Output intensity noise of lightwave transmitters employing chirp demodulation
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Abstract—Direct modulation of semiconductor lasers results in simultaneous variations of the lightwave intensity and frequency. Thus, information can be transmitted over fiber using either of both characteristics. The use of direct optical frequency modulation (FM) instead of intensity modulation (IM) requires an FM-to-IM conversion scheme which additionally converts the phase (or frequency) fluctuations into intensity, causing extra noise. In this letter, we theoretically analyze the relative-output noise of a single-mode laser followed by a Mach–Zehnder interferometer.

Index Terms—Direct modulation, Mach–Zehnder interferometer, noise, nonlinear distortion, semiconductor lasers.

A new optical transmitter configuration has been suggested a few years ago that consists of passing the output light from the laser source through a Mach–Zehnder interferometer (MZI) before it is launched into the transmitting fiber [1]. We have carried out a theoretical investigation of this technique and have shown its ability in improving the signal linearity [2]–[4]. However, the nonlinear distortion is not the only figure of merit for an optical transmitter. As mentioned earlier, the noise characteristic must be addressed if the system performance has to be completely assessed. An interferometer that converts the optical frequency modulation (FM) will additionally convert the phase noise into intensity noise, which will inevitably degrade the transmitter relative-intensity noise. The purpose of this letter is to study this effect and explicitly determine how seriously the FM-to-IM conversion scheme which additionally converts the phase (or frequency) fluctuations into intensity, causing extra noise.

The laser diode is described by the well-known single-mode rate equations including Langevin noise functions. For a device operated at a reasonable power far enough below saturation, the rate equations can be expressed in terms of normalized quantities [2] as

\[
\frac{ds}{dt} = \gamma \left[ n_{th} n + 1 - n_{th} \right] (1 - \varepsilon_n s) s - s + n_{th} \beta n + \ell_s \tag{1}
\]

\[
\frac{dn}{dt} = j - \frac{1}{n_{th}} \left( n_{th} n + 1 - n_{th} \right) (1 - \varepsilon_n s) s - n + \ell_n \tag{2}
\]

\[
\frac{d\Phi}{dt} = 2\pi \tau_e (\nu_{n0} - \nu_n) + \frac{\alpha \gamma}{2} \left[ (n_{th} n + 1 - n_{th}) (1 - \varepsilon_n s) - 1 \right] + \ell_\Phi \tag{3}
\]

where \( s \) and \( n \) are the normalized photon and electron densities, \( \tau \) is the time normalized to the electron lifetime, \( j \) is the ratio between the drive current and the threshold current, \( \nu_{n0} \) is the cavity resonance frequency, \( \nu_n \) is the lasing frequency, \( \alpha \) is the linewidth enhancement factor, \( \gamma = \tau_e / \tau_p \) is the ratio between the electron and photon lifetimes, \( n_{th} \) is the normalized carrier density at threshold, \( \varepsilon_n \) is the normalized gain compression factor and \( \beta \) is the spontaneous emission factor. The last terms in the right-hand side (RHS) of (1)–(3) represent the Langevin forces which model the spontaneous emission noise acting on the emission parameters, a shot noise contribution is additionally included in \( \ell_n \) as it will be seen later. We have previously explained why the first term in (3) must be included for an accurate modeling of the chirp [3], [4]. Since the system has no memory, the dynamic chirp should be present only when a modulation is applied to the laser diode and should vanish (after having undergone damped oscillations) if the modulation is switched off. This condition is not reached using the chirp-power relation given in [5] where the chirp is seen to present a constant component.

The parameter \( k_{in} \) in the RHS of (3) is introduced to account for the influence of the refractive index nonlinearities. The nonlinear index of refraction originates from the fact that the lasing mode does not always coincide with the gain peak, which is often the case of many laser structures having a built-in Bragg grating such as DFB lasers. The mathematical justification of the modified rate equation for the optical phase can be obtained using the analysis presented by Agrawal a few years ago [6]. The parameter \( k_{in} \) can be derived for the situation where the laser operates well below saturation. This condition is generally met in analog optical communications systems for which the bias condition of the emitting source is often chosen in the linear portion of the power-current curve in order to minimize static distortion. The calculation of \( k_{in} \) using (14) of [6] gives \( k_{in} = \frac{d \varepsilon_n}{\alpha} \) with \( d \approx -2 \Delta \nu_{io} / \Delta \nu_{pp} \) where \( \Delta \nu_{io} \) stands for the gain bandwidth, \( \Delta \nu_{pp} \) is the difference between the lasing and gain-peak frequencies. It should be noted that,
to avoid confusion with the spontaneous emission factor, we have deliberately replaced the parameter $\beta$ used in [6] by $d$. On the other hand, it should be mentioned that $k_0$ can be negative or positive depending on wether the laser operates in the red- or blue-shifted area. For devices emitting at the gain peak, $d$ cancels out and the phase equation does not involve index nonlinearities. On the other hand, a nonzero value of $d$ will result in a correction of $1 - d/\alpha$ to the second term in the chirp versus power relation given previously [3]–[5]. Therefore, in the general case, the more appropriate $k_0$ should be used for an accurate prediction of the characteristics.

To derive the noise characteristics, the rate equations are firstly linearized and subsequently solved in the frequency domain using Fourier analysis. Denoting the Fourier transforms of the random deviations of the variables by $\hat{s}_\omega, \hat{h}_\omega, \hat{\Phi}_\omega$ (with $\hat{v}_\omega = jf\hat{\Phi}_\omega$), and calling $\hat{L}_s, \hat{L}_n, \hat{L}_\Phi$, the Fourier transforms of the Langevin noise terms, the calculation yields:

$$s_\omega = \frac{1}{\omega_0^2 f(\omega)} [(m_{22} + j\omega) \hat{L}_s - m_{12} \hat{L}_n]$$  \hspace{1cm} (4)

$$h_\omega = \frac{1}{\omega_0^2 f(\omega)} [-m_{21} \hat{L}_s + (m_{11} + j\omega) \hat{L}_n]$$  \hspace{1cm} (5)

$$\Phi_\omega = \frac{1}{j\omega_0^2} [m_{32} \hat{L}_s + m_{32} \hat{L}_n + \hat{L}_\Phi]$$  \hspace{1cm} (6)

where $\omega_0 = (\gamma s_0)^{1/2}$, the $m_{ij}$'s are constant coefficients resulting from the linearization of the initial rate equations:

$$m_{11} = -\gamma [(n_{th}n_0 + 1 - n_{th})(1 - 2\varepsilon_{n,s_0}) - 1]$$

$$m_{12} = -\gamma n_{th} [s_0 (1 - \varepsilon_{n,s_0}) + \beta]$$

$$m_{21} = \frac{1}{n_{th}} (n_{th}n_0 + 1 - n_{th})(1 - 2\varepsilon_{n,s_0})$$

$$m_{22} = 1 + s_0 (1 - \varepsilon_{n,s_0})$$

$$m_{32} = -\frac{1}{2} k_n \alpha_\gamma (n_{th} n_0 + 1 - n_{th})$$

$$m_{32} = \frac{1}{2} \alpha n_{th} \gamma (1 - k_n s_0)$$

where $s_0$, $n_0$ denote the steady state average values of the variables.

The function $f(\omega)$ in (4) and (5) is given by $f(\omega) = 1 - \omega^2 / \omega_0^2 + 2\eta \omega / \omega_0$ in which $\eta = (1 + s_0 + \gamma s_0 + \beta \gamma n_{th} n_0 / (2s_0))$. The term involving the spontaneous emission factor in this expression can be neglected if the laser is biased high enough above its threshold for oscillation, but we keep it in case that its effect becomes important.

The noise behavior of the system will be modified by passing the laser output beam through an FM-to-IM MZI that additionally converts the phase noise into intensity noise. Therefore, the classical relative-intensity noise (RIN) is no longer suitable to account for intensity noise. To incorporate the MZI’s effect into the noise model, we defined a relative-output noise (RON) by $\text{RON}(\omega) = \langle |\hat{F}(\omega)|^2 \rangle / \langle \hat{F}(\omega) \rangle^2$ where the numerator denotes the power spectral density of the output intensity fluctuations. Such a definition is of great interest in that it permits us to go on expressing the system carrier-to-noise ratio (CNR) as usual. The calculation of the RON based on the small-signal consideration of the output signal yields the following relationship in which the coefficient $T_r$ describes the MZI whose transmission is labeled $T(\nu)$:

$$\text{RON}(\omega) = \text{RIN}(\omega) + T_r^2 \langle |\hat{F}(\omega)|^2 \rangle + \frac{2}{\omega_0} T_r T_{\text{R}} \text{R}_{\text{e}} \langle \langle \hat{s}_\omega \hat{s}_\omega^* \rangle \rangle$$  \hspace{1cm} (10)

in which $\text{R}_{\text{e}}$ denotes the real part, the star symbol represents the complex conjugate and $T_r = T(1/T_{\text{R}})$ where $T(\gamma) = (\partial f/\partial\gamma)^2 T(\nu_i, \gamma = 0, 1, \cdots)$ is the classical relative-intensity noise.

The use of (4)–(6) to compute the RON as defined in (10) requires specification of $\langle \hat{s}_\omega \hat{s}_\omega^* \rangle$ with $q, r \in \{s, n, \Phi\}$. It is commonly assumed that the three noise terms have zero mean and are delta-correlated (Markovian assumption). Analysis of the Langevin noise sources in the framework of these assumptions has led to a white spectrum described through the relationship $\langle \hat{s}_\omega \hat{s}_\omega^* \rangle = 2d_{ss}$, where the $d_{qq'}$'s are the normalized diffusion coefficients. These can be expressed by the following formulas derived from the results of [6], [7] where they were given in terms of number of photons and electrons:

$$d_{ss} = \tau_e (\beta \gamma n_{th} n_0 s_0)$$

$$d_{ss} = \frac{1}{4} \tau_e \gamma n_{th} n_0 s_0, \quad d_{ss} = 0$$

$$d_{ss} = \Gamma \gamma n_{th} n_0 (\beta n_0 s_0 + \beta_0 n_0)$$

$$d_{ss} = -\tau_e \gamma \beta n_{th} n_0 s_0, \quad d_{ss} = 0$$

where $\Gamma$ is the optical confinement factor and $\beta_0$ the gain slope.

Equation (10) shows that the RON is a function of the classical RIN plus a term proportional to the power spectral density of the frequency noise and a term resulting from the cross correlation between the two noise sources. Clearly, relation (10) predicts the noise characteristic to degrade in SCM systems as a result of the conversion of phase noise into intensity noise. The degree to which this extra noise may impair the system performance is the subject that will now be discussed through illustrative curves.

The simulations are achieved using $\tau_e = 1$ ns, $\tau_p = 1.21$ ps, $\beta_0 = 0.562 \times 10^{-5}$ GHz, $\Gamma = 0.3$, $\beta = 10^{-4}$, $\gamma = 6.45 \times 10^{-3}$, $n_{th} = 1.4235$ and $\alpha = 6$. The RON spectrum is reported in Fig. 1 as decibels versus frequency when the MZI is assumed to operate in quadrature. The parameter of the curves is the free-spectral range (FSR), which is most conveniently defined as the frequency offset between a maximum transmission and the next-neighboring minimum [2]. The noise is seen to show a resonant behavior. This was expected due the involvement of the IM transfer function $|1/f(\omega)|^2$ in (4)–(6). The classical RIN exhibits the well-known characteristic [7], that is, a flat pedestal in the low-frequency side, which is relatively low (below $-150$ dB/Hz). It further moves gradually to larger values above 0.1 GHz and attains a maximum at the resonant frequency. With MZI, the flat portion of the curve is seen to extend, increasing from about 0.3 to 1 GHz. Unfortunately, inspection of Fig. 1 shows notably increased levels of noise as the frequency offset between a maximum transmission and the next-neighboring minimum [2]. The noise is seen to show a resonant behavior. This was expected due the involvement of the IM transfer function $|1/f(\omega)|^2$ in (4)–(6). The classical RIN exhibits the well-known characteristic [7], that is, a flat pedestal in the low-frequency side, which is relatively low (below $-150$ dB/Hz). It further moves gradually to larger values above 0.1 GHz and attains a maximum at the resonant frequency. With MZI, the flat portion of the curve is seen to extend, increasing from about 0.3 to 1 GHz. Unfortunately, inspection of Fig. 1 shows notably increased levels of noise as the free-spectral range is reduced. In the low frequency region the increase is almost 20 dB if FSR is reduced by a factor 10. Because a rather small FSR is required to improve signal linearity [8], this does illustrate a tradeoff between distortion and noise. This result has prompted us to examine the influence...
of the MZI’s operating point when the free spectral range is fixed to a constant value. This purpose is depicted in Fig. 2 for an FSR = 2 GHz. It is seen that the noise level increases for a given frequency when the operating point of the MZI is shifted so that the output power is lower. As shown in [3], the same operation of the MZI improves the signal linearity. Although one may check this result by reference to [3], we have found it convenient to report the third-order intermodulation distortion for explicit demonstration. So, the curves in Fig. 3 are plotted using the same parameters as in Fig. 2. The comparison of IM and FM-to-IM distortion levels shows cross-over points after which the third-order distortion levels are lower in the FM-to-IM case. The best linearity is obtained for an MZI operating well below the half-power area. As this operating area produces the opposite effect on noise (see Fig. 2), the tradeoff between signal linearity and noise is clearly illustrated.

Because spurious frequencies of the type described in Fig. 3 will inevitably fall within each signal band in SCM systems, a compromise has to be reached in practice. Nevertheless, the above analysis shows that if the MZI is specifically configured to minimize nonlinear distortion, the resultant phase noise will remain high. A related conclusion is that at large received powers, the end-of-line CNR will be limited, to a large extent, by intensity noise. This is in disagreement with previous works in which the receiver shot noise was considered to be the dominant contribution [1]. Since the calculation presented in [1] was based on assumption and not on demonstration, we believe that our results are more illustrative. Although experimental verification is required, this analysis is useful in weighting the full benefits expected in SCM systems employing either conventional direct-detection or the present optical FM–demodulation technique. Because of the strong tradeoff among the parameters of merit, we believe that a suitable processing will be required to achieve a good signal linearity together with an acceptable noise characteristic. How perfect such a processing is to be will be dictated by the system under consideration. Of course, the requirements will be more easily met in those that are less sensitive to distortion and less demanding on CNR. The noise effect is predicted to be large in analog modulation systems, and the signal processing cannot be by-passed if the requirements are to be reached. For this purpose, optical feedback may be suitably developed into a technique for reducing the phase noise [9]. The detailed treatment of the processing issue is reserved for future work.

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