Surface Three-Body Recombination in Spin-Polarized Atomic Hydrogen

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We calculate the surface dipolar recombination rate $L_s$ for spin-polarized hydrogen absorbed on $^4$He surfaces at temperatures in the 0.2- to 0.6-K regime and for magnetic fields up to 30 T. For a magnetic field of 7.6 T normal to the surface and 0.4 K we find $L_s = 1.3(3) \times 10^{-25} \text{ cm}^4 \text{ s}^{-1}$ increasing by 10%/T in the range of experimental interest. The anisotropy of the magnetic field is considerably smaller than in the case of the surface dipolar relaxation.

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The recent observation\(^1\) of three-body phenomena in high-density spin-polarized hydrogen ($\text{H}_\uparrow$) has focused considerable attention on a very interesting class of thresholdless recombination processes, first described by Kagan, Vartanyants, and Shlyapnikov.\(^2\) Detailed understanding of these processes is of vital importance for $\text{H}_\uparrow$ research as they appear to limit the highest densities that may be achieved experimentally. In a recent publication Hess and co-workers\(^3\) pointed out that effects previously attributed to an anomalously large surface two-body nuclear relaxation rate\(^4\) could be accounted for by a surface analog of the Kagan process. In their analysis the surface rate was estimated by a scaling argument taken from Ref. 2.

We took up this interesting suggestion and present here the first detailed calculation of the three-body surface recombination rate $L_s$. We analyze the nature of the Kagan dipole mechanism and discuss the differences between recombination on a $^4$He surface and in the bulk. We find that the scaling argument, which results from a model in which the relative motion of the $\text{H}$ atoms on the surface is assumed to be identical to that in the volume, is not supported by detailed theory. It leads to an overestimate of the surface rate by an order of magnitude. We calculate $L_s = 1.3(3) \times 10^{-25} \text{ cm}^4 \text{ s}^{-1}$ for a magnetic field $B = 7.6 \text{ T}$ normal to the surface and temperature $T = 0.4 \text{ K}$, to be compared\(^2\) with an experimental value $L_s = 2.0(6) \times 10^{-24} \text{ cm}^4 \text{ s}^{-1}$ obtained\(^3\) at the same field. In the range of experimental interest our results show an increase of the rate of recombination with growing field although this trend is weaker than theory predicts for the bulk process. Experiments show a decreasing behavior for growing fields.\(^5,6\) The anisotropy of the Kagan mechanism is found to be less than that of the two-body surface dipolar relaxation.\(^7\) This feature is in common with a very recent experimental analysis of the surface rates by Bell et al.\(^8\) but seems to contradict earlier low-temperature results obtained by Sprik et al.\(^6\) using $^3$He surfaces. We point out that in particular the large difference in absolute value indicates that the existing discrepancy\(^4\) between theory and experimentally observed decay rates remains unresolved.

We also calculated the bulk dipolar recombination process and find a rate which at 10 T is in agreement with results obtained by Kagan, Vartanyants, and Shlyapnikov,\(^2\) although our field dependence is slightly weaker. Our value is $L_s = 8.5 \times 10^{-39} \text{ cm}^4 \text{ s}^{-1}$ ($B = 10 \text{ T}$ and $T \rightarrow 0$).

At low temperatures ($T \lesssim 1 \text{ K}$) the available number of recombination channels for a system of $\text{H}$ atoms is vastly reduced. Resonance recombination, dominant at room temperature, may be excluded entirely as the energies of the resonances are too elevated to permit thermal population.\(^9\) The first description of a low-temperature recombination mechanism for $\text{H}$ was given by Greben, Thomas, and Berlinsky.\(^9\) This exchange-recombination process requires a collision between a pair of $\text{H}$ atoms with singlet character in their initial state. A third body is required to conserve energy and momentum in the process. Besides $\text{H}$ other atoms or surfaces may be effective as third body. One of the most fascinating features of the $\text{H}_\uparrow$ system is that the above mechanism implies (in combination with slow magnetic relaxation in high fields) preferential recombination and depletion of the “mixed” $a$ state ($a$, $b$, $c$, and $d$ are the hyperfine states in order of increasing energy). This process results in a gas of atoms in the “pure” $b$ state, double-polarized hydrogen ($\text{H}_\uparrow \uparrow$) in which both

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electron and proton spins are polarized.

The Kagan process is the only recombination mechanism presented in the literature which may limit the stability of $H^+$. This process involves a combined relaxation-recombination mechanism which is thresholdless and in which the dipolar interaction between the electronic spins of the $s$-state atoms causes the spin flip required for recombination. We distinguish between single- and double-spin-flip processes and will show that the double-spin-flip process is dominant at low fields, whereas it may be suppressed entirely by application of a field $B \geq 24 \, \text{T}$.

If we divide the triple of atoms in a $bbb$ incoming state into a recombining pair (atoms 1 and 2) and a third body (atom 3), we note that one may neglect the electronic dipolar interaction between the atoms 1 and 2 as this interaction cannot cause triplet-singlet transitions. In principle the electronic-nuclear dipolar interaction may do so, but this process is believed to be much weaker. As a result only the difference in magnetic field experienced by the recombing atoms due to the third atom is effective in the recombination process. This causes the remarkable feature that even in the presence of an abundance of third bodies provided by the He surface a third H atom is required. In principle the interaction with a magnetic surface impurity may be present and may cause a similar process with a second-order character.

We write the transition amplitude $f$ for recombination of atoms 1 and 2 as

$$f = \frac{3}{2} m_H^2 \sum_{p} \langle \psi_f | V_d^i + V_d^j | \sum_{p} P \psi_i \rangle .$$

(1)

Here, $m_H$ is the mass of the hydrogen atom, $V_d^i$ represents the dipolar interaction between atoms $i$ and $j$, while the initial state $\sum_{p} P \psi_i$ is a symmetrized three-atom $bbb$ state, $P$ being a permutation operator. Following Kagan we approximate the initial state by only taking into account the spatial correlations between the atoms of the recombining pair and between the atoms interacting via the dipolar interaction. For instance, for the 13 term the initial state is written as

$$\psi_f = \phi_0(z_1) \phi_0(z_2) \phi_0(z_3) \psi_{k_{12}}^f (\vec{p}_{12}) \psi_{k_{13}}^f (\vec{p}_{13}) |bbb \rangle .$$

(2)

For each of the atoms we use a bound-state wave function $\phi_0(z) \sim z \exp(-\alpha z)$. For $\alpha = 0.2 a_0^{-1}$, $\phi_0$ resembles the bound-state wave function in a Stwalley-type potential reproducing the experimental adsorption energy, while for $\alpha = 0.15 a_0^{-1}$ it resembles the Manetz and Edwards wave function. The error bar for our $L_s$ value corresponds with these values for $\alpha$. In Eq. (2) $\psi_k^f$ describes the relative motion of a pair of H atoms along the surface distorted by the triplet interaction averaged over the $z$ motion ("$2\frac{1}{2}$-dimensional" model) and normalized with plane-wave part $\exp(i \vec{k} \cdot \vec{p})$. Here $\vec{k}$ and $\vec{p}$ are two-dimensional momentum and position vectors. The final state $\psi_f$ is assumed to be identical to that used by Kagan in the volume case, but expressed in cylindrical coordinates: the product of a final spin state $\sigma_f = \frac{1}{2}$ or $-\frac{1}{2}$ of atom 3, a plane wave with three-dimensional momentum $\hbar \vec{q}_f(B, v, j, \sigma_f)$ for the motion of this atom relative to atoms 12, and a 12 molecular singlet state with vibrational and rotational quantum numbers $vjm$.

In view of the rather high He+$H_2$ relative kinetic energy we neglect completely the influence of the helium surface on the final state, which at the same time reduces the expressions to a form manageable numerically. With this approximation we neglect a reduction of the available final-state phase space and a possible energy transfer to the center-of-mass motion or to the helium. These effects are estimated to be small.

We note that only ortho $(j = \text{odd})$ final states are allowed, as the proton spins are unaffected by the process. Furthermore, we note that in the matrix element of Eq. (1) the spatial integration is over relative coordinates. The essential difference from the volume case is the lack of translational invariance in the $z$ direction, which causes the result to depend on the center-of-mass coordinate $Z$ in this direction. The recombination rate is obtained by summing $|f|^2$ over final states, integrating over $Z$, and thermal averaging over initial momenta along the surface:

$$L_s = \sum_{vjm \sigma_f} \left| \frac{\hbar q_f}{4 m_H} \int dZ \int d\vec{p} f_{vjm \sigma_f} (\vec{q}_f, Z) \right|^2 \frac{\delta}{\delta \theta_{\text{thermal}}}. $$

(3)

Notice that in two dimensions a $T \to 0$ approximation cannot be made. Instead we use a low-energy logarithmic $k_{12}, k_{13}$ dependence of $f$ following from two-dimensional effective-range theory, using the value $2.3 a_0$ for the scattering length. It is appropriate to point out here that the same logarithmic character of $f$ probably contributes to the failure of the above-mentioned scaling argument.

To evaluate $L_s$ we reexpress the spin wave functions using the surface normal as the new quantization axis. If we represent the transfer of angular
momentum from the spin system to the orbital system along this axis by $\mu k$, we find an expression for $L_4$ as an incoherent sum over $\mu$:

$$L_4(\bar{B}) = \sum_{\sigma_f} \sum_{\mu=-2}^{2} I_4^{\mu\sigma_f}(B)[d^2_{\sigma_f+3/2,\mu}(\theta)]^2 = \sum_{\sigma_f} \sum_{n=0,2,4} A_{n\sigma_f}^{4}(B) P_n(\cos \theta),$$

where the $d$ functions are reduced Wigner functions and $\theta$ is the angle between $\bar{B}$ and the surface normal. We note that the double-spin-flip contribution tends to dominate over the single-spin-flip one because of the relation

$$L_4^{\mu\uparrow, +1/2}(B) = 4L_4^{\mu\downarrow, -1/2}(2B).$$

For the dominant states $v=14$, $j=3$ and $j=1$ (all other molecular states contribute at a negligible level), and various $|\mu|$ and $|m|$ combinations, we calculated the behavior of $|f|^2$ as a function of $Z$ and of the projection $q_{\parallel}$ of $q_f$ along the surface. For $q_{\parallel} \rightarrow q_f$, all $|f|^2$ surfaces show a strong decrease. Physically this is due to the absence of high relative momenta along the surface in the initial state. Essentially this is the same feature which gives rise to the strong $B$ dependence of volume recombination. Because this argument applies in the surface case only for two coordinate directions instead of three, the $B$ dependence of $L_4$ is weaker than that of $L_3$. In Fig. 1 we show the functions $A_{n\sigma_f}^{4}(B)$ for $T=0.4$ K. The coefficients $A_{0\sigma}(B)$ represent the recombination rate averaged over the direction of $\bar{B}$. The coefficients $A_{2\sigma}$ and $A_{4\sigma}$ express the anisotropy as a function of the field direction. The $A_4$ coefficients are negligibly small, whereas the $A_2$ are small for the single-spin-flip contributions ($\sigma_f = -\frac{1}{2}$) and at most half the $A_0$ value for the double-spin-flip process ($\sigma_f = +\frac{1}{2}$).

Although the absence of a strong anisotropy is in common with experimental indications, both the $B$ dependence and the absolute magnitude of $L_4$ seem to be at variance with the experimental data, although it would be desirable to extend the measurements to the double-spin-flip cutoff at 24 T. The extreme sharpness of the bends at this cutoff is due to the above-mentioned low-energy approximation and is similar to the behavior in the volume $T \rightarrow 0$ limit. We find a rate which is growing by 70% from $B=4$ to 9 T, whereas the experiments show a decrease by about the same amount. For $\theta=0$ we calculate $L_4=1.3(3) \times 10^{-25}$ cm$^4$ s$^{-1}$ at $B=7.6$ T. Experimentally a larger value will be observed because of the large probability for the $c$ atom, originating from the double spin-flip process, to recombine in a subsequent collision. This implies that the experimental value given by Hess and co-workers has to be scaled down by approximately a factor $2 \times 0.87$, where 0.87 is the double spin-flip fraction. This leads to $L_4=1.1(4) \times 10^{-24}$ cm$^4$ s$^{-1}$. An angular average reduces our theoretical $L_4$ value by 25%. The calculated values show an increase by roughly a factor of 2 in the temperature range $0.2-0.6$ K.

We stress that to evaluate the surface dipolar recombination process rather substantial approximations had to be imposed. Hence, our present results do not provide the same level of accuracy as the results for surface dipolar relaxation. However, we are convinced that refinements of the theory are unlikely to resolve the large discrepancy with experiment.

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2Yu. Kagan, I. A. Vartan’yants, and G. V. Shlyapnikov, Zh. Eksp. Teor. Fiz. 81, 1113 (1981) [Sov. Phys. JETP 54, 590 (1981)]; Yu. Kagan, G. V. Shlyapnikov, I. A. Vartan’yants, and N. A. Glukhov, Pis’ma Zh. Eksp. Teor. Fiz. 35, 386 (1982) [Sov. Phys. JETP Lett. 35, 477 (1982)]. Note that Kagan et al. calculate the rate of direct three-body events, which is twice as small as the rate of loss of atoms in these events, for which we quote our results. Like Kagan et al. we do not consider the enhanced recombination probability of the third body discussed in Ref. 3. We thank T. J. Greytak for drawing our attention to these points and for sharing with us his private communication on this subject with Kagan.


7A. Lagendijk, Phys. Rev. B 25, 2054 (1982); see also references in Ref. 4.


