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The stability of the self-consistently determined current of a double-barrier resonant-tunneling diode

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Double-barrier resonant-tunneling devices belong to a novel class of nanoelectronic devices with great potential applications. In these devices, the self-consistent buildup of charge due to resonant carriers in the well may lead to bistability and hysteresis. To investigate aspects of dynamical (in)stability, a simple set of equations is derived from an extension of the static theory. These dynamic equations adequately describe small and slow (< 100 GHz) deviations from the stationary state. This approach is viewed more as being more satisfactory than an equivalent-circuit analysis, but its limitations are also discussed.

I. INTRODUCTION

Double-barrier resonant-tunneling (DBRT) structures have attracted much attention, because they exhibit, among many other interesting features, intrinsic bistability in the I - V characteristic. Experimentally, this bistability is encountered as a hysteresis,^{1,2} whereas theoretically a voltage interval with triple-valued current (a Z-shaped I - V curve) is predicted.^{3,4} One way to harmonize these different data, is to declare one of the three calculated branches in the I - V characteristic unstable, whereas the other curves are assumed to be completed in a hysteresis way.⁴ Another approach is to speak of intrinsic tristability, assuming the third solution to be hard to access experimentally.⁵

As the discrepancy between theory and experiments is still too large to justify a complete identification of the hysteresis interval with the triple-current interval, a stability criterion based on a theoretical analysis of the DBRT is desirable. In this paper, we present such a stability analysis, resulting from minor extensions to our static model.^{4,6} First, we release the coupling between the charge densities in the emitter and the well, thus attaining a dynamical system with two independent variables. Second, we include a model of the leads connecting the double-barrier structure to the battery, yielding a fair description of the way in which fluctuations are compensated in this part of the circuit. In the present study, only the simplest case, that of a purely Ohmic contact, is considered.

II. STATIC MODEL

In some models of the DBRT,^{3,4} the bistability in the I - V curve corresponds to a voltage interval where the volt-

age V_s across the structure is related to three values of the current density J . Hence, there exists no single-valued function $J(V_s)$ in the bistable regime, though we can give a parametric description of the I - V curve. This is done by taking the Fermi energy E_F in the emitter as a parameter: $V_s(E_F)$, $J(E_F)$, where both $V_s(E_F)$ and $J(E_F)$ are single-valued (but nonmonotonic, hence noninvertible) functions. Expressions for $V_s(E_F)$ and $J(E_F)$ result from a combined solution of the Poisson equation and the Schrödinger equation: the charge densities calculated via the wave functions from the Schrödinger equation enter the Poisson equation for the electron potential, while this potential is used in the Schrödinger equation for the wave functions. Thus, $V_s(E_F)$ and $J(E_F)$ are determined in an iterative scheme, so that on convergence the ultimate electron potential and charge density are qualified as "self-consistent."

For $V_s(E_F)$ the above method can easily be worked out. In the double-barrier structure, charge buildup takes place in the emitter, well, and collector. Since the three charge densities are interrelated due to the demand of overall charge neutrality, we need only consider two of them. Thus, the Poisson equation yields a function $V_s(\Sigma_e, \Sigma_w)$, where Σ_e and Σ_w are the areal charge densities in the emitter and well, respectively. Considering Σ_e to result from impinging electrons in the energy range 0 - E_F , and Σ_w from electrons in the same energy range tunneling into the well, we can find expressions for $\Sigma_e(E_F)$ and $\Sigma_w(E_F)$. Now we can write for the voltage over the structure $V_s(E_F) = V_s[\Sigma_e(E_F), \Sigma_w(E_F)]$, where it is the Poisson equation that relates V_s to Σ_e and Σ_w , and the time-independent Schrödinger equation that makes these charge densities in turn related to E_F . If we abandon the latter equation (as we should in a dynamical analysis), $V_s(E_F)$ can be generalized

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straightforwardly to a function $V_s(\Sigma_e, \Sigma_w)$ of two, now independent, variables.

The current density $J(E_F)$ needs a different approach. Since J is defined within the stationary wave-function formalism, i.e., for self-consistent solutions only, giving up the time-independent Schrödinger equation means giving up our expression for the current density. In this case the generalization, necessary for the dynamic model, is not straightforward, nor even unique. Let us start from the expression for the self-consistent current density:⁴

$$J(E_F) = N_c^{2/3} \mathcal{F}_0 \left(\frac{E_F - E_{\text{res}}}{kT} \right) \times g_s e \frac{(1 - R_1)(1 - R_2)}{2(1 - R_1 R_2)} \frac{\sqrt{(2E_0/m)}}{w} \quad (1)$$

Here, $N_c = (mkT/2\pi\hbar^2)^{3/2}$ is the effective number of states in the conduction band per unit volume, $\mathcal{F}_0(\epsilon)$ is the zeroth-order Fermi-Dirac integral, E_F and E_{res} are the Fermi energy and the resonance energy with respect to the conduction band of the emitter; kT is the Boltzmann constant times the temperature. In the second factor, $g_s = 2$ takes into account the spin degeneracy, e is the electron charge, and R_1 and R_2 are the reflection coefficients on the first and second barrier at the resonance energy. Finally, m is the effective mass of electrons in the conduction band, w is the width of the well, and E_0 is the resonance energy with respect to the conduction band in the well. Note that E_{res} and E_0 are different quantities that refer to the same energy level. The first factor in Eq. (1),

$$N_0 = N_c^{2/3} \mathcal{F}_0[(E_F - E_{\text{res}})/kT], \quad (2)$$

has an easy interpretation: it is the number of points in k space per square meter, for which $|\mathbf{k}|^2 \leq 2mE_F/\hbar^2$ and $k_x = k_{\text{res}}$. In other words, N_0 is the areal density of resonant electrons in the emitter. In order to interpret the remaining factors, a little rearrangement is helpful. The second factor multiplied by $(1 + R_2)/(1 - R_2)$ yields

$$q_0 = g_s e \frac{(1 - R_1)(1 + R_2)}{2(1 - R_1 R_2)}, \quad (3)$$

which is the charge per resonant electron located in the well. If the second barrier were impenetrable, i.e., $R_2 = 1$, the q_0 would be $g_s e$, as expected. Thus, the reflection coefficients in Eq. (3) take into account the leaky nature of the well. The product of the first two factors, $N_0 q_0$, is the areal charge density in the well, Σ_w . The last factor in Eq. (1) now becomes

$$\frac{1}{t_d} = \frac{(1 - R_2)}{(1 + R_2)} \frac{\sqrt{(2E_0/m)}}{w}, \quad (4)$$

and its reciprocal t_d is called the dwell time of electrons in the well. This dwell time is equal to the time needed for an electron to traverse the well, $w/\sqrt{(2E_0/m)}$, times $(1 + R_2)/(1 - R_2)$, that takes into account the multiple reflections within the well. The quotient q_0/t_d represents the current I_0 that each resonant electron contributes to the total current density. Summarizing, it is found that

$$J(E_F) = N_0 q_0 (1/t_d) = (\Sigma_w/t_d) = N_0 I_0, \quad (5)$$

which offers us two starting points for generalizing the current density in the non-self-consistent situations. First, we can write $J(\Sigma_e, \Sigma_w) = N_0(\Sigma_e, \Sigma_w) \times I_0(\Sigma_e, \Sigma_w)$, in which N_0 is the areal density of resonant electrons and I_0 the current that each electron carries. Second, we can write $J(\Sigma_e, \Sigma_w) = \Sigma_w/t_d(\Sigma_e, \Sigma_w)$, as the quotient of the charge density in the well and the time electrons spend there. For self-consistent solutions, the functions $\Sigma_e(E_F)$ and $\Sigma_w(E_F)$ are such that both generalizations yield the same relation $J(E_F) = J[\Sigma_e(E_F), \Sigma_w(E_F)]$. However, when Σ_e and Σ_w are not coupled self-consistently, the two generalizations differ. In the following section, where the dynamic model is presented, it will be seen that both formulas are needed.

III. EXTENSIONS TO STATIC MODEL

The static model of the DBRT outlined in the previous section, amounts to handling the Poisson equation and the Schrödinger equation simultaneously. Since both equations are time independent, no dynamics is included. In order to investigate the stability, however, we have to deal with time-dependent charge and current densities, so we cannot adhere to this set of equations. Nevertheless, it suffices to give up the time-independent Schrödinger equation. This is because the electron potential adjusts itself to changes in the charge distribution effectively instantaneously, i.e., with the speed of light.

Instead of replacing the time-independent Schrödinger equation with its time-dependent version (which is the formally correct method), we opt for a different, approximate, and intuitive, approach: the time-independent equation effectuated a coupling between Σ_e and Σ_w , so that abandoning this equation amounts to considering Σ_e and Σ_w as henceforth independent variables. Since in the dynamic case the current density has nonzero divergence, we need to specify it for various positions in the structure: Introducing the current densities J_L in the lead, and $J_1(J_2)$ through the first (second) barrier (see Fig. 1), we write for the rates of areal charge density in the emitter and well

$$\frac{d\Sigma_e}{dt} = J_L - J_1, \quad (6a)$$

$$\frac{d\Sigma_w}{dt} = J_1 - J_2. \quad (6b)$$

The third equation, for the charge density in the collector, is equivalent to the condition of overall charge neutrality, and contains no new information.

If the simplest example of a lead, a purely Ohmic contact, is considered, we can specify the current density J_L by the quotient

$$J_L(t) = \frac{V_b(t) - V_s[\Sigma_e(t), \Sigma_w(t)]}{\rho l}, \quad (7a)$$

where ρ is the specific resistance of the lead and l is its length. $V_b(t)$ is the externally applied battery voltage, and

FIG. 1. Conduction-band minimum in the DBRT structure as a function of position z . Below: schematic picture of charge buildup and current densities.

$V_s[\Sigma_e(t), \Sigma_w(t)]$ is the potential drop across the double-barrier structure as obtained from the Poisson equation.

In order to specify the current densities J_1 and J_2 , we turn to the two generalizations of the previous section. A natural and appealing choice is

$$J_1(t) = N_0[\Sigma_e(t), \Sigma_w(t)] \times I_0[\Sigma_e(t), \Sigma_w(t)] \quad (7b)$$

and

$$J_2(t) = \frac{\Sigma_w(t)}{t_d[\Sigma_e(t), \Sigma_w(t)]}. \quad (7c)$$

Equation (7b) represents the current density as seen from the emitter, and Eq. (7c) as seen from the well. In Eq. (7b) the instream of electrons from the emitter to the well is proportional to the areal density of resonant electrons in the emitter, but the current they carry depends on the actual amount of space charge in the well. In Eq. (7c) the outstream of electrons from the well to the collector is proportional to Σ_w , but the time constant t_d depends on the actual amount of space charge in the emitter. This intuitive reasoning substitutes a formal derivation.

Equations (6) and (7) are an approximate model for the dynamics of the DBRT structure. It may seem that the dynamics is described by a set of truly nonlinear equations. However, the way in which the expressions for J_1 and J_2 are obtained, prevents us from taking Eqs. (7b) and (7c) too seriously for large deviations from the self-consistent solution. Therefore, a linearized version of Eq. (7) will be investigated in the following section.

IV. LINEARIZATION OF DYNAMIC EQUATIONS

Let us suppose that at $t = 0$ the DBRT structure has a self-consistent electron potential and corresponding charge distribution. A small fluctuation in the battery voltage $V_b(t)$, e.g., a pulse function of small amplitude and finite duration, will initiate a dynamical evolution of charge and current densities, that may or may not be bounded at infinite time. If all quantities remain bounded, the system originated from a stable point in the I - V characteristic. If, on the other hand, there is an unbounded quantity, the

starting pointing is called unstable. The boundedness is most easily checked by performing Laplace transforms of the linearized dynamic equations.

Let for all quantities the difference between the value at time t and the value at $t = 0$ be indicated with the corresponding lower-case notation, e.g., $v_s(t) = V_s(t) - V_s(0)$, $\sigma_w(t) = \Sigma_w(t) - \Sigma_w(0)$, etc. In terms of these differences, and leaving out all higher-order terms, Eqs. (7) read as follow:

$$j_L(t) = \frac{v_b(t) - A_e \sigma_e(t) - A_w \sigma_w(t)}{\rho l}, \quad (8a)$$

$$j_1(t) = B_e \sigma_e(t) + B_w \sigma_w(t), \quad (8b)$$

$$j_2(t) = C_e \sigma_e(t) + C_w \sigma_w(t), \quad (8c)$$

where the coefficients A_e and C_w are partial derivatives in the original self-consistent solution:

$$A_e = \frac{\partial V_s}{\partial \Sigma_e}, \quad A_w = \frac{\partial V_s}{\partial \Sigma_w},$$

$$B_e = \frac{\partial N_0}{\partial \Sigma_e} I_0 + N_0 \frac{\partial I_0}{\partial \Sigma_e}, \quad B_w = \frac{\partial N_0}{\partial \Sigma_w} I_0 + N_0 \frac{\partial I_0}{\partial \Sigma_w},$$

$$C_e = -\frac{\Sigma_w(0)}{t_d^2} \left(\frac{\partial t_d}{\partial \Sigma_e} \right), \quad C_w = \frac{1}{t_d} + \frac{\Sigma_w(0)}{t_d^2} \left(\frac{\partial t_d}{\partial \Sigma_w} \right).$$

In these expressions, t_d and $\partial t_d / \partial \Sigma_e$ are to be understood as $t_d[\Sigma_e(0), \Sigma_w(0)]$ and $\partial t_d[\Sigma_e(0), \Sigma_w(0)] / \partial \Sigma_e$, etc. Substituting Eq. (8) in Eq. (6), we obtain the linearized rate equations

$$\frac{d\sigma_e}{dt} = \frac{v_b(t)}{\rho l} + \left(-\frac{A_e}{\rho l} - B_e \right) \sigma_e(t) + \left(-\frac{A_w}{\rho l} - B_w \right) \sigma_w(t) \quad (9a)$$

and

$$\frac{d\sigma_w}{dt} = (B_e - C_e) \sigma_e(t) + (B_w - C_w) \sigma_w(t), \quad (9b)$$

which are most easily solved using the Laplace transformation. Denoting the Laplace transform of $\sigma_e(t)$ by $\tilde{\sigma}_e(s)$, etc., we can cast Eq. (9) in matrix form:

$$\begin{pmatrix} s + \frac{A_e}{\rho l} + B_e & -\frac{A_w}{\rho l} + B_w \\ -B_e + C_e & s - B_w + C_w \end{pmatrix} \begin{pmatrix} \tilde{\sigma}_e \\ \tilde{\sigma}_w \end{pmatrix} = \begin{pmatrix} \tilde{v}_b / \rho l \\ 0 \end{pmatrix}, \quad (10)$$

which is solved by inverting the matrix. Thus we find

$$\tilde{\sigma}_e = \frac{s - B_w + C_w}{(s - s_1)(s - s_2)} \left(\frac{\tilde{v}_b}{\rho l} \right), \quad (11a)$$

$$\tilde{\sigma}_w = \frac{B_e - C_e}{(s - s_1)(s - s_2)} \left(\frac{\tilde{v}_b}{\rho l} \right), \quad (11b)$$

where s_1 and s_2 obey

$$s_1 + s_2 = -(A_e / \rho l) - B_e + B_w - C_w, \quad (12a)$$

$$s_1 s_2 = \frac{A_w(B_e - C_e) - A_e(B_w - C_w)}{\rho l} + (B_e C_w - B_w C_e). \quad (12b)$$

Equation (12) is of utmost importance for the stability, as can be seen from the inverse transforms of Eq. (11):

$$\sigma_e(t) = \int_0^t dt' \left(\frac{s_1 - B_w + C_w}{s_1 - s_2} e^{s_1 t'} + \frac{B_w - C_w - s_2}{s_1 - s_2} e^{s_2 t'} \right) \times \frac{v_b(t - t')}{\rho l}, \quad (13a)$$

$$\sigma_w(t) = \int_0^t dt' \frac{B_e - C_e}{s_1 - s_2} (e^{s_1 t'} - e^{s_2 t'}) \frac{v_b(t - t')}{\rho l}. \quad (13b)$$

If s_1 or s_2 have a positive real part, the exponential functions in the integrands in Eq. (13) increase unboundedly as t goes to infinity. Hence, the criterion for stable charge densities $\sigma_e(t)$ and $\sigma_w(t)$ must be that both $\text{Re}(s_1) < 0$ and $\text{Re}(s_2) < 0$, or, since the sum and product of s_1 and s_2 are real, that $s_1 + s_2 < 0$ whereas $s_1 s_2 > 0$. Combining this with Eq. (12) yields the stability criterion

$$(A_e/\rho l) + B_e - B_w + C_w > 0, \quad (14a)$$

$$\frac{A_w(B_e - C_e) - A_e(B_w - C_w)}{\rho l} + (B_e C_w - B_w C_e) > 0. \quad (14b)$$

In the following section, the consequences of this stability result for the I - V characteristic will be investigated.

V. STABILITY AND IMPEDANCE

In order to appreciate the internal stability conditions (14), we will relate them to the impedance of the circuit. For this purpose, we derive some useful relations between the coefficients from A_e and C_w in Eq. (8).

Since we assumed that at $t = 0$ the system was in a self-consistent point of the I - V curve, the $t = 0$ charge densities $\Sigma_e(0)$ and $\Sigma_w(0)$ are interrelated in such a way that $J_1 = J_2$ [see Eqs. (7a) and (7b)]. We use this equation to derive a relation for the B and C coefficients in Eq. (8). At $t = 0$, we have $N_0 I_0 = \Sigma_w/t_d$ or $\Sigma_w = N_0 I_0 t_d$. Then for the total derivative $d\Sigma_w/d\Sigma_e$ we find

$$\frac{d\Sigma_w}{d\Sigma_e} = \frac{\partial}{\partial \Sigma_e} (N_0 I_0 t_d) + \frac{d\Sigma_e}{d\Sigma_w} \left(\frac{\partial}{\partial \Sigma_e} \right) (N_0 I_0 t_d)$$

or

$$\frac{d\Sigma_w}{d\Sigma_e} = \frac{\partial}{\partial \Sigma_e} (N_0 I_0 t_d) \left[1 - \frac{\partial}{\partial \Sigma_e} (N_0 I_0 t_d) \right].$$

The partial derivatives can be worked out and replaced by the B and C coefficients, yielding

$$\frac{d\Sigma_w}{d\Sigma_e} = - \frac{B_e - C_e}{B_w - C_w}. \quad (15)$$

The total derivative in Eq. (15) describes the change of Σ_w with Σ_e along the self-consistent curve. With the help of this result, two more identifications can be made. Introducing the abbreviations

FIG. 2. I - V characteristic of the asymmetric DBRT structure (for structure parameters see Sec. VI), and the loadline ($\rho l = 10^8 \Omega \text{ m}^2$). Between P_1 and P_2 the differential resistance is negative. P_2 is the point of contact between the I - V curve and the loadline, and the turning point from stability to instability.

$$\Delta_1 \equiv B_w C_e - B_e C_w,$$

$$\Delta_2 \equiv A_e(B_w - C_w) - A_w(B_e - C_e),$$

$$\Delta_3 \equiv -B_e + B_w - C_w,$$

we can express the changes in V_s and J_1 along the self-consistent curve as

$$\frac{dV_s}{d\Sigma_e} = \frac{\partial V_s}{\partial \Sigma_e} + \frac{d\Sigma_w}{d\Sigma_e} \frac{\partial V_s}{\partial \Sigma_w} = A_e - \left[\frac{B_e - C_e}{B_w - C_w} \right] A_w = \frac{\Delta_2}{B_w - C_w} \quad (16a)$$

and

$$\frac{dJ_1}{d\Sigma_e} = \frac{\partial J_1}{\partial \Sigma_e} + \frac{d\Sigma_w}{d\Sigma_e} \frac{\partial J_1}{\partial \Sigma_w} = \frac{B_w C_e - B_e C_w}{B_w - C_w} = \frac{\Delta_1}{B_w - C_w}. \quad (16b)$$

As a matter of course, $dJ_2/d\Sigma_e$ is identical with $dJ_1/d\Sigma_e$. The quotient of Eqs. (16a) and (16b),

$$r_d \equiv \frac{dV_s/d\Sigma_e}{dJ_1/d\Sigma_e} = \frac{\Delta_2}{\Delta_1},$$

is the differential resistance of the DBRT structure (times unit area). We see that the top of the (I - V) curve (P_1 in Fig. 2) corresponds to $dJ_1/dV_b = 0$, hence $\Delta_1 = 0$. The point on the I - V curve where the tangent is vertical (P_2 in Fig. 2) corresponds to $dJ_1/dV_b = -\infty$, hence $\Delta_2 = 0$. Between P_1 and P_2 we have $\Delta_1 > 0$ and $\Delta_2 < 0$, so that r_d is negative here (NDR region). The negative differential resistance r_d is present in the rewritten stability conditions (14):

$$1 - \rho l \frac{\Delta_3}{A_e} > 0, \quad (17a)$$

$$\frac{\Delta_2}{A_e} + \rho l \frac{\Delta_1}{A_e} = \frac{\Delta_1}{A_e} (r_d + \rho l) < 0, \quad (17b)$$

which can now be easily related to the impedance of the circuit.

We define the small signal impedance $Z(s)$ as the quotient of $\tilde{v}_b$ and $\tilde{j}_L$. This is obtained by combining Eqs. (8a) and (12) yielding

$$Z(s) = \tilde{v}_b(s)/\tilde{j}_L(s) = \rho l + \frac{A_e s - \Delta_2}{s^2 - \Delta_3 s - \Delta_1}. \quad (18)$$

The first term of this equation is the contribution of the leads. Since we have assumed purely Ohmic contacts, it is simply ρl . For more realistic leads, it is to be replaced by some $Z_{\text{lead}}(s)$. The second term of Eq. (18) is the impedance of the "bare" DBRT structure, $Z_{\text{DBRT}}(s)$. It has a simple zero $s = \Delta_2/A_e$, so that stability is ensured if $\Delta_2/A_e < 0$. Since Δ_2 changes sign at P_2 (see Fig. 2), a stable solution for the bare DBRT can be found throughout the NDR region. This agrees fully with the conditions (17) for $\rho l = 0$, thus reading $\Delta_2/A_e < 0$ [Eq. (17a) being trivial for $\rho l = 0$].

For $\rho l \neq 0$, i.e., for the DBRT in circuit, the total impedance must be considered. An extra zero is introduced, and the two stability criteria of Eq. (17) result again. It is clear, that for ρl not too large, the second criterion (17b) is decisive. For points on the I - V curve that are before P_1 (see Fig. 2), Eq. (17b) is satisfied since here $\Delta_1 < 0$ and $\rho l + r_d > 0$. Just after P_1 Eq. (17b) is still satisfied because both factors change sign at the top of the I - V curve. However, at P_2 we have $\rho l + r_d = \rho l > 0$ whereas still $\Delta_1 > 0$, so that the stability criterion is no longer met. Therefore, the turning point from stability to instability must be between P_1 and P_2 , in the NDR region. A more precise position can be specified. Since the factor responsible for the change of sign is $(\rho l + r_d)$, the end of the stability interval can be characterized by $r_d = -\rho l$. The slope of the load line $J_L(V_s)$ being $-1/\rho l$ [see Eq. (7a)], the turning point is found to be the point of contact between the I - V curve and this loadline.

From Eq. (18) a resistance and reactance for the bare structure are defined as the real and imaginary part of $Z_{\text{DBRT}}(i\omega)$:

$$R_{\text{DBRT}}(\omega) \equiv \text{Re}[Z_{\text{DBRT}}(i\omega)] = \frac{\Delta_2(\Delta_1 + \omega^2) - \omega^2 A_e \Delta_3}{(\Delta_1 + \omega^2)^2 + \omega^2 \Delta_3^2}, \quad (19a)$$

$$\begin{aligned} X_{\text{DBRT}}(\omega) &\equiv \text{Im}[Z_{\text{DBRT}}(i\omega)] \\ &= -\omega \frac{A_e(\Delta_1 + \omega^2) + \Delta_2 \Delta_3}{(\Delta_1 + \omega^2)^2 + \omega^2 \Delta_3^2}. \end{aligned} \quad (19b)$$

From the condition $R_{\text{DBRT}}(\omega_{\text{co}}) = 0$, we define for the bare structure a cutoff frequency $\omega_{\text{co}}^2 = -\Delta_1 \Delta_2 / (\Delta_2 - A_e \Delta_3)$. Analogously, a self-resonance frequency $\omega_{\text{sr}}^2 = -\Delta_1 - \Delta_2 \Delta_3 / A_e$ is found from the demand that the reactance $X_{\text{DBRT}}(\omega_{\text{sr}}) = 0$. The cutoff frequency is positive in the NDR region, as expected. Here, the DBRT structure can be used as an oscillator for frequencies up to ω_{co} . The series resistance ρl will lower the cutoff frequency. Since our model does not contain any inductive elements, only the addition of an external inductance can yield a positive

ω_{sr}^2 . At low frequencies the impedance of the structure becomes purely resistive and approaches the differential resistance $r_d = \Delta_2 / \Delta_1$.

VI. NUMERICAL RESULTS FOR ASYMMETRIC GaAs-AlGaAs STRUCTURE

The above theory has been worked out numerically for a GaAs-Al_xGa_{1-x}As DBRT device with different widths for the two barriers. For the purpose of calculations we choose $x = 0.4$, barrier height 0.44 eV, well width 5 nm, and barrier widths 5.6 and 8.4 nm, respectively. The effective mass of GaAs has been used for all the layers. Assuming contact layers of 50 nm (doping 2×10^{16}) plus 1 μm (doping 2×10^{18}) on both sides of the DBRT structure, we can estimate ρl to be of the order $10^{-10} \Omega \text{ m}^2$.⁷

In Fig. 3, the real and imaginary part of the impedance is shown as a function of the Fermi level E_F in the left-hand-side reservoir, for $\omega = 10^9 \text{ rad/s}$. We know that for $\omega = 0$ we have $R_{\text{DBRT}} = r_d$ and $X_{\text{DBRT}} = 0$, so that the impedance at zero frequency is merely the derivative of $V_s(J_1)$ shown in the I - V curve of Fig. 2. Hence, $R_{\text{DBRT}}(E_F)$ has a vertical asymptote when $\Delta_1(E_F) = 0$, and a zero when $\Delta_2(E_F) = 0$. As we see from Fig. 3(a) this picture is not changed very much for frequencies up to a few GHz. The vertical asymptote of $R_{\text{DBRT}}(E_F)$ now corresponds to $(\Delta_1 + \omega^2)^2 + \omega^2 \Delta_3^2 = 0$ and is accompanied by a sharp negative peak in the reactance $X_{\text{DBRT}}(E_F)$. The zero of $R_{\text{DBRT}}(E_F)$ is shifted too [see Eq. (19a)], but both shifts are small, so that $R_{\text{DBRT}}(E_F)$ still resembles the differential resistance r_d very well.

Figure 4 shows the impedance as a function of frequency at fixed value of E_F ($E_F = 0.1 \text{ eV}$). As expected from Eq. (19), $X_{\text{DBRT}}(\omega)$ is zero for $\omega = 0$ and $\omega \rightarrow \infty$, whereas $R_{\text{DBRT}}(\omega)$ is r_d for $\omega = 0$ and zero for $\omega \rightarrow \infty$. Around $\omega/2\pi \approx 0.3 \text{ GHz}$, the imaginary part shows a peak where the real part increases to zero. This is the cutoff frequency ω_{co} that separates the amplified oscillations from those that are damped by the DBRT structure. The cutoff frequency is real for E_F values in the NDR region only. For the bare structure, ω_{co} is zero at P_1 and P_2 (see Fig. 2), and increases to infinity when $\Delta_2 - A_e \Delta_3 = 0$. Its typical value is found to be $\omega_{\text{co}}/2\pi \sim 5 \text{ GHz}$. Decreasing the barrier width by 10% leads to an increase of this value to $\sim 15 \text{ GHz}$. Both tendency and magnitude agree quite well with other models.⁸

VII. CONCLUSION

Comparison of our results with experimental findings is complicated by the fact that most impedance measurements were done on symmetric structures showing no bi-stability. Besides, our simple model yields a negative differential conductance that decreases from 0 to $-\infty$, whereas in real devices (due to nonresonant tunneling) the conductance remains finite. Nevertheless, we find a qualitative agreement between our calculated impedance as a function of bias and frequency (Figs. 3 and 4) and the measured curves presented in Refs. 9–11.

FIG. 3. (a) The real and (b) imaginary part of the impedance of the asymmetric DBRT structure as a function of Fermi level E_F at fixed frequency of 10^9 rad/s.

Zarea *et al.*⁹ studied a GaAs-AlGaAs structure with a 5-nm well and 5.6-nm barriers (cf. Sec. VI), for which they present the real and imaginary part of Z as function of bias. These curves show the same trends as our Fig. 3. To be able to compare the horizontal scales we use $V_b \sim 4E_F/e$ (see Fig. 2), giving a nice agreement. For the comparison of the vertical scales the impedances of Ref. 9 are multiplied by the mesa area. It is then found that the scales differ by a factor of 10. This is, however, due to our static model (cf. Fig. 2 and Fig. 1 of Ref. 9) which overestimates the current, and not to the dynamical extensions.

The cutoff frequency for the device of Ref. 9 was measured to be 1.10 GHz. Gering *et al.*¹⁰ found a value of 88.2 GHz for a similar structure with slightly smaller barriers (5 nm). The difference is mainly due to a difference in series resistance (4 and 0.17 Ω , respectively). Our calculated typical value of 5 GHz has the correct order of magnitude. Moreover, we find an exponential decrease of ω_{co} with barrier thickness, in agreement with the results of Ref. 10.

Lippens and Mounaix¹¹ measured the impedance of a structure similar to the one of Ref. 10 as a function of frequency, corresponding to our Fig. 4 (but with linear horizontal scale). Again we see the same trends in the curves. The real part starts at the negative differential re-

FIG. 4. (a) The real and (b) imaginary part of the impedance of the asymmetric DBRT structure as a function of frequency at fixed Fermi level of 0.1 eV.

sistance to become quite small at higher frequencies; the imaginary part shows a negative peak. The inductive behavior at higher frequencies is, of course, not reproduced by our calculations. Comparing the horizontal scale we find that our frequencies are smaller by a factor of 3.

We conclude that our dynamical equations are a fair description of the ac behavior of DBRT structures in the GHz regime. This is partly due to the fact that the impedance is strongly related to the differential resistance. Hence a reasonable I - V characteristic will yield a reasonable impedance. Thus the agreement with experiments is partly owing to our static model. The essentially new element is the generalized expression for the current density [Eqs. (7b) and (7c)], making it a time- and position-dependent quantity. This distinguishes our approach from experimental equivalent-circuit fits, and enables a purely theoretical analysis of the DBRT dynamics.

Yet one should bear in mind that this double-current definition poses some limitations to the applicability of the model. Its close relation with the static description leads to a restriction in the frequency domain to values of ω for which the adiabatic approach is valid. A simple estimation shows that the THz regime is beyond the scope of our approach. Also, only small-amplitude oscillations in the applied voltage V_b are allowed, thus excluding interesting

nonlinear behavior. Finally, Eqs. (7) cannot describe any inductive effects of the DBRT structure that may nevertheless be present.¹⁰

In fact, Eqs. (7) amount to a dynamic description of the DBRT structure in terms of a couple of shunted capacitors. If there is some charge buildup in the emitter and well, we expect a connection between the time constants $1/|s_1|$ and $1/|s_2|$ of Eq. (12), and the typical times for charging these two regions, λ_e/E_F and t_d , respectively. Here, λ_e is the emitter screening length, v_F is the Fermi velocity, and t_d the dwell time already discussed in Eq. (4). Indeed, where the coupling is small, s_1 and s_2 have the correct order of magnitude. In the general case, the complete matrix equation (10) must be solved, thus making the task nontrivial.

In summary, we have been able to describe the low-frequency small-signal part of the DBRT dynamics by minimal extensions of the static model. From a theoretical point of view, this approach is an improvement with regard to the equivalent-circuit method. A more complete account of the dynamics requires the consideration of the time-dependent Schrödinger equation and the nonlinear aspects.¹²

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