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Elastic Scattering in a Normal-Metal Loop Causing Resistive Electronic Behavior

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A model treatment of a 1D ring-shaped wire, in which noninteracting electrons experience an induced emf and weak elastic scattering, reveals the occurrence of self-induced randomization of phases in the electronic wave functions. This novel phenomenon leads to resistive behavior in spite of the inherent time reversibility of the system.

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A quasi-1D ring-shaped system with circumference \( L \) enclosing a magnetic flux is about the simplest model system one can think of in the theory of electronic conduction.\(^1\) Such 1D loops have therefore become important tools in discussing fundamental and conceptual properties associated with quantum-coherence effects in size-restricted configurations.\(^2\) Here we will study the 1D loop in order to discuss the fundamental question as to whether resistance can be assigned to quantum systems in which electrons experience only elastic scattering.

A theory of electronic conduction at \( T=0 \) \( K \) in a 1D disordered system with periodic boundary conditions was developed in 1981 by the present authors.\(^5\) We recognized Zener tunneling through minigaps as a possible mechanism for obtaining metallic-like conduction and "Ohmic" behavior. However, the necessary randomization mechanism remained obscure and our final conclusions were rather intuitive. In fact, differing predictions were put forward in 1983 by Büttiker, Imry, and Landauer\(^6\) who claimed that the absence of phase-randomizing reservoirs would guarantee the occurrence of quantum-coherence effects such as Josephson-type oscillations in response to a dc electric field. However, their arguments are based upon complete neglect of Zener tunneling, which may be questionable in the presence of an electric field of realistic strength.

Zener tunneling in small loops has been discussed recently by Landauer\(^7\) in the context of dissipative behavior. Landauer shows that Zener-tunneling transitions can be undone by reversal of the electric field direction, and concludes from that property that there cannot be dissipation nor resistance. Büttiker\(^4\) supports this point of view stating that electrical resistance is indissolubly connected to inelastic scattering.

We will show here that, in spite of time reversibility, the dynamics of a single electron cause quasirandomization of phases in the time-dependent wave function of the electron. With respect to electrical transport, this quasirandomization is shown to be equally effective in washing out coherences and correlations as randomization due to the interaction with some reservoir. The only requirement for this self-induced phase randomization to occur is that the electrons experience some elastic scattering.

Let us sketch the origin of the effect by discussing the principles rather than presenting the complete expressions. A more detailed treatment including many numerical results will be published elsewhere.\(^6\) The one-electron eigenenergy spectrum consists of minibands \( E_{nk} \) with typically small Brillouin zones of width \( \Delta k=2\pi/L \), with \( L \) the circumference of the ring.\(^2\) Each miniband can be occupied by two electrons at most. Adjacent bands are separated by minigaps. The precise forms of bands and the magnitudes of minigaps are fully determined by the potential which the electrons in the loop experience. A potential is said to be weak if, for energies near the Fermi energy, the gaps are much smaller than the width of minibands. In the case of a weak single \( \delta \)-function potential all gaps have equal width.

According to the general theory,\(^5\) the wave number \( k \) will move for \( t>0 \) as \( k(t)=eFt/h \), where the electric field \( F \) is related to the electromotive force \( E_{mf} \) by \( FL=E_{mf} \), and \( E_{mf} \) is assumed to be switched on at \( t=0 \). Simultaneously, the field induces transitions between minibands, but we restrict ourselves to the case of sufficiently small gaps such that these transitions take place between adjacent bands mainly.

Let us first see what happens to free electrons experiencing no scattering at all. The energy spectrum then consists of intersecting bands given by \( (\hbar^2/2m)(\pi n/L \pm k)^2 \), where \( n=1,2,3,\ldots \) (see Fig. 1). An electron at time \( t_1 \) represented by point \( P \) in Fig. 1 will be represented by \( R \) at time \( t_1+4\tau \). The phase difference with respect to \( P \) is then given by

\[
\phi(PR) = \phi_R - \phi_P = -\left(\frac{\hbar}{2m}\right) \int_{0}^{4\tau} dt \left[ \left( n - \frac{1}{2} \right) \frac{\pi}{L} + eFt/\hbar \right].
\]

(1)

This phase difference may be quite substantial, as can be seen from the following numerical evaluation. With \( n=2000 \), \( L=10^{-6} \) m, \( E_{mf} = FL = 10^{-7} \) V, and

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\[ \tau = \pi \hbar / eFL = 2 \times 10^{-8} \text{ s}, \text{ we find } \phi(PA_2) = 4 \times 10^7. \]

However, phases are not relevant unless waves interfere and this happens only in the presence of scattering. Indeed, the most important effect of a weak potential is to make other bands in Fig. 1 accessible too, thus enlarging the total "phase space" available for one electron. This implies that certain points, e.g., \( Q \), can be reached from \( P \) along various trajectories, each of which has its own phase contribution.

Let us write down the phase contributions collected

\[
\phi(PA_1B_1C_1Q) = \phi_1(n) + \phi_1(n-1) + \phi_1(n) + \phi_1(n+1),
\]

\[
\phi(PA_2B_2C_1Q) = \phi_1(n) + \phi_1(n+1) + \phi_1(n) + \phi_1(n+1),
\]

\[
\phi(PA_2B_2C_2Q) = \phi_1(n) + \phi_1(n+1) + \phi_1(n+2) + \phi_1(n+1).
\]

The essential point to realize is that \( \{\phi_1(n)\} \) and \( \{\phi_1(n)\} \), for varying \( n \) and when reduced to the interval \( [0, 2\pi] \), form quasirandom sets of angles. This follows from the fact that \( n \) is a large number (2000 or so), while the prefactor in (2), i.e., \( \pi \hbar^2 / 2meFL^3 \approx 10^{-15} / FL^3 \), is typically larger than unity \( \approx 10 \) for the data below (1)). In Eq. (3) we have only three different trajectories connecting \( P \) and \( Q \). For larger values of \( \tau \) many more phase contributions are to be collected in a given point in \( E-k \) space. These phase contributions cannot be distinguished from randomly distributed angles between 0 and \( 2\pi \).

In calculating one-electron properties such as the current at a given time, one needs all occupation numbers \( |c_Q|^2 \) for the respective \( Q \) points at that particular time. Here, the set of amplitudes \( c_Q \) specifies the electronic wave function at that time. Each \( c_Q \) can be written \( c_Q = \sum_i m_i \), where \( i \) runs over all different trajectories arriving at \( Q \) and \( m_i \) is the complex number specifying the contribution to \( c_Q \) originating from trajectory \( i \). The occupation number in \( Q \) can thus be written as

\[
|c_Q|^2 = \sum_i |m_i|^2 + \sum_{i \neq j} m_i^* m_j.
\]

Since all phases in the respective \( m_i \) form a quasirandom set, an effective washing out of the second term in (4) will take place. Hence, with regard to the effect on current, we can write \( |c_Q|^2 = \sum_i |m_i|^2 \), that is, the precise phase relations play no role. Note, by the way, that the above-described self-randomization guarantees statistical independence such as needed when applying Boltzmann’s transport theory (Stosszahlansatz). It seems, therefore, that Boltzmann’s equation can indeed be applied to the systems considered here.\(^5,7\)

Until now, nothing special has been said about the nature of the scattering potential, except that it shall be weak. We stress the point that there is no need for the potential itself to be of random type. Indeed, one easily checks that the self-induced randomization was discussed in terms of free-electron-type phase relations, while the main role of the scattering potential was to induce step-over possibilities between intersecting bands. In view of this our numerical results have all been obtained within the model with one single \( \delta \)-function potential.

In Fig. 2 we show four different curves, each of which gives the current response to a dc field switched on at \( t = 0 \). In these calculations all Zener-tunneling processes, as well as all dynamical phase effects, have been taken into account properly. The precise formulas and the complete description of the numerical pro-

FIG. 1. Free-electron energy bands in a 1D loop, represented as a function of time (through \( k = eFl/\hbar \)). The energy values are given by \( E(n) = \pi \hbar^2 n^2 / 2mL^2 \). The bands are intersecting but an electron will follow the band that it originally occupies. Transitions from one band to the other become possible at the intersections only after introduction of a weak potential. In that case, an electron may follow many different trajectories.
FIG. 2. Current-response curves for four different values of $\gamma_{200}$ as indicated. Along the horizontal axis stands the number of time steps, where one time step equals $\pi\ell/eF$. The field has been switched on at $q = \ell$. The dotted curve corresponds to one of the very special cases for which no saturation occurs. The three other curves correspond to field values in the ratio $1:2:3$. Also drawn are the average saturation levels (horizontal straight lines).

procedure can be found in Ref. 6. All four curves in Fig. 2 correspond to the same model system but differ in strength of the applied electric field $F$. The currents are due to two electrons only, which started at $t = 0$ in the 199th and 200th level (counted from below), respectively. The parameter values used in the calculations of Fig. 2 (and Fig. 3) can be given in terms of the dimensionless quantities, introduced in Ref. 6,

$$x_n = \frac{\pi^2 h^2 n}{m L^2 g}, \quad \gamma_n = \frac{m L g^2}{2 \pi e F h^2 n},$$

(5)

where $n = 1, 2, 3, \ldots$ labels the minibands from below and $g$ is the width of the energy gaps. In Fig. 2 we have for all curves $x_{200} = 100$, while the values of $\gamma_{200}$ are as indicated. The dotted curve is very special and shows no tendency to saturate. This is due to a very special choice of the prefactor in (2), that is,

$$\pi^2 h^2 / 2 e m F L^3 = x_{200} \gamma_{200} / 200 = \pi / 2,$$

corresponding to the occurrence in (2) of four equidistant phase angles $\text{mod}(2\pi)$. However, the existence of such special cases is an artifact of the single $\delta$-function potential rather than a general property, since most potentials will introduce additional contributions to the phases (2). The other curves in Fig. 2 do show saturating, or resistive, behavior.

Figure 3 demonstrates linearity of the saturated current level versus field strength. Each dot represents the time-averaged current level in the saturation regime, obtained at the value of $\gamma_{200}$ indicated on the horizontal axis. Since all values correspond to the same $N = 200$ and $x_{200} = 100$ values, the $\gamma_{200}$ values in Fig. 3 scale with $F$. Figure 3 therefore strongly suggests a linear relationship between current and field over a full decade of field values. Clearly, the loop with one single $\delta$-function potential can be assigned resistance.

We have checked that self-randomization is not a consequence of round-off errors in the numerical procedure. After a given number of time steps the direction of the electric field was reversed. The current was then observed to follow precisely the original time development in the opposite direction. When returned to the starting point, the total occupation probability collected in all minibands which ought to be strictly empty did not exceed the estimated value based on the ordinary accumulation of numerical errors.

In conclusion, we have dealt with about the simplest model system one can think of when studying electrical conduction from a quantum-mechanical viewpoint. For this system of noninteracting electrons, experiencing only elastic scattering, we have simulated the current response to an external electric field. Evidence has been obtained for self-induced phase randomization causing saturating behavior of the current. By demonstrating the time reversibility of the system, we have shown that the dissipation-like behavior has nothing to do with irreversibility or memory loss, but merely with increasing complexity of the phase information. Though these conclusions are based on numerical results obtained for a single $\delta$-function potential, we do not expect different qualitative behavior in the case of a less regular potential or in the case of more-dimensional systems. On the contrary, because of the increasing complexity in such systems we expect an even faster (quasi)randomization of phases.

In this work the term chaos has been avoided, although one observes many similarities, the most striking of which is the purely deterministic and time-
reversible, but randomlike, development of phases. This would be an interesting point for further investigations. Other suggestions for further study are to what extent Boltzmann's equation and Landauer's conductance formula can be applied to our system.

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3R. Landauer, “Zener tunneling and dissipation in small loops” (to be published).


