions. Some parameters will be discussed individually.

**Bubble characteristics**

Bubble frequency

In general, the bubble frequency decreased with increasing height, due to coalescence, and increased with increasing superficial velocity.

**Mean pierced length**

The pierced length was calculated from two factors being \( t_b \) and \( t_a \) [eqs (16) and (17)] and therefore the accuracy will be lower and the variance larger than for the bubble frequency (which is measured directly). As was expected \( \bar{k} \) (the radially averaged mean pierced length) increased with increasing measuring height \( h \) and superficial gas velocity \( U \). Particle size had a minor effect at a given \( U - U_m \).

Many correlations to estimate the bubble diameter have been developed, e.g. Mori and Wen (1975), Darton et al. (1977) and Horio and Nonaka (1987), and most of these correlations lead to similar results. Because of the underlying theoretical principles the correlation of Darton et al. (1977) was used to get an estimate of the ratio of bubble diameter to mean pierce length \( d_b / \bar{k} \).

According to Darton et al. (1977) the ratio between the equivalent bubble diameter and the maximum

<p>| Table 1(c). Results for the 230 µm quartz sand powder (( U_m = 1.4 ) cm/s) |
|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|</p>
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<th>( U ) (cm/s)</th>
<th>( h ) (cm)</th>
<th>( k ) (s(^{-1}))</th>
<th>( \bar{k} ) (cm)</th>
<th>( \rho ) (m/s)</th>
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<td>( ~ 0.03 )</td>
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<p>| Table 1(d). Results for the 587 µm quartz sand powder (( U_m = 21.3 ) cm/s) |
|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|</p>
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<th>( h ) (cm)</th>
<th>( k ) (s(^{-1}))</th>
<th>( \bar{k} ) (cm)</th>
<th>( \rho ) (m/s)</th>
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<td>0.22</td>
<td>0.04</td>
<td>1.03</td>
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<td>0.79</td>
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<td>0.05</td>
<td>( ~ 0.07 )</td>
</tr>
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<td>( ~ 0.07 )</td>
</tr>
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<td>1.83</td>
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<td>1.29</td>
</tr>
<tr>
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<td>( ~ 0.07 )</td>
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<td>1.14</td>
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</table>
pierced length is about 1.6 for a spherical cap bubble. Because the bubbles are pierced in an arbitrary place the average $E[I]$ value will be smaller than the maximum $E[I]$ value at one height and so the ratio of $d_b$ (Darton) / $E[I]$ has to be larger than 1.6. From Fig. 12(a) it can be seen that this ratio is about 2.5-3. From Fig. 12(b) (data for 106 and 165 pm) this ratio was estimated to be about 2.7. Again the deviation of the values was smaller for the smaller particles.

A plot of $d_b$ (Darton) vs $E[I]$ yielded a linear relation which implies that the measured $E[I]$ values are related to the process variables as predicted as by the relation given by Darton et al. (1977). Furthermore, it implied that the $E[I]$ values could be used to investigate the bubble growth in height.

**Stable bubble height $h^*$**

Plotting the $E[I]$ values for the 106 um powder vs the measuring height $h$, it can be seen that $E[I]$ appears to reach a maximum value at a given bed height of about 60 cm (Fig. 13). This is the so called stable bubble height $h^*$ beyond which bubbles do not grow further: an equilibrium between coalescence and splitting is reached. This height $h^*$ has been introduced first by Werther (1978). For the other powders the bed height was not high enough to reach $h^*$, which indicates that there might be a particle-size dependency of $h^*$.

**Bubble velocity**

The bubble velocity $u_b$ increased with increasing measuring height $h$ and increasing superficial velocity $U$. This was in agreement with results of other authors (Clift and Grace, 1985). The total bed height appeared to be of no influence.

Figure 14 shows a plot of $(E[I]g)^{1/2}$ vs $u_b$, where the data points could be described by $u_b = 1.5(E[I]g)^{1/2}$.

There appeared to be a minor particle-size effect, which could be due to the fact that it is more difficult to determine a bubble boundary for the larger particles, because of the more gradual change discussed before. There is however another effect: for the smaller particles the bubble frequency and therefore the probability of coalescence were larger at a given $E[I]$. It is a well-known effect that the bubble rise velocity is influenced when a bubble is in the vicinity of other bubbles (see, for instance, Clift and Grace, 1985). This again could be an explanation for the fact that the bubble rise velocity is somewhat larger for the smaller particles at a given $E[I]$. Therefore, a particle-size effect cannot be ruled out nor can it be confirmed.

The measured bubble velocity could be described by

$$u_b \approx 1.5 \cdot \sqrt{E[I]g}.$$  \hspace{1cm} (22)

This relation can be compared to the relation of Werther (1978):

$$u_b = 1.6 \cdot D^{0.4} \sqrt{d_b g} \quad (\text{for} \ 0.1 \ m < D < 1 \ m)$$

$$= 0.92 \cdot \sqrt{d_b g} \quad (\text{with} \ D = 0.25 \ m)$$  \hspace{1cm} (23)
Table 2. Stable bubble height $h^*$ as a function of particle size $d_p$.

<table>
<thead>
<tr>
<th>$d_p$ ($\mu$m)</th>
<th>$h^*$ (cm)</th>
<th>Author</th>
</tr>
</thead>
<tbody>
<tr>
<td>~30</td>
<td>0</td>
<td>Werther (1983)</td>
</tr>
<tr>
<td>70</td>
<td>25</td>
<td>This work</td>
</tr>
<tr>
<td>106</td>
<td>60</td>
<td>This work</td>
</tr>
<tr>
<td>165/230/587</td>
<td>&gt;100</td>
<td>This work</td>
</tr>
</tbody>
</table>

where $D$ is the bed diameter. Equations (22) and (24) can be combined to give

$$u_b \approx 0.92 \cdot \sqrt{2.7 \cdot \frac{E[1]}{D} \cdot g}.$$  \hspace{1cm} (25)

The same ratio for $\frac{d_p}{E[1]}$ ($\approx 2.7$) is found as from the comparison with the relation given by Darton et al. (1977).

---

**The visible bubble gas flow $\dot{V}_b$**

In general, our measured values of the local visible bubble gas flow $\dot{V}_b$ were the same as those obtained by...
Werther (1972). With increasing bed height bubbles moved towards the bed center and away from the wall. This effect became more outspoken at larger \( U/U_{mf} \) ratios. The local maximum \( \bar{V}_b \) values were about 2 to 4 times the radial averaged \( \bar{V}_b \) value. The bed height, the superficial gas velocity and the particle size had no detectable effect on this ratio. At the reactor wall the visible bubble gas flow was virtually zero. This indicates a circulation of powder, which was generally observed by several investigators (e.g. Werther, 1972). Examples of \( \bar{V}_b \) vs \( r \) and \( h \) for the 106 and 165 \( \mu m \) powders are given in Figs 15 and 16.

The through flow factor \( q \)

The bubbles grow with increasing height which means that the \( q \) factor should decrease with increasing height because more bubbles will be 'visible' for the probe. This was indeed the case [see Tables 1(a) - (d)]. The \( q \) values obtained from measurements at different total bed heights but equal superficial velocities were virtually the same for the same particle size at equal measuring height. This indicated the negligible effect of total bed height on \( q(h) \).

Linear extrapolation of \( q(h) \) to \( h = 0 \) always gave \( q \approx U/U_{mf} \). An example is given in Fig. 17. This could be expected, because a porous plate was used as a gas distributor: the initial bubbles are that small that they cannot be detected and therefore \( V_b \) will be equal to zero at \( h = 0 \). Equation (20) then shows that \( q \) should indeed be equal to \( U/U_{mf} \) at \( h = 0 \).

**NOTATION**

- \( d_b \): average equivalent bubble diameter, cm
- \( d_p \): average particle size, \( \mu m \)
- \( D \): bed diameter, m
- \( E(t) \): mean pierced length, cm
- \( E_{pt} \): radial averaged mean pierced length, cm
- \( f(x) \): probability distribution of value \( x \)
- \( F(t) \): cumulative probability distribution for \( t \)
- \( g \): acceleration constant due to gravity, m/s\(^2\)
- \( h \): measuring height, cm
- \( h^* \): stable bubble height, m, cm
- \( k \): bubble frequency, l/s
- \( l_i \): local pierced length, cm
- \( M \): real average
- \( n \): total number of bubbles counted, dimensionless
- \( r \): radial position, cm
- \( R \): radius of fluid bed reactor, cm
- \( s \): distance between two probe points (=10 mm)
- \( S \): real deviation
- \( t \): real time, s
- \( T \): total measuring time, s
- \( t_a \): bubble rising time, s
- \( t_b \): bubble contact time, s
- \( t_{rig} \): trigger time, s
- \( t_0 \): time at which bubble hits lower probe, s
- \( t_m \): time at which maximum capacity change is reached, s
- \( t_3 \): time at which probe hits 'bottom' of bubble, s
- \( t_4 \): time at which bubble has passed probe completely, s
- \( u_{b, i} \): local bubble velocity, cm/s
- \( u_b \): averaged bubble velocity, cm/s
- \( U \): superficial gas velocity, m/s
- \( U_{mf} \): minimum fluidization velocity, m/s
- \( V_b \): local visible bubble gas flow, cm\(^3\)/cm\(^2\) s
- \( V_{bav} \): radial averaged visible bubble gas flow, cm\(^3\)/cm\(^2\) s
- \( V_i \): invisible bubble gas flow, cm\(^3\)/cm\(^2\) s
- \( V \): total bubble gas flow, cm\(^3\)/cm\(^2\) s

We estimated the bubble diameter to be about 0.7 times the mean pierced length \( E(t) \).

The results on stable bubble height \( h^* \) seemed to indicate that there is a relationship between this \( h^* \) and the average powder particle size. With increasing particle size \( h^* \) increases.

Based on the results presented here it seems reasonable to conclude that the deviations from the two-phase theory are not so large as is often assumed [e.g., Clift and Grace (1985) for a review]. The conclusions reported in literature are often based on experimental techniques with which it is not possible to measure the total bubble (and therefore dense phase) gas flows. The real dense-phase gas flow can only be measured with collapse experiments, and this is only possible with powders having a dense-phase expansion (A-type powders). It is therefore suggested to use the two-phase theory of Toomey and Johnstone (1952) or the \( n \)-type theory to describe the gas flow division in a gas fluidized bed.

**CONCLUDING REMARKS**

Information on the bubble characteristics was obtained from measurements based on electrical capacity changes. Four different powders were used.

The time averaged signal could not be used to obtain all necessary parameters for calculating \( \bar{V}_b \). Therefore, a statistical method was developed which gave good, reproducible and consistent results. It was possible to obtain information on the deviation in rising time and bubble contact time, which can be helpful for a statistical description of a fluidized bed.
x value of probe signal, V
y value of probe signal, V
y_{max} maximum value of probe signal, V

Greek letters
z slope of trapezium V/s
δ bubble hold up, dimensionless
θ starting value in probability distribution f(x)
μ average value of log-normal distribution
σ deviation of log-normal distribution
σ_{u} deviation of bubble rising time τ
τ time difference between two probe signals, s
φ density phase through flow factor, dimensionless
φ_{xy} cross-correlation of signals x and y

REFERENCES