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A perceptual error measure for sampled and interpolated complex colour images

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Abstract

Error measures are needed for optimizing display parameters. Most of the existing error measures are inappropriate since they correlate poorly with the perceived impairment of the reproduced images. The reason is that these measures, often distance measures, do not or do not sufficiently incorporate properties of the human visual system. We therefore present a perceptual error measure that uses the distance in a perceptual space spanned by the sampling and interpolation artifacts periodic structure and blur, to predict the impairment of sampled and interpolated images. It is shown how the sensorial strengths of periodic structure and blur that are along the axes of this perceptual space can be determined from the image. In addition, we demonstrate that after the perceptual error measure is calibrated to a particular observer, it can predict experimentally determined subjective image quality of sampled and interpolated colour images, and may thereby be used to optimize display parameters.

Keywords: Perceptual image quality; Sampling; Interpolation

1. Introduction

The majority of electro-optical display systems, including liquid-crystal video displays and CRTs, produce sampled and interpolated images that are visually impaired compared to the originals i.e. images produced by a hypothetical ideal display. Measurements of perceptual image quality, i.e. the degree of excellence of an image [1], should be used to evaluate sampled and interpolated images produced by displays. Perceptual image quality also enables the perceptual optimization of the physical parameters specifying the sampling and interpolation process.

In general, perceptual image quality values predicted by error measures such as the mean-square error do not correlate well with the results of subjective tests because these measures do not incorporate properties of the human visual system e.g. Pratt [2] and Mannos and Sakrison [3].

At present, there is a growing interest in error measures employing one or more properties of the visual

system. One of these properties is the modulation transfer function of the eye (e.g. Campbell and Robson [4]) which specifies the threshold modulation depth of sine gratings as a function of spatial and temporal frequencies of the sine grating. It is a popular description in the frequency domain that can easily be included in error measures using a Fourier description of the sampling and interpolation problem (e.g. Watson, Ahumada and Farrell [5]). However, such measures generally neglect the fundamental property that perceptual image quality is a global percept having many underlying perceptual attributes.

In this paper we specify a quantitative perceptual error measure predicting the perceptual quality of sampled and interpolated colour images relative to the originals. The measure explicitly states the underlying perceptual attributes as well as how these attributes are combined into the global image quality percept. The relations between the physical display parameters and the strengths of the perceptual attributes are formulated using properties of the human visual system. After the perceptual parameters of the perceptual error measure have been determined, we study the performance of the error measure and finally show how the measure can be used to optimize display parameters.

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Fig. 1. Schematic diagram of the computational steps of the perceptual error measure that relates the perceptual image quality (Q) or the total perceptual impairment (I) to the physical parameters (Φ_i), using an intermediate perceptual space with the sensorial strengths (S_i) or the impairments (I_i) of the most dominant sampling and interpolation artifacts, periodic structure (p) and blur (b), along the (orthogonal) axes.

2. Perceptual error measure

In this section we review the perceptual error measure for sampled and interpolated images presented in [6,7]. The diagram in Fig. 1 shows its computational steps that determine the perceptual image quality (Q) from the physical parameters (Φ_i) specifying both the image and the sampling and interpolation process. The perceptual error measure is distinguished from conventional error measures by a perceptual space with the sampling and interpolation artifacts as (orthogonal) dimensions. Thus, the perceptual error measure explicitly incorporates the hypothetical dimensions underlying the visual impairment of the images. Typical artifacts that may occur are, for example, periodic structure, blur, 'blockiness', staircase, moiré and colour errors. We restrict ourselves here to periodic structure and blur since these are usually the most dominant artifacts.

The perceptual image quality (Q) is related linearly to the total perceptual impairment (I) [8,9]:

$$I = 1 - Q \quad (1)$$

For computational convenience, the perceptual attributes are defined in the interval $[0, 1]$.

Besides judging total perceptual impairment, people can also distinguish between the various underlying sampling and interpolation artifacts and are able to judge their perceptual impairments separately. Minkowski metrics with an exponent of two can be used to combine the (M) underlying perceptual impairments into the total impairment I [9,10]:

$$I^2 = \sum_{i=1}^M I_i^2 \quad (2)$$

which reduces to

$$I^2 = I_p^2 + I_b^2 \quad (3)$$

if we consider only the impairments of the two most dominant artifacts periodic structure (I_p) and blur (I_b). The total impairment can therefore be interpreted as a Euclidean distance in a perceptual space.

The perceptual impairment of an underlying artifact (I_i) was found to be linearly related to the perceptual

strength of the artifact (S_i):

$$I_i = a_i S_i \quad (4)$$

The constants a_i represent the relative weights of the artifacts in the total perceptual impairment.

According to Eqs. (3) and (4) the total perceptual impairment of a sampled and interpolated image can be written as

$$I^2 = a_p^2 S_p^2 + a_b^2 S_b^2 \quad (5)$$

or, alternatively, as

$$I^2 = a_p^2 [S_p^2 + \lambda_b S_b^2] \quad (6)$$

The perceptual parameter $\lambda_b = a_b^2/a_p^2$ expresses the observer's weight of the blur artifact relative to the periodic structure artifact. Eq. (6) shows that the optimization of the sampling and interpolation problem is equivalent to minimizing the cost function

$$S_p^2 + \lambda_b S_b^2 \quad (7)$$

Minimization of cost functions is a well-known variational regularization solution method for ill-posed problems [11]. The solution is a compromise between conflicting demands i.e. between the absence of periodic structure and the absence of blur. The regularization parameter λ_b controls the relative importance of the constraints. It tells us something about an observer's weight of periodic structure compared to the observer's weight for blur. Note that the value of λ_b may depend both on the observer and on image content.

On the basis of Eq. (1) the difference in perceptual quality between an original image and a sampled and interpolated version of this image simply equals the total impairment caused by sampling and interpolation. Under the assumption that $I = 0$ for the original image, this difference can be written as (Eq. (6))

$$I = a_p \sqrt{S_p^2 + \lambda_b S_b^2} \quad (8)$$

The interpretation of the error measure as a distance measure in a perceptual space is straightforward. If both the periodic structure and blur artifacts are visible, the resulting loss in perceptual quality is proportional to the length of a vector ($S_p, \sqrt{\lambda_b} S_b$) in a perceptual plane spanned by these artifacts.

3. Sensorial strength functions

The sensorial strengths of the periodic structure and blur artifacts are determined from the physical display parameters. In [6] we derived and experimentally verified explicit expressions for these strengths for a columnar sampling structure and Gaussian interpolation. In the case of a columnar structure the image is sampled in the horizontal direction only. The Gaussian interpolation can for instance be implemented by an optical filter

placed in front of the display. The expressions are:

$$S_b = 1 - \frac{1}{\left(\left(\frac{\sigma}{\sigma_0}\right)^2 + 1\right)^{0.25}} \quad (9)$$

$$S_p = c \frac{\left(1 + \left(\frac{m}{m_0}\right)^3\right)^{1/3} - 1}{\left(\frac{m}{m_0}\right)^\beta} \quad (10)$$

where σ is the spread parameter of the Gaussian interpolation filter, m is the modulation depth of the periodic structure, $\beta \approx 0.7$ and c is a constant such that $0 \leq S_p \leq 1$. The parameter m_0 is a periodic structure threshold parameter and σ_0 is the spread parameter of a Gaussian interpolation function representing the intrinsic blur of the early-visual pathway and including the optics of the eye. These last two parameters depend on the observer and must be determined experimentally.

Fig. 2 shows a part of the luminance profiles along a horizontal line for columnar sampling structures for black-and-white and colour displays. The former will be referred to as the columnar structure with grey columns whereas the latter is referred to as the RGB-triplet columnar sampling structure. Note that the RGB-triplet structure can also be used to display black-and-white images.

We use Fourier theory to determine the modulation depth m . The modulation depth of the first harmonic of the luminance profiles in Fig. 2 is taken as the modulation depth of the grey-column and RGB-triplet structures because the first harmonic is a good predictor for the visibility of any periodic structure [4]. Note that the first harmonic of the RGB-triplet structure is the sum of

Fig. 2. Luminance profiles along a horizontal line of a columnar sampling structure with sampling distance d and column width $w = d/3$. (a) Grey-columns, (b) RGB-triplet columns. Note that if the average luminance of a displayed image is equal for both structures, then the luminance of the grey-column must be equal to the sum of the red, green and blue luminances of the corresponding RGB-triplet. If black-and-white images are displayed using the RGB-triplet structure, then the ratio of the red, green and blue luminances is fixed. For colour images this ratio may vary from one RGB-triplet to another.

the three first harmonics of the red, green and blue columns having the same frequency, different phases and in most cases unequal amplitudes (L_R , L_G and L_B).

The modulation depth depends on three factors. The first accounts for the width of the columns relative to the sampling distance. The second accounts for the colour i.e. the ratio of the luminances L_R , L_G and L_B . For computational purposes we used the value corresponding to the D_{65} illuminant (PAL white reference). The third factor represents the attenuation of the modulation depth due to the Gaussian interpolation filter.

The RGB-triplet columnar structure may cause both a periodic luminance variation and a periodic chrominance variation. In this paper we consider only periodic luminance variations. Periodic chrominance structures are neglected for two reasons. First, we note that the resolution for chrominance gratings such as red–green and blue–yellow gratings is roughly a factor of three smaller than the resolution for luminance gratings such as a green monochromatic grating [12]. Second, the periodic chrominance structure is perceived best in the absence of luminance variations. In practice, however, chrominance variations often occur simultaneously with relatively large luminance variations.

4. Experimental validation

Various experiments were executed to check (parts of) the model under several conditions [6]. This paper focuses on a test of the complete model for complex colour images. The necessary experimental data were gathered in the ‘complex colour experiment’. In this experiment subjects rated the perceptual image quality of sampled and interpolated images. In the experiment we used various colour as well as black-and-white images and varied both the spatial arrangement of the RGB-triplets of a columnar sampling structure and the spread parameter of a Gaussian interpolation filter. The experiment is described in full detail in Appendix A.

In the sequel, the perceptual error measure of Eqs. (1), (8), (9) and (10) with parameters σ_0 , m_0 and λ_b is fit to the experimental data for six images and six subjects and the ‘average’ subject, using the m and σ values corresponding to the stimuli. The parameters σ_0 , m_0 and λ_b are unique for each subject and λ_b is also unique for each combination of image and sampling structure. The σ_0 and m_0 values are estimated only from the experimental data of the single-edged black-and-white image.

4.1. Parameters of the sensorial strength functions

In this section we determine the values of the intrinsic blur of the early-visual pathway (σ_0) parameter and the periodic structure threshold (m_0) parameter in Eqs. (9)

and (10). To this end we only use the data of the stimuli of the single-edged¹ black-and-white image.

We have two groups that contain some of these stimuli. The first group is used for determining the value of the intrinsic blur parameter. The five stimuli in this group are the non-sampled original single-edged black-and-white image and the four filtered versions of this image (see Appendix A). In these stimuli only the blur artifact is visible at the edge. The sensorial strength of periodic structure is zero and hence, according to Eq. (5), $I^2 = a_b^2 S_b^2$. Therefore and on the basis of Eq. (1), experimentally determined quality values are related linearly to the sensorial strength of blur. Since the stimuli are completely specified, we can make a graph of the calculated strength of blur (Eq. (9)) versus the ‘experimental’ blur values. The value for the intrinsic blur parameter σ_0 is taken such that the points in this graph are closest to a straight line. More specifically: The value for the parameter σ_0 is taken such that the coefficient of determination (e.g. Chatfield [13]) of the regression line² of I_m^2 on S_b^2 is closest to unity. Here S_b is the calculated strength and I_m stands for the measured impairment which, according to Eq. (1), can easily be calculated from the quality values measured in the complex colour experiment.

The second group contains six stimuli of the single-edged black-and-white image. Besides the non-sampled, non-filtered original it includes the sampled version with the grey-column structure as well as its four corresponding extra stimuli that are filtered in the horizontal direction only (see Appendix A). Since there is no filtering in the vertical direction, no (extra) blur is visible at the edge. Only the modulation depth and hence the sensorial strength of periodic structure is varied in these extra stimuli and therefore, according to Eq. (5), $I^2 = a_p^2 S_p^2$. Hence, experimentally determined quality values are related linearly to the sensorial strength of periodic structure. We can now plot the calculated sensorial strength of periodic structure (Eq. (10)) versus the ‘experimental’

strength values and determine the values of the periodic structure threshold m_0 and intrinsic blur σ_0 parameters in Eq. (10) for which the points in this graph are closest to a line. More formally we can say that the periodic structure threshold parameter and the intrinsic blur parameter in the sensorial strength function for periodic structure (S_p) are determined in such a way that the coefficient of determination of the regression line of I_m^2 on S_p^2 is closest to unity.

Tables 1 and 2 list the values of the intrinsic blur and periodic structure threshold parameters. For each subject and for the data averaged over subjects we have estimated two values for the intrinsic blur parameter. For each subject and for the data averaged over subjects we use the average of these two values as the value of the intrinsic blur parameter.

4.2. Regularization parameter

In this section we determine a unique value for the regularization parameter for blur $\lambda_b = a_b^2/a_p^2$ in Eq. (6) for each combination of subject, image and sampling structure. We use Eqs. (9) and (10) with the values of the periodic structure threshold and intrinsic blur parameters as determined above, to calculate the sensorial strengths of blur and periodic structure for each stimulus in the experiment. If these strength values together with the impairment values (calculated from the experimentally determined quality) are substituted into Eq. (5), we get an equation in a_p^2 and a_b^2 i.e. the squares of the relative weights of the artifacts. Since there are $6 \times 5 \times 5 = 150$ different stimuli for each subject (see Appendix A) we arrive at a set of 150 simultaneous equations for each subject. We subdivide this set into 18 partially overlapping subsets. For each of the six images used in the experiment there are three subsets; the first for the colour versions of the images and columnar sampling with RGB-triplets, the second for the black-and-white versions with RGB-triplet columnar sampling and the third for the black-and-white versions with the grey-column columnar sampling. Each subset includes the

¹ In the experiments we often use the ‘single-edged’ image that is formed by two adjacent uniform regions with different luminances which are separated by a horizontal straight edge. The image is completely specified by these two luminance values (L_1 and L_2) or alternatively by the average luminance $((L_1 + L_2)/2)$ and the Michelson contrast ($C = (L_1 - L_2)/(L_1 + L_2)$). Besides the fact that the image content of the single-edged image is completely specified by only two parameters, the single-edged image offers additional advantages if we make appropriate choices for the sampling and interpolation process. For columnar sampling and Gaussian interpolation only the artifacts’ periodic structure and blur can occur. Furthermore, the strengths of these artifacts can be controlled independently since (1) the structure is perpendicular to the edge, and (2) we can control the degree of interpolation in the horizontal and vertical direction separately.

² We use the regression line of I_m^2 on S_b^2 instead of the regression line of I_m on S_b thus avoiding the value of the parameter σ_0 being influenced by any other artifact that may be present. Other artifacts only lead to a shift of the regression line ($I^2 = a_b^2 S_b^2 + \sum_i I_i^2$).

Table 1
Values for the intrinsic blur parameter (σ_0) and the standard deviation (s_{σ_0}) in the perceptual strength of blur function determined in the complex colour experiment for data from the individual subjects and for data averaged over subjects

Subject	σ_0 (arc min)	s_{σ_0} (arc min)
FB	0.34	0.45
GS	0.25	0.20
HR	0.93	0.15
MN	0.25	0.08
MV	0.59	0.35
RH	1.06	0.55
AVERAGE	0.49	0.19

Table 2

Values for the intrinsic blur parameter (σ_0), the periodic structure threshold parameter (m_0) and the standard deviations (s_{σ_0} and s_{m_0}) in the perceptual strength of periodic structure function determined in the complex colour experiment for data from the individual subjects and for data averaged over subjects

Subject	m_0	s_{m_0}	σ_0 (arc min)	s_{σ_0} (arc min)
FB	0.02	0.04	0.77	0.41
GS	0.09	0.20	0.77	0.76
HR	0.01	0.13	0.74	2.35
MN	0.01	0.27	0.64	5.88
MV	0.01	0.10	0.68	2.04
RH	0.01	0.44	0.57	10.43
AVERAGE	0.02	0.09	0.73	0.88

sampled image, its four filtered versions, the non-sampled image and the four filtered versions of the non-sampled image. For each subject and for the data averaged over subjects we solved these subsets for the squares of relative weights a_p^2 and a_b^2 and calculated the regularization parameter for blur $\lambda_b = a_b^2/a_p^2$.

The regularization parameters for blur are plotted in Figs. 3 and 4. The first figure shows the regularization parameters for the black-and-white images with the grey-column sampling as a function of the observer. There is a separate curve for each image. In Fig. 4 the regularization parameters for blur for the data averaged over subjects are plotted as a function of image content.

We conclude that the regularization parameter for blur depends neither on image content (trivial exceptions excluded) nor on the spatial arrangement of the columns (grey-columns or RGB-triplet column). However, the regularization parameter depends on the observer. If the perceptual error measure is used to determine optimal display parameters, we must therefore study the

Fig. 3. Regularization parameters for blur of the black-and-white images with grey-columns of the complex colour experiment as a function of the observer with image content as a parameter. (□) 'Wanda', (○) 'Terrasgeel', (△) positive contrast pseudo text, (+) negative contrast pseudo text, (×) 'Mondrian' and (◇) single-edged. The error bar is twice the standard deviation in λ_b for the data averaged over subjects.

Fig. 4. Regularization parameters for blur for the data of the complex colour experiment averaged over subjects as a function of image content. (□) Black-and-white images and grey-columns, (○) black-and-white images and RGB-triplet columns, (△) colour images and RGB-triplet columns. The error bar is twice the standard deviation in λ_b for the data averaged over subjects.

effect of changes in the regularization parameter on the optimal display parameters.

4.3. Performance of the perceptual error measure

In this section we compare the measured impairments (I_m) with the impairments (I_c) calculated with the perceptual error measure (Eqs. (1), (8), (9) and (10)). We use the parameter values determined in the previous sections. Hence all parameters (σ_0 , m_0 and λ_b) are unique for each subject and λ_b is also unique for each of the 18 combinations of image (six) and sampling structure (three).

The direct comparison in Fig. 5 for all stimuli of the data averaged over subjects shows that the calculated and measured impairments correlate satisfactorily.

Fig. 5. Squared measured perceptual impairments versus squared calculated perceptual impairments for the data of the complex colour experiment averaged over subjects. The dashed line indicates points with $I_c = I_m$.

A more detailed comparison is found in Fig. 6. In this figure we have plotted one minus the impairment, i.e. quality, versus the spread parameter (σ) of the optical Gaussian interpolation filter for a specific image for the data averaged over subjects. Fig. 7 contains a similar plot but now for the data of a specific subject. Again we conclude that the perceptual error measure performs well.

Figs. 6 and 7 illustrate the effect of the columnar sampling structure and the optical Gaussian filter on the perceptual image quality. For the non-filtered stimuli ($\sigma = 0$), image quality drops if sampling is applied. Image quality of the non-sampled images decreases as the filtering increases. The effect of filtering on the sampled images is twofold. First, the perceptual quality of the sampled images increases as the spread parameter increases until it reaches some maximum. Beyond this point the quality decreases as the filtering increases. The first part of the curve corresponds to a decrease of the perceptual strength of periodic structure while the sensorial strength of blur remains relatively small. In the second part of the curve the perceptual strength of blur increases while the perceptual strength of periodic structure hardly decreases.

5. Perceptually optimal sampling and interpolation

A perceptually optimal combination of sampling and interpolation minimizes the total perceptual impairment which satisfies Eq. (2):

$$I^2 = I_p^2 + I_b^2 + \sum_{i=3}^M I_i^2 \quad (11)$$

Fig. 6. Comparison of measured (dashed curves) and calculated (continuous curves) perceptual quality. The perceptual quality averaged over subjects is plotted versus the spread parameter (σ) of the optical Gaussian interpolation filter for the 'Terrasgeel' image of the complex colour experiment. From top to bottom, the five pairs of curves are for ($\square$) the non-sampled black-and-white images, ($\times$) the non-sampled colour images, ($\triangle$) the sampled black-and-white images with RGB-triplet columns, ($+$) the sampled colour images with RGB-triplet columns and ($\circ$) the sampled black-and-white images with grey-columns.

Fig. 7. Perceptual quality of subject RH for the 'Terrasgeel' image of the complex colour experiment. See the caption to Fig. 6 for further details.

and expresses the fact that the total impairment is a combination of the impairments of the individual artifacts' periodic structure, blur and others such as staircase and 'blockiness'.

For a columnar sampling structure with sampling distance d and Gaussian interpolation with spread parameter σ we found it to be sufficient to consider only the impairments of periodic structure and blur. Thus, according to Eqs. (6), (9) and (10), we should minimize

$$I^2 = I_p^2 + I_b^2 = a_p^2[S_p^2(\sigma, d) + \lambda_b S_b^2(\sigma)] \quad (12)$$

The optimization problem thus reduces to finding a (σ, d) combination that minimizes the cost function $S_p^2(d, \sigma) + \lambda_b S_b^2(\sigma)$.

Let us assume that for economic and technical limitations the sampling distance is fixed and non-zero so that periodic structure is visible. In this case the σ value that minimizes the total perceptual impairment is a solution of

$$\frac{\partial}{\partial \sigma} [S_p^2(d, \sigma) + \lambda_b S_b^2(\sigma)] = 0. \quad (13)$$

Although Eq. (13) can be solved analytically, the perceptually optimal σ value which satisfies this equation is determined graphically from Fig. 8. This figure shows a plot of $\sqrt{\lambda_b} S_b$ versus S_p for $\lambda_b = 2$. The spread parameter σ of the Gaussian interpolation filter is a parameter of the points on the curve. The sensorial strength functions and the values of the parameters correspond to the complex colour experiment. Note that the interval $[0, 1]$ on either axis in Fig. 8 comprises the whole range from a perfect image to a completely mutilated image. The distance between the origin and a point on the curve is equal to the total perceptual impairment for the σ value corresponding to the point. Iso-impairment curves are circles with the origin as their centre. The smaller the radius of the circle the smaller the total impairment. The σ value

Fig. 8. Standard parametric curve of points $(S_p(d, \sigma), \sqrt{\lambda_b} S_b(\sigma))$ with σ as a parameter for $\lambda_b = 2$. The dashed circle is an iso-impairment curve. The $\times$ indicates the point on the curve that is closest to the origin. The sensorial strength functions for periodic structure (S_p) and blur (S_b) as well as the parameter values are taken from the complex colour experiment. The σ_0 and m_0 values are $\sigma_0 = 0.61$ arc min and $m_0 = 0.022$. The sampling distance $d = 2.72$ arc min, $\alpha_c = 0.55$, $\alpha_p = 0.74$ and the viewing distance is 4m.

that corresponds to the point on the parametric curve closest to the origin is therefore the optimal σ value which minimizes the cost function. The point corresponding to a minimal total perceptual impairment is indicated by the symbol $\times$.

Plots similar to that in Fig. 8 can be used to study the influences of variations in the regularization parameter for blur (λ_b) and the viewing distance on the optimal σ value [6].

6. Discussion

In this paper we presented a model that satisfactorily predicts the perceptual image quality of columnar sampled and Gaussian interpolated complex colour images. This holds for different images and for individual subjects as well as for the average subject.

The model was, however, only tested for one fixed sampling distance. In another experiment, the complex black-and-white experiment [6], we used three different sampling distances. This experiment has an experimental set-up that is similar to the set-up of the complex colour experiment. Fig. 9 shows the data for the 'Terrasgeel' image. The shape of the curves for the data of the complex black-and-white experiment is similar to the shape of the curves in Figs. 6 and 7. For smaller values of the sampling distance the maxima in the curves become more pronounced, i.e. the slopes of the rising and falling parts of the curves become steeper.

The perceptual error measure for sampled and interpolated images is a specific example from a class

Fig. 9. Comparison of measured ($\square$) and calculated ($\times$) perceptual quality. The perceptual quality averaged over subjects is plotted versus the spread parameter (σ) of the optical Gaussian interpolation filter for the 'Terrasgeel' image of the complex black-and-white experiment. From top to bottom the four pairs of curves are for the non-sampled images and the sampled images with $d = 1.81$ arc min, $d = 2.72$ arc min and $d = 3.63$ arc min.

of general image quality models predicting perceptual image quality from physical parameters. An important feature of these models is a perceptual space intermediate between the physical parameter space and perceptual image quality. The sensorial strengths of the perceptual attributes underlying perceptual image quality are along the axis of this space. In our case we chose periodic structure and blur as the relevant dimensions since these attributes were dominantly visible when we looked at sampled and interpolated images. This choice is supported by the predictions of the model. The general model is similar to the model proposed by Engeldrum [14].

The sensorial strength functions model the front-end of the visual system. The bottom-up processing of images in this part of the visual system is relatively independent of image content and basically the same for different observers. We therefore expect small variations in the perceptual parameters (σ_0 and m_0) of the strength functions. High-level vision is modelled by the Minkowski metric for the sensorial strengths. Top-down processing in this part enable observers to express their personal preference which may depend on image content. Hence, we expect larger variations in the perceptual parameter (λ_b) in the combination rule.

The main reasons for the larger standard deviations in σ_0 and especially m_0 is that the complex colour experiment was primarily designed to determine the regularization parameter for blur in a variety of conditions rather than to determine the intrinsic blur and periodic structure threshold parameters. Despite the larger standard deviations the parameter values are similar to those found in other experiments [6].

Calculations show that the influence of variations in

the σ_0 and m_0 parameters on the value of the regularization parameter for blur is only minor. The relatively large spread in the intrinsic blur σ_0 and periodic structure threshold m_0 parameters is therefore only partly the reason for the relatively large standard deviation of the regularization parameter. The larger part of the variation in the regularization parameter is caused by the individual preferences of the observers.

We would like to stress that the variations in the regularization parameter are not a shortcoming of the model but an explicit modelling of variations that exist among observers. The current model uses only one regularization parameter to account for the individual differences. The variations put a certain constraint on the optimization of image reproduction techniques, namely that optimal solutions should also possess a certain robustness. Based on the results with sampling and interpolation, we think that the structure of the general image quality model is an adequate reflection of the perceptual process that leads to image quality. Hence, we expect it also to work for other image processing problems.

In order to increase the practical value, display designers must solve two types of problem. The first is to handle the variation in the regularization parameter. A possible solution is to use the value of the average observer. The second problem is the specification of new sensorial strength functions for non-Gaussian interpolation filters and two-dimensional or spatio-temporal sampling structures. We found [6] that the model can also be used for two-dimensional horizontal-vertical sampling structures by combining the strengths of the two individual periodic structures into an overall strength using a Minkowski metric with a power of four.

Appendix A. Complex colour experiment

A.1. Equipment

In the experiment we used both colour and black-and-white images. Images were displayed using a Gould DeAnza IP 8400 image-processing system. Each image consisted of 512 by 512 pixels with eight bits per primary colour per pixel.³ The viewing distance was 4 m and the pixel pitch was 0.45 arc min. During the experiment only the centre 496 by 496 pixels (size 0.26 m by 0.26 m or 3.7° by 3.7°) were shown on a Barco CCID 7351B

high-resolution 70 Hz interlace colour monitor. The colour coordinates $(x_R, y_R) = (0.62, 0.35)$, $(x_G, y_G) = (0.30, 0.59)$ and $(x_B, y_B) = (0.15, 0.07)$ of the monitor's primary colours red, green and blue did not deviate much from the standard PAL colour coordinates $(x_R, y_R) = (0.64, 0.33)$, $(x_G, y_G) = (0.29, 0.60)$ and $(x_B, y_B) = (0.15, 0.06)$. The display was calibrated in such a way that for each of the primaries there was a power-law relation $L \propto g^\gamma$, with an exponent $\gamma = 2.5$, between the eight-bit pixel value (g) and the luminance L of a pixel. The D_{65} illuminant $(x_w, y_w) = (0.31, 0.33)$, adopted as a white reference for PAL systems, was obtained for equal pixel values of the primaries. In this case the ratio of the red, green and blue luminances was $L_R : L_G : L_B = 0.230 : 0.689 : 0.081$ as against $L_R : L_G : L_B = 0.222 : 0.707 : 0.071$ for standard PAL. For black and white images the pixel values of the three primaries were equal.

A.2. Images

Both colour and black-and-white images were used in the experiment. Some of these images are shown in Fig. 10. Six different black-and-white images were used in this

Fig. 10. Photographs of some of the stimuli used in the complex colour experiment.

³ The term 'pixel' is restricted to indicating the elements of the combination of the image-processing system and the monitor used to display the stimuli. The area of a pixel is approximately equiluminant. Pixels lie next to each other and the distance between them (pixel pitch) is equal to the pixel's size. Pixels are sufficiently small to ensure that artifacts such as periodic structure and 'blockiness' are not visible. In addition, no moiré effects are introduced during stimulation since sampling distances of the simulated sampling structures are multiples of the pixel pitch.

experiment: The complex portrait of a woman, 'Wanda', the complex terrace scene, 'Terrasgeel', a bright pseudo text on a dark background (labelled positive contrast pseudo text), a dark identical text on a bright background (labelled negative contrast pseudo text), an abstract 'Mondrian' image and a single-edged image consisting of two uniform regions separated by a horizontal edge. The luminances of the regions of the single-edged image were 14.2 cd/m^2 and 0.44 cd/m^2 . The 'Mondrian' image consisted of seven partially overlapping uniform rectangles of varying size and luminance. The brightest and darkest rectangles had the same luminances as the regions of the single-edged image. The six colour images were coloured versions of the black-and-white images. The luminances of the colour and the black-and-white images were identical. Unlike in the 'Wanda' and 'Terrasgeel' images, colours had to be specified for the pseudo text, 'Mondrian' and single-edged images. The low and high luminance regions of the single-edged image were coloured magenta $((x, y) = (0.23, 0.11))$ and yellow $((x, y) = (0.48, 0.46))$, respectively. The positive contrast pseudo text image had bright yellow $((x, y) = (0.48, 0.46))$ text on a dark blue $((x, y) = (0.15, 0.07))$ background. The negative contrast pseudo text image had dark red $((x, y) = (0.62, 0.35))$ text on a bright green $((x, y) = (0.30, 0.59))$ background. The colours of the 'Mondrian' rectangles were red $((x, y) = (0.62, 0.35))$, green $((x, y) = (0.30, 0.59))$, blue $((x, y) = (0.15, 0.07))$, magenta $((x, y) = (0.23, 0.11))$, yellow $((x, y) = (0.50, 0.44))$, cyan $((x, y) = (0.17, 0.12))$ and grey $((x, y) = (0.31, 0.33))$. Chromatic aberration of the eye lens may cause retinal blur at the edge of two rectangles with different colours. Although chromatic aberration also occurs with other images, the effects are annoying in the 'Mondrian' image. The reason is that, unlike in most natural complex images, primary colours occur side by side. To avoid the visibility of aberration effects in the 'Mondrian' image, the transitions between the rectangles were replaced by black lines, six pixel pitch units wide, of zero luminance in both the colour and the black-and-white version of the 'Mondrian' image.

A.3. Stimuli

Both sampled and non-sampled versions of the colour and black-and-white images were used in the experiment. The columnar structure of Fig. 2.a was applied only to the black-and-white images. The columnar RGB-triplet structure of Fig. 2.b was used for both the colour and the black-and-white images. An overview is given in Table 3. The sampling distance of the columnar structure was 6 pixel pitch units (2.72 arc min) and the width of all columns (red, green, blue and grey) was 2 pixel pitch units (0.91 arc min).

The stimulus set also contained filtered versions of the stimuli specified above. Besides the non-filtered version

Table 3

Overview of the sampling structures that were applied to colour and black-and-white versions of all six images used in the complex colour experiment

Sampling structure	Image type	
	black-and-white	colour
non-sampled	×	×
grey-columns	×	
RGB-triplet	×	×

there were four filtered versions for each of the five combinations of image type and sampling structure in Table 3. The filter was an optical Gaussian filter. It was simulated on the Gould DeAnza by transforming pixel values into luminance values, consecutive filtering in the horizontal and vertical direction with a binomial impulse response of length l [15] and finally transforming the filtered luminance values into pixel values. The filter length is related to the spread parameter σ of the Gaussian filter by $\sigma = \frac{1}{2}\sqrt{l-1}$. The filter lengths were 3, 7, 13 and 22 pixel pitch units. The spread parameter of the non-filtered image is $\sigma = 0$. To facilitate the determination of the σ_0 and m_0 parameter values, twelve extra stimuli of the single-edged image were added. There were four extra filtered versions for each of the three combinations in Table 3 corresponding to a sampling structure. The extra stimuli were only filtered in the horizontal direction. Hence only the modulation depth of the periodic structure was varied while the edge was not altered. The horizontal filter lengths were 3, 7, 13 and 22 pixel pitch units.

In practice, the luminance profiles of the sampled images did not exactly match the profile drawn in Fig. 2. The rectangular corners of the profiles were rounded. Consequently, the modulation depths of the sampling structures are somewhat lower than the modulation depths corresponding to the profiles in Fig. 2. This effect is included in the factor accounting for the width of the columns relative to the sampling distance. For the profiles in Fig. 2 this factor is equal to $\text{sinc}(w/d) = \text{sinc}(1/3) \approx 0.827$. We measured some of the displayed profiles and calculated the value of this factor. For the grey-column structure the factor is 0.71 and for the RGB-triplet structure it is 0.74. The factor turned out to be relatively independent of the peak luminance of the profile.

A.4. Procedure

In two sessions perceptual quality of the displayed images was rated on a 10-point numerical category scale ranging from one to ten, by one female and five male subjects between 27 and 43 years of age. Subjects had normal or corrected-to-normal vision and a visual acuity between 1.25 and 2, measured on a Landolt chart. Although two subjects (FB and GS) had a slight red-

green deficiency, their results did not differ significantly from those of other subjects. Subjects received an instruction form in which the quality was defined as depending only on periodic structure and blur of the displayed images. In order to adjust the sensitivity of their scale, subjects judged a test series of 18 stimuli containing the extreme stimuli before the start of the actual experiment. All 162 stimuli were presented twice in each of the two sessions except for the three sampled, non-filtered single-edged images, which were presented four times. The sequence of the images was random. However, in consecutive trials there were never two non-sampled colour stimuli, two non-sampled black-and-white stimuli, two sampled colour stimuli, or sampled black-and-white stimuli with the same sampling structure. Images were presented for five seconds and followed by a white adaptation field with a luminance of 12 cd/m^2 which lasted until subjects pressed a key but had a minimum duration of two seconds. The viewing conditions satisfied CCIR recommendation 500 [16] except for the viewing distance, which was 4 m.

All category data were transformed into an interval scale on the psychological continuum using Thurstone's law of categorical judgement. We applied a class I model involving replications over trials within a single subject with condition D constraints [17]. These constraints limit the number of model parameters by assuming that the correlation between the momentary position of stimuli and category boundaries as well as the dispersion of both category boundaries and stimuli are constant. Prior to the Thurstone correction, data were processed in accordance with Edward's method [18] in order to correct scale values of the extreme categories. It was found that the trends in the Thurstone-corrected data of the individual subjects were similar. It was therefore permissible to average the Thurstone-corrected data over subjects.

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