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Analysis of a Biphase-Based Servo Format for Hard-Disk Drives

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Abstract—Biphase modulation in an embedded-servo format for hard-disk drives is investigated. It is shown that for biphase, at the low linear densities typical of servo information, near-maximum-likelihood performance can be attained by a simple bit detector consisting of a full-response linear equalizer and a binary slicer. Compared to the commonly used method of dibit coding, a signal-to-noise ratio gain of some 4 dB is achieved. The same equalizer may be used as the basis for near-maximum-likelihood position error signal amplitude estimation and timing recovery. Simulations of a practical servo demodulator based on a fifth-order analog filter show that at typical linear densities, this ideal performance is closely approached. The equalizer has a band-pass character and yields excellent suppression of the effects of thermal asperities and magneto-resistive head asymmetry.

Index Terms—Biphase modulation, disk drive, position error signal, servo.

I. INTRODUCTION

EMBEDDED-SERVO systems are commonly used in hard-disk drives to determine read/write-head position. In such systems, disk area is divided into narrow servo sectors interspersed with user-data sectors. Head position is then determined by processing the output of the read head as it passes over the servo sectors. Coarse position information is obtained by reading track addresses written in servo-data fields, whereas fine position information is obtained as the head passes over a number of position error signal (PES) bursts. The latter consist of periodic magnetization patterns offset radially across the width of the tracks. Examples are split-burst and null patterns [1], with which read-head position (relative to track center) may be determined by demodulating the amplitude of the replay signal.

A common method for writing numerical information, such as a track address, in servo-data fields is shown in Fig. 1 (top) and will be referred to as dibit coding [2], [3]. It is also known as return-to-zero (RZ) coding [4]. Here, a logical “1” is represented by a dibit, i.e., two transitions spaced $T/2$ seconds apart, and a logic “0” by a constant magnetization. Bit cells are $T$-seconds long, which ensures a minimum spacing of $T/2$-seconds between transitions.

During the replay process, each logical “1” will produce a dibit response, i.e., two partially overlapping, Lorentzian-like pulses of opposite polarity, whereas a logical “0” produces no output at all. Because only one binary symbol “carries” energy (in the form of transitions), this is clearly suboptimum from the point of view of SNR. In this respect, biphase modulation, another well-known signaling method is more suitable. As it turns out, biphase has recently been proposed for use in embedded-servo applications [5], [6]. The present work, however, was carried out independently, before the issuance of these publications.

In biphase modulation, the second half of each bit cell is simply the inverse of the first half, as shown in Fig. 1 (bottom), where bit cells are again $T$-seconds long. Using this rule, two unique bit cell patterns may be generated, corresponding to the two binary symbols. The minimum interval between transitions is the same as for dibit coding; however transitions are now generated by both binary symbols, indicating a gain in SNR. The periodic magnetization pattern required for the PES bursts may also be regarded as a biphase modulated sequence, obtained, for instance, by biphase modulating the all ones sequence, i.e., “1, 1, 1 · · · 1.” The resulting burst has a period of $T$-seconds.

Another useful property of biphase modulation is the fact that every signaling pulse has a midbit transition. This means that timing recovery may continue throughout the servo sector, except for where the replay signal is significantly attenuated by cross-talk from neighboring tracks. Timing recovery is therefore not restricted to a special preamble located at the beginning of a servo sector. The length of such a preamble may thus be shortened (as far as the requirements for timing recovery are concerned), thus, reducing the servo-sector area.

Corruption of the replay signal by noise is not the only issue that impacts the correct demodulation of servo information.

Fig. 1. Comparison of dibit coding and biphase modulation.
Short-term media defects will cause dropouts and drop-ins in the replay signal [7]. The presence of magneto-resistive (MR) head asymmetry will distort the replay signal, whereas events such as thermal asperities and preamp mode-switching cause significant base-line shifts [2]. Yet another issue is the presence of phase incoherence among magnetization patterns written on adjacent servo tracks, which is caused by imperfections in the servo-writing process. This causes pulse broadening and shouldering in the replay signal [8] when the read-head straddles adjacent servo tracks. It also causes significant phase differences between the preamble and the various PES bursts [1].

This paper discusses the implications of a servo format based on biphase modulation on servo bit-detection, PES amplitude estimation, and timing recovery. It is assumed that the various magnetization patterns required by the format are accurately recorded during a servo-writing step. In Section II, performance bounds on bit detection for dibit coding and biphase modulation in the presence of additive white noise are presented. It is shown that for biphase, near-optimal performance is achieved by a simple demodulator based on a full-response linear equalizer (FRLE) and binary slicer. Also presented is the performance of a practical biphase demodulator based on a fifth-order analog filter. In Section III, performance bounds for optimal PES amplitude estimation are compared with the performance of a practical demodulator based on the biphase FRLE. A near-optimal timing recovery scheme is described in Section IV. The ability of the biphase FRLE to reject disturbances due to thermal asperities and MR-head asymmetry is briefly assessed in Section V. Finally, in Section VI, two alternative, high-density formats are described, one being a modified biphase format, whereas the other employs an interleaved variant of biphase known as quad-phase.

II. BIT DETECTION

A. Dibit Coding

The dibit-coded magnetization pattern \( m_d(t) \) shown in Fig. 1 may be thought of as being obtained by linear pulse modulation of a binary data sequence \( a_k \) according to

\[
m_d(t) = \sum_{k=-\infty}^{\infty} a_k c_d(t-kT) .
\]

Here, \( a_k \) is a binary data sequence, as shown in Fig. 1, whereas the function \( c_d(t) \) represents the basic shape of a dibit signaling pulse and is given by

\[
c_d(t) = \begin{cases} 
2 & \text{for } 0 < t < T/2, \\
0 & \text{elsewhere}. 
\end{cases}
\]

For a Lorentzian head/media system, the transition response \( g(t) \) is given by

\[
g(t) = \frac{1}{1 + \left( \frac{2t}{t_{50}} \right)^2}
\]

where \( t_{50} \) denotes the isolated pulsewidth at half-amplitude. To qualify the spacing between transitions, we define the normalized information density, \( D = \frac{t_{50}}{T} \). For robustness, servo data is typically recorded at lower normalized information densities than user data [4]. Using linear superposition (valid at low densities), the response of the head/media system to the dibit signaling pulse \( c_d(t) \), which we call the dibit symbol response \( h_d(t) \), is given by

\[
h_d(t) = g(t) - g(t-T/2).
\]

An optimal method of dibit detection in the presence of additive white noise is to use a filter matched to the dibit symbol response \( h_d(t) \), [9]. As shown in Fig. 2, the matched filter produces a single, symmetrical pulse \( q_d(t) \) in response to each transmitted dibit.

It may also be seen that \( q_d(t) \) is almost a Nyquist-1 pulse, i.e., that \( q_d(kT) \approx 0 \) for all \( k \neq 0 \). Only the first precursors and postcursors \( q_d(\pm T) \) deviate significantly from zero and cause some intersymbol interference (ISI), but even so, the eye pattern at the filter output will be almost perfectly binary. These observations lead to the synchronous detector architecture shown in Fig. 3. Here, the replay signal \( r(t) \), corresponding to a binary data sequence \( a_k \), is applied to a servo filter, with impulse response \( p(t) \), that realizes the matched-filter. The output \( y(t) \) of this filter consists of a series of pulses (corresponding to recorded dibits), which is sampled at instants \( t = kT \) to generate the discrete time sequence \( y_k \). This sequence is applied to a binary slicer (with a nonzero threshold) to produce near-optimum decisions \( \hat{a}_k \).

B. Biphase Modulation

The biphase magnetization pattern \( m_b(t) \) shown in Fig. 1 may also be obtained by linear pulse modulation of a binary data sequence \( a_k \) according to

\[
m_b(t) = \sum_{k=-\infty}^{\infty} a_k c_b(t-kT).
\]

This paper discusses the implications of a servo format based on biphase modulation on servo bit-detection, PES amplitude estimation, and timing recovery.
for a logical “0.” The function $c_0(t)$ represents the basic shape of a biphase signaling pulse and is given by

$$c_0(t) = \begin{cases} 
1 & \text{for } -T/2 < t < 0, \\
-1 & \text{for } 0 < t < T/2, \\
0 & \text{else.}
\end{cases} \quad (5)$$

Denoting the biphase symbol response, i.e., the response of the head/media system to the basic biphase signaling pulse $c_0(t)$, by $h_b(t)$, then, the impulse response of the corresponding biphase matched-filter is $h_b(-t)$, provided that noise is white. The output of this filter in response to $h_b(t)$ is again a single pulse $q_b(t)$, as shown in Fig. 4. The filter output $y(t)$ in response to the biphase modulated data sequence $a_k$ will be

$$y(t) = \sum_{k=-\infty}^{\infty} a_k q_b(t-kT) \quad (6)$$

i.e., a linear superposition of positive and negative pulses $q_b(t-kT)$. From Fig. 4, it may be seen that $q_b(t)$ is almost a Nyquist-1 pulse, and so the Fig. 3 architecture may be re-used to produce near-optimum decisions $\hat{a}_k$. The servo filter now realizes the biphase matched-filter, i.e., $h_b(-t)$, and the slicer has a threshold at zero to detect the polarity of the bipolar samples $y_k$. A similar architecture is described in [5]. There, however, the linear equalizer is designed to meet a partial-response target and is thus not optimal for biphase. With such an equalizer, improved performance may be obtained by using a Viterbi detector instead of a simple slicer [6].

C. Performance Comparison

For both formats, the performance of the Fig. 3 detector may be evaluated by computing predetection SNR at the input of the binary slicer. This has been done analytically (Appendix I and II) for the case in which an isolated symbol is transmitted (i.e., ISI is neglected), and noise, at the input of the matched filter, is white. The results represent the matched-filter bound (MFB) on bit-detector performance. In general, however, there will be some residual ISI at the input of the slicer. This may be suppressed by modifying the servo filter’s impulse response such that $q(t)$ is a Nyquist-1 pulse. In this case, the servo filter is effectively a full-response linear equalizer (FRLE) and $q(t)$ will be referred to as the equalized system response.

Predetection SNR may be computed numerically for the FRLE with minimum noise enhancement [10], and is compared in Fig. 5 with the MFB for various normalized information densities, $D$. The 0-dB level in Fig. 5 is the MFB for dibit coding as density tends to zero. For the computations, a fixed value for $T_{50}$ is assumed and $D = T_{50}/T$ is varied by changing the bit period $T$. From Fig. 5, it may be seen that the performance of the optimum biphase FRLE virtually coincides with the MFB. Furthermore, for practical servo densities, e.g., in the range $D \in [0.15 \cdots 0.6]$, biphase SNRs are about 4 dB better than are those for dibit coding.

Realization of the FRLE with minimum noise enhancement is somewhat impractical. For this reason, the performance of a limited-complexity detector, dubbed “RF-biphase detector,” that seems particularly attractive for a low-power mixed-signal implementation is also shown in Fig. 5. This detector is described in Appendix III. This version of Fig. 5 uses a fifth-order analog filter. Over the density range of interest, the performance of this detector is superior even to the MFB for dibit coding.

III. PES Demodulation

Read-head position (relative to track center) will typically be determined by demodulating the amplitude of the replay signal as the head passes over a number of PES bursts [12]. Optimum amplitude estimation of the PES burst is in principle possible via a filter matched to the entire PES burst. This is not a practical proposition. Noting that the PES burst may be regarded as the biphase modulated all-ones sequence, it is shown in Appendix IV that a completely equivalent PES amplitude estimate may be produced by integrating the product of the (a priori known) data and the sampled output of a filter matched to the biphase symbol response, over the duration of the burst. The resulting demodulator structure is shown in Fig. 6 and is closely akin to the Fig. 3 detector.

For a Lorentzian head/media system with additive white noise, the performance of servo filters matched either to the entire burst or only to its fundamental frequency has been computed analytically [11]. These matched-filter and first harmonic bounds (denoted by MFB and FHB, respectively) are shown in Fig. 7. SNR is maximized at an optimum burst period $T_{PES} \approx 3.281 T_{50}$. Also shown is the simulated performance of a demodulator that uses the servo filter of the RF-biphase
Fig. 6. PES demodulator for biphase.

Fig. 7. Signal-to-noise ratios of the PES amplitude estimate for various demodulators. The PES burst consists of 22 cycles, the first and last cycles of which are not demodulated. Solid lines: MFB and FHB; circles: RF-biphase servo filter; crosses: area detector.

Fig. 8. Amplitude-frequency characteristics at $D = 0.3$ of the biphase symbol response (solid line) and of the RF-biphase servo filter (dashed lines).

detector described in Appendix III. The performance of this demodulator is virtually identical to the FHB. This is because the filter has a bandpass characteristic (see Fig. 8) and, thus, rejects the odd harmonics of the PESs fundamental frequency. As density decreases, these harmonics increase in magnitude, resulting in increasing performance loss with respect to the MFB.

A commonly used method of PES amplitude estimation is area detection, [8], [12]. Here, the low-pass filtered replay signal is asynchronously rectified and integrated. Area detection may also be applied at the output of the RF-biphase servo filter, which then functions simply as a bandpass filter. The SNR of the PES amplitude estimate may be improved somewhat by employing synchronous rectification [7], [9]. This is however at the expense of increased sensitivity to phase incoherence between the PES burst and the preamble (during which timing is acquired), which may arise during servo-writing.

In Fig. 7, the performance of an area detector consisting of a fifth-order Bessel low-pass filter with cut-off frequency $1.8/T$, followed by a synchronous rectifier, is also shown. The low-pass filter effectively suppresses the higher harmonics of the PES burst, resulting in performance close to the FHB.

IV. TIMING RECOVERY

The bit detection and PES demodulation schemes described above require accurate knowledge of sampling phase. If noise at the input of the receiver is white, this knowledge may be obtained in an optimum manner by a timing-recovery loop based on the biphase matched-filter. The basic topology is shown in Fig. 9. A similar scheme for dibit-coded data is described in [9].

Here, a servo filter with impulse response $h_b(t)$ realizes the biphase matched-filter. Besides a main output $y_b(t)$, used to produce bit decisions $\hat{a}_k$ as in Fig. 3, the filter also has a second output that provides the derivative $\dot{y}_b(t)$ of $y_b(t)$. This derivative is sampled and multiplied with the bit-decisions $\hat{a}_k$. The resulting cross-product excites a loop filter (LF) that provides the control signal for a voltage-controlled oscillator (VCO). A suitable initial phase for the VCO may be obtained at the beginning of the preamble, for example, by a zero-phase start circuit operating on $y_b(t)$.

Roughly speaking, the scheme of Fig. 9 attempts to find the sampling phase at which $y_b(t)$ assumes its extremal values, or, equivalently, where $\dot{y}_b(t)$ is zero. The scheme is of the maximum-likelihood variety and is optimum in the sense that it extracts all timing information that is fundamentally present in the replay signal $r(t)$ [9], [10]. This is true irrespective of the data $a_k$, provided only that the bit-decisions $\hat{a}_k$ are correct.

During the servo data-fields and PES bursts, the amplitude at the slicer’s input is a function of head position and may be (near) zero. Under these circumstances, a binary slicer will yield unreliable decisions. The usual solution involves operating the timing-recovery loop only during the preamble, where the data are known a priori. An alternative solution replaces the binary slicer in Fig. 9 with a ternary slicer. This generates erasures when the amplitude of $y_b(t)$ falls below a fixed threshold. A detected erasure will, thus, not alter the VCO-state, permitting the loop to “coast” through such events.

Due to their similarity, the biphase FRLE may be used instead of the biphase matched filter in Fig. 9 at a small cost in performance. The resulting scheme is near-optimal for biphase, permits the simple use of erasure information, and is a natural extension of the biphase bit detector and PES amplitude-estimator described previously.
At the sampling instants $t_k = (k + \Delta)T$, where $\Delta$ is the sampling phase error expressed in units $T$, the loop is driven by the cross product (in the absence of decision errors)

$$a_k q'(t_k) = a_k \sum_{n=-\infty}^{\infty} a_n q'(t_k - nT) + a_k(n + p)(t_k)$$

(7)

where $q'(t)$ is the equalized system response defined earlier. This cross product has a noise component $u_k = a_k(n + p)(t_k)$ that induces random jitter in the loop and a data-dependent component $z_k = a_k \sum_{n=-\infty}^{\infty} a_n q'(t_k - nT)$ that provides the desired control information. Clearly

$$z_k = a_k \sum_{j=-\infty}^{\infty} a_{k-j} q'((j + \Delta)T).$$

(8)

This signal will be averaged by the loop; thus, for uncorrelated random data, only the $j = 0$ term provides control information, and the loop will attempt to force $q'(\Delta T)$ to zero; i.e., it will settle at the sampling phase for which $q'(t)$ assumes its peak. As shown in Fig. 10, where the time-axis is chosen such that $q'(t) = 0$ at $t = 0$, $q'(t)$ is approximately linear for small phase errors.

During the preamble and PES bursts, $a_k = 1$ and, thus, $z_k = \sum_{j=-\infty}^{\infty} q'(j + \Delta T)$. The desirable requirement that the loop settle at the same sampling phase as for random data is met when $q'(t)$ is antisymmetric about $t = 0$. This is true for the matched filter (see Fig. 10), and, by design, also holds for the RF-biphase servo-filter. For random data, however, the fact that $q'(jT) \neq 0$ when $j \neq 0$ implies that there will be some pattern-dependent jitter in the loop even when it is in tracking mode.

To assess the impact of this jitter, simulations were carried out on a first-order timing-recovery-loop employing the RF-biphase servo filter at a density of $D = 0.3$. The loop is approximately linear for small phase errors and so has an exponential step response with time constant $1/K_F$, where $K_F$ is the loop gain. At the start of a servo sector, the timing loop should acquire lock during the preamble, which typically may be 30 to 40 cycles long, and therefore, loop time-constants of about 10 symbol intervals are of practical interest. The preamble is usually followed by several bits of random data (e.g., track address, a synchronization word, etc.), which will cause pattern-dependent jitter. As shown in Fig. 11, this jitter is a strong function of loop gain, and becomes negligible for time constants of practical interest.

Noise at the input of the timing recovery loop will also cause loop jitter. A reasonable requirement for servo data, not protected by an error-correcting code, is that the bit-error rate (BER) be less than $10^{-8}$. For white noise at the input of a bit detector based on the RF-biphase servo filter, this corresponds to a predetection SNR of 15.8 dB. For an appropriate choice of loop gain (see Fig. 11), total loop jitter at this noise level is only slightly limited by the effects of pattern-dependent jitter.

V. THERMAL ASPERITY AND MR-HEAD ASYMMETRY HANDLING

The RF-biphase servo filter has a double zero at DC and as such may be expected to effectively suppress thermal asperities. This is confirmed in Fig. 12, which shows the response of the filter to a $4 \times$ thermal asperity with rise and fall time constants of 25 and 800 ns, respectively, at a servo frequency of 25 MHz and a normalized information density $D = 0.3$.

MR-head asymmetries can be roughly modeled by assigning distinct amplitudes $A_p$ and $A_n$ to the positive and negative transition responses of the head/media system. The degree of asymmetry $|A_p - A_n|/|A_p + A_n|$ can reach some 30% [2]. It is clearly desirable that the demodulator be insensitive to such asymmetries. The RF-biphase servo filter approximates a matched filter, which concentrates the compound effect of all transition responses within each bit cell at a single sampling phase. This concentration process largely eliminates the effect of asymmetries. Noise-free eye patterns after the RF-biphase servo filter are shown in Fig. 13 for a normalized information density $D = 0.3$. Although some asymmetry is still visible, the eye remains sufficiently open to permit reliable bit detection in the presence of noise.
VI. ALTERNATIVE HIGH-DENSITY FORMATS

As described in Section I, the PES burst may be regarded as the biphase modulated sequence $a_k = 1, 1, 1, \cdots$. An alternative format results if the PES burst is regarded as the modulated sequence $a_k = 1, 0, 1, 0, \cdots$. In terms of symbol rate $T$, the resulting burst period $T_{\text{PES}} = 2T$. For a given PES period (chosen to maximize SNR), the two possible formats will be referred to as low- and high-density formats, respectively, reflecting the difference in symbol rates. Use of the high-density format implies a doubling of the servo-data information-density. This halves the length of servo-data field(s), at the expense of a substantial decrease in predetection SNR (see Fig. 14).

Improved performance at higher densities may be obtained by using a somewhat more complex signaling format known as quad-phase, [13]. Quad-phase is essentially an interleaved variant of biphase and is DC-free. Each pair of bits results in one or more transitions; i.e., the presence of timing content is still guaranteed. As for high-density biphase, a PES burst with period $T_{\text{PES}} = 2T$ may be generated, by quad-phase encoding the all ones (or all zeros) sequence. The MFB for quad-phase is computed in Appendix V and is shown in Fig. 14. Over the density range of interest, i.e., $D_{\text{PES}} \in [0.45, 0.6]$, quad-phase offers performance similar to that of dibit signaling but at twice the linear density and with guaranteed timing content. Near-optimum receiver structures for quad-phase are similar to the ones developed here but are somewhat more complicated; see, for instance, [14] and [15].

VII. FINAL REMARKS

A servo format based on biphase modulation has been presented. At the low normalized information densities typically employed for servo information, near-maximum-likelihood PES demodulation and bit-detection are possible with a simple receiver based on an equalizer approximately matched to the biphase symbol response. The equalizer also provides excellent suppression of the effects of MR asymmetries and thermal asperities. When compared with a format based on dibit coding, significant gains in predetection SNR (some 4 dB) and timing content are attained. Simulations of a receiver based on a fifth-order analog filter show that such performance may be closely approximated in practice.

Servo-data densities may be doubled by adopting a high-density biphase format; this, however, leads to a significant loss in SNR with respect to dibit signaling. Improved performance is obtained with a quad-phase-based format, but at the cost of increased receiver complexity.

APPENDIX I

DIBIT CODING: SNR ANALYSIS

A system model for matched-filter bit detection is shown in Fig. 15.

Here, binary data symbols $a_k \in \{1, 0\}$ are conveyed via a linear pulse modulator with symbol response

$$c_d(t) = \begin{cases} 2 & \text{for } 0 < t < T/2 \\ 0 & \text{elsewhere} \end{cases}$$

(9)

Noise $n(t)$ is assumed to be white with power spectral density $N_0$. The dibit response of the head/media system is $h_d(t) = 2[g(t) - g(t-T/2)]$, where $g(t)$ is the Lorentzian pulse defined in (3). Then, the Fourier transform $H_d(\Omega)$ of $h_d(t)$ is

$$H_d(\Omega) = \pi T_{\text{dib}} \left[ 1 - e^{-j\pi \Omega T} \right] e^{-j\pi \Omega T/2}$$

(10)

where $\Omega$ is a normalized measure of frequency, with $\Omega = 1$ corresponding to the signaling rate $1/T$. In response to a transmitted dibit, the output of the matched filter is a symmetric pulse
with peak value \( A \), which using Parseval’s Theorem may be expressed as
\[
A = \int_{-\infty}^{\infty} |h_d(t)|^2 \, dt = \frac{1}{T} \int_{-\infty}^{\infty} |H_d(\Omega)|^2 \, d\Omega. \tag{11}
\]
The noise variance at the output of the matched filter is then
\[
\sigma^2 = \frac{N_0}{T} \int_{-\infty}^{\infty} |H_d(\Omega)|^2 \, d\Omega. \tag{12}
\]
Noting that there is no output when the symbol “0” is transmitted, bit decisions may be made with a binary slicer with threshold \( A/2 \). The predetection SNR \( \text{SNR}_d \) at the input of the slicer is then
\[
\text{SNR}_d = \frac{(A/2)^2}{\sigma^2} = \frac{1}{4N_0T} \int_{-\infty}^{\infty} |H_d(\Omega)|^2 \, d\Omega. \tag{13}
\]
And after some simplification, we obtain
\[
\text{SNR}_d = \frac{\pi t_{50}}{2N_0} \frac{1}{1 + (2D)^2}, \tag{14}
\]

**APPENDIX II**

**BIPHASE MODULATION: SNR ANALYSIS**

Using (5), the biphase symbol response \( h_b(t) \) is given by
\[
h_b(t) = [g(t + T/2) - 2g(t) + g(t - T/2)] \tag{15}
\]
and its Fourier transform \( H_b(\Omega) \) is
\[
H_b(\Omega) = -4\pi t_{50} \sin^2 \left( \frac{\pi \Omega}{2} \right) e^{-\pi D|\Omega|}. \tag{16}
\]
The output of the biphase matched-filter consists of both positive and negative pulses, corresponding to the biphase symbols for logical “1” and “0.” Thus, the binary slicer should have a threshold at zero. In this case, the predetection SNR \( \text{SNR}_b \) is given by
\[
\text{SNR}_b = \frac{A^2}{\sigma^2} = \frac{1}{N_0T} \int_{-\infty}^{\infty} |H_b(\Omega)|^2 \, d\Omega. \tag{17}
\]
This may be evaluated with the aid of integral tables to obtain
\[
\text{SNR}_b = \frac{\pi t_{50}}{2N_0} \frac{1}{1 + (2D)^2} \frac{3}{1}. \tag{18}
\]

**APPENDIX III**

**AN RF-BIPHASE DETECTOR**

The detector developed in this appendix seems well suited for a low-power mixed-signal IC implementation. It is based on the observation that biphase may be regarded as an “RF-modulated” version of NRZ, as shown in Fig. 16. Here, an NRZ signal \( d(t) \) is multiplied by a synchronous, binary, clock signal \( c(t) \) of period \( T \) to yield the biphase signal \( m(t) \).

This interpretation of biphase indicates that we can perform demodulation/detection in the following manner [16]. First, the received biphase signal is “down-converted” by multiplication with the clock \( c(t) \), and then bit-detection is performed in a manner adapted to NRZ. The latter step basically requires an integrate-and-dump filter. This leads to the detector topology of Fig. 17. Here, the replay signal \( r(t) \) is filtered by a servo filter with impulse response \( w_3(t) \). The filter output is multiplied with the in-phase clock \( c(t) \) and integrated across successive, \( T \)-second–wide intervals. At the end of each interval, the sign of the integrand is determined and serves as the bit-decision.

Mathematically, we may express the decision variable \( z_{2,k} \) as
\[
z_{2,k} = \int_{(k-0.5)T}^{(k+0.5)T} y_2(t) \chi(t) \, dt
\]
\[
= \int_{(k-0.5)T}^{(k+0.5)T} y_2(t) \, dt - \int_{(k-0.5)T}^{(k+0.5)T} y_2(t) \, dt \tag{19}
\]
where the instants \( kT \) correspond to signaling-pulse centers. This may be further simplified in terms of the basic biphase symbol \( c(t) \) defined in (5). Then
\[
z_{2,k} = \int_{-\infty}^{\infty} y_2(t) \chi(t - kT) \, dt,
\]
\[
= (y_2 * c_k)(kT) = (r * u_2 * c_2)(-t))(kT) \tag{20}
\]
where * denotes convolution. This analysis shows that the Fig. 17 topology is equivalent to that of Fig. 3, provided that the impulse responses \( y_2(t) \) and \( p(t) \) are related by
\[
p(t) = (y_2 * c_2)(-t))(t).
\]
When this condition holds, the Fig. 17 topology (slicer excluded) may be seen to be an alternative way of realizing a servo filter with impulse response \( p(t) \). This equivalent servo filter will be referred to as the RF-biphase servo filter.

If the servo filter is to be matched to the biphase symbol response, i.e., if \( p(t) = h_b(-t) \), then equivalence of the two topologies is obtained when \( u_2(t) = f(-t) \), where \( f(t) \) is the impulse response of the head/media system. Performance of the Fig. 17 topology will then depend on the accuracy to which \( u_2(t) \) may be realized, because the other blocks may be realized relatively accurately. This task is, however, considerably less complex than that of directly realizing \( h_b(t) \). One
way of realizing $\phi_2(t)$ is with a fifth order analog filter made of a first-order high-pass filter and two second-order low-pass filters. Filter parameters were determined by a computer program that optimizes predetection SNR via a simplex search. The values found are shown in Table I, where $Q$ denotes quality factor and $\Omega_c$ denotes the cutoff and resonance frequencies of the first-order and second-order sections, respectively. The latter are normalized such that $\Omega = 1$ at the signaling rate $1/T$.

### APPENDIX IV

**OPTIMUM PES DEMODULATION**

The magnetization pattern $m(t)$ of a PES burst may be expressed as

$$m(t) = \sum_{k=0}^{N-1} a_k c(t-kT)$$  \hspace{1cm} (21)

where

- $a_k$ (bipolar) data sequence used to generate the PES burst;
- $N$ number of cycles in the PES burst;
- $c(t)$ biphase or dibit symbol response.

The corresponding replay signal $r(t)$ is a filtered and noisy version of $m(t)$ according to $r(t) = Ax(t) + n(t)$, where $A$ is the burst amplitude and

$$x(t) = \sum_{k=0}^{N-1} a_k h_k(t-kT).$$  \hspace{1cm} (22)

Optimum PES demodulation is possible by applying $r(t)$ to a filter whose impulse response is matched to the entire PES burst. If $n(t)$ is white, this implies that $d(t) = x(-t)$. The matched-filter output $v(t)$ is sampled at instant $t = 0$ to obtain an optimum PES estimate $\hat{v}(0)$. Then

$$v(0) = \int_{-\infty}^{\infty} r(t)d(-t)dt = \int_{-\infty}^{\infty} r(t)x(t)dt$$  

$$= \sum_{k=0}^{N-1} \int_{-\infty}^{\infty} r(t)a_k h_k(t-kT)dt$$  

$$= \sum_{k=0}^{N-1} a_k y(kT)$$  \hspace{1cm} (23)

where $y(t)$ may be recognized as the output of a filter with impulse response $h(-t)$ operating on $r(t)$. This is just the matched filter described in Section II.

### APPENDIX V

**QUAD-PHASE MODULATION: SNR ANALYSIS**

Consider a binary stream $a_k$ with symbol rate $2/T$, which is subdivided into even and odd streams $d^e_k = a_{2k}$ and $d^o_k = a_{2k+1}$. Then, the quad-phase encoded binary data signal $b_n$ at the data rate $4/T$ is given by

$$b_{kn} = a^0_{mn}, \quad b_{kn+1} = a^1_{mn},$$  

$$b_{kn+2} = a^0_{m\bar{n}}, \quad b_{kn+3} = a^1_{m\bar{n}}.$$  \hspace{1cm} (24)

The basic shape of a quad-phase signaling pulse, denoted by $c_{qp}(t)$, may then be expressed as

$$c_{qp}(t) = \begin{cases} 
1 & \text{for } -3T/8 < t < -T/8, \\
-1 & \text{for } T/8 < t < 3T/8, \\
0 & \text{else.}
\end{cases}$$  \hspace{1cm} (25)

The quad-phase symbol response $h_{qp}(t)$, i.e., the response of the channel to a quad-phase signaling pulse, is given by

$$h_{qp}(t) = g(t+3T/8) - g(t+T/8) - g(t-T/8) + g(t-3T/8)$$  \hspace{1cm} (26)

and its Fourier transform $H_{qp}(\Omega)$ is

$$H_{qp}(\Omega) = \pi t_{\pi\Omega}(\cos(3\pi\Omega/4) - \cos(\pi\Omega/4))e^{-\pi D|\Omega|}.$$  \hspace{1cm} (27)

As for biphase (Appendix II), the predetection SNR $SNR_{qp}$ is given by

$$SNR_{qp} = \frac{A^2}{\sigma^2} = \frac{1}{N_0 T} \int_{-\infty}^{\infty} |H_{qp}(\Omega)|^2 d\Omega$$  \hspace{1cm} (28)

which, after some simplification, results in

$$SNR_{qp} = \frac{3\pi t_{\pi\Omega}}{N_0} \frac{1}{1+(2D)^2} \frac{1}{1+4(2D)^2} \frac{3+32D^2}{9+4(2D)^2}.$$  \hspace{1cm} (29)

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### REFERENCES


