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Technical Note:
Equalization of Run-out Times under High Demand and Lost Sales

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In this note we study the behavior of the well-known approach for sequencing which equalizes the run-out times of each of the products being produced on a single installation. We consider a single machine situation with a number of non-identical products being manufactured and different requirements in service levels for each of the products. The installation is characterized by a very high utilization of capacity and non-preemption of production runs (i.e. production runs must be completed before the next run will start). Furthermore, backorders do not exist, so any demand which is not delivered out of stock, gets lost. This situation is based on a number of typical companies in flow process industries.

The objective of the note is to show that the individual service levels cannot be controlled in a situation with excess demand by equalization of run-out times as a sequencing procedure. A fixed sequence is proven to provide better results. Equalization of run-out times is frequently used in industry as a sequencing procedure without explicitly considering its behavior under high demand levels.

We consider a single machine, multi-product installation. The demand per period is distributed normally. The coefficient of variation in demand is constant per period and identical for each product. Any demand which cannot be delivered out of stock, gets lost. Demand in consecutive periods is statistically independent. The analyzed lot-sizing and
sequencing procedure is basically a two-step procedure. First, the order-up-to-levels are determined for each of the products. These levels are determined using the cycle time procedure provided by Doll and Whybark (1973). Using these target cycle times, the actual lot-size can be determined once a product is selected for production. The actual selection of the products takes place with the objective of equalizing the run-out times (Hax and Meal 1975). The procedure is summarized below:

Notation:

\[ c_i \] Set-up time for product \( i \)

\[ p_i \] Production rate of product \( i \)

\[ I_i \] Inventory level of product \( i \) at the start of the current period

\[ d_i \] Average demand rate for product \( i \)

\[ RO_i \] Expected run-out time for product \( i \)

\[ T^* \] Target fundamental cycle length.

\[ T_i^* \] Target cycle time for product \( i \) \((T_i^*=k_iT^*, k_i \text{ a positive integer})\).

**Step 1.** Calculate target cycle times, according to Doll & Whybark (1973). Make a feasibility check to insure that

\[
T^* > \frac{\sum_{i=1}^{n} \frac{c_i}{k_i}}{1 - \sum_{i=1}^{n} \frac{d_i}{p_i}}
\]

If the calculated \( T^* \) violates this inequality, then \( T^* \) is set equal to the right hand side of this inequality.
Step 2. Select the product with the shortest run-out time $RO_i$:

$$RO_i = \frac{I_i}{d_i}$$

Step 3. Produce the selected product in the following quantity:

$$d_i T_{i^*} \cdot I_i$$

Step 4. When a production run has been finished, return to step 2.

The target cycle time basically ensures that production in more or less optimal lot-sizes takes place, taking into account long-term cost objectives. The objective of the run-out time procedure is to ensure that the service levels for each of the individual products are as high as possible by reacting to short term demand fluctuations.

It may be expected that under the procedure mentioned above, the service levels for each of the products will be similar, since the sequencing procedure will equalize the run-out times of each of the individual products. It is important that can be predicted what the effects of this procedure on the individual product service levels will be, because predicted service levels will result in commitments towards either internal customers (such as the sales department) or external customers. However, due to the differences between the products in terms of demand level, uncontrolled service levels result. A run-out based sequencing procedure favors a product with a higher demand level over a product with a lower demand level. This can be explained using the following example. Suppose two products are made on a single installation. The production rate is the same for both products. One product (A) consumes more than half of the available capacity, the other product (B) consumes less than half of the available capacity. Suppose that
during a cycle both products are produced once, up to the level which is sufficient to fill
the average demand over the cycle. In the following cycle, we consider the probability
that, just after the production run of A has been completed, the run-out time of A is less
than the run-out time of B. The expected run-out time of A is related to the time span
since the last production start of A. We know that at the expected start of the new run
of product B, more than half of the cycle time of product A has been completed. Now
consider the probability that, again in the next cycle, product B will have a shorter run-
out time than product A. At this moment in time, less than half of the cycle time of
product B has passed, due to the larger share of product A in the total capacity con-
sumption.

This example illustrates the phenomenon that in a situation where multiple products are
produced on a single machine, and where some products have a considerably larger
demand than other products, these products tend to pull their production runs forward
at the expense of the smaller products. This will be demonstrated analytically in the two-
product case. More general observations, based on simulation experiments, can be found
in Fransoo (1993).

Again, we consider a single machine production system with two products. The following
notation is used.

\[ i = \text{product index, } i = \text{A or B} \]
\[ d_i(t) = \text{demand of product } i \text{ during } t \text{ periods, } d_i(t) \sim N(\mu_i, \sigma_i^2) \]
\[ d_i = \text{expected demand of product } i \text{ for 1 period} \]
\[ d_A = \frac{(s-1)p}{s}, \quad d_B = \frac{p}{s} \]

s = auxiliary variable to indicate the reciprocal of the portion of capacity consumed by product B, s > 1

\( \sigma_i \) = standard deviation of product i for 1 period

\( \frac{\sigma_i}{d_i} = c \), for both i

p = production rate, the same for both i

T = cycle time (time between the start of two production runs of the same product; we assume that there is no set-up time)

\[ \text{Figure 1. Production sequence} \]

Suppose that during one cycle both products are produced once, up to the level which
is sufficient to deliver the average demand over the cycle. What is the probability that, in the next cycle, product $A$ will be produced before $B$?

$$P(RO_A < RO_B \text{ at the decision moment } m_1 \text{ in Figure 1}) = P_1$$

What is the probability, under similar conditions, that product $B$ will be produced before product $A$?

$$P(RO_B < RO_A \text{ at the decision moment } m_2 \text{ in Figure 1}) = P_2$$

Lemma: $P_1 > P_2$, for all $s > 2$.

Proof:

1. $P_1 > P_2$
2. $P(RO_A(m_1) - RO_B(m_1) < 0) > P(RO_1(m_2) - RO_2(m_2) < 0)$
3. $P \left[ \frac{(s-1)p}{s} T - d_A(\text{runlength}_A) - \frac{p}{s} T - d_B(T) \left( \frac{(s-1)p}{s} \right) \right] > 0$
4. $P \left[ \frac{p}{s} T - d_B(\text{runlength}_B) - \frac{(s-1)p}{s} T - d_A(T) \left( \frac{(s-1)p}{s} \right) \right] < 0$

5. $P \left[ - \frac{s}{p(s-1)} d_A \left[ \frac{s-1}{s} T \right] + \frac{s}{p} d_B(T) < 0 \right] > P \left[ - \frac{s}{p} d_B \left[ \frac{1}{s} T \right] + \frac{s}{p(s-1)} d_A(T) < 0 \right]$

If $k$ is a stochastic variable with a standard normal distribution, then
This is true if and only if

\[
\frac{1}{p} Td_A - s \frac{Td_B}{p} > \frac{1}{p} Td_B - s \frac{Td_A}{p(s-1)}
\]

\[
\sqrt{\frac{s}{p(s-1)} T\sigma_A^2 + \left(\frac{s}{p}\right)^2 T\sigma_B^2} > \sqrt{\frac{s}{p(s-1)} T\sigma_B^2 + \left(\frac{s}{p}\right)^2 T\sigma_A^2}
\]

Since \(d_A = \frac{s-1}{s} p, d_B = \frac{p}{s}, \sigma_A = c \frac{s-1}{s} p, \) and \(\sigma_B = c \frac{p}{s},\) we may transform this inequality into

\[
\frac{-c^2 \frac{s-1}{s} T + c^2 T}{s} > \frac{(1-s)T}{s} \quad \Rightarrow \quad \sqrt{T} > \frac{\sqrt{T}}{s \sqrt{\frac{c^2 \frac{s-1}{s} T + c^2 T}{s}}}
\]

\[
- sc \frac{s-1}{s} + 1 < \frac{sc}{1-s} \frac{1}{s} + 1
\]

\[
- sc \frac{s-1}{s} + 1 < \frac{sc}{1-s} \frac{1}{s} + 1
\]

Since \(sc>0:\)

\[
\frac{2s-1}{s} < \frac{1}{s} \sqrt{\frac{1+s}{1-s}}
\]
\[ \Rightarrow \sqrt{\frac{2s - 1}{s}} > \frac{1}{s-1} \sqrt{\frac{1+s}{s}} \]

This is true if:
\[
\left[ \sqrt{\frac{2s - 1}{s}} > \frac{1+s}{s} \right] \land \left[ \frac{1}{s-1} < 1 \right]
\]
\[ \Rightarrow (2s - 1 > 1+s) \land (s-1 < 1) \]
\[ \Rightarrow s > 2 \]
q.e.d.

Single machine systems are quite common in flow process industries. Usually, the number of products is limited and it is an important requirement for the production planning system to control the service levels of the individual products. The analysis in this paper has demonstrated that a sequencing mechanism which is based on run-out times does not lead to this required control. Fransoo (1993) shows some suggestions for developing a sequencing rule which takes long-term service objectives into account. More research in this direction is however required.
References

