I. INTRODUCTION

For decades, display researchers have been looking for alternatives to the conventional bulky cathode ray tube displays. Recently, two alternative technologies have emerged for large size television monitors, which both make use of plasma. The first and most common is the plasma display panel technology, using the ultraviolet emission from a plasma to excite phosphors.\(^1\) The other is the plasma addressed liquid crystal (PALC) technology.\(^2\)–\(^4\) This technology, invented at Tektronics, uses microdischarges as electrical switches for the addressing of a liquid crystal layer.

Liquid crystal (LC) is an electro-optic material: It can alter the state of polarization of light passing through it. The polarization changing effect depends on the electric field applied to the LC. In liquid crystal displays (LCDs), light from a uniform backlight passes first through a polarizing layer, then through a layer of LC, and finally through a second polarizing layer. The percentage of light transmitted by the second polarizer depends on the electric field in the LC, and unaffected by any change in the data voltages. The electric fields in the LC layer are produced by depositing surface charge on top of the microsheet. It continues to build up this surface charge until the fields in the channel have vanished and the data voltages stand entirely across the LC layer and the microsheet. After the plasma has completely decayed, the surface charge is fixed and unaffected by any change in the data voltages. The resulting transmission through the channel.

The image on the display is written row by row, i.e., all pixels of one picture row are addressed at the same time. In order to address a certain row, a discharge is created in the corresponding channel by applying a short dc voltage pulse to the channel electrodes. During the afterglow of the discharge, small voltages—representing the data to be written on the pixels of the row—are applied to the data electrodes with respect to the channel electrodes. The decaying plasma in the channel screens itself from the resulting electric fields by depositing surface charge on top of the microsheet. It continues to build up this surface charge until the fields in the channel have vanished and the data voltages stand entirely across the LC layer and the microsheet. After the plasma has completely decayed, the surface charge is fixed and unaffected by any change in the data voltages. The electric fields in the LC (and the resulting transmission through the second polarizer) of the picture row will now remain nearly unchanged until the next discharge pulse is applied to the channel.

For successful application of the PALC discharges, it is essential to understand the physics underlying their behavior. The discharges in question are in the glow regime: they are characterized by a very low ionization degree (\(<10^{-6}\)) and space charge densities governing the electric field. Although
the physical principles of glow discharges are nowadays quite well understood, their exact implications for the PALC operation are unclear. Issues of special interest are the charging of the microsheet, which must be accurate and homogeneous, and the decay time of the plasma in the channels, which must be as short as possible. In view of the small size of the discharges, experimental results are usually coarse and hard to interpret, so that discharge modeling is an indispensable tool in the development of the PALC technology. Modeling work on PALC has been reported previously in Ref. 5. In this article we present a two-dimensional discharge model, which is suitable for the simulation of the PALC operation.

The article is organized as follows: Sec. II describes the physical and numerical principles of the model, as well as the input data that have been used in the calculations. In Sec. III we present several examples of PALC simulation results, illustrating the usefulness of the model.

II. DESCRIPTION OF THE MODEL

This section discusses the physical and numerical principles of the model. Input data are presented for pure helium and helium–hydrogen mixtures.

A. Fluid model

We pursue the well-known fluid approach and describe the behavior of reactive particle species by the first few moments of the Boltzmann equation: the continuity equation, the momentum transport equation, and the energy transport equation. Since this approach has been described previously in numerous articles,\textsuperscript{6–17} we will only outline it briefly here.

Given the low ionization degree, the density and temperature of the gas particles are assumed to be constant and unaffected by the plasma. For every plasma particle species \( p \) the density results from a continuity equation

\[
\frac{\partial n_p}{\partial t} + \nabla \cdot \Gamma_p = S_p,
\]

where \( n_p \) is the density, \( \Gamma_p \) the flux, \( S_p \) the source term, and the index \( p \) can indicate: electrons (\( p = e \)), an ion species, or a neutral species. The flux is given by the momentum transport equation, which we approximate by the drift–diffusion equation

\[
\Gamma_p = \text{sgn}(q_p) \mu_p E n_p - D_p \nabla n_p. \tag{2}
\]

\( E \) is the electric field, \( q_p \) the particle charge, \( \mu_p \) the mobility, and \( D_p \) the diffusion coefficient. The first term gives the flux due to the electric field (drift) and the second term represents the flux due to concentration gradients (diffusion). Particle inertia is neglected.

The source term \( S_p \) is determined by the reactions occurring in the plasma. It consists of positive contributions from the reactions in which a particle of species \( p \) is created and negative contributions from those in which such a particle is lost:

\[
S_p = \sum_r c_{p,r} R_r. \tag{3}
\]

The index \( r \) refers to a reaction; \( c_{p,r} \) is the net number of particles of species \( p \) created in one reaction of type \( r \), and it can be negative as well as positive. The reaction rate \( R \) is proportional to the densities of the reacting particles:

\[
R = kn_1 n_2 \tag{4}
\]

for two-body reactions, and

\[
R = kn_1 n_2 n_3 \tag{5}
\]

for three-body reactions. The proportionality constant \( k \) is the reaction rate coefficient. Similarly, the rate of spontaneous decay processes is

\[
R = kn. \tag{6}
\]

where \( k \) is the decay frequency.

Transport equations (1) and (2) require the input of reaction rate coefficients \( k \) and transport coefficients \( \mu \) and \( D \). In general these quantities depend on the energy distribution of the considered particles. We use the following approximations concerning these dependencies:

We assume that the charged particle transport coefficients satisfy the Einstein relation

\[
D = \frac{k_B T \mu}{e}, \tag{7}
\]

where \( k_B \) is the Boltzmann constant, \( e \) is the elementary charge, and \( T \) is the particle temperature, corresponding to the energy of the random particle motion.

For ions we use the local field approximation, which assumes a direct relation between the particle energy distribution and the electric field. Transport and rate coefficients are regarded as functions of the electric field:

\[
\mu = \mu(E), \quad D = D(E), \quad k = k(E). \tag{8}
\]

These relations can be found in the literature as results of experiments and classical theories. In particular, the ion diffusion coefficients are found from the mobilities by the Einstein relation (7), in which the ion temperature is related to the electric field by\textsuperscript{18,19}
\[ k_B T = k_B T_g + \frac{m + m_g}{5m + 3m_e} m_e (\mu E)^2, \]  

where \( T_g \) is the gas temperature and \( m \) and \( m_g \) are the ion and gas particle mass, respectively.

For electrons, however, the local field approximation often leads to unsatisfactory modeling results, as a result of the huge mass difference. Rather than using the relations (8), we therefore assume the electron transport coefficients and the rate coefficients of electron impact reactions to be functions of the electron mean energy, as in Refs. 9 and 11:

\[ \mu_e = \mu_e (\bar{e}), \quad D_e = D_e (\bar{e}), \quad k = k (\bar{e}), \]  

where the subscript \( e \) refers to electrons, and the electron mean energy \( \bar{e} \) is calculated as a function of time and space from an energy balance equation

\[ \frac{\partial n_e}{\partial t} + \nabla \cdot \left( \frac{5}{3} \bar{e} \Gamma_e + q \right) = S_e. \]  

In this equation \( n_e \) is the electron energy density

\[ n_e = n_e \bar{e}, \]  

and \( q \) is the heat flux, which we assume to be proportional to the electron mean energy gradient according to

\[ q = -\frac{2}{5} n_e D_e \nabla \bar{e}. \]  

The source term for electron energy is given by

\[ S_e = -e \Gamma_e E - n_e \sum_t \bar{e}_t k_t n_t, \]  

where the two terms represent heating by the electric field and energy loss in collisions, respectively. The summation in the loss term is only over the electron impact reactions, with \( n_t \) the density of the target particles and \( \bar{e}_t \) the threshold energy.

The functions (10) are obtained from cross sections, assuming a Maxwellian electron energy distribution function, or—which usually gives better results—using the electron energy distribution function resulting from uniform-field Monte Carlo or Boltzmann calculations. In the above equations, the electron mean energy is supposed to result mainly from random motion, so that it is consistent to use

\[ T_e = \frac{2}{3} \bar{e} \]  

with the Einstein relation (7) to find the electron diffusion coefficient.

Finally, the electric field depends on the space charge density according to Poisson’s equation

\[ \nabla \cdot (\varepsilon E) = -\nabla \cdot (\varepsilon \nabla V) = \rho, \]  

where \( \varepsilon \) is the dielectric permittivity, \( V \) the electrostatic potential, and \( \rho \) the space charge density

\[ \rho = \sum_p q_p n_p. \]  

B. Boundary conditions

Since the boundary conditions we apply are slightly different from the boundary conditions found in most articles, we will discuss them somewhat more elaborately. A full derivation and discussion of our boundary conditions for particle transport is given in Ref. 20.

The transport equations for heavy particles are solved for the boundary condition of zero particle influx. A microscopic analysis yields the following expression for the particle flux toward the wall:

\[ \Gamma \cdot n = a \sgn(q) \mu E \cdot nn + \frac{1}{2} v_{th} n - \frac{1}{2} D \nabla n \cdot n, \]  

where \( n \) is the normal vector pointing outward and \( v_{th} \) is the thermal velocity

\[ v_{th} = \sqrt{\frac{8 k_B T}{\pi m}}. \]  

The number \( a \) is set to one if the drift velocity is directed toward the all and to zero otherwise:

\[ a = \begin{cases} 1, & \sgn(q) \mu E \cdot n > 0 \\ 0, & \sgn(q) \mu E \cdot n \leq 0 \end{cases}. \]  

The first term on the right-hand side of Eq. (18) represents the flux due to the electric field; the last two terms are the diffusion flux, due to the random motion of the particles. Most authors\(^6\)–\(^10\),\(^15\),\(^17\) apply Eq. (18) without the last term, which they wrongly ignore; this term reflects the fact that the random motion flux involves all particles within a certain mean free path from the wall, not just the local particles at the wall. In order to incorporate the last term without running into numerical difficulties in accurately evaluating the density gradient it contains, we rewrite Eq. (18) as follows:

Imposing Eq. (18) as a boundary condition for the drift–diffusion Eq. (2) implies that the following equation must hold at the boundary:

\[ \sgn(q) \mu E \cdot nn - D \nabla n \cdot n = a \sgn(q) \mu E \cdot nn \]  

Note that although both members of this equation contain similar terms, their nature is very different: the left member is a continuum expression, which in principle can be used anywhere in space, but only has physical meaning inside the plasma volume, whereas the right member is a kinetic expression for the flux at the boundary. From Eq. (21) we find an expression to replace the last term in (18). Substitution of that expression gives

\[ \Gamma \cdot n = (2a - 1) \sgn(q) \mu E \cdot nn + \frac{1}{2} v_{th} n, \]  

which is an appropriate boundary condition for heavy species.

The boundary condition for electrons is similar, but includes influx by secondary electron emission. It is important to realize that the electrons emitted from the surface all have velocities directed away from the wall, and are described rather poorly by the drift–diffusion approximation. The most serious artifact arising from the poor description of these electrons is an unrealistic flux back to the surface by random...
motion. In order to find an appropriate boundary condition, which avoids this undesired effect, we distinguish between two electron groups at the wall: γ electrons, emitted from the surface, and α electrons, coming from the bulk. Both groups are treated equally and indistinguishably with the drift–diffusion equation, but have different boundary conditions. To the α electrons we apply the boundary condition (22):

$$\Gamma_\alpha \mathbf{n} = -(2a_e - 1)\mu_e \mathbf{E} \cdot \mathbf{n} n_\alpha + \frac{1}{2} v_\text{inh} n_e n_\alpha,$$

(23)

where $\Gamma_\alpha$ and $n_\alpha$ are the flux and density of the α electrons. In contrast, the γ electrons do not flow (back) to the wall;

$$\Gamma_\gamma \mathbf{n} = -(1-a_e) \sum_p \gamma_p \Gamma_p \mathbf{n},$$

(24)

where $\Gamma_\gamma$ is the flux of γ electrons, and the factor $(1-a_e)$ is included to cancel the flux in case the electric field is directed away from the wall. Using Eq. (24) as a boundary condition for the drift-diffusion equation for γ electrons gives

$$-\mu_e \mathbf{E} \cdot \mathbf{n} n_\gamma - D_e \nabla n_\gamma \cdot \mathbf{n} = -(1-a_e) \sum_p \gamma_p \Gamma_p \mathbf{n}.$$ (25)

For the regions where secondary electron emission is the most important, it is justified to neglect the diffusion term in Eq. (25) in any case, which yields the following density for the γ electrons:

$$n_\gamma = (1-a_e) \sum_p \gamma_p \Gamma_p \mathbf{n} \overline{\mu_e \mathbf{E} \cdot \mathbf{n}}.$$ (26)

Adding the fluxes (23) and (24), and using Eq. (26), now leads to a suitable boundary condition for the total electron flux

$$\Gamma_e \mathbf{n} = -(2a_e - 1)\mu_e \mathbf{E} \cdot \mathbf{n} n_e + \frac{1}{2} v_\text{inh} (n_e - n_\gamma) - 2(1-a_e) \sum_p \gamma_p \Gamma_p \mathbf{n}.$$ (27)

where $n_\gamma$ is once again given by Eq. (26). For the sake of numerical stability, it is useful to approximate $n_\gamma$ (26) in this equation by

$$n_\gamma \approx (1-a_e) \frac{1}{\mu_e} \sum_p \gamma_p \left[(2a_e - 1)\overline{\text{sgn}(q_p)} + \frac{1}{2} \sqrt{\frac{16(m_p + m_e)}{3\pi(5m_p + 3m_e)m_g}} \mu_p \mu_p \right].$$ (28)

For this approximation we assumed the electric field to be perpendicular to the wall, and used Eqs. (22), (19), and (9).

The boundary condition for the electron energy transport is very similar to the one for electron transport (27):

$$\Gamma_e \mathbf{n} = -(2a_e - 1)\frac{5}{3} \mu_e \mathbf{E} \cdot \mathbf{n} n_e + \frac{1}{2} v_\text{inh} (n_e - n_{e,\gamma}) - 2(1-a_e) \sum_p \gamma_p \overline{\varepsilon_p} \Gamma_p \mathbf{n}.$$ (29)

where

$$n_{e,\gamma} = (1-a_e) \sum_p \gamma_p \overline{\varepsilon_p} \Gamma_p \mathbf{n} \overline{\mu_e \mathbf{E} \cdot \mathbf{n}},$$ (30)

and $\overline{\varepsilon_p}$ is the mean initial energy of electrons emitted by incidence of species $p$. Expression (30) can then be approximated analogous to (28).

The electrode potentials are boundary conditions for Poisson’s equation. Dielectric materials surrounding the

---

**Table I. Reactions in helium–hydrogen mixtures. The second column gives the mean electron energy lost in the reaction.**

<table>
<thead>
<tr>
<th>No.</th>
<th>Reaction</th>
<th>Energy (eV)</th>
<th>Rate coefficient</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$e + \text{He} \rightarrow e + \text{He}^+$</td>
<td>20.215</td>
<td>Fig. 3(a)</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>$e + \text{He}^+ \rightarrow 2e + \text{He}^+$</td>
<td>4.365</td>
<td>Fig. 3(b)</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>$\text{e} + \text{He} \rightarrow 2e + \text{He}^+$</td>
<td>24.58</td>
<td>Fig. 3(c)</td>
<td>29</td>
</tr>
<tr>
<td>4</td>
<td>$e + \text{He}^+ \rightarrow e(20.2 \text{ eV}) + \text{He}$</td>
<td>-20.215</td>
<td>$2.9 \times 10^{-9}$ cm$^3$/s</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>$\text{He}^+ + 2\text{He} \rightarrow \text{He}_2^+$ + $\text{He}$</td>
<td>-15.0</td>
<td>$8.7 \times 10^{-9}$ cm$^3$/s</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>$\text{He}^+ + \text{He}^+ \rightarrow e(15.0 \text{ eV}) + \text{He}^+ + \text{He}$</td>
<td>-17.4</td>
<td>$2.03 \times 10^{-9}$ cm$^3$/s</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>$\text{He}^+ + \text{He}^+ \rightarrow e(17.4 \text{ eV}) + \text{He}_2^+$</td>
<td>11.7</td>
<td>Fig. 3(f)</td>
<td>29</td>
</tr>
<tr>
<td>8</td>
<td>$e + \text{He}_2 \rightarrow e + 2\text{H}$</td>
<td>15.4</td>
<td>Fig. 3(g)</td>
<td>29</td>
</tr>
<tr>
<td>9</td>
<td>$e + \text{He}_2 \rightarrow 2e + \text{H}_2^+$</td>
<td>0</td>
<td>Fig. 3(h)</td>
<td>29</td>
</tr>
<tr>
<td>10</td>
<td>$e + \text{He}_2^+ \rightarrow 3\text{H}$</td>
<td>0</td>
<td>Fig. 3(i)</td>
<td>29</td>
</tr>
<tr>
<td>11</td>
<td>$e + \text{He}_2^+ \rightarrow \text{He}^+(\nu^+) + \text{H}$</td>
<td>2.1 $\times 10^{-9}$ cm$^3$/s</td>
<td></td>
<td>32</td>
</tr>
<tr>
<td>12</td>
<td>$\text{He}_2^+ + \text{He} \rightarrow \text{He}_2^+ + \text{H}$</td>
<td>3.1 $\times 10^{-9}$ cm$^3$/s</td>
<td></td>
<td>32</td>
</tr>
<tr>
<td>13</td>
<td>$\text{He}_2^+ + \text{H} \rightarrow \text{He}_2^+ + \text{He}$</td>
<td>3.1 $\times 10^{-9}$ cm$^3$/s</td>
<td></td>
<td>32</td>
</tr>
<tr>
<td>14</td>
<td>$\text{He}^+ + \text{He} \rightarrow e + \text{He}^+$</td>
<td>-4.815</td>
<td>$1.38 \times 10^{-11}$ cm$^3$/s</td>
<td>33</td>
</tr>
<tr>
<td>15</td>
<td>$\text{He}_2^+ + \text{He}_2 \rightarrow 2e + \text{He}$</td>
<td>4.1 $\times 10^{-10}$ cm$^3$/s</td>
<td></td>
<td>34</td>
</tr>
<tr>
<td>16</td>
<td>$\text{He}^+ + \text{He} \rightarrow e + \text{He}^+$</td>
<td>-6.615</td>
<td>$1.1 \times 10^{-9}$ cm$^3$/s</td>
<td>35</td>
</tr>
<tr>
<td>17</td>
<td>$e + \text{He}_2^+ + \text{He}^+(\nu^+)$</td>
<td>12.0</td>
<td>Fig. 3(e)</td>
<td>29</td>
</tr>
<tr>
<td>18</td>
<td>$e + \text{He}_2^+(\nu^+) \rightarrow \text{He}^+ + 2\text{H}$</td>
<td>0</td>
<td>Fig. 3(i)</td>
<td>29</td>
</tr>
<tr>
<td>19</td>
<td>$\text{H}^+ + \text{He} \rightarrow e + \text{H}_2^+$</td>
<td>7.6 $\times 10^{-11}$ cm$^3$/s</td>
<td></td>
<td>32</td>
</tr>
<tr>
<td>20</td>
<td>$\text{H}^+ + \text{He}_2 \rightarrow 2\text{H}_2$</td>
<td>2 $\times 10^{-7}$ cm$^3$/s</td>
<td></td>
<td>36</td>
</tr>
</tbody>
</table>
plasma do not naturally impose boundary conditions for the electric field, so that it is necessary to solve Poisson’s equation in the entire dielectric surroundings of the plasma. The effect of surface charge on top of a dielectric wall is described by Gauss’s law

\[ \varepsilon_{\text{wall}} \mathbf{E}_{\text{wall}} \cdot \mathbf{n} - \varepsilon_{\text{0}} \mathbf{E}_{\text{plasma}} \cdot \mathbf{n} = \sigma, \]

where \( \mathbf{E}_{\text{wall}} \) and \( \mathbf{E}_{\text{plasma}} \) are the electric fields at the surface, respectively, inside and outside the dielectric material. The surface charge density \( \sigma \) results from plasma currents striking the wall. We assume that this wall charge does not diffuse along the surface, but stays in the very spot it is deposited by the plasma:

\[ \sigma = \int j \cdot \mathbf{n} \, dt, \]

where \( j \) is the plasma current density

\[ j = \sum_p q \Gamma_p. \]

The current through an electrode is given by

\[ I = \int \int_{\text{electrode surface}} \left( j \cdot \mathbf{n} - \epsilon \frac{\partial}{\partial t} \mathbf{E} \cdot \mathbf{n} \right) d^2S. \]

The second term of this equation is the displacement current; it corresponds to changes in total amount of charge present at the surface of the electrode. An external electric circuit, involving back coupling from the discharge current to the electrode voltage, is not included in the model: The electrode potentials are imposed at all times.

**C. Numerical implementation**

The numerical method used for solving the system of equations is described elaborately in Ref. 23, and will therefore only be outlined here. A unique feature of this numerical method is the time integration scheme: When integrating the system of equations numerically with respect to time, the explicit evaluation of coupled quantities necessitates strong time step restrictions, resulting in an enormous slowdown of the calculation. The strongest time step restriction is caused by an explicit treatment of the coupling between the charged particle transport and the space charge field. In order to circumvent this constraint, we use a semi-implicit technique, similar to the method reported in Refs. 25 and 26. When using this technique however, the explicit handling of the dependence of the electron energy source term on the mean electron energy becomes limiting for the time step. Therefore, we have developed a new technique for the implicit treatment of the electron energy source term, based on linearization with respect to electron mean energy. This approach makes it possible to increase the time step by several

---

**FIG. 2.** Electron mobility in 97% helium–3% hydrogen. The data come from Ref. 21 (<1 eV) and a BOLSIG calculation (>1 eV).

**FIG. 3.** Rate coefficients of electron impact reactions in 97% helium–3% hydrogen, as calculated with BOLSIG (Ref. 29). (a) total excitation He(2 3S) and He(2 1S) (I-1), (b) ionization from He* (I-2), (c) direct ionization (I-3), (d) direct ionization H (I-9), (e) 20% total excitation H(4 1P) and H(5 1S) (I-17), (f) dissociation H (I-8), (g) recombination H (I-9), (h) recombination H (I-10), (i) dissociative attachment (I-18).

**FIG. 4.** Ion mobilities in helium. The data are taken from Ref. 18 (for He , He , H , H , and H ), Ref. 19 (for H ), and Ref. 37 (for He at high field).
orders of magnitude, thus giving a tremendous speedup of the calculation. Details and a discussion of the implicit technique can be found in Ref. 23.

The spatial numerical grid is two-dimensional, equidistant, and Cartesian. The discretization of the equations is based on the widely used Sharfetter–Gummel scheme, as in Ref. 7. This scheme, which is derived from the analytical solution for a constant drift–diffusion flux between two grid points, supports large density gradients, as opposed to the more straightforward central difference scheme. All discretized equations are solved by the modified strongly implicit technique published in Ref. 28, which we slightly modified in order to ensure convergence for all cases. Besides being simple to implement, we found this method to be extremely efficient for solving the plasma equations.

The model is flexible with respect to the discharge geometry. Material properties are defined in every separate grid cell: a cell can be filled with discharge gas, with electrode material at a certain voltage, or with dielectric material with a certain permittivity. In this way, arbitrarily shaped discharge channels, surrounded by an arbitrary configuration of electrodes and dielectric materials, can be defined. The plasma equations (1), (2), and (11) are solved only within the gas area’s. Poisson’s equation (16) is solved on the entire grid, taking into account the effect of possible surface charge on the channel walls. At the edges of the simulation domain, the electric field inside the dielectrics is taken to be parallel to the edges.

D. Input data

The simulations presented in this article concern pure helium and helium hydrogen mixtures. The reactions taken into account are listed in Table I; this reaction scheme has been optimized for PALC discharges, and is only valid for mixtures with a small percentage of hydrogen (<5%). Since all the important metastable helium states—He(2\(^3\)S), He(2\(^1\)S), and He\(_2\)(2\(^s\)\(^3\)Σ\(_u\))—participate in similar reactions with very similar rate coefficients, we have grouped them together into one species: He\(^*\). Highly vibrationally excited hydrogen molecules (v > 6) are treated as one separate species: H\(_2\)\(^v\).

The electron mobility and the rate coefficients of the electron impact reactions are obtained with the Boltzmann code BOLSIG, which calculates the electron energy distribution in uniform electric fields. Figures 2 and 3 show these coefficients for a mixture of 97% helium and 3% hydrogen; for other mixing ratios they are slightly different. The mobilities and diffusion coefficients of all heavy species are...
assumed to be determined merely by the helium gas. Ion mobilities are shown in Fig. 4. The diffusion coefficients of the neutral species are all taken from Ref. 38: \( D \times p = 450 \, \text{cm}^2 \, \text{Torr/s} \) for \( \text{He}^+ \), \( D \times p = 2102.9 \, \text{cm}^2 \, \text{Torr/s} \) for \( \text{H} \), and \( D \times p = 1156.7 \, \text{cm}^2 \, \text{Torr/s} \) for \( \text{H}_2(v') \), where \( p \) is the gas pressure. We use constant secondary electron emission coefficients: 0.2 for helium ions and 0.01 for hydrogen ions. Secondary electron emission by neutral species is neglected. The mean initial energy is taken to be 5 eV for the secondary electrons emitted by helium ions and 1 eV for those emitted by hydrogen ions.

The metastable–metastable ionization processes \( \text{I}^{-6} \) and \( \text{I}^{-7} \) produce electrons at 15 and 17.4 eV of energy, which is not enough to excite or ionize helium atoms, but at the same time too much to be efficiently lost in elastic collisions. As a result, these electrons are not thermalized and treated incorrectly by the transport and rate coefficients calculated with BOLSIG. In pure helium, where metastable–metastable ionization plays an important role, we consider them as a separate species: monoenergetic electrons of 15 eV, which do not take part in any reaction. This approach is supported by Monte Carlo calculations. For the mobility of these electrons we use \( \mu \times p = 6.8 \times 10^5 \, \text{cm}^2 \, \text{V/s} \), the diffusion coefficient is taken to be \( D \times p = 6.8 \times 10^6 \, \text{cm}^2 \, \text{Torr/s} \). In helium–hydrogen mixtures, we use only one electron group.

The importance of \( \text{H}^- \) is unclear. The reactions (I-17,18) only represent the most probable mechanism for its production. Details and reliable values for rate coefficients are not known. The rate coefficient for dissociative attachment (I-18) increases by orders of magnitude with the increasing vibrational level of \( \text{H}_2 \), so that only states \( \text{H}_2(v > 6) \) may contribute significantly to the production of \( \text{H}^- \). It has been suggested that these high vibrational levels are populated mainly through radiative decay of higher singlet electronic states excited by energetic electrons. Since the exact vibrational distribution resulting from these processes is unknown, the effective rate coefficient for reaction (I-18) is more than a rough estimate. The rate coefficient of reaction (I-17) is taken to be proportional to the excitation rates of \( \text{H}_2(C^1\Pi_u) \) and \( \text{H}_2(B^1\Sigma^+_u) \); the proportionality constant, however, is only a conjecture. In view of the large uncertainties concerning the production of \( \text{H}^- \), we choose to omit the species \( \text{H}^- \) and \( \text{H}(v') \), as well as the last four reactions (I-17-20), from the reaction scheme, unless there is evidence for the importance of \( \text{H}^- \) in PALC discharges.

III. RESULTS

This section discusses examples of simulation results, illustrating the validity and usefulness of the model. All calculations presented here are based on the standard geometry shown in Fig. 5, which represents a cross section through three discharge channels of a PALC display. Three consecutive channels are taken into account in order to be able to study crosstalk effects: electrical influences of adjacent channels on each other. In addition, a large region of the bottom glass plate is taken into account in order to obtain the correct electric field at the bottom wall of the discharge channels. Typical PALC simulations take 1 min—1 h of CPU time on a Pentium II PC.

![FIG. 7. Particle densities and electron mean energy in a dc discharge in a mixture of 97% helium and 3% hydrogen. These plots correspond to the middle channel of the geometry shown in Figure 5. The gas pressure is 105 Torr. The cathode (2) is set to $-320 \, \text{V}$, all other electrodes are grounded. The resulting current is 3.0 mA/cm. The increment of the contours is 1/10 times the maximum value indicated in the top right corner of each plot. The darkest regions correspond to this maximum value.](http://jap.aip.org/jap/copyright.jsp)
We first present the simulation of a discharge pulse (Sec. III A) and an afterglow (Sec. III B) in a PALC channel. Then we present the calculation of the surface charge and the resulting transmission that develops during a series of discharge pulses and afterglows in three consecutive channels, corresponding to a realistic addressing procedure (Sec. III C).

A. Discharge pulse

A PALC discharge is generated by a dc voltage pulse on the discharge electrodes. Figures 6 and 7 show the calculated steady state particle densities and electron mean energy under typical conditions at the end of the pulse, for helium and a mixture of 97% helium and 3% hydrogen.

The calculations clearly show a so-called cathode fall: a region in front of the cathode where the total positive ion density exceeds the electron density by orders of magnitude. The strong positive space charge density of the cathode fall screens the remainder of the channel largely from the applied electric field. This remaining region contains a quasineutral plasma. In the cathode fall region an extremely high electric field heats the electrons to a mean energy of about 50 eV, resulting in high electron impact ionization and excitation rates. As a result particle species which are mainly created by electron impact reactions reach their maximum density around the cathode fall region, whereas species created in heavy particle reactions, such as He$_2^+$, are mainly present in the plasma region.

As can be seen comparing Fig. 6 to Fig. 7, the cathode fall in the helium–hydrogen mixture is very similar to the one in helium, but in the plasma region the role of the helium ions is taken over by the hydrogen ions.

By performing steady state dc discharge calculations for different discharge voltages, it is possible to calculate current–voltage (I–V) curves, which can be compared to experimental data. Exact quantitative agreement cannot be expected, in view of the fact that the I–V curves are ex-

FIG. 8. Calculated and measured I–V curves of pure helium at 150 Torr, for different values of the channel height. The simulated discharge configurations are all similar to the one shown in Fig. 5; the discharges are created only in the middle channel.

FIG. 9. Decay of the numbers of particles in the afterglow in pure helium, per centimeter of channel length. Two electron groups are shown separately in this figure: $e$ are the (thermalized) electrons produced by electron impact ionization, $e^*$ are the (nonthermalized) electrons originating from metastable–metastable ionization. (See Sec. II D) The discharge configuration is shown in Fig. 5; all electrodes are grounded. The gas pressure is 150 Torr. This afterglow belongs to the discharge shown in Fig. 6.

FIG. 10. Decay of the numbers of particles in the afterglow in a mixture of 97% helium and 3% hydrogen, per centimeter of channel length. The discharge configuration is shown in Fig. 5, all electrodes are grounded. The gas pressure is 105 Torr. This afterglow belongs to the discharge shown in Fig. 7.
tremely sensitive to many delicate parameters, such as secondary emission coefficients. However, trends in calculated I–V curves turn out to be in very good agreement with trends in measured I–V curves. As an example, Fig. 8 presents a study of the influence of the channel height on the I–V curve of a pure helium discharge. The figure shows that channel height does not affect the I–V curve as long as it is well above 200 μm. For smaller channel heights the cathode fall region is somewhat squeezed by the channel walls, which leads to an increase in the discharge voltage.

B. Afterglow

During the period immediately following the discharge pulse—the so-called afterglow—the plasma in the channel decays. In the simulations we assume the electrode voltage to go down abruptly to 0 V at the beginning of the afterglow; this assumption seems to be a reasonable approximation of the reality. Figures 9 and 10 show the simulated decay of the different particle species in the afterglow in pure helium, and in a mixture of 97% helium and 3% hydrogen, respectively.

Initially, the densities of most charged species increase somewhat due to the sudden decrease in drift losses. During the first part of the afterglow the electrons and ions are coupled together by space charge fields, resulting in ambipolar diffusion. As the plasma density decreases, the space charge fields become weaker until the ambipolar diffusion breaks and the electrons run off leaving the ions behind.

In pure helium however, the decay is not determined by diffusion alone. In this gas the decay of charged species is enormously slowed down by metastable–metastable ionizations (I-6) and (I-7), which continue far into the afterglow. As can be seen in Fig. 9, the electrons produced by these processes leave the channel very quickly, indifferent to the ambipolar coupling, due to their relatively high energies. (See Sec. II.D.) The produced ions then lead to enhanced trapping of the less energetic thermalized electrons remaining from the discharge. In helium–hydrogen mixtures the helium metastables are strongly quenched by Penning reactions (I-14) and (I-16) so that the metastable–metastable ionizations have no influence.

C. Addressing

In true PALC operation, small voltages are applied to the data electrodes during the discharge pulse and the afterglow, which leads to the buildup of surface charge on the microsheet. The surface charge invokes an electric field in the...
LC layer, resulting in a certain transmission of the (second) polarizing layer. The percentage of light transmitted by the polarizer as function of the total voltage across the LC layer and the microsheet is given by a so-called transmission–voltage curve. Using the transmission–voltage curve shown in Fig. 11, it is possible to calculate transmission profiles of simulated PALC channels, which can be compared to experimental results. Transmission measurements on real PALC panels, such as shown in Fig. 12, have raised the suspicion that the buildup of surface charge in a certain channel is influenced by the surface charge present in the adjacent channels (electrical crosstalk). A proper calculation of the surface charge and the resulting transmission profile therefore requires the full simulation of not only the channel in question, but also of the two neighboring channels.

We used the three channel geometry of Fig. 5. Following a realistic addressing procedure, the channels were addressed one by one from the left to the right, every time inverting the sign of the voltage on the data electrodes (row inversion). The exact addressing scheme is represented by Fig. 13. We applied a data voltage of $\pm 20 \text{ V}$, corresponding to the maximum slope of the transmission–voltage curve, where the transmission is most sensitive to charging errors. The calculated electric potential at the end of the three channel addressing scheme is plotted in Fig. 14. It can clearly be seen, that the surface charge field in the LC layer and microsheet is directed oppositely for the consecutive channels, due to the row inversion technique. Note that the outer sides of the first and last channel are not properly modeled, given the artificial boundary conditions for the electric field at the edges of the simulation domain; only the middle channel has been treated correctly. Figure 15 shows the final transmission profile of this middle channel. In the center of the channel a homogeneous transmission has been achieved, but near the inter-channel walls (ribs), transmission inhomogeneities occur. The charging is incorrect especially on the right-hand side of the channel. This asymmetric effect is in full agreement with the experimental observations shown in Fig. 12. It results from electrical crosstalk: At the moment the surface charge in the middle channel is established, the right neighbor channel contains a repelling surface charge with the same polarity, whereas the left neighbor channel has an attracting surface charge with opposite polarity. In fact, the phenomenon depends on the addressing procedure; if the channels are addressed in reverse order (so: from right to left), the main charging errors occur on the other side of the channel.

Figure 15 also illustrates that a minimum of addressing time is required for a proper charging: If the addressing time is too short for the plasma to decay, the row inversion will partially erase the surface charge, leading to an increase of the transmission. According to Fig. 15, at least 30 $\mu$s of addressing time are needed for pure helium channels. The minimal required addressing time is determined by the decay rate of the plasma; one can define a plasma decay time as the addressing time corresponding to a 1% increase in transmission in the center of the channel. Figure 16 shows a comparison of calculated and measured decay times for helium at different pressures. The helium decay time goes up as the pressure increases, due to an increase of the metastable density and a decrease of the diffusion coefficients. In Fig. 17 the calculated and measured decay times for different helium–hydrogen mixtures are compared. Adding hydrogen to helium strongly decreases the decay time due to the quenching of the helium metastables. For all calculations the agreement with experiments is excellent.

**IV. CONCLUSIONS**

We have developed a two-dimensional (2D) fluid model of PALC discharges. The model comprises continuity equations and drift–diffusion equations for plasma particle species, a balance equation for the electron energy, and Poisson’s equation for the electric potential. The boundary conditions for electron transport include a correction for the directed motion of the electrons emitted by secondary emission. The implementation of the model allows for arbitrary...
2D geometries and arbitrary gas mixtures. Reaction schemes and data are presented for pure helium and helium–hydrogen mixtures.

Using this model, it is possible to simulate the full PALC operation. We have reproduced a series of discharge pulses and afterglows in three consecutive PALC channels, filled with pure helium or helium–hydrogen mixtures. The simulations show that at the end of a discharge pulse a cathode fall and a plasma region are present in the channel in question. Calculated I–V curves of the PALC discharges are in good agreement with measurements. The simulations reproduce nonuniformities in the charging of the microsheet, known from experiments, and related to electrical crosstalk between adjacent channels. Calculated plasma decay times are in good agreement with measured decay times. In pure helium, the decay is slowed down enormously by a continuous plasma production through metastable–metastable ionization; in helium–hydrogen mixtures the helium metastables are quenched so that this effect does not take place.

**ACKNOWLEDGMENT**

This work was supported by the Philips Research Laboratories in Eindhoven, The Netherlands.


**FIG. 17.** Decay times for different He–H₂ mixtures. Here the decay time is defined as the addressing time corresponding to a transmission error of 1% (see text). For all calculations the gas pressure is 105 Torr, the discharge current 3.0 mA/cm, and the data voltage ~20 V.

---

R. R. Newton, Phys. Rev. 73, 570 (1948).
BOLSG, Boltzmann Solver for the SIGLO Series 1.0 (CPA Toulouse & Kinema Software, 1996).