A numerical comparison between multiple-scales and FEM solution for sound propagation in lined flow ducts

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**A Numerical Comparison between Multiple-Scales and
Fem Solution for Sound Propagation in Lined Flow
Ducts**

by

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A NUMERICAL COMPARISON BETWEEN MULTIPLE-SCALES AND FEM SOLUTION FOR SOUND PROPAGATION IN LINED FLOW DUCTS

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Abstract

An explicit, analytical, multiple-scales solution for modal sound transmission through slowly varying ducts with mean flow and acoustic lining, is tested against a numerically “exact” finite element solution. The test geometry taken is representative of a high-bypass turbo fan aircraft engine, with typical Mach numbers of 0.5–0.7, circumferential mode numbers m of 10–40, dimensionless wave numbers of 10–50, and both hard and acoustically treated inlet walls of impedance $Z = 2 - i$. Of special interest is the presence of the spinner, which incorporates a geometrical complexity which could previously only be handled by fully numerical solutions. The results in predicted power attenuation loss show in general a very good agreement. The results in iso-pressure contour plots show good comparison in the cases where scattering into many higher radial modes can occur easily (high frequency, low angular mode), and again a very good agreement in the other cases.

Introduction

The calculational complexities of the multiple-scales solution for modal sound transmission through slowly varying ducts with mean flow and acoustic lining (presented in [Rienstra (1999)]), are no more than for the classical modal solution for a straight duct. The multiple-scales solution is an approximation utilizing the axial slope of the duct walls as small parameter. This slope is for aerodynamical reasons indeed invariably small in any aero-engine duct.

Therefore, this multiple-scales solution provides an interesting alternative in aero-engine applications, as it both allows for the advantages of the analytical approach (speed of calculation and relative simplicity of programming), and variable geometries including spinner and mean flow variation.

The final approximation error in realistic geometries, however, is difficult to determine, except for an order of magnitude estimate saying that it scales on this slope. It is therefore of interest to directly compare the analytical approximation with a state of the art fully numerical solution of the same physical model. This is the subject of the present paper.

As a first step towards exploring the possibilities, a series of tests are carried out, comparing the analytical results with results of the finite element solution, given in [Danda Roy & Eversman (1995)], of a compressible inviscid isentropic irrotational mean flow, superimposed by linear acoustic perturbations.

Physical model

We consider a circular symmetrical duct with a compressible inviscid perfect isentropic irrotational gas flow, consisting of a mean flow and acoustic perturbations. To the mean flow the duct is hard-walled, but for the acoustic field the duct is lined with an impedance wall. In view of the adopted aero-engine geometry, the inner wall (corresponding to the spinner) will be hardwalled, without lining.

We make dimensionless: spatial dimensions on a typical duct radius R_∞ , densities on a reference value ρ_∞ , velocities on a reference sound speed c_∞ , time on R_∞/c_∞ , pressure on $\rho_\infty c_\infty^2$, and velocity potential on $R_\infty c_\infty$. Note that the corresponding reference pressure p_∞ satisfies $\rho_\infty c_\infty^2 = \gamma p_\infty$, where $\gamma = 1.4$ is the (constant) ratio of specific heats at constant pressure and volume. The fluid in the duct is described by

$$\tilde{\rho}_t + \nabla \cdot (\tilde{\rho} \tilde{\mathbf{v}}) = 0, \quad \tilde{\rho}(\tilde{\mathbf{v}}_t + \tilde{\mathbf{v}} \cdot \nabla \tilde{\mathbf{v}}) + \nabla \tilde{p} = 0, \quad \gamma \tilde{p} = \tilde{\rho}^\gamma, \quad \tilde{c}^2 = \frac{d\tilde{p}}{d\tilde{\rho}} = \tilde{\rho}^{\gamma-1} \quad (1a, b, c, d)$$

(with boundary and initial conditions), where $\tilde{\mathbf{v}}$ is particle velocity, $\tilde{\rho}$ is density, $\tilde{p}$ is pressure, $\tilde{c}$ is sound speed (all dimensionless).

Since we assumed the flow to be irrotational, we may introduce a velocity potential $\tilde{\phi}$, such that $\tilde{\mathbf{v}} = \nabla \tilde{\phi}$, and the above momentum equation may be integrated to a variant of Bernoulli's equation

$$\frac{\partial \tilde{\phi}}{\partial t} + \frac{1}{2} |\tilde{\mathbf{v}}|^2 + \frac{\tilde{c}^2}{\gamma - 1} = \text{const.} \quad (2)$$

This flow is split up into a stationary mean flow part, and an acoustic perturbation. This acoustic part varies harmonically in time with circular frequency ω , and with small amplitude to allow linearization.

In the usual complex notation we write then

$$\tilde{\mathbf{v}} = \mathbf{V} + \mathbf{v} e^{i\omega t}, \quad \tilde{\phi} = \Phi + \phi e^{i\omega t}, \quad \tilde{\rho} = D + \rho e^{i\omega t}, \quad \tilde{p} = P + p e^{i\omega t}, \quad \tilde{c} = C + c e^{i\omega t}.$$

Substitution and linearization yields:

- mean flow field

$$\nabla \cdot (D\mathbf{V}) = 0, \quad \frac{1}{2} |\mathbf{V}|^2 + \frac{C^2}{\gamma - 1} = E, \quad C^2 = \gamma P/D = D^{\gamma-1}; \quad (3a, b, c)$$

- acoustic field

$$i\omega\rho + \nabla \cdot (D\nabla\phi + \rho\mathbf{V}) = 0, \quad i\omega\phi + \mathbf{V} \cdot \nabla\phi + \frac{p}{D} = 0, \quad p = C^2\rho, \quad (4a, b, c)$$

where E is a constant, and the acoustic perturbation of $\tilde{c}$ is further ignored. The integration constant in equation (4b) may be absorbed by ϕ . For the mean flow the duct wall is solid, so the normal velocity vanishes at the wall. The subsonic mean flow is determined by conditions of uniformity upstream, the constant E , and an axial mass flux πF .

For the acoustic part the outer duct wall is a locally reacting impedance wall with complex impedance Z . The pertaining boundary condition is for a point near the wall but still (just) inside the mean flow. For arbitrary mean flow along a (smoothly) curved wall, with normal $\mathbf{n}$ directed into the wall, this was given by Myers [Myers (1980)], eq. 15, as

$$i\omega(\mathbf{v} \cdot \mathbf{n}) = \left[i\omega + \mathbf{V} \cdot \nabla - \mathbf{n} \cdot (\mathbf{n} \cdot \nabla \mathbf{V}) \right] \left(\frac{p}{Z} \right). \quad (5)$$

Geometry

The reference values taken for non-dimensionalization are at the source plane $x = 0$, including the outer radius for length scale.

The outer radius R_2 and inner radius R_1 are described by the following formulas

$$R_2(x) = 1 - 0.18453x'^2 + 0.10158 \frac{e^{-11(1-x')} - e^{-11}}{1 - e^{-11}}, \quad (6)$$

$$R_1(x) = \max[0, 0.64212 - (0.04777 + 0.98234x'^2)^{1/2}], \quad (7)$$

where $x' = x/L$ and $L = 1.86393$ is the length of the duct; see figure (1)

The mean flow is selected such that at the source plane $x = 0$ the Mach number is equal to -0.5 , and the dimensionless density equal to 1. The corresponding axial Mach number and dimensionless density variation (based on the quasi-one dimensional mean flow solution; see below) is depicted in figure (2).

Multiple scales solution

For the success of the analytical solution it is essential that the mean flow and the acoustic field are approximated on the same footing. An arbitrary, ad-hoc, mean flow field would not allow the present explicit solution. So the mean flow used for the multiple scales solution is not exactly the same as the one used for the finite element solution. They are, however, in terms of approximation of the same level. Therefore, we give here the mean flow and the acoustic field together.

The approximation is based on the assumption that geometry and mean flow vary slowly, *i.e.* on a length scale much larger than a typical duct diameter or wave length. This is, of course, for aerodynamical reasons the case inside an aero-engine inlet duct. We introduce the ratio between a typical diameter and this length scale as the small parameter ε , and rewrite the duct surface (in radial coordinates (x, r, θ))

$$r = R_1(X), \quad r = R_2(X), \quad X = \varepsilon x. \quad (8)$$

Figure 1: Geometry

Figure 2: Mach number and density

By rewriting $R_{1,2}$ as a function of slow variable X , rather than x , we have made our formal assumption of slow variation explicit in a convenient and simple way. Although in the final result ε will play no explicit rôle, a representative value of ε will be necessary for an order of magnitude estimate of the approximation error.

By assuming that the mean flow is nearly uniform with axial variations in X only, we find that small axial mass variations can only be balanced by a small radial flow, so

$$\mathbf{V} \simeq U_0(X)\mathbf{e}_x + \varepsilon V_1(X, r)\mathbf{e}_r$$

and similarly are $P \simeq P_0(X)$, $D \simeq D_0(X)$, and $C \simeq C_0(X)$ to leading order only dependent on X . It follows that

$$U_0(X) = \frac{F}{D_0(X)(R_2^2(X) - R_1^2(X))}$$

with V_1 , D_0 , P_0 and C_0 are given by the well known one dimensional gas flow equations (see *e.g.* [Rienstra (1999)]).

The acoustic field is assumed to be described by mode-like solutions of the form

$$\phi(x, r, \theta; \varepsilon) = A(X, r; \varepsilon) e^{-im\theta - i\varepsilon^{-1} \int^X \mu(\xi) d\xi} \quad (9)$$

After expanding $A = A_0 + \varepsilon A_1 + O(\varepsilon^2)$ and $\mu = \mu_0 + O(\varepsilon^2)$ (any possible μ_1 can be absorbed by A_1), and substitution in equations and boundary conditions, we find for A_0 a Bessel-type equation in r , so we obtain the slowly

varying mode

$$A_0 = N(X)J_m(\alpha(X)r) + M(X)Y_m(\alpha(X)r) \quad (10)$$

where J_m and Y_m are the m -th order Bessel function of the first and second kind. Radial eigenvalue α and M/N are determined by the boundary condition, while

$$\alpha^2 + \mu^2 = \Omega^2/C_0^2, \quad \Omega = \omega - \mu U_0.$$

The crux of the solution is the determination of amplitude $N(X)$, as a function of X . This is determined by the next order equation for A_1 . It is, however, not necessary to solve this complicated equation. A solvability condition [Rienstra (1999)] is enough to generate a differential equation in X for N , which can be solved exactly. The general solution for the hollow cylinder ($R_1 \equiv 0$, $M \equiv 0$) is given by

$$\left(\frac{Q_0}{N}\right)^2 = \left(\frac{D_0\omega\sigma R_2^2}{2C_0}\left(1 - \frac{m^2 - \zeta_2^2}{\alpha^2 R_2^2}\right) + \frac{D_0 U_0}{\Omega}\zeta_2\right) J_m(\alpha R_2)^2 \quad (11)$$

where Q_0 is an integration constant, and $\zeta_2 = \Omega^2 D_0 R_2 / i\omega Z_2$, $\sigma^2 = 1 - (C_0^2 - U_0^2)\alpha^2/\omega^2$. The solution for the annular cylinder is more complicated, although explicit, and can be found in [Rienstra (1999)]

Finite element solution

A numerical model for sound propagation is based on a finite element discretization of the steady flow field equations (3a-c) and the acoustic field equations (4a-c). The weak formulation of equation (3a) for the steady compressible flow in the duct is in terms of the steady flow velocity potential Φ and steady flow density D ,

$$\iiint_V \nabla W \cdot (D \nabla \Phi) dV = \iint_S W (D \nabla \Phi) \cdot \mathbf{n} dS. \quad (12)$$

Figure 3: Computational domain FEM

Weighting functions W are from the class of continuous functions on the volume V of the duct bounded by the duct surface S , which includes the duct walls and source and exit planes. A solution for Φ is sought in the class of continuous functions. The unit normal is directed out of the duct. The duct geometry and the steady flow field are axially symmetric favoring the introduction of a cylindrical coordinate system with x axis coincident with the axis of symmetry, r axis in the source plane at $x = 0$, and the angular coordinate θ locating the r axis in the $x = 0$ plane (see figure 3 for the computational domain). The steady flow field is represented in an (x, r) -plane, and is two-dimensional. A standard finite element formulation of equation (12) is based on eight node isoparametric serendipity elements.

Equations (3b) and (3c) are subsidiary relations that are used in an iterative solution in which at each stage the finite element discretization of equation (12) is solved with a density and speed of sound field derived from the previous iteration step. The boundary integral on the right hand side, which is a natural boundary condition, specifies the mass flow rate on the source plane. A forced boundary condition setting the level of the potential is required on the exit plane (figure 3). It is assumed that the source plane and exit plane are located remotely enough from regions of non-uniformity in the duct so that at the source and exit planes the flow velocity is uniform, permitting the natural and

forced boundary conditions to be easily implemented. In practical calculations in the type of duct considered this turns out not to be restrictive.

The mean flow in the duct given by figure 3, with a uniform Mach number at the source plane $M = -0.5$, is directed from right to left (an inlet flow). The duct shape defined by equations (6) and (7) begins at the source plane $x = 0$ and is extended beyond the nominal termination used in the analytical development in a uniform duct to allow the flow field to become uniform at the exit plane. No extension is used at the source end and the extension at the exit end is probably longer than necessary. $\nabla\Phi$, C , D are required data for the FEM solution for acoustic propagation.

A finite element model for acoustic propagation is based on a weak formulation of equations (4a-c). Acoustic perturbations in pressure, density and velocity potential are harmonic in time with frequency ω and harmonic in the angular coordinate θ of the form $p(x, r)$, $\rho(x, r)$, and $\phi(x, r)$ times the complex exponent $e^{i\omega t - im\theta}$. The weak formulation [Danda Roy & Eversman (1995)] is

$$\iiint_V \{ \nabla W \cdot (D \nabla \phi + \rho \nabla \Phi) - i\omega W \rho \} dV = \iint_S W (D \nabla \phi + \rho \nabla \Phi) \cdot \mathbf{n} dS \quad (13)$$

The weighting functions are taken as $W(x, r) e^{im\theta}$. Angular harmonics proportional to $e^{-im\theta}$ represent the decomposition of the solution periodic in θ in a Fourier Series. The angular mode number m is a parameter of the solution. The surface integral is over all surfaces bounding the domain. The unit normal for the surface integral is out of the domain at the surface in question. The weak formulation continues with the linearized momentum equation (4b) and linearized equation of state (4c), to rewrite equation (13) in the form

$$\begin{aligned} \iiint_V \frac{D}{C^2} \{ C^2 \nabla W \cdot \nabla \phi - (\mathbf{V} \cdot \nabla W)(\mathbf{V} \cdot \nabla \phi) + i\omega [W(\mathbf{V} \cdot \nabla \phi) - (\mathbf{V} \cdot \nabla W)\phi] - \omega^2 W \phi \} dV \\ = \iint_S \frac{D}{C^2} \{ C^2 W \nabla \phi - \mathbf{V} W (\mathbf{V} \cdot \nabla \phi) - i\omega \mathbf{V} W \phi \} \cdot \mathbf{n} dS \end{aligned} \quad (14)$$

Note that the local steady flow dimensionless velocity $\mathbf{V}$ is equivalent to the reference Mach number $\mathbf{M}_r$ which in fact is local steady flow velocity divided by the speed of sound at the source plane.

The surface integral on the right hand side of equation (14) is the natural boundary condition. On the duct walls this provides the boundary condition for either rigid walls (the integral vanishes) or for a normally reacting lining with an impedance specified by equation (5). In the present FEM implementation equation (5) is simplified by the elimination of the term involving $\mathbf{n} \cdot (\mathbf{n} \cdot \nabla \mathbf{V})$ on the right hand side. In the duct geometry studied here this term, which depends on the nonuniformity of the steady flow field, is of little importance in affecting attenuation (even though it is asymptotically small but not negligible, and crucial in completing the analytical formulation).

Details of the FEM procedure for discretization of equation (14), and other references, can be found in [Danda Roy & Eversman (1995)]. A discussion of the implementation of the impedance boundary condition is available in [Eversman & Okunbor (1998)].

On the source plane and exit plane the natural boundary condition is used to introduce the noise source and non-reflecting boundary conditions. On these planes the acoustic potential is recast via an eigenmode expansion such that the acoustic potential is given in terms of the complex amplitudes of the right and left propagating acoustic duct modes appropriate for the geometry and flow conditions which prevail there [Eversman (1991)]. On the source plane, $x = 0$ in the present study, right propagating modal amplitudes at the source plane are specified via a forced boundary condition. Left running (reflected) modal amplitudes at the source plane and right running modal amplitudes at the exit plane are unknown and part of the solution. Left running modal amplitudes at the exit plane are forced to vanish, imposing a non-reflecting boundary condition. Details of the modal boundary condition are available in [Eversman & Baumeister (1986)].

Finite element discretization for acoustic propagation is carried out on the same grid with the same element type as used in the steady flow model. Required data generated in the steady flow representation is transferred directly to the acoustic analysis. Mesh density is governed by the demands of the acoustic problem and is substantially more refined than would be required in the steady flow analysis.

The FEM solution proceeds with the computation of the acoustic potential field. Post-processing by the use of equation (4b) generates the acoustic pressure field. The solution also includes reflected modal amplitudes and transmitted modal amplitudes. Acoustic power reflection and transmission characteristics are computed directly from

the input modal amplitudes and computed reflected and transmitted modal amplitudes. Reciprocity characteristics of the scattering matrices and acoustic power balances are also monitored as a check of computational accuracy in the case of no acoustic treatment on the duct walls [Eversman (1999)].

Post processed acoustic pressures are represented on iso-pressure amplitude contour plots superimposed on the duct geometry. Comparison of FEM and multiple scales results is based on visual comparison of these contours, but perhaps more importantly on the basis of computed power transmission coefficients.

Differences between Multiple Scales and FEM Formulations

There are minor differences between the multiple scales solution and the finite element model. The field equations (3a-c) and (4a-c) are exactly the same in both cases. The implementation of the impedance boundary condition in the FEM formulation neglects the $\mathbf{n} \cdot (\mathbf{n} \cdot \nabla \mathbf{V})$ -term in equation (5). This is done principally to simplify the FEM implementation of this difficult boundary term (it requires a gradient of the steady flow velocity which is already the gradient of the steady flow potential). For a cylindrical duct this term is to leading order in ε equal to $\frac{\partial}{\partial r} V \simeq \varepsilon U R_X / R$. So it is small, but asymptotically not smaller than any other effect due to duct variation. Nevertheless, we found as yet no indication that the effect on attenuation predictions is significant.

The FEM formulation includes the propagation of many modes and therefore scattering is an integral part of the solution. This is manifested by reflection of the incident mode and other modes which are not incident as well as the transmission of modes which are not incident. This will be clearly seen in examples which are presented. The multiple scales method utilizes the fact that in the smooth parts of the duct any scattering into other modes is normally negligible. The principal manifestation of the propagating sound is still mode-like, albeit not in the strict sense of self similar straight duct modes, but mode-like solutions with slowly varying amplitude and phase. At abrupt changes in geometry, like the inlet plane scattering into other radial modes may be included (not done here) by traditional methods like mode-matching. This is, however, not included in the present implementation. Scattering is therefore not a feature of the multiple-scales solution.

The FEM solution requires the source to be represented in terms of input modal amplitudes for eigenmodes for a duct with hard walls. The source is always located in a section of the duct which has rigid walls. Acoustic treatment is not present exactly at the source plane, although it can be initiated an arbitrarily small distance from the source. This is done because properties of the rigid wall duct modes (namely, orthogonality) are important in the implementation of the source boundary conditions. Eigenmodes in a duct with an impedance boundary are not orthogonal in the usual sense. The net effect is that there is always a transition from a rigid wall to an impedance wall at both ends of the duct in the FEM model. This has implications for scattering which are not readily quantifiable because it is undoubtedly dependent on frequency, impedance of the boundary, and the number of modes used to represent the source.

In the multiple-scales analysis the general solution is built up from a summation over slowly varying modes. The natural way to test its validity is therefore to study a single, soft-wall, mode. In order to generate an equivalent input in the FEM model it is necessary to represent the soft wall eigenmode as an eigenmode expansion of hard wall eigenmodes. Since this sound field distribution, presented to the lined duct, essentially “fits” directly into one soft-wall mode, only little reflection at the source plane is to be expected. A single mode multiple-scales analysis is therefore simulated by a multiple mode FEM solution.

Finally, it is noted that the FEM model requires conditions at the source and exit planes which give rise to reflected modal amplitudes and to the reflection free termination (or specified or computed reflection characteristics). These conditions do not play an immediate role in the analytical solution, with left and right running waves already given explicitly. Although inherent in any practical application, we have not tried to model these conditions in the analytical part of the present tests in order not to obscure the comparison and to restrict the sound field to that of a single, right-running, mode.

It is not possible to make FEM and multiple-scales models exactly equivalent, nor should it be, since the multiple-scales solution is an approximation based on well documented assumptions. It is a goal of the numerical comparisons to be given here to investigate how successfully the multiple-scales solution represents the more exact FEM model.

Results

The cases considered are grouped as the following 4 series of iso-pressure contour plots: the first radial mode of

fig. 4 $m = 10, \omega = 10,$

fig. 5 $m = 10, \omega = 16,$

fig. 6 $m = 10, \omega = 50,$

fig. 7 $m = 40, \omega = 50,$

is input under the following conditions:

fig. a hard wall, no flow,

fig. b hard wall, flow,

fig. c soft wall, no flow,

fig. d soft wall, flow.

The left column of the figures is the numerical FEM solution, the right column the analytical MS (multiple scales) solution. “Soft wall” denotes a wall impedance $Z = 2 - i$. “Flow” denotes a mean flow with Mach number -0.5 at the source plane. “First radial mode” denotes in general the mode with smallest real part of radial eigenvalue α . For the soft walls the predicted attenuation ($10 \log$ of ratio of acoustic power through source and inlet plane) is given in the caption of the figures. For the hard walls the attenuation is either zero (mode is cut on) or infinite (mode is cut-off and reflection is negligible).

The selected cases do not show turning point behaviour (hard-wall cut-on, cut-off transition).

The possible differences between FEM and MS are due to the following errors or modelling discrepancies.

1. The approximation error of $O(\varepsilon^2)$. For this we need an estimate of ε . Suitable is a typical value of $\frac{d}{dx} R_2 = \varepsilon \frac{d}{dx} R_2 = O(\varepsilon)$. From formula (6) it appears that R'_2 varies between -0.12 and 0.12 along $[0, 1.75]$, but increases to 0.4 in the lip region $[1.75, L]$. If we take $\varepsilon = 0.1$, the estimated approximation error is a few percent.
2. Small but inherent reflection in FEM at inlet plane and lip region.
3. Not exactly the same source in soft wall cases, since FEM uses a source defined by an expansion in a finite number (15) of hard wall modes, a small distance ($\frac{1}{100}L$) away from the lined section.
4. Slightly different impedance definition in flow cases.
5. Slightly different mean flow. In MS the mean flow field is approximated to the same level as the acoustic field.

For highly attenuated or only cut-off modes ($m = 10, \omega = 10$) of fig. 4a-d, the agreement is almost perfect. None of the above errors seem to play a role.

For the series ($m = 10, \omega = 16$) of fig. 5, with 1 (no flow) or 2 (flow) modes cut-on, the agreement is good in the iso-contour plots, and almost perfect in attenuation. Some wiggles are visible in fig. 5b what is probably error type 1, due to interference with the other cut-on radial modes.

The high frequency series ($m = 10, \omega = 50$) of fig. 6 has very low acoustic pressure values near the duct axis, and many radial modes cut on (at source plane, no flow: 9, flow: 11). We see strong interference with these higher modes. The most important region near the outer wall, however, is in very good agreement, and for no flow the attenuation agrees exactly. With flow the 1st and 2nd radial soft wall modes, *i.e.* μ_1 and μ_2 , happen to be rather close to each other, and a residual second mode (due to error 3) leads to a slightly (0.5 dB) different attenuation.

The high m , high frequency ($m = 40, \omega = 50$) series of fig. 7 has 2 (no flow) and 3 (flow) radial modes cut on, giving rise to some wiggles in figs. 7a,b. Effects due to error 3 are probably visible in fig. 7d, although the predicted attenuation agrees very well. The (academic!) difference in attenuation of fig. 7c (195 and 210 dB) is no numerical round-off error, but due to the fact that the plotted mode is least attenuated at the inlet but not at the source plane. ($\Im \mu_1$ and $\Im \mu_2$ change order.) Residual modes due to error 3 are likely to dominate in the FEM solution near the source plane, leading to a different attenuation.

Conclusions

Any selection of test cases is necessarily limited. It would have been easy to create a more or a less favourable comparison, by making a suitable selection of geometry and parameters. This is not done here. We have defined the test runs entirely on the basis of their relevance to turbo fan engine inlet duct applications, and we have not skipped unfavourable cases afterwards. The only restriction we made was that no cut-on/off transition in the hard walled duct be present. This phenomenon is not yet implemented in the analytical solution, while at the same time, of course, it is absent in any lined duct.

So considering the fact that the cases are likely to be a representative cross section of reality, we think the conclusion is justified that the MS and FEM solutions compare favourably, both in iso-pressure contours and in predicted attenuation. Principle differences are related to scattering at inlet plane, and to input mode synthesis. The best results

are obtained with lining (reducing importance of reflection) and when the modal structure permits few or no cut-on scattered modes. The attenuation differs in general no more than a few tenths of a dB.

The correlation shows that MS is definitely useful in applications for assessing liner performance in realistic geometries. Both extending the theory, and further comparison with FEM, for example with an MS implementation that includes a complete modal spectrum and open end reflection, is therefore to be recommended.

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Figure 4a: $m = 10$, $\omega = 10$, hard wall, no flow

Figure 4b: $m = 10$, $\omega = 10$, hard wall, flow

Figure 4c: $m = 10$, $\omega = 10$, soft wall, no flow;
att. 120 dB (MS)

Figure 4d: $m = 10$, $\omega = 10$, soft wall, flow;
att. 164 dB (MS)

Figure 5a: $m = 10$, $\omega = 16$, hard wall, no flow

Figure 5b: $m = 10$, $\omega = 16$, hard wall, flow

Figure 5c: $m = 10$, $\omega = 16$, soft wall, no flow;
att. 51.57 dB (FEM), 51.64 dB (MS)

Figure 5d: $m = 10$, $\omega = 16$, soft wall, flow;
att. 27.2 dB (FEM), 27.1 dB (MS)

Figure 6a: $m = 10$, $\omega = 50$, hard wall, no flow

Figure 6b: $m = 10$, $\omega = 50$, hard wall, flow

Figure 6c: $m = 10$, $\omega = 50$, soft wall, no flow;
att. 3.49 dB (FEM), 3.49 dB (MS)

Figure 6d: $m = 10$, $\omega = 50$, soft wall, flow;
att. 1.49 dB (FEM), 0.92 dB (MS)

Figure 7a: $m = 40$, $\omega = 50$, hard wall, no flow

Figure 7b: $m = 40$, $\omega = 50$, hard wall, flow

Figure 7c: $m = 40$, $\omega = 50$, soft wall, no flow;
att. 195.6 dB (FEM), 210.4 dB (MS)

Figure 7d: $m = 40$, $\omega = 50$, soft wall, flow;
att. 28.4 dB (FEM), 28.6 dB (MS)