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Theory and Methodology

Novel types of sensitivity analysis for additive MCDM methods

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Abstract

Three novel types of sensitivity analysis are presented. They determine the following: 1) the sensitivity of a ranking to specific changes in the evaluations of all alternatives on certain criteria; 2) the influence of specific changes in certain criterion-scores of an alternative; 3) the minimum modification of the weights required to make an alternative ranked first. The use of these instruments is demonstrated by the results of a simulation experiment. Results show that the first and second type of sensitivity analysis enable to apply multiple criteria decision making (MCDM) methods in dynamic circumstances. The third type of sensitivity analysis is demonstrated to be a tool to analyze the total weight space.

Keywords: Multiple criteria; Sensitivity analysis

1. Introduction

In multiple criteria decision aid the assessment of the data (e.g. the evaluation table, the weights of the criteria and the type and parameters of the preference functions) plays a crucial role. The results obtained by application of a multiple criteria decision aid method are strongly related to the actual values assigned to these data. Since uncertainties may be present, great care has to be taken when the results of such a method are interpreted. To facilitate this task, a number of methods have been proposed in literature, mainly focused on the assessment and influence of the weights. Three approaches have been considered [10,11]: 1) the use of weaker information on the criteria; 2) the use of weight specification methods; 3) the use of sensitivity analyses to study the consequences on the results of modifications of the initially specified weights.

In the work presented in this paper the first and third approach are adapted and extended into novel types of sensitivity analysis. Although the analyses are focused on and elaborated for the PROMETHEE methods [1–3,12] the basic ideas behind them are thought to be generally applicable for additive MCDM methods [11], including additive utility theory. These ideas are:

1) to determine the sensitivity of a ranking to changes in the data of all alternatives on certain criteria;

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2) to determine the influence of changes in the scores of a specific alternative on certain criteria;

3) to determine the minimum modification of the weights required to make a specific alternative ranked first.

The first type of sensitivity analysis is important in case uncertainties are present in certain criterion-scores. This is, for instance, the case if one has to deal with uncertain economic circumstances. The second type of sensitivity analysis is of importance if uncertainties arise in the scores of just one alternative (e.g. in case a decision maker is expecting a grant for that specific alternative). With respect to the third type of sensitivity analysis it can be said that in currently applied weight sensitivity analysis only the stability of a ranking is determined, i.e. boundaries are derived within which the values of (combinations of) the weights are allowed to vary without modifying the ranking [10,11]. Although this provides sufficient information on the stability of the ranking itself, it does not give insight in the way the ranking is changed if these boundaries are exceeded. The type of weight sensitivity analysis proposed in this paper supports decision makers in gaining this insight, by exploring the total weight space, meanwhile taking into account specific requirements on the variations of the weights.

For the first type of sensitivity analysis, a set of scenarios has to be defined in order to incorporate the uncertainties. The second type is studied by iteration. To perform the third type of sensitivity analysis, a linear objective function and a number of constraints is derived. It is demonstrated that within this mathematical framework the decision maker is able (by introducing cost coefficients in the objective function) to state that the weights of certain criteria are more likely to change than others. Considerations like the relative importance of two criteria remaining constant, or a set of criteria keeping the same absolute importance, are taken into account by adding additional constraints to the linear programming (LP) model. Weight intervals specified by the decision maker can also be taken into consideration.

As an example of additive multiple criteria methods, the PROMETHEE methods are introduced in Section 2. Subsequently, the mathematical formulation needed for the third type of sensitivity analysis is derived in Section 3, and extended in Section 4. To illustrate the proposed sensitivity analyses, a demonstration problem is formulated in Section 5. The results of this experiment are presented and discussed in Section 6.

2. The PROMETHEE methods

Consider the set A of n alternative alternatives a_i $(i=1,\dots,n)$ that have to be ranked, and k criteria f_j $(j=1,\dots,k)$ that have to be maximised. Each alternative from A is evaluated with respect to all criteria. The resulting multiple criteria decision problem has the following form:

$$\text{Max}\ \{f_1(a),\dots,f_j(a),\dots,f_k(a) \mid a \in A\}. \qquad (2.1)$$

The basic PROMETHEE methods build a valued outranking relation which is used to obtain either a partial (PROMETHEE I) or a complete (PROMETHEE II) ranking on A. For this purpose, the notion of preference function is introduced. The preference function $P_j(a, b)$ $(a, b \in A)$ associated to criterion f_j gives the degree of preference, expressed by the decision maker, for alternative a with respect to alternative b on criterion f_j. It is normalised, such that

$$0 \le P_j(a, b) \le 1 \qquad (2.2)$$

and:

- $P_j(a, b) = 0$ if there is no preference of a over b, or indifference between them;
- $P_j(a, b) \approx 0$ if there is a weak preference of a over b;
- $P_j(a, b) \approx 1$ if there is strong preference of a over b;
- $P_j(a, b) = 1$ if there is strict preference of a over b.

Usually, P_j is a non-decreasing function of the deviation $d = f_j(a) - f_j(b)$. Six specific shapes have been proposed by the authors of the PROMETHEE methods [1-3]. These seem sufficient in practice, and can easily be selected by the decision maker.

Given weights w_j $(j=1,\dots,k)$ representing the relative importance of the criteria, a multiple criteria preference index π is defined:

$$\pi(a, b) = \sum_{j=1}^{k} w_j P_j(a, b) \quad \forall a, b \in A. \tag{2.3}$$

Weights are defined on a ratio scale so that it is always possible to consider normalised weights ($\Sigma w_j = 1$). In this case, $\pi(a, b)$ has the following property:

$$0 \le \pi(a, b) \le 1. \tag{2.4}$$

In order to rank the alternatives of A, the following outranking flows are defined:

- the leaving flow (or positive flow):

$$\phi^+(a) = \sum_{b \in A} \pi(a, b), \tag{2.5}$$

- the entering flow (or negative flow):

$$\phi^-(a) = \sum_{b \in A} \pi(b, a), \tag{2.6}$$

- the net flow:

$$\phi(a) = \phi^+(a) - \phi^-(a). \tag{2.7}$$

Each of these flows induces a complete ranking on A. The higher $\phi^+(a)$, the better a. The higher $\phi^-(a)$, the weaker a. The intersection of the rankings resulting from ϕ^+ and ϕ^- defines the PROMETHEE I partial ranking. It includes possible incomparabilities between pairs of alternatives: two alternatives are incomparable if they have conflicting relative rankings. This often occurs when one alternative is fairly better on a subset of criteria and the other one is better on other criteria. Information about incomparabilities is very valuable for decision making.

The PROMETHEE II complete ranking is obtained by considering the net flow ϕ. This ranking does not include incomparability, but allows to rank the alternatives from the best to the worst one.

3. Sensitivity analysis: Mathematical formulation

To facilitate the interpretation of the results, the use of sensitivity analyses has been promoted in the PROMETHEE methods. This enables to study the consequences of modifications of the initially specified weights on the results. These sensitivity analyses include the determination of weight stability intervals, polygons and areas [10,11]. Although these analyses provide sufficient information on the stability of the ranking, they do not give insight in the way the ranking changes if the stability boundaries are exceeded. Therefore, it is interesting to know the minimum modification of the weights required to modify the ranking in a certain way. To find the minimum modification of the weights required to make a certain alternative ranked first, an LP model is formulated.

Consider the set A of n alternatives evaluated on k criteria. Given a set of weights w_j $(j = 1,\dots,k)$ and preference functions, the PROMETHEE II complete ranking is determined as in Section 2.

If a set of modified weights w_j^* has to be determined such that alternative a^* is ranked first, the following inequality has to be met:

$$\phi(a^*) \ge \phi(a) \quad \forall a \in A, \tag{3.1}$$

or by developing the expression of the net flow (2.7):

$$\sum_{j=1}^{k} \left[\phi_j(a^*) - \phi_j(a)\right] w_j^* \ge 0 \quad \forall a \in A, \tag{3.2}$$

with

$$\phi_j(a) = \sum_{b \in A} \left[P_j(a, b) - P_j(b, a)\right],$$

$$j = 1,\dots,k, \tag{3.3}$$

$$\sum_{j=1}^{k} w_j^* = 1, \tag{3.4}$$

$$w_j^* \ge 0. \tag{3.5}$$

Relations (3.2)–(3.5) define a polyhedron in the space of the weights. Each set of weights w_j^* satisfying these relations, is such that alternative a^* is ranked first. To determine the minimum modification of the weights w_j, the following objective function is introduced to measure the distance between the initial weights and the modified ones:

$$\text{Min} \sum_{j=1}^{k} | w_j^* - w_j |. \tag{3.6}$$

(3.6) cannot be optimised directly by applying LP. This problem can be overcome by defining the modified weights in the following way:

$$w_j^* = w_j + d_j^+ - d_j^- \tag{3.7}$$

where d_j^+ and d_j^- represent respectively the increase or decrease of w_j. Using (3.7), the mathematical programme resulting from (3.6) subject to (3.2)–(3.5) can be transformed into

$$\text{Min} \sum_{j=1}^{k} (d_j^+ + d_j^-) \tag{3.8}$$

subject to

$$\sum_{j=1}^{k} [\phi_j(a^*) - \phi_j(a)](w_j + d_j^+ - d_j^-) \geq 0$$

$$\forall a \in A, \tag{3.9}$$

$$\sum_{j=1}^{k} (d_j^+ - d_j^-) = 0, \tag{3.10}$$

$$d_j^+ - d_j^- \geq -w_j, \quad j = 1, \ldots, k, \tag{3.11}$$

$$d_j^+ \geq 0, \qquad d_j^- \geq 0, \quad j = 1, \ldots, k. \tag{3.12}$$

The LP model defined by (3.8)–(3.12) includes $2k$ variables and $n + 3k$ constraints, and can be solved easily by a standard LP code.

4. Extensions of the model

1) *Constraints on the absolute importance of specific criteria*

In most sensitivity analyses, it is assumed that all weights are independent variables. However, in practice this might not always be the case. For instance, a decision maker can state that a certain subset of criteria has to determine the result of the multiple criteria evaluation to a certain fixed degree, i.e. that the sum of the normalised weights of the criteria from the subset has to remain constant. This requirement can be included in the formulation developed in the previous section. Let C be the set of criteria, and let S be a subset of C. If the sum of the weights of the criteria of S has to remain constant:

$$\sum_{j \in S} w_j = \sum_{j \in S} w_j^* \tag{4.1}$$

and, using (3.7), the following constraint is added to the LP model:

$$\sum_{j \in S} (d_j^+ - d_j^-) = 0. \tag{4.2}$$

2) *Constraints on the relative importance of specific weights*

The decision maker can also state that the relative importance of two criteria or subsets of criteria must remain constant. This may be expressed, for two criteria with indices j and j', as

$$\frac{w_j}{w_{j'}} = \frac{w_j^*}{w_{j'}^*}. \tag{4.3}$$

This relation can also be transformed into a linear constraint and included in the LP model as follows:

$$w_j(d_{j'}^+ - d_{j'}^-) = w_{j'}(d_j^+ - d_j^-). \tag{4.4}$$

3) *Cost coefficients in the objective function*

In deriving the objective function (3.8) it is assumed that all weights are equally likely to change. Consequently their contributions to the objective function are equal. However, in practice a decision maker can state that some weights are more likely to change than others, or that a certain weight is more likely to increase than to decrease. To take such effects into account, cost coefficients can be included in (3.8). The objective function then becomes

$$\text{Min} \sum_{j=1}^{k} (c_j^+ d_j^+ + c_j^- d_j^-) \tag{4.5}$$

The coefficients c_j^+ and c_j^- are positive numbers, representative of the cost or likeliness of respectively increasing and decreasing the value of w_j: a higher/lower value indicates a higher/lower cost or a lower/higher likeliness.

4) *Bounds on the values of the weights*

In some cases, the decision maker may specify bounds on the values of the weights. For instance, for criterion j:

$$w_j^{lo} \le w_j \le w_j^{hi}. \tag{4.6}$$

This requirement is introduced in the LP model by adding the following constraints:

$$d_j^- \le w_j - w_j^{lo} \tag{4.7}$$

$$d_j^+ \le w_j^{hi} - w_j. \tag{4.8}$$

In this way it becomes possible to test whether an alternative can be ranked first with the weights of the criteria limited to the specified intervals.

5. Simulation experiment

In industrial production systems, passive heat recovery by using heat exchangers, is a well known and widely applied way to conserve energy. The problem of designing (near-)optimal heat exchanger networks has been studied thoroughly, and several methods to solve this problem have been proposed [4–9]. The result of these methods generally is not a single optimal heat exchanger network, but a family of feasible near-optimal networks that can be constructed for different values of the assumed minimum temperature difference. Moreover, numerous non-optimal feasible solutions exist (for instance solutions with less heat exchangers at the cost of more energy use). None of the methods mentioned above is able to incorporate (strategic) managerial aspects like complexity and net present value, or the preferences of the designer/decision maker, into the 'optimal' solution. Moreover, due to relatively large unexpected variations in the economic circumstances of a heat exchanger network (e.g. variations in the price of fossil fuel or in the cost of cooling water), it is impossible to take a decision on a heat exchanger network purely based on an economic evaluation of the alternatives. In [13] it is illustrated that the use of a multiple criteria evaluation method (viz. the PROMETHEE methods) supports a decision maker in the strategic process of selecting a heat exchanger network.

To demonstrate the proposed sensitivity analyses, a reduced version of the problem introduced in [13] is treated. 10 alternative heat exchanger networks a_i $(i = 1, 2, \ldots, 10)$ for a specific problem defined in [9] are evaluated with respect to 10 criteria C_j $(j = 1, 2, \ldots, 10)$: C_1, number of active matches; C_2, number of heat exchangers; C_3, number of active heaters; C_4, number of active coolers; C_5, total heat exchanging area; C_6, required amount of external hot utility; C_7, total cost of the heat exchanging network; C_8, payback time (discounted); C_9, net present value over the total lifetime; C_{10}, complexity. Operational definitions of these criteria can be found in [4,5,6,8,9,13]. In Table 1 the weights w_j, the type and parameters (q, p, σ) of the preference function P_j, and the objective that are specified for each criterion are presented.

As stated above, the economic circumstances under which a heat exchanger network has to operate are very uncertain, which in this case reveals itself in the costs of hot and cold utility (required for heating and cooling purposes respectively). The evaluations of the alternatives on the economic criteria C_8 (payback time) and C_9 (net present value) are determined by these costs,

Table 1
Weights, type and parameters of P_j, and objective of the criteria

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
w_j	0.025	0.025	0.050	0.050	0.025	0.075	0.125	0.250	0.250	0.125
P_j-type	4	2	4	4	6	5	6	3	3	4
q	0.5	3.5	0.5	0.5	–	15	–	–	–	0.65
p	1.5	–	1.5	1.5	–	50	–	0.7	100	1.15
σ	–	–	–	–	75	–	50	–	–	–
Obj.	Min.	Min.	Max.	Max.	Min.	Min.	Min.	Min	Max.	Min.

Table 2
Annual increase in the hot (ϵ_1) and cold (ϵ_2) utility costs

	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5	Scen. 6
ϵ_1 [%]	0.00	0.02	0.04	0.06	0.08	0.10
ϵ_2 [%]	0.00	0.02	0.04	0.06	0.08	0.10

since each kilowatt of saved energy reduces the demand for both hot and cold utility. It is expected that the costs of both will increase in near future, mainly due to environmental policy regulations focused on, for instance, a decrease of CO_2 emissions by energy conservation, or replacement of CFC as refrigerant.

To test the stability of the ranking to changes in the costs of hot and cold utility, a set of 6 scenarios is defined. Each scenario is characterised by a specific annual increase (%) in the hot and cold utility costs, denoted by respectively ϵ_1 and ϵ_2. The data of these scenarios are presented in Table 2.

At this moment the cost of hot and cold utility are respectively 80 \$/kWyr and 20 \$/kWyr. To calculate the discounted pay back time and the net present value over the total lifetime (15 years), an internal rate of return of 7.5% is used.

The 10 alternatives are evaluated with respect to the 10 criteria, for each of the six scenarios. The evaluations are presented in Table 3. Note that in this table the evaluations of the alternatives on criterion C_8 and C_9 are not presented. However, they can be calculated for each scenario, given the investment cost (C_7) and the amount of saved energy. The latter is equal to the demand for hot utility *without* a heat exchanger network (487.5 kW) minus the demand with a heat exchanger network (C_6).

For the purpose of the third type of sensitivity analysis, the following additional constraints are specified:

$$d_1^+ - d_2^+ - d_1^- + d_2^- = 0, \tag{5.1}$$

$$d_2^+ - d_5^+ - d_2^- + d_5^- = 0, \tag{5.2}$$

$$d_3^+ - d_4^+ - d_3^- + d_4^+ = 0, \tag{5.3}$$

$$2d_7^+ - d_8^+ - 2d_7^- + d_8^- = 0, \tag{5.4}$$

$$2d_7^+ - d_9^+ - 2d_7^- + d_9^- = 0, \tag{5.5}$$

$$d_1^+ + d_2^+ + d_5^+ - d_1^- - d_2^+ - d_5^- = 0, \tag{5.6}$$

$$d_7^+ + d_8^+ + d_9^+ - d_7^- - d_8^- - d_9^- = 0. \tag{5.7}$$

Relations (5.1)–(5.5) and (5.6)–(5.7) express that, respectively, the relative and absolute importance of the involved criteria has to remain constant. To take into account that some weights are more likely to vary than others, cost coefficients c_j are specified ($c_j = c_j^+ = c_j^-$). These are presented in Table 4. The bounds on the values of the weight are specified to be $w_j \pm 25\%$.

Table 3
The evaluations of the alternatives a_i on the criteria C_j

	C_1	C_2	C_3	C_4	C_5 [m^2]	C_6 [kW]	C_7 [k\$]	C_{10}
a_1	3	3	1	0	454.73	67.5	148.7	1.25
a_2	3	3	2	1	304.54	107.5	105.2	0.75
a_3	3	3	1	1	293.43	116.6	101.3	0.75
a_4	2	2	2	1	309.77	133.5	89.8	0.50
a_5	3	3	1	1	296.81	105.0	110.2	0.75
a_6	2	2	2	1	306.27	150.0	87.3	0.50
a_7	3	3	1	2	249.24	162.5	85.0	1.00
a_8	3	3	1	0	372.04	67.5	131.3	2.00
a_9	4	4	1	1	405.67	69.0	125.9	1.50
a_{10}	2	2	2	1	265.12	151.5	76.8	0.50

6. Numerical results and discussion

A. Sensitivity analysis 1

The alternative heat exchanger networks are ranked completely by means of the PROMETHEE methods, using the weights and preference functions specified in Table 1. This is done for all six scenarios. The results of this complete ranking are presented in Fig. 1.

In Fig. 1 it can be seen that alternatives a_1, a_6, a_7, and a_8 remain ranked in the last four positions for all six scenarios. This may lead to the exclusion of these alternatives, which would facilitate the selection process. Alternatives a_2 and a_9 go up in the ranking very fast from scenario 1 to scenario 6. From Table 3 it follows that these two alternatives are relatively good on criterion C_6, the required amount of hot utility. Consequently they become 'better' alternatives as the annual increase in hot and cold utility costs is higher. For the same reason, alternatives a_3, a_4 and a_{10} go down in ranking because they score relatively bad on criterion C_6.

In Table 5 the ϕ-values corresponding to the ranks shown in Fig. 1 are presented. Note that the difference between the highest and lowest ϕ-value decreases with annually increasing costs of hot and cold utility.

B. Sensitivity analysis 2

The second type of sensitivity analysis is studied by means of iteration. For this purpose a number of alternatives is selected. For each of these alternatives it is determined how much the investment cost C_7 has to be reduced (e.g. as a result of a grant on a specific alternative) to make this alternative ranked first. Since a reduction in investment cost also affects the pay back time C_8 and the net present value C_9, these are corrected too. The required reduction is determined for each scenario for alternative a_1, a_2, a_8 and a_9. The results are presented in Table 6. Since this type of sensitivity analysis is studied by means iteration, only intervals containing the required reduction can be determined.

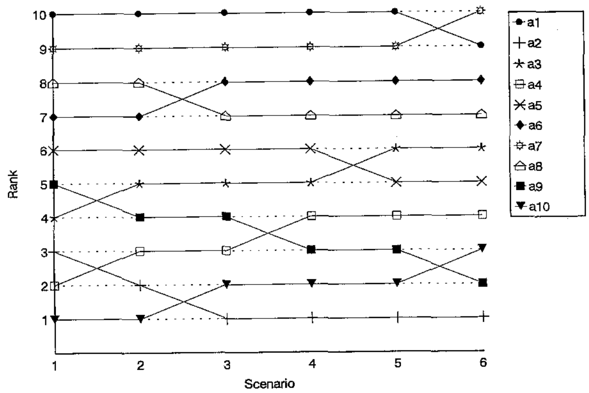

Fig. 1. The complete ranking of the alternatives for the 6 scenarios.

Table 6 shows that the reduction in investment cost required to make the alternatives ranked first, decreases very slowly as the annual increase in the cost of hot and cold utility is higher. Furthermore it can be seen that it is possible to make alternative a_9 ranked first by giving it a relatively very low grant. Note that in the first type of sensitivity analysis a_9 never became ranked first, although it is a very energy efficient alternative.

C. Sensitivity analysis 3

The third type of sensitivity analysis is performed for all alternatives. Five cases are considered; in the first case no additional constraints and no cost coefficients are added to the LP

Table 4
The cost coefficients c_j

	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$
c_j	15	15	6	6	10	50	9	10	6	40

Table 5
The ϕ-values of the alternatives for the six scenarios

	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5	Scen. 6
a_1	−0.33259	−0.27903	−0.23155	−0.19652	−0.16848	−0.15033
a_2	0.12167	0.12326	0.11859	0.11433	0.11233	0.10833
a_3	0.07016	0.04542	0.04475	0.04093	0.03627	0.03405
a_4	0.14054	0.11702	0.09547	0.07865	0.06687	0.06343
a_5	0.01886	0.02812	0.03008	0.03408	0.03726	0.03712
a_6	−0.01222	−0.04351	−0.05196	−0.05599	−0.05988	−0.05944
a_7	−0.13617	−0.14557	−0.15380	−0.15839	−0.16672	−0.17117
a_8	−0.10422	−0.06811	−0.04637	−0.03637	−0.02863	−0.02522
a_9	0.04203	0.06614	0.07766	0.08054	0.08173	0.08529
a_{10}	0.19194	0.15628	0.11713	0.09872	0.08924	0.07794

problem which is solved for each of the six scenarios. In the second case relations (5.1)–(5.7) are added as additional constraints. In the third case the specified weight intervals are taken into account. In the fourth and fifth case the cost coefficients are introduced in the objective function, and the LP problem is solved respectively without and with relations (5.1)–(5.7) as additional constraints. The second, third, fourth and fifth case are solved for scenario 1.

In Table 7 the results of the analysis for the first case are presented. It appears that for all scenarios no set of weights $\{w_j^*\}$ exists for which alternative a_3 is ranked first. This is also the case for alternative a_6 in 4 of the 6 scenarios. This may lead to the exclusion of these alternatives.

From the Table 7 it becomes clear that the calculated minimum modification is relatively low for high ranked alternatives (e.g. a_2, a_9), and relatively high for low ranked ones (e.g. a_1, a_7, a_5). Despite of this, ranking of the alternatives based on the minimum modification, does not yield the same result as in Fig. 1. From this it is concluded that a high ranked alternative might be 'further' away than a lower ranked alternative. This is for instance the case with alternative a_4 and a_9 in scenario 1, which may lead the decision maker to the conclusion that a_9 might be a better alternative than a_4.

From Table 7 it also follows that the highest value of the objective function decreases from scenario 1 to 6. This is in agreement with Table 5 which reveals an identical decrease of the difference between the highest and lowest ϕ-value.

The second case that is considered is the introduction of relations (5.1)–(5.7) as additional constraints. In the first column of Table 8 the results that are obtained in this case are presented. It can be seen that the introduction of the additional constraints makes the LP problem more restricted, as a result of which most alternatives cannot be ranked first. From this it is concluded

Table 6
Reduction in investment cost required to make an alternative ranked first

	Scen. 1 Red. [k$]	Scen. 2 Red. [k$]	Scen. 3 Red. [k$]	Scen. 4 Red. [k$]	Scen. 5 Red. [k$]	Scen. 6 Red. [k$]
a_1	28.7–33.7	28.7–33.7	23.7–28.7	23.7–28.7	23.7–28.7	23.7–28.7
a_2	2.7– 5.2	0.2– 2.7	0	0	0	0
a_8	16.3–21.3	16.3–21.3	11.3–16.3	11.3–16.3	11.3–16.3	11.3–16.3
a_9	5.9–10.9	5.9–10.9	0.9– 5.9	0.9– 5.9	0.9– 5.9	0.9– 5.9

Table 7
The values of the objective function (first case)

	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5	Scen. 6
a_1	0.84181	0.80050	0.70665	0.60114	0.52222	0.48397
a_2	0.10227	0.04885	0.00000	0.00000	0.00000	0.00000
a_3	-- [a]	--	--	--	--	--
a_4	0.25365	0.21295	0.14716	0.17257	0.20770	0.17466
a_5	0.79149	0.75818	0.68381	0.58499	0.55114	0.50996
a_6	--	--	--	1.44953	1.44898	--
a_7	0.55900	0.55213	0.52982	0.51317	0.51188	0.50235
a_8	0.42701	0.35899	0.30635	0.27827	0.25778	0.24920
a_9	0.13844	0.09281	0.06150	0.04946	0.04460	0.04946
a_{10}	0.00000	0.00000	0.00217	0.02211	0.03294	0.04431

[a] -- = impossible.

Table 8
The values of the objective function in the second, third, fourth and fifth case

	Case 2	Case 3	Case 4	Case 5
a_1	-- [a]	--	16.10594	--
a_2	--	0.11058	0.92310	--
a_3	--	--	--	--
a_4	--	--	2.90110	--
a_5	--	--	8.50293	--
a_6	--	--	--	--
a_7	--	--	3.35401	--
a_8	--	--	5.95399	--
a_9	0.16832	0.15569	1.12835	5.58380
a_{10}	0.00000	0.00000	0.00000	0.00000

[a] -- = impossible.

that introducing additional constraints like relations (5.1)–(5.7) facilitates the selection process.

In the second, third and fourth column of Table 8, the results obtained from the third, fourth and fifth case are presented. From the results of the third case it can be seen that if the weights are allowed to vary in intervals of $\pm 25\%$ around the values originally specified, only a_2 and a_9 can become ranked first, which reduces the selection procedure drastically.

If the results of the fourth case are compared to the results of the first case, it becomes clear that in the fourth case the values of the objective function are changed relatively to each other (e.g. a_1 and a_2). This means that the objective function with non-equal cost coefficients discriminates more among the alternatives.

In Table 9 the values of d_j^+, d_j^- and w_j^* are presented for a_1 in the first and fourth case (scenario 1). It can be seen that by introducing the cost coefficients other weights have to be

Table 9
The values of d_j^+, d_j^- and w_j^* (a_1, scenario 1) in the 1st and 4th case

j	w_j	Case 1			Case 4		
		d_j^+	d_j^-	w_j^*	d_j^+	d_j^-	w_j^*
1	0.025	0.1347	0.0000	0.1597	0.0000	0.0000	0.0250
2	0.025	0.0000	0.0000	0.0250	0.5844	0.0000	0.6094
3	0.050	0.0000	0.0000	0.0500	0.0000	0.0500	0.0000
4	0.050	0.0000	0.0000	0.0500	0.0000	0.0500	0.0000
5	0.025	0.0000	0.0000	0.0250	0.0000	0.0250	0.0000
6	0.075	0.2716	0.0000	0.3466	0.0421	0.0000	0.1171
7	0.125	0.0000	0.0000	0.1250	0.0000	0.1250	0.0000
8	0.250	0.0000	0.2500	0.0000	0.0000	0.2500	0.0000
9	0.250	0.0000	0.1709	0.0791	0.0000	0.1265	0.1335
10	0.125	0.0146	0.0000	0.1396	0.0000	0.0000	0.1250

changed, i.e. another longer path has to be taken in the weight space. If the results of the fifth case are compared to the results of the second case, the same observations hold true.

7. Conclusions

Three novel types of sensitivity analysis for additive MCDM methods have been presented. The first and second type demonstrate that it is possible to apply MCDM methods in cases where inaccuracies in the data assessment procedure are unavoidable, e.g. in case of uncertain economic circumstances. It has been demonstrated that by defining a set of scenarios for those parameters that directly affect the evaluations of all alternatives on certain criteria, it is possible to facilitate the selection of alternatives placed in dynamic circumstances. Furthermore it has been demonstrated that it is possible to study the influence of changes in the scores of a specific alternative on certain criteria by iteration.

The third type of sensitivity analysis determines the minimum modification of the weights required to make a specific alternative ranked first, meanwhile taking into account specific requirements on the weight variations. It is demonstrated that this is a tool to analyze the total weight space, and thus to eliminate a number of alternatives. Furthermore it enables to determine whether an alternative can reasonably be selected, given the requirements on the weight variations specified by the decision maker. The minimum weight modification that is determined, enables to define a proximity ranking. Thus it can be studied which alternative is closer (and consequently more likely) to being ranked first, given an initial set of weights. Results show that the proximity ranking does not fully correspond to the complete ranking of the alternatives. This indicates that some lower ranked alternatives (in the complete ranking) are more likely to being ranked first than higher alternatives.

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