The isentropic exponent in plasmas

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I. INTRODUCTION

The isentropic relations for a flowing gas system, relating pressure, density, and temperature to each other, are usually used in situations of atmospheric pressure and of local thermal equilibrium (LTE). Local thermal equilibrium means that the electron temperature may not deviate from the heavy particle (i.e., atoms and ions) temperature (the plasma is so-called equithermal) and that ionization may not differ from Saha’s ionization equilibrium (LSE). The isentropic relations can be used instead of more complex energy equations when viscosity and heating play no significant role. The Debye length must be small and the Reynolds number sufficiently high, typically in the order of 100 or even larger. The isentropic exponent as used in these relations is a constant, making the isentropic relations a convenient tool to analyze many systems with gas flow.

In this paper we try to extend the isentropic relations to atmospheric plasmas. The main difference between gases and plasmas is that in plasmas ionization needs to occur to sustain the plasma. Plasmas are not in the LTE, which implies that we need to distinguish a temperature for the electrons and one for the heavy particles, and that the actual neutral density does not equal the Saha neutral density. An important question is how much the isentropic exponent deviates from its LTE value due to ionization as well as due to equithermal disequilibrium.

In order to stress the influence of ionization and the disequilibria, we will use an alternative description in which the plasma pressure, the ionization degree, and two nonequilibrium parameters, $b_1$ for deviations from Saha’s relation and $\theta$ for equithermal disequilibrium are used, instead of the electron temperature, the electron density, the heavy particle temperature, and the heavy particle density.

The isentropic exponent is usually investigated as a function of the temperature. However, in this paper we will show how the isentropic exponent behaves as a function of the ionization degree (ratio), which is the parameter that shows best the difference between gases and plasmas. It should be stressed that the ionization degree (ratio) and the electron density in plasmas vary over many orders of magnitude, whereas the electron temperature varies over a relatively small range, e.g., for atmospheric plasmas the electron temperature is always in the range of 1–2 eV.

Next we will consider three plasmas in particular; a monoatomic LTE plasma, a monoatomic non-LTE plasma and a diatomic non-LTE plasma.

II. THE ISENTROPIC RELATION

In gas dynamic theory it is common to use the isentropic relation relating pressure and density, which reduces the complexity of the hydrodynamic description. Although the isentropic condition is not always valid, it is often used as a first approximation of complex gas dynamical systems. In this paper we discuss this isentropic relation for plasmas in order to have the same tool to describe plasmas and gases. The isentropic relation relating the pressure, $p$, or the temperature, $T$, and the mass density, $\rho$, reads

$$\frac{p}{\rho^\gamma} = C, \quad \frac{T}{\rho^{\gamma-1}} = C', \quad (1)$$

The isentropic exponent in plasmas

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where $C$ (and $C'$) is a constant and $\gamma$ is the so-called isentropic exponent, which is only a function of how energy is distributed internally in the considered fluid, i.e., of the degrees of freedom. Note that by determining expression (1) the relation for pressure vs temperature is also determined.

In thermodynamics expression (1) is called Poisson’s relation. The isentropic exponent equals $5/3$ for monoatomic gases, which have three kinetic degrees of freedom (e.g., an argon gas), and equals $7/5$ for diatomic gases, which can also rotate and vibrate (e.g., nitrogen and hydrogen gases).

A plasma has extra degrees of freedom because of the occurrence of ionization. In ordinary plasmas, heat is used to ionize and create the plasma, which is an extra freedom in the energy distribution inside a plasma. The isentropic exponent of a plasma will therefore differ from that of a gas. To calculate the isentropic exponent $\gamma$, we will use the definition in gas dynamics; the isentropic exponent equals the ratio of the heat capacity at constant pressure, $c_p$, and the heat capacity at constant volume, $c_V$. For a full derivation of the isentropic exponent of monoatomic and diatomic gases we refer to Matsuzaki.\(^{1}\)

Further, we will consider only singly ionized ions and make use of quasineutrality, i.e., the ion density equals the electron density $n_i = n_e$.

The heat capacity at constant pressure, at constant volume, and the isentropic exponent are defined as

$$c_p = \left(\frac{\partial H}{\partial T}\right)_p = T \left(\frac{\partial S}{\partial T}\right)_p,$$

(2)

$$c_V = \left(\frac{\partial e}{\partial V}\right)_T = T \left(\frac{\partial S}{\partial T}\right)_T = e - 1 \left(\frac{\partial p}{\partial T}\right)_p,$$

(3)

$$\gamma = \left(\frac{\partial S}{\partial \ln p}\right)_s = \frac{\rho}{p} \left(\frac{\partial p}{\partial T}\right)_s,$$

(4)

with $H$ the specific enthalpy, $e$ the internal energy, and $S$ the entropy. At constant entropy,

$$c_p = -T \left(\frac{\partial S}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_v,$$

$$c_V = -T \left(\frac{\partial S}{\partial \rho}\right)_T \left(\frac{\partial \rho}{\partial T}\right)_s,$$

which we use to derive

$$\gamma = \frac{c_p \rho}{c_V \frac{\partial \rho}{\partial T}} \frac{1}{T}.$$

(5)

For a plasma $c_p$, $c_V$, and $\gamma$ are functions of the ionization degree, $\alpha$. The ionization degree is defined as

$$\alpha = \frac{n_i}{n_h}$$

(6)

in which $n_h$, the heavy particles number density, is equal to the sum of the atom density, $n_a$, the ion density, $n_i$, and (if not equal to zero) the molecule number density, $n_m$. Note that we use number densities, which equals the number of particles per molecule times the density of that molecule.

### III. THE ISENTROPIC EXPONENT

#### A. The monoatomic LTE plasma case

For plasmas we need to focus on the contribution of ionization in the specific enthalpy and the internal energy. We will first discuss the specific enthalpy and the internal energy before we derive how the temperature influences the ionization degree. We need this latter result in the derivation of the heat capacity at constant pressure, at constant volume, and the isentropic exponent [see formulas (2), (3), and (5)]. We will see that due to ionization $\gamma$ decreases towards 1.16 for a plasma in comparison to $\gamma = 5/3$ or $\gamma = 7/5$ in case of an ideal monoatomic, respectively, diatomic gas.

The following set of plasma conditions is considered first:

1. monoatomic plasma (like for, e.g., an argon plasma);
2. the electron temperature may not deviate from the heavy particle (i.e., atoms and ions) temperature, i.e., local equi-uthermal equilibrium (LEE); and
3. ionization may not differ from the Saha equilibrium, i.e., local Saha equilibrium (LSE).

Later on we will discuss non-LTE and diatomic plasmas.

The basic equations governing the thermodynamic properties of the plasma are the specific enthalpy and the internal energy (including ionization). The specific enthalpy can be obtained by using the methods of statistical mechanics.\(^{2}\) For most of the monoatomic plasmas (like an argon plasma), this includes only the translational energy and ionization energy, since the excitational energy is negligible.\(^{3}\) The specific enthalpy, $H$, can be written as

$$H = \frac{5}{2}RT(1 + \alpha) + \alpha \frac{e^{\text{ion}}}{m_a},$$

(7)

with $R$ the gas constant, $e^{\text{ion}}$ the ionization energy, and $m_a$ the atom mass. The internal energy, $e$, is related to the specific enthalpy by

$$e = H - \frac{p}{\rho} = H - RT(1 + \alpha).$$

(8)

Expressions (7) and (8) are similar to the expressions for gases when the ionization degree is set equal to zero.

Next we will derive the expression for the isentropic exponent in a plasma starting from the Saha balance\(^{4}\) and Dalton’s law. The Saha equation (S) gives the relation between the electron density, $n_e$, the ion density, $n_i$, and the density of the ground level neutrals, $n_s$,

$$S = \frac{n_e n_i}{n_s} = \frac{2g_i}{g_a} \left(\frac{2\pi m_e kT_e}{\hbar^2}\right)^{3/2} \exp \left(-\frac{e^{\text{ion}}}{kT_e}\right),$$

(9)

where $m_e$ equals the electron mass, $h$ the Planck constant, $T_e$ the electron temperature and $g_i$ and $g_a$ are the statistical weights of the ion and atomic ground state.

Notice that

$$\frac{\partial S}{\partial T_e} = \frac{3}{2} \frac{S}{T_e} + \frac{e^{\text{ion}} S}{kT_e^2},$$

(10)

and in LTE $T = T_e$ and $n_a = n_s$. Dalton’s law in LTE reads
\[ p = (n_e + n_i + n_a)kT = n_a kT (1 + \alpha). \]  

Dalton’s law and quasineutrality together with the Saha balance (9) gives

\[
\left( \frac{n_e^2}{S} + 2n_e \right) = \frac{p}{kT}. \tag{12}
\]

After solving this quadratic equation in \( n_e \), we get

\[ \alpha = \left( \sqrt{1 + \frac{p}{S kT}} \right)^{-1} \tag{13} \]

or

\[ \frac{p}{S kT} = \frac{1 - \alpha^2}{\alpha^2}. \tag{14} \]

Taking the derivative of the ionization degree \( \alpha \) with respect to the temperature \( T \), leaving pressure constant,

\[
\frac{\partial \alpha}{\partial T} = \frac{\alpha^2}{2} \left( \frac{1/2 S^2}{1 + (1/2 S)(p/kT)} \right) \frac{\partial S}{\partial T} + \frac{p}{S kT} \left( \frac{\alpha}{2} - \frac{\alpha^3}{2} \right) + \frac{1}{T}.
\]

This yields, conform Refs. 1 and 5 using expressions (10) and (14),

\[
\frac{\partial \alpha}{\partial T} = \frac{\alpha}{2T} (1 - \alpha)(1 + \alpha) \left( \frac{5}{2} + \frac{e_{\text{ion}}}{kT} \right). \tag{16}
\]

We need expression (16) in the derivation of the heat capacities with respect to the temperature at constant pressure, respectively, at constant volume, which can be written as

\[
c_p = \frac{5}{2} R (1 + \alpha) + RT \left( \frac{5}{2} + \frac{e_{\text{ion}}}{kT} \right) \frac{\partial \alpha}{\partial T}. \tag{17}
\]

respectively,

\[
c_v = \frac{3}{2} R (1 + \alpha) + RT \left( \frac{3}{2} + \frac{e_{\text{ion}}}{kT} \right) \frac{\partial \alpha}{\partial T}. \tag{18}
\]

Note that the derivative of the ionization degree with respect to the temperature at constant density can be found in a similar way as expression (15) from (13), with an extra term expressing the derivative of pressure with respect to the temperature at constant density. This yields

\[
\frac{\partial \alpha}{\partial T} = \frac{\partial \alpha}{\partial T} - \frac{\alpha (1 - \alpha)}{2T} \left( 1 + \alpha + T \frac{\partial \alpha}{\partial T} \right). \tag{19}
\]

Combining expressions (16) and (19) yields

\[
\frac{\partial \alpha}{\partial T} = \frac{\alpha (1 - \alpha)}{T (1 - 2\alpha)} \frac{3}{2} + \frac{e_{\text{ion}}}{kT}. \tag{20}
\]

Hence the heat capacity at constant pressure becomes for a monoatomic LTE plasma,

\[
c_p = \frac{5}{2} R (1 + \alpha) + R \frac{\alpha (1 - \alpha)(1 + \alpha)}{2} \left( \frac{5}{2} + \frac{e_{\text{ion}}}{kT} \right)^2. \tag{21}
\]

and the heat capacity at constant volume,

\[
c_v = \frac{3}{2} R (1 + \alpha) + R \frac{\alpha (1 - \alpha)}{2} \left( \frac{3}{2} + \frac{e_{\text{ion}}}{kT} \right)^2. \tag{22}
\]

The isentropic exponent from expression (5) can now be analyzed further. By making use of

\[
\left( \frac{\partial p}{\partial \rho} \right)_T = \frac{\partial}{\partial \rho} \left( \frac{p}{RT (1 + \alpha)} \right)_T
\]

and from expressions (13) and (14),

\[
\left( \frac{\partial \alpha}{\partial \rho} \right)_T = \frac{\alpha (1 - \alpha)(1 + \alpha)}{2p},
\]

we can write for the isentropic exponent of a monoatomic LTE plasma,

\[
\gamma = \frac{c_p}{c_v} \frac{2}{2 + (1 - \alpha)(1 + \alpha)} = \frac{c_p}{c_v} \frac{1}{1 + (1 - 1/2\alpha)(1 + 1/2\alpha)} \tag{23}
\]

as in agreement with Matsuzaki,\textsuperscript{1} using thermodynamic methods, and Owczarek,\textsuperscript{6} using the method of characteristics. Notice that the expressions for the heat capacities and the isentropic exponent are similar to the expressions for gases when the ionization degree is put equal to zero. We can calculate the isentropic exponent from expression (23) with use of the above expressions for \( c_p \) and \( c_v \).

\section*{B. The monoatomic plasma in non-LTE}

In this section we allow the plasma to deviate from the two types of equilibria we considered. We have the following set of plasma conditions:

1. monoatomic plasma (like an argon plasma);
2. the electron temperature, \( T_e \), may deviate from the heavy particle (i.e., atoms and ions) temperature, \( T_h \) (i.e., non-LTE); and
3. ionization may differ from the Saha equilibrium (i.e., non-LSE).

For high density plasmas, which are not too far from local thermodynamic equilibrium (LTE), the commonly used thermodynamic plasma variables are the electron temperature, electron density, heavy particle temperature, and heavy particle density (or pressure). In this section we use an alternative description in which the plasma pressure, the ionization degree (or electron density) and two nonequilibrium parameters, \( b_1 \) and \( \theta \), conform Schram \textit{et al.}\textsuperscript{7}

\[
b_1 = \frac{n_e}{n_e} \text{ and } \theta = \frac{T_h}{T_e},
\]

are used. Note that in local thermodynamic equilibrium \( b_1 \) and \( \theta \) are both equal to one. This alternative description will turn out to be more convenient, as we will see when we discuss the results in the next section. In this paper, \( b_1 \) and \( \theta \) are considered as independent variables.

We start again with the expressions for the specific enthalpy and the internal energy, which are
Dalton’s Law and quasineutrality together with the Saha balance (9) now gives
\[ \Theta b_1 n_e^2 \frac{2}{S} + (1 + \Theta) n_e - \frac{p}{kT_e} = 0. \] (27)

Solving this quadratic equation in \( n_e \), we get
\[ \alpha = \Theta \left( \frac{\Theta - 1}{2} + \frac{1}{2} \sqrt{(1 + \Theta)^2 + 4 \frac{\Theta b_1}{S} \frac{p}{kT_e}} \right)^{-1} \] (28)
or
\[ \Theta b_1 p \frac{2}{SKT_e} = \frac{\Theta(1 - \alpha)(\Theta + \alpha)}{\alpha^2}. \] (29)

Taking the derivative of the ionization degree with respect to the electron temperature \( T_e \), leaving pressure constant, as we did before in expression (15), yields
\[ \left( \frac{\partial \alpha}{\partial T_e} \right)_p = \alpha \left( \frac{1 - \alpha}{2} \frac{\Theta(1 - 1 - \Theta)(\Theta + \Theta)}{\Theta(2 + (1 - \Theta)\alpha)(1 + (1 - \Theta)\alpha)} \left( \frac{5 + e^{\text{ion}}}{2 + kT_e} \right) \right)^{-1} \] (30)

To find expression (30) we used expressions (10) and (29). Note that the nonequilibrium parameters, \( b_1 \), disappears in expression (30), since we made use of expression (29).

In a similar way, we find
\[ \left( \frac{\partial \alpha}{\partial T_e} \right)_p = \alpha \left( \frac{1 - \alpha}{2} \frac{\Theta(1 - 1 - \Theta)(\Theta + \Theta)}{\Theta(2 + (1 - \Theta)\alpha)(1 + (1 - \Theta)\alpha)} \left( \frac{3 + e^{\text{ion}}}{2 + kT_e} \right) \right)^{-1} \] (31)

Hence the heat capacity at constant pressure becomes in non-LTE,
\[ c_p = \left( \frac{\partial H}{\partial T_h} \right)_P = \frac{5}{2} \frac{R}{\Theta(\Theta + \alpha)} + \frac{\alpha R}{\Theta} \left( \frac{1 - \alpha}{2} \frac{\Theta(1 - 1 - \Theta)(\Theta + \Theta)}{\Theta(2 + (1 - \Theta)\alpha)(1 + (1 - \Theta)\alpha)} \left( \frac{5 + e^{\text{ion}}}{2 + kT_e} \right)^2 \right). \] (32)

respectively, the heat capacity at constant volume,
\[ c_V = \left( \frac{\partial e}{\partial T_h} \right)_P = \frac{3}{2} \frac{R}{\Theta(\Theta + \alpha)} + \frac{\alpha R}{\Theta} \left( \frac{1 - \alpha}{2} \frac{\Theta(1 - 1 - \Theta)(\Theta + \Theta)}{\Theta(2 + (1 - \Theta)\alpha)(1 + (1 - \Theta)\alpha)} \left( \frac{3 + e^{\text{ion}}}{2 + kT_e} \right)^2 \right) \times \left( \frac{3}{2 + kT_e} \right) \] (33)

Note that the nonequilibrium parameters, \( b_1 \), remains to be disappeared.

For analyzing the isentropic exponent in non-LTE we use
\[ \left( \frac{\partial p}{\partial \theta} \right)_T = \left( \frac{\partial}{\partial \theta} \left( \frac{p}{RT_e(\Theta + \alpha)} \right) \right)_T, \]
\[ = \frac{1}{RT_e(\Theta + \alpha)} \left[ 1 - \frac{p}{\Theta(\Theta + \alpha)} \left( \frac{\partial \alpha}{\partial \theta} \right)_T \right], \]

and from expression (28),
\[ \left( \frac{\partial \alpha}{\partial \theta} \right)_T = - \frac{\alpha}{p} \frac{(1 - \alpha)(\Theta + \alpha)}{2(1 - \Theta)(\Theta + \alpha)}, \]

The isentropic exponent [expression (5)] for monoatomic non-LTE plasma becomes
\[ \gamma = \frac{c_p}{c_v} \frac{2(1 - 1 - \Theta)(\Theta + \Theta)}{2(1 - \Theta)(\Theta + \Theta)\alpha(1 + (1 - \Theta)\alpha)}. \] (34)

Notice that the expressions for the heat capacities and the isentropic exponent are identical to the corresponding expressions in LTE when \( \theta \) is set equal to one. Like before, we can calculate the isentropic exponent from expression (34) with use of the above expressions for \( c_p \) and \( c_v \).

C. The diatomic plasma in non-LTE

In this section we consider a diatomic plasma (e.g., hydrogen) which deviates from the two types of equilibria, i.e., deviations from LEE and LSE.

The expressions for the specific enthalpy and the internal energy [expressions (24) and (25)] do not differ from the monoatomic case after rotation and vibration are added,
\[ H = \frac{5}{2} R(T_h + \alpha T_e) + RT_h + \alpha \frac{e^{\text{ion}}}{m_a}, \] (35)

respectively,
\[ e = H - R(T_h + \alpha T_e). \] (36)

Dalton’s law in the diatomic molecular case differs in the number density,
\[ p = (n_a + n_i + n_m)kT_h + n_kT_e. \] (37)

Dalton’s Law and quasineutrality together with the Saha balance gives an expression, similar as expression (28) before, for the ionization degree,
\[ \alpha = \Theta \left( \frac{\Theta - 1}{2} + \frac{1}{2} \sqrt{(1 + \Theta)^2 + 4 \frac{\Theta b_1}{S} \frac{p}{kT_e - n_m \Theta}} \right)^{-1}. \] (38)

Also expression (29) is similar,
\[ \Theta b_1 \left( \frac{p}{kT_e - n_m \Theta} \right) = \Theta(1 - 1 - \Theta)(\Theta + \Theta) \frac{1}{\alpha^2}. \] (39)

Taking the derivative of the ionization degree with respect to the electron temperature \( T_e \), leaving pressure constant, respectively, density, yields again similar expressions as in the monoatomic case. Expression (30) respectively, (31) becomes

\[ \left( \frac{\partial \alpha}{\partial T_e} \right)_p = \alpha \left( \frac{1 - \alpha}{2} \frac{\Theta(1 - 1 - \Theta)(\Theta + \Theta)}{\Theta(2 + (1 - \Theta)\alpha)(1 + (1 - \Theta)\alpha)} \left( \frac{5 + e^{\text{ion}}}{2 + kT_e} \right)^{-1} \right)^{-1} \] (30)

respectively, the heat capacity at constant pressure becomes in non-LTE,
\[ c_p = \left( \frac{\partial H}{\partial T_h} \right)_P = \frac{5}{2} \frac{R}{\Theta(\Theta + \alpha)} + \frac{\alpha R}{\Theta} \left( \frac{1 - \alpha}{2} \frac{\Theta(1 - 1 - \Theta)(\Theta + \Theta)}{\Theta(2 + (1 - \Theta)\alpha)(1 + (1 - \Theta)\alpha)} \left( \frac{5 + e^{\text{ion}}}{2 + kT_e} \right)^2 \right). \] (32)

respectively, the heat capacity at constant volume,
\[ c_V = \left( \frac{\partial e}{\partial T_h} \right)_P = \frac{3}{2} \frac{R}{\Theta(\Theta + \alpha)} + \frac{\alpha R}{\Theta} \left( \frac{1 - \alpha}{2} \frac{\Theta(1 - 1 - \Theta)(\Theta + \Theta)}{\Theta(2 + (1 - \Theta)\alpha)(1 + (1 - \Theta)\alpha)} \left( \frac{3 + e^{\text{ion}}}{2 + kT_e} \right)^2 \right) \times \left( \frac{3}{2 + kT_e} \right) \] (33)

Note that the nonequilibrium parameters, \( b_1 \), remains to be disappeared.
Hence the heat capacity at constant pressure becomes in non-LTE,

\[
\left( \frac{\partial \alpha}{\partial T_e} \right)_p = \frac{\alpha}{T_e} \frac{(1 - \alpha)(\Theta + \alpha)}{(2 \Theta + (1 - \Theta) \alpha)} \left( \frac{5}{2} + \frac{\epsilon^{\text{ion}}}{kT_e} + \frac{kT_h^2}{p} \left( \frac{\partial n_m}{\partial T_h} \right)_p \right).
\]

\[
\left( \frac{\partial \alpha}{\partial T_e} \right)_p = \frac{\alpha}{T_e} \frac{(1 - \alpha)(\Theta + \alpha)}{(2 \Theta + (1 - \Theta) \alpha) + (1 - \alpha) \alpha} \times \left( \frac{3}{2} + \frac{\epsilon^{\text{ion}}}{kT_e} + \frac{kT_h^2}{p} \left( \frac{\partial n_m}{\partial T_h} \right)_p \right).
\]

(40)

(41)

respectively, the heat capacity at constant volume,

\[
c_v = \frac{3}{2} \frac{R}{\Theta} (\Theta + \alpha) + R + \frac{\alpha R}{\Theta} \frac{(1 - \alpha)(\Theta + \alpha)}{(2 \Theta + (1 - \Theta) \alpha) + (1 - \alpha) \alpha} \times \left( \frac{3}{2} + \frac{\epsilon^{\text{ion}}}{kT_e} \right) \left( \frac{5}{2} + \frac{\epsilon^{\text{ion}}}{kT_e} + \frac{kT_h^2}{p} \left( \frac{\partial n_m}{\partial T_h} \right)_p \right).
\]

(42)

The isentropic exponent [expression (5)] for a diatomic non-LTE plasma equals the one for a monoatomic plasma,

\[
\gamma = \frac{c_p}{c_v} = \frac{2 \Theta + (1 - \Theta) \alpha}{2 \Theta + (1 - \Theta) \alpha + (1 - \alpha) \alpha}.
\]

(44)

Again, we can calculate the isentropic exponent from expression (44) with use of the above expressions for \(c_p\) and \(c_v\).

IV. RESULTS

From the expressions for the heat capacities and the isentropic exponent, in the previous section, it is noted that these variables are only functions of the ionization degree, the ionization energy, the electron temperature, the ratio of the heavy particle temperature and the electron temperature, and the mass (via \(R\)), but not of the pressure. In this paper we used an alternative description in which the plasma pressure, the ionization degree (or electron density) and two nonequilibrium parameters, \(b_1\) and \(\theta\) conform Schram et al.\(^7\) are used. In the alternative description we make use of the lack of pressure influence on the heat capacities and on the isentropic exponent, and of the relatively weak dependence of the isentropic exponent on the electron temperature together with the small range of relevant electron temperatures over which a plasma can exist.

In computer calculations considering high density argon plasmas, we found that the isentropic exponent hardly depends on the pressure and the nonequilibrium parameter \(b_1\). However, some dependence on the ionization degree and on the nonequilibrium parameter \(\theta\) were found.

![Electron density as a function of the electron temperature](image1)

**FIG. 1.** Electron density (\(m^{-3}\)) as function of the electron temperature (\(K\)) \((b_1 = 1.0, \theta = 1.0)\).

In the simulations, the relation between the electron temperature and the electron density must be calculated first by solving the Saha balance and the pressure balance simultaneously. To solve these equations, in which the pressure is assumed to be given, the bisection method is used. The result is depicted in Fig. 1.

The results for the isentropic exponent from the computer calculations are depicted below in Figs. 2, 3, and 4.

We conclude from Figs. 2–4 that the isentropic exponent of an argon plasma is almost always equal or close to 1.16. Under rather extreme plasma conditions [like very low (<5%) and very high (>80%) ionization degrees], the isen-
tropic exponent may increase or decrease somewhat, but in general the 1.16 value is a very good first estimate.

When we consider other plasmas than the argon plasma only the ionization energy and statistical weights differ in the expression for the isentropic exponent, subject to situations in which

$$\frac{3}{2} + \frac{e^{\text{ion}}}{kT_c} + \frac{kT_h}{p} \left( \frac{\partial n_m}{\partial T_h} \right)_p \approx \frac{3}{2} + \frac{e^{\text{ion}}}{kT_c}. \quad (45)$$

Expression (45) will not hold in plasmas that are dominated by (wall) association or dissociation. It holds only for plasmas with a relative small molecule density.

Subject to expression (45), all plasmas give similar result as expressed in Figs. 2–4 for the argon plasma. Only the value of 1.16 as an estimation of the isentropic exponent in all plasma conditions must be adjusted, but can be expected to be close to 1.16 in most cases. The ionization energy ranges typically from 13.6 to 15.76 eV for hydrogen, oxygen, nitrogen, and argon atoms, respectively. Therefore an isentropic exponent of 1.16 is a reasonable value for common plasmas used in daily practice.

V. CONCLUSIONS

The isentropic exponent for a plasma has been derived and its behavior as a function of the ionization degree, the pressure, and the nonequilibrium parameters, $b_1$ (i.e., with respect to the heavy particle density) and $\theta$ (i.e., with respect to the temperature), have been investigated. The isentropic exponent of a plasma is lower than that of a gas, which is due to ionization. For ionization degrees between 5%–80% common plasmas of about 1 eV have an isentropic exponent of 1.16.