Hierarchical mathematical programming for operational planning in a process industry *

W.G.M.M. Rutten

Graduate School of Industrial Engineering and Management Science, Eindhoven University of Technology, Eindhoven, Netherlands

Received June 1990; revised November 1990

Abstract: Scheduling techniques for discrete manufacturing industries do not fit in many process industries. A technique that often is used in process industries is mathematical programming. However, mathematical programming is used mostly for strategic or tactical planning. This article describes a problem at the operational planning level in a process industry. The (large) problem is decomposed into smaller problems, which can be solved separately with use of mathematical programming. The resulting models are less complex and require less calculation time than a large model. The decomposition is based on the uncertainties for raw materials and customer orders that exist for future time periods. In this way a Decision Support System is developed which has an acceptable calculation time.

Keywords: Process industry; Uncertainty; Operational planning; Mathematical programming

1. Introduction

Production planning in a firm can be divided into three hierarchical levels: the strategic, the tactical and the operational level. In many discrete manufacturing industries, MRP II has been developed as an overall concept for production control, with MRP I being used as a technique for material coordination at the operational level (Wight and Landvater, 1976).

In process industries, however, this MRP approach sometimes is not appropriate. For instance, the price of raw materials can cause variations in the Bill Of Material (BOM) (Taylor et al., 1981) That is, cheap raw materials are preferred to expensive ones, as long as they can be processed into products of the required quality. Also raw materials often have variable qualities which lead to variations in the BOM (Cokins, 1988; May, 1984; Taylor et al., 1981). This is contradictory to the MRP logic, where the (fixed) BOM ultimately determines what should be bought. In conclusion, the MRP technique cannot be used in some process industries because of the variable Bill Of Material.

A technique that often is used in this type of process industries is mathematical programming; it is used to compute recipes (BOMs) and product flows, considering product quality specifications and available capacities. This technique is used mostly at the strategic or tactical level. No cases are known which describe a mathematical...
programming model for operational planning in process industries.

This article describes the development of a set of models for operational planning in a division of a dairy firm. In the next section, the manufacturing process of the dairy firm is presented and in Section 3 the operational planning is introduced. These sections are mainly retrieved from Rutten (1989). In Section 4, the problems encountered in the operational planning are analyzed and the first solution (a large mathematical programming model) is described. Also the problems that arise by using the large model are given. In Section 5, the large model is hierarchically decomposed and the adjustment of the large model to several smaller models is explained. In the last section, some conclusions are presented.

2. The manufacturing process

The firm manufactures milk replacers for calves. A milk replacer is a powder which can replace mother's milk when it is dissolved in water. Approximately 100 different milk replacers are being made. A milk replacer is produced by blending a number of powders. Every product has some constraints on raw materials and on their ingredients (e.g. fat and protein). In most cases, a recipe uses six or seven raw materials. The final recipe for an order is computed with use of linear programming. The results of this computation (quantities of raw materials) are used by the production computer to control the production units in the factory.

The factory contains four mixers (volume equals 25000 kg each), sixty silos and a transportation system. In a mixer raw materials are homogenized and products are blended. In the silos raw materials are stocked for use in the production process. Twelve of the sixty silos are end product silos. Six of these twelve silos are attached to a rotating bagger and the other six silos are used for bulk products. The transportation system moves powders from a silo to a mixer and vice versa.

Raw material arrives at the factory as bulk or it can be put up in bags. The bulk powder is stored in a silo and the packed powder is stored in the stockroom. When the packed powder is needed, the bags are cut in a bag cutting machine and the powder is blown into a mixer. Raw materials are homogenized in a mixer and analysed before they can be processed. A limited amount of raw materials can be processed directly (without homogenization) in a product. The total production process is illustrated in Figure 1.

3. Operational planning

In the past, the first step of production planning was to make a production schedule every week. Customer orders were translated to production orders (batches of maximum 25000 kg) and these production orders were allocated to a production day in the next week. No attention was paid to capacity constraints or availability of raw materials. The next step consisted of multiplying the production orders by standard recipes which resulted in a (rough) requirements plan for the raw materials. Based on this requirements plan, decisions were made to buy new raw materials and to take in some raw materials from

![Figure 1. General flows of material. Raw materials arrive in two different forms; bulk or packed. Raw materials are homogenized in a mixer before they can be used for production and then are stored in a silo. To produce a product, the computed quantities of raw materials are collected in a mixer and blended. The product can be delivered in two different forms; bulk or packed](image)
the stockroom into the silos. But, the purchasing leadtime of some raw materials was larger than the planning horizon of one week (maximum leadtime is two weeks). Thus, some raw materials had to be purchased on estimated requirements. The last planning step was to release the production orders for the next week to the operators in the manufacturing department. During the week, the operators decided when to start each new production order.

Every raw material contains some ingredients (e.g. fat and protein) and the amount of these ingredients in a raw material varies. To account for seasonal effects, once a month standard recipes are computed, based on a (updated) standard composition of ingredients in the raw materials. But, every receipt of raw material has an unique composition of ingredients. Since customers demand a certain (fixed) quality for their products, one can only use the currently available raw materials for calculating a final recipe for an order because the composition of coming raw materials is not certain. Therefore, each recipe for an order needs to be optimized before actual manufacturing can start. Just before manufacturing started, the recipe for an order was optimized by the following LP model:

$$\text{Minimize } \sum_{r \in R} CR_r \times q_u_r$$

subject to

$$\sum_{r \in R} q_u_r = TQ \quad (1)$$

$$\sum_{r \in R} q_u_r \times PI_{r,i} \geq LI_i \times TQ \quad \text{for every } i, \quad (2)$$

$$\sum_{r \in R} q_u_r \times PI_{r,i} \leq UI_i \times TQ \quad \text{for every } i, \quad (3)$$

$$q_u_r \geq LR_r \times TQ / 100\% \quad \text{for every } r, \quad (4)$$

$$q_u_r \leq UR_r \times TQ / 100\% \quad \text{for every } r, \quad (5)$$

$$q_u_r \leq AQ_r \quad \text{for every } r \quad (6)$$

where:

- $R$: denotes the set of raw materials that are allowed to be used in the product.
- $CR_r$: denotes the cost of raw material $r$.
- $q_u_r$: denotes the (variable) quantity of raw material $r$ which is used for production of the product.
- $TQ$: denotes the total quantity to be produced.
- $PI_{r,i}$: denotes the percentage of raw material $r$ that consists of ingredient $i$.
- $LI_i$: denotes the lower bound percentage of ingredient $i$ in the product.
- $UI_i$: denotes the upper bound percentage of ingredient $i$ in the product.
- $LR_r$: denotes the lower bound percentage of raw material $r$ in the product.
- $UR_r$: denotes the upper bound percentage of raw material $r$ in the product.
- $AQ_r$: denotes the total available quantity (could be the tonnage in a silo) of raw material $r$.

The objective is to minimize the total costs of raw materials. The sum of the quantities used equals the ordered quantity (constraint (1)). The second and third constraint present the demanded amount of ingredients in the product. Constraints (4) and (5) denote the lower and upper bounds on raw materials in the product. The last constraint gives an upper bound to assure that the solution does not use a larger quantity of a raw material than available.

Actual recipes for orders were computed that differed completely from the standard recipes, because of variations in availability and composition of raw materials. This disturbed the requirements plan. In addition, it was possible that some orders could not be delivered in time, because capacity constraints were not considered in the planning process. The materials manager could not foresee this situation.

4. The first model

On analyzing this situation, the first problem is that the final recipe for an order needs to be calculated every time before actual production can start. This disturbs the requirements plan. A second problem is the fact that capacity usage is not controlled during the planning process. The last problem we mention is the purchasing leadtime which is two weeks for some raw materials while the planning horizon is one week. On the other hand, about 80–90% of the customer orders are known two weeks in advance. In conclusion, there are three problems: 1) uncertainty exists in the composition of ingredients in raw
materials, 2) capacity usage is not controlled and 3) the purchasing leadtime is larger than the operational planning horizon.

The first solution we applied was to cover all these problems at once by developing a mathematical programming model which optimizes all variables of all orders over the purchasing leadtime (classical approach). A model was developed which optimizes all production orders for ten days simultaneously instead of per order (as is the case in the old situation). By optimizing simultaneously, the most economical allocation of the raw materials to orders is computed. (By optimizing sequentially, the first order will use raw materials which cannot be used by subsequent orders. Consequently, successive orders can become very expensive or even infeasible.) Also, capacity constraints were added to control the effective use of the mixers, rotating bagger, bag cutting machine and the bulk product silos. The model is described in the Appendix.

This model is very complex and too large to handle (over 12000 variables). The calculation time was too long compared to the planning horizon (approximately one day calculation time for a planning horizon of ten days). In practice, this implicates that no alternatives can be compared. Furthermore, the impact of information at the end of the horizon on current decisions decreases; the balance between detail and usage of information and effort to obtain the information should be considered here (Carlson, Beckman and Kropp, 1982; Baker, 1977). When the parameters are stochastic (uncertainty), the marginal impact of future information decreases even more. In conclusion, the large mathematical model is not useful in practice.

5. Three levels of uncertainty

Basically, there are two approaches to production control. First, there is the classical approach, which covers the total problem with use of one, large model. Second, there is the hierarchical approach, which decomposes a problem into subproblems which can be managed separately. In the literature, this hierarchical approach is preferred more and more to the classical approach of total optimization (Bitran and Hax, 1977; Hax and Bitran, 1979; Hax and Meal, 1975; Wester, Wijngaard and Zijm, 1989). In the beginning, we applied the classical approach, which resulted in a large model with some serious drawbacks (e.g. enormous calculation time, very complex). Therefore, we changed to the hierarchical approach and decomposed the problem into smaller problems, based on the uncertainty that exists for future time periods (to create a balance between detail of information and effort to obtain the information).

Three levels of uncertainty can be distinguished for this production situation (Rutten, 1989):

1. More than ten days ahead; not all production orders are known and all of the raw materials still have to arrive. This level is very uncertain. When optimizing at this level, it mostly will result in the standard recipe.
2. Three up to ten days ahead; all production orders are known, but not all raw materials are available yet. The raw materials which are not yet available will deviate in their composition of ingredients and total quantity when they arrive. At this level there is some uncertainty.
3. One and two days ahead; all production orders are known and all raw materials have to be present. At this level there is approximately 100% certainty.

Notice that the problems mentioned earlier only affect the lower two ‘uncertainty levels’. At the first level optimization is not useful. At this level the (estimated) order quantities are multiplied by standard recipes, which results in a rough requirements plan for the raw materials. In accordance to the two lowest levels of uncertainty, the large model as mentioned in Section 4 was divided into two parts.

For the second level (days three up to ten), the large model described in the Appendix is adjusted as follows:

- The model schedules orders at the optimal production day considering the capacity constraints.
- The objective is effective use of available capacity. The objective function is modified: capacity is used as soon as possible, thus remaining capacity occurs at the end of the planning period. This creates flexibility towards customers.
- No optimization of recipes takes place. The recipe constraints are deleted. Standard, fixed recipes will be used in the optimization.
• Because the models are used on a rolling horizon basis, the orders that are calculated at day three are a suggestion of the set of orders that can be shifted to the next level.

For the third level (days one and two), the large model described in the Appendix is adjusted as follows:

• The orders that flow from the second to the third level will never exceed the available capacity, because of the capacity constraints at the second level; the capacity constraints are deleted.

• The production day of an order in the third level is taken fixed: day one or day two. The sequence of orders during a day is decided by the operators.

• The materials manager can add some extra constraints, for example, minimum use of a raw material over all orders in a specified time period.

• To handle the variations in the composition of ingredients in raw materials, new, more rigid constraints are developed to assure that the final recipe for an order remains close to the standard recipe. The new constraints were determined in order that the total remaining flexibility of the recipe is sufficient to handle variations in the composition of ingredients in raw materials, but also in order that less variations in recipes occur (for example, the original constraint allowed raw A up to 100% in the product; the new constraint allows raw A between 42% and 48%; this will remain close to 45% of raw A as is the case in the standard recipe).

These smaller models have more acceptable calculation times (nearly half an hour). Other benefits are:

• More accurate forecasts of requirements of raw materials can be made. Therefore, the information for the supply department is more reliable.

• Besides control on the composition of ingredients in the product, the product is organoleptically (taste, aroma, etc.) and physically more constant because of the more rigid recipe constraints. This improves the quality of the product.

• Information about future time periods becomes sooner available because of the rolling plan.

The links between the different levels are made by the materials manager. The materials manager decides when production orders enter the next (lower) level. The addition of human control transforms these models into a real Decision Support System.

6. Conclusion

It was demonstrated in this case that it is possible to split up a large mathematical programming model into smaller ones based on decomposition of the planning horizon. The large model is of no practical use because of the enormous calculation time needed; the smaller models have acceptable calculation times, but they do not give the (total) optimal solution. This also indicates the main difference between the hierarchical approach and the classical approach. Since one should balance the practical value of information against computational effort needed to obtain the information, it can be concluded that in this case decomposition of the problem can be justified.

Appendix

The following notation is used in the model description:

- \( p \): Indices for the product. \( p \) can be a packed product or a bulk product. The set \( P(a) \) denotes the set of products which are packed; \( P(b) \) denotes the set of products which are sold as bulk.

- \( r \): Indices for raw materials. The set \( R(p) \) denotes a set of raw materials which are allowed to be used in product \( p \).

- \( i \): Indices for ingredients.

- \( t \): Indices for days. \( t \) ranges from 1 to \( T \) (in this case \( T = 10 \)).

The variables in the model are notated in lowercase.

- \( C_r \): Cost of raw material \( r \).

- \( tq_{p,t} \): Total quantity of product \( p \) to be produced at day \( t \). Equals either zero or the total order quantity.

- \( OQ_p \): Total order quantity of product \( p \).

- \( \beta_{p,t} \): Binary variable which indicates the production day of product \( p \).

- \( qu_{r,p,t} \): Quantity of raw material \( r \) for product \( p \) at day \( t \) which is stored in a silo.
qd\_{r,p,t}::\text{Quantity of raw material } r \text{ which is directly used from packed form in a product } p \text{ at day } t.

UD_{p}::\text{Maximum quantity of raw materials that can be used from packed form in product } p.

PI_{r,i}::\text{Percentage of ingredient } i \text{ in raw material } r.

UI_{p,i}::\text{Maximum percentage of ingredient } i \text{ in product } p.

LI_{p,i}::\text{Minimum percentage of ingredient } i \text{ in product } p.

UR_{r,p}::\text{Maximum percentage of raw material } r \text{ in product } p.

LR_{r,p}::\text{Minimum percentage of raw material } r \text{ in product } p.

CP_{p}::\text{Capacity use factor of product } p \text{ on the mixer.}

CR_{r}::\text{Capacity use factor of raw material } r \text{ on the mixer for homogenization.}

CD_{r}::\text{Capacity use factor of directly used packed raw material } r \text{ on the mixer.}

CC_{r}::\text{Capacity use factor of packed raw material } r \text{ on the bag cutting machine.}

CA_{p}::\text{Capacity use factor of product } p \text{ on the rotating bagger.}

CB_{p}::\text{Capacity use factor of product } p \text{ on the bulk silos.}

AM_{t}::\text{Available capacity of the mixers during day } t.

AC_{t}::\text{Available capacity of the bag cutting machine during day } t.

AA_{t}::\text{Available capacity of the rotating bagger during day } t.

AB_{t}::\text{Available capacity of the bulk silos during day } t.

tm_{t}::\text{Overtime of the mixers during day } t.

tc_{t}::\text{Overtime of the bag cutting machine during day } t.

ta_{t}::\text{Overtime of the rotating bagger during day } t.

AQ_{r}::\text{Available quantity of raw material } r.

AT(r)::\text{Arrival day of raw material } r.

\gamma_{r,t}::\text{Binary input value to trigger the arrival day; } \gamma_{r,AT(r)} = 1.

xa_{r}::\text{Extra needed quantity of raw material } r, \text{ after arrival of raw material.}

xb_{r}::\text{Extra needed quantity of raw material } r, \text{ before arrival of raw material.}

XCR_{r}::\text{Cost of raw material } r, \text{ when use before arrival of the raw material.}

\text{Minimize } \sum_{t=1}^{T} \left[ \sum_{p \in P(a+b)} \sum_{r \in R(p)} \left[ qu_{r,p,t} + qd_{r,p,t} \right] \times C_{r} + \sum_{r \in R(p)} xa_{r} \times CR_{r} + \sum_{r \in R(p)} xb_{r} \times XCR_{r} \right] 

\text{subject to}

1. General constraints:

\begin{align*}
t q_{p,t} &= OQ_{p} \times \beta_{p,t} \quad \text{for every } p,t, \\
\sum_{t=1}^{T} \beta_{p,t} &= 1 \quad \text{for every } p, \\
\sum_{t=1}^{T} \left[ qu_{r,p,t} + qd_{r,p,t} \right] &= t q_{p,t} \quad \text{for every } p,t, \\
\sum_{t=1}^{T} \sum_{r \in R(p)} qd_{r,p,t} &\le UD_{p} \times OQ_{p}/100\% \quad \text{for every } p, \\
\sum_{t=1}^{T} \sum_{r \in R(p)} qu_{r,p,t} &= x b_{r} \quad \text{for every } r, \\
\sum_{t=1}^{T} \sum_{r \in R(p)} qu_{r,p,t} &\le AQ_{r} + xa_{r} \quad \text{for every } r, \\
\sum_{t=1}^{T} \sum_{r \in R(p)} qu_{r,p,t} &\le LR_{r,p} \times OQ_{p} \quad \text{for every } p, \\
\sum_{t=1}^{T} \sum_{r \in R(p)} qu_{r,p,t} &\ge UR_{r,p} \times OQ_{p} \quad /100\% \quad \text{for every } p, \\
\sum_{t=1}^{T} \sum_{r \in R(p)} qu_{r,p,t} &\ge LR_{r,p} \times OQ_{p} \quad /100\% \quad \text{for every } p,
\end{align*}

2. Recipe constraints:

\begin{align*}
\sum_{t=1}^{T} \sum_{r \in R(p)} \left[ qu_{r,p,t} + qd_{r,p,t} \right] \times PI_{r,t} &\le UI_{p,t} \times OQ_{p} \quad \text{for every } p, \\
\sum_{t=1}^{T} \sum_{r \in R(p)} \left[ qu_{r,p,t} + qd_{r,p,t} \right] \times PI_{r,t} &\ge LI_{p,t} \times OQ_{p} \quad \text{for every } p, \\
\sum_{t=1}^{T} \sum_{r \in R(p)} \left[ qu_{r,p,t} + qd_{r,p,t} \right] &\le UR_{r,p} \times OQ_{p} \quad /100\% \quad \text{for every } p, \\
\sum_{t=1}^{T} \sum_{r \in R(p)} \left[ qu_{r,p,t} + qd_{r,p,t} \right] &\ge LR_{r,p} \times OQ_{p} \quad /100\% \quad \text{for every } p,
\end{align*}

3. Capacity constraints:

\begin{align*}
\sum_{p \in P(a+b)} CP_{p} \times t q_{p,t} \\
+ \sum_{p \in P(a+b)} \sum_{r \in R(p)} CD_{r} \times qd_{r,p,t} \\
+ \sum_{r} CR_{r} \times AQ_{r} \times \gamma_{r,t} &\le AM_{t} + tm_{t}
\end{align*}
for every \( t \),
\[
\sum_{p \in P(a+b)} \sum_{r \in R(p)} CC_r \times q_{d,r,p,t} \leq AC_t + tc_t
\]
for every \( t \),
\[
\sum_{p \in P(a)} CA_p \times t_{q,p,t} \leq AA_t + ta_t \quad \text{for every} \ t,
\]
\[
\sum_{p \in P(b)} CB_p \times t_{q,p,t} \leq AB_t \quad \text{for every} \ t.
\]

References


