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AN ASSOCIATIVE BLOCK DESIGN ABD(8,5)*

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To Maja, on the occasion of her seventeenth birthday.

Abstract. An associative block design is a certain balanced partition of a hypercube into smaller hypercubes. We construct such a design, thus settling the smallest open case.

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An ABD(k, w) is a $b \times k$ matrix (where $b = 2^w$) with entries from $\{0, 1, *\}$ such that (i) the stars form a 1-design: each row has $k-w$ stars and each column $b(k-w)/k$ stars, and (ii) the rows represent disjoint subsets of $\{0, 1\}^k$. Here a row represents the set of binary vectors of length $k$ obtained by replacing its stars in all possible ways by 0s and 1s.

This concept was introduced in 1974 by Rivest [4, 5, 6] in order to find a hash function with good worst-case behavior with respect to partial-match queries. For example, the eight rows

| 00*0 | 11*1 |
| 100* | 011* |
| *100 | *011 |
| 1*10 | 0*01 |

form an ABD(4, 3).

In order to save space, let us extend our alphabet with the minus sign, where a row containing $r$ minus signs stands for the $2^r$ rows obtained by replacing these minus signs in all possible ways by 0s and 1s. Then the only other ABD(4, 3) is the following:

| *000 |
| *111 |
| -*10 |
| -0*1 |
| -*10 |

The theory is as follows (see [1, 2, 3, 6]).

Proposition 0.1. (i) ([6]) An ABD(k, w) has exactly $bw/(2k)$ 0s and $bw/(2k)$ 1s in each column. In particular, $bw/(2k)$ is an integer.

(ii) ([1]) In an ABD(k, w) with $w > 0$ any given star pattern occurs in an even number of rows. Moreover, among the rows with a given star pattern there are as many with an even number of 1s as with an odd number of 1s.

(iii) For $w \leq 4$ the only ABD(k, w) are the trivial ones with $w = 0$ or $w = k$ (represented, respectively, by a single row of stars or minus signs only) and the two examples shown above.

(iv) ([2]) If $w > 3$, then $k \leq w(w-1)/2$.
There is no ABD(10, 5).

If ABD($k_i, w_i$) exist for $i = 1, 2$, then there also is an ABD($k_1k_2, w_1w_2$).

Suppose that $k \geq w > 0$ and $k' \geq w' > 0$ and $k' \geq k$ and $w'/k' \geq w/k$. Then if an ABD($k, w$) exists, and $2^w w'/(2k')$ is an integer, then ABD($k', w'$) also exists.

One may use generating function arguments to get more detailed information on the possible star patterns. See [1].

The purpose of this note is to show that an ABD(8, 5) exists:

\begin{verbatim}
-0000*** *01*10*0
-0001*** -*1*1*11
-001*0** **11*001
---1010* *10*00*0
*0*1*110 *1*0*001
***01*111 *100**11
---1110* -1*0*10*
-1*00*0 *1**0110
*010**01 *1**1000
-*1*0*11 -1***1*10
*010*1*0 *101*0*1
\end{verbatim}

Now the smallest open case is the question of whether an ABD(12, 6) exists.

Acknowledgment. This note was inspired by a letter from Knuth, who asked whether there had been any progress on ABDs since 1976 and in particular whether the existence of an ABD(8, 5) was still open.

REFERENCES


