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Day-to-day needs-based activity-travel dynamics and equilibria in multi-state supernetworks[☆]

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ABSTRACT

This study proposes a new framework for studying activity-travel dynamics in multi-state supernetworks based on a needs-based theory. The framework postulates that temporal fluctuations in human needs are the source of dynamically generated activities, which in turn are manifested in the choice of day-to-day activity-travel patterns (ATPs) and dynamics in day-to-day traffic. Specifically, the net utility of a daily ATP is determined jointly by the degree of satisfying particular needs and the traffic conditions when conducting the ATP. Needs are updated after completing the activities and so is the net utility of the ATP, which means that the net utility that is derived from conducting a particular ATP varies across time. Thus, the choice of a daily ATP is determined not only by the needs reflected in past ATP choices but also on current traffic conditions and usage of facilities. We formulate these dynamics in terms of an activity-travel adjustment process, through which the proposed needs-based activity-travel dynamic system converges to a steady state, coinciding with the Wardropian user equilibrium and the equivalent mixed-strategy Nash equilibrium. This formalism integrates activity generation, activity-travel scheduling, and traffic flow evolution, and offers appealing explanations for day-to-day traffic dynamics and equilibria.

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1. Introduction

1.1. Day-to-day activity generation

It has been widely recognized that activity-based modeling (ABM) has become a dominant research paradigm in travel demand forecasting and analysis. A complete activity-based travel demand modeling framework normally encompasses three components, namely, activity generation, activity-travel scheduling, and activity-travel execution. These three components have been treated in isolation in many previous studies. Several comprehensive activity-based travel demand systems have been developed by integrating activity generation and activity-travel scheduling components (e.g., DAS, Bowman and Ben-Akiva, 2001; ALBATROSS, Arentze and Timmermans, 2004; MATSim, Balmer et al., 2006). However, to consider the aggregated effects, these systems need external traffic assignment or simulation models for the execution stage. Most applications rely

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on single step integration. Others involve an iterative process in which the simulated travel times, based on traffic assignment models, serve as input to the next iteration of the activity-based model until an equilibrium is reached. A few activity-travel assignment models have integrated components of activity-travel scheduling and execution (e.g., Lam and Yin, 2001; Lam and Huang, 2002; Li et al., 2010, 2014; Chow and Djavadian, 2015; Fu and Lam, 2014, 2018). Particularly, to incorporate multi-dimensional choice facets simultaneously, supernetwork representations have been adopted to achieve activity-travel pattern (ATP) based user equilibria. Based on multi-state supernetworks (Liao et al., 2010, 2011, 2013), Liu et al. (2015, 2016) represent a recent elaboration of dynamic user equilibrium that can capture multi-modal and multi-activity trip chaining at a high level of detail. To a large extent, these models considered activities within a one-day time frame.

However, as for activity generation, increasingly more scholars have recognized the necessity for extending the time-frame of ABM from one-day to multiple days, shifting to dynamic modeling of activity-travel behavior. Arentze and Timmermans (2009) made a concrete step in this direction and provided a theoretical framework of the concept of “need”, stating that activities are conducted to satisfy particular needs, which fluctuate over time. After completing an activity, the corresponding need drops and then increases again over time until it appears again on the agenda. The nature and speed of the changes depend on the activity. Owing to the heterogeneity of people’s preferences, activities may meet the needs at different levels with varying frequencies and may also increase or decrease the needs of other activities as side-effects. Patabhiraman (2012) developed a conceptual framework of the relationship between needs and activities based on inventory theory. This behavioral needs-based framework enhanced conventional activity generation models. In that sense, the specific physical and psychological needs of a day have effects on the choice of activities, and thus daily activities are supposed to exhibit day-to-day dynamics. This seminal idea has been applied to address activity generation (e.g., Nijland et al., 2012, 2014) and activity scheduling (e.g., Märki et al., 2014; Chow and Nurumbetova, 2015) within a multi-day time frame or an open planning horizon. Nevertheless, these models in principle need external modules that give feedback on the travel dynamics at the aggregate level.

1.2. Day-to-day traffic dynamics

Day-to-day traffic dynamics models predict the evolution of travel choices and traffic flows over time. These models can be classified along various dimensions: continuous time processes (e.g., Smith, 1984) versus discrete time processes (e.g., Cantarella and Cascetta, 1995; Watling, 1999); deterministic processes (e.g., Smith, 1983) versus stochastic processes (e.g., Watling and Hazelton, 2003; Parry and Hazelton, 2013). Under a rational adjustment process (RAP), as link flows evolve day-to-day, the aggregate travel costs of the traffic system based on the link travel costs fluctuate; and the flow patterns should be equivalent to those at a user equilibrium state if link flows become steady. To avoid path overlap and flow non-uniqueness, day-to-day traffic assignment models that directly deal with link flow variables have been proposed and analyzed (He et al., 2010; Han and Du, 2012). In addition, many path-based dynamical systems approaches have been provided. Yang and Zhang (2009) classified the path-based RAP into five major categories, i.e., the simplex gravity flow dynamics (e.g., Smith, 1984), the network tatonnement process (e.g., Friesz et al., 1994), the route swapping adjustment process (e.g., Smith and Wisten, 1995), the projected dynamical system (e.g., Zhang and Nagurney, 1996), and evolutionary traffic dynamics (e.g., Wang et al., 2013). Furthermore, the route swapping approach has been classified into three major categories, i.e., the swapping towards the least costly route (e.g., Huang and Lam, 2002), pairwise swapping (e.g., Mounce, 2006), and swapping from above average cost routes to below average cost routes (e.g., Mounce and Carey, 2011).

Recently, some studies further extended the day-to-day model by considering departure time choice, stochastic contexts, bounded rationality (e.g., Cantarella and Watling, 2016; Guo et al., 2018; Ye et al., 2018) and even multi-modal traffic (e.g., Cantarella et al., 2015; Li and Yang, 2016; Liu and Geroliminis, 2017). In addition, some studies incorporated the effects of travel information on day-to-day traffic dynamics (e.g., Bifulco et al., 2016; Liu et al., 2017; Li et al., 2018; Zhang et al., 2018). All these models focus on the day-to-day evolution process of traffic flows from non-equilibria to equilibria. However, these day-to-day traffic models only examine the traffic characteristics with fixed trip-based OD pairs over time. The adjustment processes are mainly channeled by the relation of travel cost and traffic flow, which after all can only be categorized as static processes from the view of activity-travel generation.

1.3. Day-to-day activity-travel dynamics

As discussed, existing traffic models tend to discard varying activity-travel demand. Therefore, the aim of this study is to propose a new framework for dynamic activity-based travel demand analysis in multi-state supernetworks based on needs-based theory. The framework postulates that temporal fluctuations in human needs are the source of dynamically generated activities, which in turn are manifested in the choice of day-to-day ATPs and day-to-day traffic dynamics. To capture the evolution of activity-travel flows, we formulate a general activity-travel rational adjustment process (AT-RAP). Specifically, the net utility of executing an ATP on a particular day is determined jointly by the utility of satisfying the needs on that particular day and the travel disutility on the ATP. On the next day, needs are updated and tend to result in a higher utility, and so the net utilities of ATPs are updated. It implies that executing the same ATP on two consecutive days may result in different net utilities, even everything else being equal. To accommodate the dynamic generation of ATPs, a multi-state supernetwork representation is introduced. Given day-to-day dynamics are the focuses of attention, within-day dynamics are not considered in the present study. AT-RAP combines dynamic activity generation, activity-travel scheduling, and traffic

flow evolution in a holistic framework. AT-RAP consistently considers ATP as the unit of analysis and seamlessly bridges the three phases of ABM. It offers an appealing explanation why traffic flows fluctuate on a day-to-day basis.

To that end, the remainder of this paper is organized as follows. Section 2 introduces the basic settings of multi-state supernetworks. Section 3 presents the postulated mechanism of dynamic needs and the flow adjustment process. Section 4 formulates activity-travel link (dis)utilities and ATP net utilities. Section 5 analyzes the convergence and steady state of the dynamic needs-based activity-travel system. Section 6 illustrates the proposed modeling framework with a numerical example. Finally, a discussion and some concluding remarks are provided in Section 7.

2. The settings

2.1. Notation

The following notation will be adopted throughout this paper.

A	The set of activities
a	An activity
$\mathbf{B}$	The vector of all activity needs for all traveler classes from all home locations
B_{ia}^0	The reference need of activity a for traveler class i
$B_{ia}^{(n)}$	The need of activity a for traveler class i on day n
$B_{iha}^{(n)}$	The need of activity a for traveler class i from home h on day n
$\tilde{B}_{iha}^{(n)}$	The need of activity a for traveler class i from home h making $V_{ihrl}(\tilde{B}_{iha}^{(n)}) = \max\{V_{ihw}, \forall w \in R_h\}$
$\tilde{B}_{iha}^{(n)}$	The need of activity a for traveler class i from home h making $v_{ihrl}(\tilde{B}_{iha}^{(n)}) = 0$
B_{iha}^*	The need of activity a for traveler class i from home h making $\partial v_{ihrl}(B_{iha}^*)/\partial x_{ihrl} = 0$
$\mathbf{C}$	The vector of all ATP disutilities for all traveler classes from all home locations
$\mathbf{C}_i$	The vector of all ATP disutilities for traveler class i from all home locations
C_{ihr}	The ATP disutility incurred by traveler class i traversing r from h
$\mathbf{c}$	The vector of all link disutilities for all traveler classes
$\mathbf{c}_i$	The vector of all link disutilities for traveler class i
$c_l(\bullet)$	The link disutility function of traveler traversing link l
c_l^0	The fixed disutility of traveler traversing link l independent of flow
e_{iha}	The probability of traveler class i conducting activity a from home h
$\mathbf{F}$	The vector of the total number of all traveler classes from all home locations
$\mathbf{F}_i$	The vector of the total number of traveler class i from all home locations
F_{ih}	The total number of traveler class i from h
$\mathbf{f}$	The vector of all path flows for all traveler classes from all home locations
$\mathbf{f}_i$	The vector of all path flows for traveler class i
f_{ihr}	The flow of traveler class i on path r from h
g_l^0	The free flow traverse disutility per unit time on link l
H	The set of home locations
h	A home location
I	The set of travelers' classes
i	A traveler class
$K_{ia}(\bullet)$	The inner growth rate function of the need of activity a for traveler class i
L	The set of links in the supernetwork
L_{PVN}	The set of physical links of private vehicle network in the supernetwork
L_{PTN}	The set of physical links of public transit network in the supernetwork
L_T	The set of transfer links between traffic modes in the supernetwork
L_A	The set of activity links in the supernetwork
L_r	The set of links on path r
l	A link in the supernetwork
m, j	A traveler
n	A day
$O_B(\bullet)$	The need convergence indicator function
$O_F(\bullet)$	The flow convergence indicator function
$\mathbf{P}$	The vector of the mixed strategies of ATP choices of all traveler classes from all home locations
$\mathbf{P}_{ih}$	The vector of the mixed strategies of ATP choices of traveler class i from h
$\mathbf{P}_{ih}^m$	The mixed strategy of ATP choice by traveler m class i from h
$\mathbf{P}^{-m}$	All other travelers' strategies except traveler m
p_{ihr}	The probability of traveler class i choosing ATP r from h
p_{ihr}^m	The probability of traveler m of class i choosing ATP r from h
$\mathbf{q}$	The vector of activity participation quantity

q_{iha}	The activity participation quantity of activity a by traveler class i from h
R_h	The set of feasible ATPs from h
$\hat{R}_{ih}$	The set of ATPs with the highest utility for traveler class i from h
r, w	An ATP(path) in the supernetwork
SNK	A supernetwork
s_l	The capacity of link l
$t_l(\bullet)$	The traverse time on link l
t_l^0	The free flow traverse time of link l
$\mathbf{U}$	The vector of all ATP utilities for all traveler classes from all home locations
$\mathbf{U}_i$	The vector of all ATP utilities for traveler class i from all home locations
U_{ihr}	The ATP utility obtained by traveler class i traversing r from h
$\mathbf{u}$	The vector of all link utilities for all traveler classes from all home locations through all paths
$\mathbf{u}_i$	The vector of all link utilities for traveler class i from all home locations
$u_{ihrl}(\bullet)$	The utility function for class i traversing link l from h through path r
u_{ihrl}^0	The free flow utility for class i traversing link l from h through path r
$\mathbf{V}$	The vector of all ATP net utilities for all traveler classes from all home locations
V_{ihr}	The ATP net utility obtained by traveler class i traversing r from h
$\mathbf{v}$	The vector of all link net utilities for all traveler classes from all home locations through all paths
$v_{ihrl}(\bullet)$	The net utility for class i traversing link l from h through path r ,
$\mathbf{x}$	The vector of all link flows
x_l	The flow on link l
$y_l(\bullet)$	The positive network effect multiplier due to flow on link l
$z_l(\bullet)$	The negative network effect multiplier due to flow on link l
α_{la}	The parameter of marginal utility of activity a on link l
β_l, γ_l	The parameters of positive network effect multiplier on link l
η_l, μ_l	The parameters of negative network effect multiplier on link l
Δ	The link-path incidence matrix
δ_{hrl}	The link-path incidence for h, r and l
λ	A conversion coefficient
θ	The home-path incidence matrix
θ_{hr}	A home-path incidence for h and r
ρ_{ira}	A scaling factor of activity a for traveler class i due to the activity sequence in path r .
σ_{rw}	The vector of swapping indicators from r to w
Φ	The flow transfer vector
Φ_U	The utility-dependent flow transfer vector
Φ_C	The disutility-dependent flow transfer vector
φ_{hr}	The ATP flow transfer parameter for r from h
Ψ	The activity-path incidence matrix
ψ_{hra}	The activity-path incidence for h, r and a
Ω	A compact closed convex set
Ω_A	The set of activity participation quantities
Ω_L	The set of feasible link flows
Ω_R	The set of feasible path flows
ω	The period of adjusting the conversion

2.2. The supernetwork

We consider multi-state supernetwork SNK. As shown in Fig. 1, h and h' denote an O-D pair, which refers to the same location (i.e., home), respectively at the start and end state of the execution of a daily activity program. Thus, we ignore the very small percentage of schedules that do not start and end at home. The essence of a multi-state supernetwork is that the activity-travel scheduling process is decomposed into path choice through different activity-travel states of executing the daily activity program. By splitting the integrated multi-modal transport network into a private vehicle network (PVN) and a public transportation network (PTN), the links interconnecting PVN and PTN represent parking/picking-up private vehicles, and those interconnecting PTN and PTN represent conducting activities (see Liao et al., 2013, 2017 for a detailed explanation). Thus, a consistent ATP from h to h' goes through a series of PTNs and PVNs of reachable states. For example, the ATP formed by the bold links in Fig. 1 shows that the traveler leaves home by car to conduct an activity at a_1 with parking at d_2 , then returns home and switches to bike to conduct another activity at a_2 with parking at d_4 , and finally returns home. As seen, an activity sequence is a part of the path choice in an ATP. Let l^s denote a link in a SNK, where r and s represent the associated ATP and activity state, respectively. Specifically, an activity state is a vector representing which activities are conducted or not, i.e., $\mathbf{s} = \{s_a, a \in A\}$, where A denotes the set of activities and s_a is equal to 1 if activity a has been conducted and 0 otherwise. Hereafter, using short notation, let l represent a link.

Fig. 1. Multi-state supernetwork representation.

There are four basic link types in a *SNK*, that is, $l \in L_{PVN} \cup L_{PTN} \cup L_T \cup L_A$. L_{PVN} is the set of physical links in the PVN, which can only be accessed by private vehicles; L_{PTN} is the set of physical links in the PTN, where travelers can walk and take public transit; L_T is the set of transfer links between traffic modes; and L_A is the set of activity links. Note that there is a many-to-one mapping between activity links and an activity. For example, a shopping or entertainment activity may be conducted at one of multiple alternative locations, which correspond to different activity links in *SNK*. In view of this, let L_a be the set of links for conducting activity a . Let r denote a path, which is an ordered set of links, i.e., $\{l_1, \dots, l_m\}$, and let L_r be the set of links of path r . Logically, any path from h to h' defines an ATP, expressing the specific choice of mode/route, parking/activity location, and activity sequence. Therefore, we call ATP as a path hereafter. The set of home locations is represented by H , i.e., $h \in H$. In order to formulate home-based paths, a home location is both the origin and the final destination of a daily schedule. Let R_h denote the set of all feasible paths from h . It should be noted that the ordered set of links could be $\{0\}$, which indicates a stay-home schedule without travel. The utility of link “0” can be set to zero. Furthermore, let $\mathbf{F} = (\mathbf{F}_i^T, i \in I)^T$ be a vector of the class-specific total numbers of travelers, where $\mathbf{F}_i = (F_{ih}, h \in H)^T$ is its sub-vector for traveler class i . The vectors of class-specific path flows, link flows, and activity participation are denoted as $\mathbf{f} = (\mathbf{f}_i^T, i \in I)^T$, $\mathbf{x} = (\mathbf{x}_i^T, i \in I)^T$ and $\mathbf{q} = (\mathbf{q}_i^T, i \in I)^T$ respectively, where $\mathbf{f}_i = (f_{ihr}, h \in H, r \in R_h)^T$, $\mathbf{x}_i = (x_{il}, l \in L)^T$ and $\mathbf{q}_i = (q_{iha}, h \in H, a \in A)^T$ are their sub-vectors for traveler class i respectively. Note that activity interdependencies of household members are not considered in this study. Overall, the geometric relationships among path flow, link flow, and activity participation can be expressed as

$$\mathbf{F}_i = \Theta \mathbf{f}_i, \quad \mathbf{x}_i = \Delta \mathbf{f}_i, \quad \mathbf{q}_i = \Psi \mathbf{f}_i, \quad \forall i \in I \quad (1)$$

where $\Theta = (\theta_{hr} | h \in H, r \in R_h)$ is the home-path incidence matrix and θ_{hr} is equal to 1 if path r departs from h and 0 otherwise. $\Delta = (\delta_{hrl} | h \in H, r \in R_h, l \in L)$ is the link-path incidence matrix and δ_{hrl} is equal to 1 if path r from h uses link l and 0 otherwise; $\Psi = (\psi_{hra} | h \in H, r \in R_h, a \in A)$ is the activity-path incidence matrix and ψ_{hra} is equal to 1 if path r from h includes activity a and 0 otherwise. Note that Eq. (1) denotes the topology of a multi-state supernetwork and that the aggregation of path flows on a link is activity state dependent. We denote a compact closed convex set as $\Omega = \{(\mathbf{f}, \mathbf{x}, \mathbf{q}) | \mathbf{f}_i = \Theta \mathbf{f}_i, \mathbf{x}_i = \Delta \mathbf{f}_i, \mathbf{q}_i = \Psi \mathbf{f}_i, \mathbf{f}_i \in \mathbb{R}_+^{\sum_{h \in H} \|R_h\|}\}$, where operator $\|\cdot\|$ denotes the mapping from a set to the number of its elements. The sets of feasible path flows, link flows, and activity participation are $\Omega_R = \{\mathbf{f} | (\mathbf{f}, \mathbf{x}, \mathbf{q}) \in \Omega\}$, $\Omega_L = \{\mathbf{x} | (\mathbf{f}, \mathbf{x}, \mathbf{q}) \in \Omega\}$, and $\Omega_A = \{\mathbf{q} | (\mathbf{f}, \mathbf{x}, \mathbf{q}) \in \Omega\}$, respectively. Furthermore, since travelers want to increase their net utilities by conducting activities and meanwhile decrease their travel disutilities (generalized travel costs), we formulate the net utility of fulfilling a path by the activity utilities deducting the travel disutilities on this path, which can be expressed as

$$\mathbf{V} = \mathbf{U} - \mathbf{C}, \quad \forall (\mathbf{f}, \mathbf{x}, \mathbf{q}) \in \Omega. \quad (2)$$

In Eq. (2), $\mathbf{V} = (V_{ihr}, i \in I, h \in H, r \in R_h)^T$, $\mathbf{U} = (U_{ihr}, i \in I, h \in H, r \in R_h)^T$ and $\mathbf{C} = (C_{ihr}, i \in I, h \in H, r \in R_h)^T$ respectively are the vectors of path net utilities, activity link utilities and travel link disutilities, where V_{ihr} , U_{ihr} and C_{ihr} are the corresponding values for traveler class i traversing r from h , respectively. Activity link utilities and travel link disutilities are considered

additive under network topology Eq. (1):

$$\mathbf{U}_i = \Delta^{-1} \mathbf{u}_i, \mathbf{C}_i = \Delta^{-1} \mathbf{c}_i. \quad (3)$$

where $\mathbf{u}_i$ and $\mathbf{c}_i$ are the vector of link utilities and disutilities for traveler class i , respectively.

3. Needs-based activity-travel dynamical system

3.1. Needs adjustment process

The notion that people's daily activities are driven by their basic needs has long been recognized in the literature and put forward in several conceptual studies (e.g., Adler and Ben-Akiva, 1979; Axhausen, 2006). Arentze and Timmermans (2009) were the first to develop an analytical model of dynamic activity generation based on the assumption that individuals' activities are driven by a limited set of needs that tend to grow over time in autonomous processes. The central concept of their needs-based theory is that each activity of an individual satisfies one or more needs. Chow and Nurumbetova (2015) extended the HAPP model (Recker, 1995) to the multi-day horizon, in which every need is associated with a psychological inventory that reflects the level of satisfaction. When an activity is conducted, the corresponding need is (partly) satisfied and the psychological inventory is replenished by a quantity called activity production. When the need is low, the psychological inventory is high and vice versa. Over time, the need builds up and the inventory gets depleted. The inventory is replenished when the individual performs an activity that satisfies the need. Although needs-based models account for complex interactions between activities, needs, and persons (in a household context), we consider a simplified situation in which an individual is faced with the decision which activities to conduct on the current day given the time elapses since the activities were conducted the last time (Nijland et al., 2012, 2014). It is also assumed that an activity is performed at most once within a day. If an activity is conducted multiple times during the same day, each has a different label. In that regard, the following simplifying assumption is made.

Assumption 1. There is a one-to-one mapping between a need and an activity, and the need to conduct an activity is positively correlated with the time elapsed since it was conducted the last time.

The above assumption signifies that conducting an activity only satisfies one type of need. Once an activity is conducted, the need decreases to a lower (or base) level; otherwise, the need will increase over time. The need is formulated as

$$B_{ia}^{(n+1)} = \begin{cases} B_{ia}^{(n)} + K_{ia}(B_{ia}^{(n)}), & \text{if } a \text{ is not conducted by traveler class } i \text{ on day } n, \\ B_{ia}^0, & \text{if } a \text{ is conducted by traveler class } i \text{ on day } n, \end{cases} \quad \forall i \in I, a \in A, \quad (4)$$

where the above variables relate to traveler class i ; $B_{ia}^{(n)}$ is the need for activity a on day n , B_{ia}^0 is the reference need for activity a satisfying $B_{ia}^0 \geq 0$, and $K_{ia}(\cdot)$ is the need build-up rate of activity a as a function of the current need. Eq. (4) specifies the dynamic needs at the individual level. However, at the network level, we cannot distinguish which person implements which specific activities. To update the aggregate choices of activities and needs, an alternative way is to suggest a mixed-strategy to depict travelers' activity choices. This mixed-strategy is expressed as $(e_{iha}^{(n)}, 1 - e_{iha}^{(n)})$, where $e_{iha}^{(n)}$ is the probability of traveler class i conducting activity a from home h on day n . Then, the adjusted need can be expressed in an expected form

$$B_{iha}^{(n+1)} = e_{iha}^{(n)} B_{ia}^0 + (1 - e_{iha}^{(n)}) (B_{ia}^{(n)} + K_{ia}(B_{ia}^{(n)})), \quad \forall i \in I, h \in H, a \in A. \quad (5)$$

where $B_{iha}^{(n+1)}$ is the need for activity a for traveler class i from home h on day $n + 1$. To facilitate the presentation of the essential ideas, the following assumption is adopted.

Assumption 2. An individual is inappreciable relative to the large number of travelers at a residential area.

Under Assumption 2, according to the Weak Law of Large Numbers, it is known that for any arbitrarily small positive number ε , with a sufficiently large F_{hi} there will be a very high probability that $q_{iha}^{(n)}/F_{ih}$ will be close to $e_{iha}^{(n)}$, that is, $\lim_{F_{ih} \rightarrow \infty} \Pr\{|q_{iha}^{(n)}/F_{ih} - e_{iha}^{(n)}| > \varepsilon\} = 0 \Rightarrow q_{iha}^{(n)}/F_{ih} \cong e_{iha}^{(n)}$. This simplification is made given the focus of modeling the effect chain from traffic flow via activity to need. According to Eq. (1), $q_{iha} = \sum_{h \in H} \sum_{r \in R_h} \psi_{hra} f_{ih}^{(n)}$ is obtained. Finally, the need adjustment process is derived as

$$B_{iha}^{(n+1)} = \left(1 - \frac{\sum_{h \in H} \sum_{r \in R_h} \psi_{hra} f_{ih}^{(n)}}{F_{ih}}\right) (B_{ia}^{(n)} + K_{ia}(B_{ia}^{(n)})) + \frac{\sum_{h \in H} \sum_{r \in R_h} \psi_{hra} f_{ih}^{(n)}}{F_{ih}} B_{ia}^0, \quad \forall i \in I, h \in H, a \in A. \quad (6)$$

3.2. Activity-travel flow adjustment process

In order to unify the macroscopic traffic flow and microscopic individual travel choice, we introduce a mixed-strategy to depict multi-traveler activity-travel choice. Specifically, we consider that any traveler faces several alternative paths and chooses a mixed strategy, which can be expressed as a class-specific probability vector $\mathbf{P}_{ih} = (p_{ihr}, r \in R_h)^T, i \in I, h \in H$. p_{ihr}

is the probability of traveler class i choosing path r from home h . Under Assumption 2, it is further known that $f_{ihr} \approx p_{ihr} F_{ih}$ according to the Weak Law of Large Numbers. Then, we adopt an activity-travel rational adjustment process based on the route-swapping system proposed by [Smith and Wisten \(1995\)](#), [Nagurney and Zhang \(1997\)](#), and [Mounce and Carey \(2011\)](#). In this dynamic day-to-day system, activity-travel flows are forced to evolve to a state with higher net utilities. The swapping occurs only for paths of non-maximum net utilities to those paths of maximum net utility from the same h and i . Overall, the detailed dynamic system is formulated as

$$\mathbf{f}^{(n+1)} = \mathbf{f}^{(n)} + \lambda^{(n)} \Phi(\mathbf{f}^{(n)}), \quad (7)$$

where $\lambda^{(n)}$ and $\Phi(\mathbf{f}^{(n)})$ are respectively a conversion coefficient and the flow transfer vector on day n . The conversion coefficient reflects the travelers' adjustment scale of path choice, which depends on how sensitive travelers are to the difference between path net utilities ([Zhang and Nagurney, 1996](#)). The sequence of conversion coefficients is considered as

$$\lambda^{(n)} = \frac{\lambda^0}{\lceil n/\omega \rceil}, \quad (8)$$

where λ^0 represents the initial conversion coefficient and ω denotes the period of adjusting the conversion. $\lceil n/\omega \rceil$ means the period index of the current day. It is noted that $\omega = 1$ implies that the conversion coefficient changes every day and the larger ω the slower the conversion coefficient changes. Unless specified otherwise, throughout this paper, we assume $\lambda^{(n)}$ will not stay the same forever ($\omega < +\infty$).

Travelers try to employ an appropriate conversion coefficient based on their current day to day benefits, so as to void stagnant net utility improvement. In addition, the flow transfer vector $\Phi(\mathbf{f})$ is given by

$$\Phi(\mathbf{f}^{(n)}) = \sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} \sum_{w \in \tilde{R}_{ih}^{(n)}} \frac{f_{ihr}^{(n)} (V_{ihw}(\mathbf{f}^{(n)}) - V_{ihr}(\mathbf{f}^{(n)}))}{\|\tilde{R}_{ih}^{(n)}\|} \sigma_{\mathbf{r}\mathbf{w}}^{(n)}, \quad (9)$$

where $\tilde{R}_{ih}^{(n)} = \{w \in R_{ih} | V_{ihw}(\mathbf{f}^{(n)}) \geq V_{ihr}(\mathbf{f}^{(n)}), \forall r \in R_h\}$ is the set of best paths (with the highest net utility) for class i traveling from h on day n and $\sigma_{\mathbf{r}\mathbf{w}}^{(n)}$ is the vector of swapping indicators from path r to path w , which is set as 1 for the r th element, -1 for the w th element, and zeros for all other elements ([Mounce and Carey, 2011](#)). Behaviorally, the flow transfer vector ensures that it deducts flows from the paths with lower net utilities and adds these flows onto the paths with the maximal net utilities. The sum of the deductions is equal to the sum of the additions. This swapping ensures that the new path flows are non-negative. Substituting [Eq. \(2\)](#) into [Eq. \(9\)](#) yields

$$\Phi(\mathbf{f}^{(n)}) = \sum_{h \in H} \sum_{r \in R_h} \sum_{w \in \tilde{R}_{ih}^{(n)}} \frac{f_{ihr}^{(n)} (U_{ihw}(\mathbf{f}^{(n)}) - U_{ihr}(\mathbf{f}^{(n)}))}{\|\tilde{R}_{ih}^{(n)}\|} \sigma_{\mathbf{r}\mathbf{w}}^{(n)} + \sum_{h \in H} \sum_{r \in R_h} \sum_{w \in \tilde{R}_{ih}^{(n)}} \frac{f_{ihr}^{(n)} (C_{ihr}(\mathbf{f}^{(n)}) - C_{ihw}(\mathbf{f}^{(n)}))}{\|\tilde{R}_{ih}^{(n)}\|} \sigma_{\mathbf{r}\mathbf{w}}^{(n)}, \quad (10)$$

where the two items at the right-hand side can be seen as utility-dependent and disutility-dependent flow transfer vectors, respectively represented as $\Phi_U(\mathbf{f}^{(n)})$ and $\Phi_C(\mathbf{f}^{(n)})$. More specifically, $\Phi_U(\mathbf{f}^{(n)})$ represents the force which swaps flows from the paths with lower activity participation utility to the paths with the higher activity participation utility; $\Phi_C(\mathbf{f}^{(n)})$ represents the force which swaps flows from the paths with higher travel disutilities to the paths with the lower travel disutilities. Then, [Eq. \(10\)](#) can be simplified as

$$\Phi(\mathbf{f}^{(n)}) = \Phi_U(\mathbf{f}^{(n)}) + \Phi_C(\mathbf{f}^{(n)}). \quad (11)$$

In addition, traffic congestion, as a negative network effect, makes travel disutility increase with increasing traffic flows. Therefore, based on disutility-dependent flow transfer, the traditional day-to-day studies mainly construct negative feedback on day-to-day traffic dynamic systems. However, travelers do not only wish to decrease their travel disutilities but also want to increase their activity participation utilities. These utilities may be positively affected by the accumulated choices of others in the sense as an additional user of a good/service may have a positive value of that product to others ([Shapiro et al., 1998](#)).

Examples are joint activities to create greater value, demand-side economies of scale, choosing a restaurant according to its popularity, enjoying a lively atmosphere for leisure activities ([Han et al., 2007](#)), etc. Therefore, we try to capture this potentially positive effect and assume that the utility on an activity links increases with the increasing number of travelers using it. Note that this positive effect may not apply to all activities nor to all individuals. Under the influence of $\Phi_U(\mathbf{f}^{(n)})$, it generates a positive feedback on the flow adjustment process. Taken together, the entire feedback system is shown in [Fig. 2](#), where $y_l(\cdot)$ and $z_l(\cdot)$ are respectively related to the positive and negative supernetwork effects on link l . Note that the proposed system unifies the choice of route, mode, activity sequence, and activity location into path choices. As a result, both positive and negative feedbacks are integrated into the needs-based dynamic activity-travel system. On each day, the path flows tend to change from the current pattern towards the desired pattern based on the paths' net utility of the current day.

Fig. 2. The mechanism of needs-based activity-travel dynamical system.

4. Activity-travel utility

4.1. Positive and negative feedback effects

The accumulated choice of individuals results in aggregate state of the links of the network. Most travel demand forecasting systems have only focused on congestion and increased travel times on road networks (e.g., [Arnott et al., 1990](#)). To increase behavioral realism, a few transportation studies have also considered other negative network effects such as queuing at parking locations (e.g., [Lam et al., 2006](#); [Fu and Lam, 2014](#)) and discomfort (mainly attributed to privacy loss, uncomfortable physical proximity, risk of sexual harassment, etc.) in public transit vehicles (e.g., [Tian et al., 2007](#)). On the other hand, to the best of our knowledge, very few studies in transportation research took crowdedness and decline of quality of services at activity locations (e.g., supermarkets and restaurants) into consideration (e.g., [Han et al., 2007](#); [Liu et al., 2016](#); [Fu and Lam, 2018](#)). This study explicitly posits that the presence of others in addition to negative congestion effects may also lead to a higher utility (e.g., [Han et al., 2007](#)). In general, this occurs for joint activities and social activities ([Kim et al., 2018](#)).

To model the chain effects, we first formulate the activity-travel utility for traveler class i traversing link l from h of path r . $u_{ihrl}(\bullet)$ has the following simplified form

$$u_{ihrl}(B_{iha}, x_l) = u_{ihrl}^0 y_l(x_l), \forall i \in I, h \in H, r \in R_h, l \in L, a \in A, \quad (12)$$

where u_{ihrl}^0 is the free flow utility for traveler class i traversing link l from h of path r , $y_l(x_l)$ is the positive multiplier for link l . u_{ihrl}^0 depends on need-satisfaction in the past and the execution of activities during the present day. Based on the classic utility formulation ([Arentze and Timmermans, 2009](#)), we adopt that

$$u_{ihrl}^0 = \alpha_{la} \rho_{ira} B_{iha}, \forall i \in I, h \in H, r \in R_h, l \in L, a \in A, \quad (13)$$

where α_{la} is marginal utility of activity a on link l and ρ_{ira} is a scaling factor for activity a that depends on the activity sequence in path r for traveler class i . Specifically, α_{la} is positive if link l corresponds to activity a and 0 otherwise, which ensures that utility is zero on any non-activity link. Additionally, ρ_{ira} can be assigned different values for different activity sequences. It is noted that a path explicitly specifies the activity sequence, which can be decomposed into a progression of activity states. The combination of r and a implicitly discloses the activity state when conducting a . In that sense, the specification of ρ_{ira} includes the effects of activity states, evidenced in [Rasouli and Timmermans \(2014\)](#). For traveler class i , the condition $0 < \rho_{ira} < 1$ implies that conducting a is mutually-exclusive from the foregoing activities in r ; the condition $\rho_{ira} = 1$ implies that a is independent of the foregoing activities in r ; while the condition $\rho_{ira} > 1$ implies that a is complementary with the foregoing activities in r .

Regarding $y_l(x_l)$, without loss of generality, the following is assumed.

Assumption 3. The positive network effect multiplier for an activity link is equal to one when there is no traveler using it, while it is continuous and increasing with the flow on it, i.e., $y_l(0) = 1$ and $dy_l(x_l)/dx_l > 0$. That is to say, with other components unchanged, travelers obtain higher activity utilities with increasing activity participants.

On the other hand, although negative network effects are ubiquitous, most travel demand forecasting systems have only focused on congestion and increased travel times on road networks (e.g., [Arnott et al., 1990](#)). To increase the travel behavior realism, a few studies have also considered negative network effects as queuing at parking locations (e.g., [Lam et al., 2006](#); [Fu and Lam, 2014](#)) and discomfort (mainly attributed to the privacy loss, uncomfortable physical proximity, risk of sexual harassment, etc.) in public transit vehicles (e.g., [Tian et al., 2007](#)); while to the best of our knowledge, very few took crowdedness and decline of quality of services at activity locations (e.g., supermarkets and restaurants) into consideration (e.g., [Han et al., 2007](#); [Liu et al., 2016](#); [Fu and Lam, 2018](#)). This study explicitly posits that crowding may lead to positive utility, that is to say, individuals may obtain higher utilities as the number of activity participants increases. Above all, let $z_l(x_l)$ be the negative network effect on roads for private vehicles, in public transit vehicles, and at locations for activity and parking. It is associated with congestion delay, crowding discomfort, and decline of quality of service, and so on. Without loss of generality, we adopt the following assumption.

Assumption 4. The negative network effect multiplier for a link is equal to one when there is no traveler using it, while it is continuous and increasing with the flow on it, i.e., $z_l(0) = 1$ and $dz_l(x_l)/dx_l > 0$.

We formulate the actual link travel disutility $c_l(x_l)$ as

$$c_l(x_l) = c_l^0 + g_l(x_l)t_l(x_l), \forall l \in L, \quad (14)$$

where c_l^0 is fixed disutility independent of link flow, capturing factors such as transit ticket fare, parking fare and payment of using private vehicle in equal shares to link trip; $g_l(x_l)$ and $t_l(x_l)$ respectively are the disutility per unit time and link traverse time/activity duration (hereinafter referred to as link traverse time) on link l , which is composed of the variable part of link travel disutility. To be more specific, $g_l(x_l)$ can be calculated by

$$g_l(x_l) = \begin{cases} g_l^0, & \text{if } l \in L_{PVN} \cup L_T, \\ g_l^0 z_l(x_l), & \text{if } l \in L_{PTN} \cup L_A, \end{cases} \quad (15)$$

where g_l^0 is the free flow traverse disutility per unit time on link l and $z_l(x_l)$ is the negative network effect multiplier measuring discomfort due to crowdedness or decline of quality of services. As driving by private vehicle or transferring between modes is often in private space, we assume that its traverse cost per unit time is independent of link flow. However, in public transit vehicle or at an activity location, the traverse disutility per unit time is dependent on crowdedness or decline of quality of services. In addition, the link traverse time is formulated as

$$t_l(x_l) = \begin{cases} t_l^0 z_l(x_l), & \text{if } l \in L_{PVN} \cup L_T, \\ t_l^0, & \text{if } l \in L_{PTN} \cup L_A, \end{cases} \quad (16)$$

where t_l^0 is the free flow traverse time and $z_l(x_l)$ measures delay due to congestion. It is noted that the free flow traverse time on a public transit link already contains the waiting time due to the discrete departure intervals. We consider public transit in-vehicle times and activity durations are all fixed and thus link traverse times are independent of the flows. However, when driving on a road by private vehicles or transferring between modes, the traverse time is dependent on congestion. Substituting Eqs. (15) and (16) into Eq. (14), we have a unified function to model link disutility for all types of links as

$$c_l(x_l) = c_l^0 + g_l^0 t_l^0 z_l(x_l), \forall l \in L. \quad (17)$$

Under Assumption 4, it is known that $z_l(0) = 1$ and $c_l^0 + g_l^0 t_l^0$ is the free flow travel disutility, which is independent of link flow while $z_l(x_l)$ is dependent on link flow.

4.2. Path net utilities

As mentioned above, travelers want to increase their activity utilities and decrease their travel disutilities. Then, the link net utility is equal to the activity utility (if any) deducting the travel disutility (if any), expressed as

$$v_{ihrl} = u_{ihrl} - c_l, \forall l \in L_A. \quad (18)$$

On the path level, according to Eqs. (1)–(3), (12), (13) and (17), the path net utility vector is formulated by

$$\mathbf{V}(\mathbf{B}, \mathbf{f}) = \Delta^{-1} \mathbf{u}(\mathbf{B}, \Delta \mathbf{f}) - \Delta^{-1} \mathbf{c}(\Delta \mathbf{f}), \quad (19)$$

where $\mathbf{u}(\mathbf{B}, \Delta \mathbf{f}) = (u_{ihrl}, \forall i \in I, h \in H, r \in R_h, l \in L)^T$ is the vector of activity utilities with $||I|| \sum_{h \in H} \sum_{r \in R_h} ||L_r||$ dimensions, $\mathbf{B} = (B_{iha}, i \in I, h \in H, a \in A)^T$ is the vector of activity needs, and $\mathbf{c}(\mathbf{x}) = (c_l(x_l), l \in L)^T$ is the vector of link travel disutilities. Based on the above setup, the following proposition is derived.

Proposition 1. The path net utilities are continuous with the path flows.

Proof. $\mathbf{V}(\mathbf{B}, \mathbf{f})$ can be regarded as an $||I|| \sum_{h \in H} ||R_h||$ -dimensional vector value function. Under Assumption 4, according to Eqs. (3) and (16), it is obtained that path disutility C_{ihrl} is continuous with link flows $\mathbf{x}$. According to Eq. (1), it is further obtained that link flows $\mathbf{x}$ is continuous with path flows $\mathbf{f}$. Therefore, C_{ihrl} is continuous with $\mathbf{f}$. On the other hand, substituting Eqs. (6), (12) and (13) into Eq. (3), we have

$$U_{ihr}^{(n)} = \sum_{l \in L} \delta_{ihrl} \alpha_{la} \rho_{ira} \left(\left(1 - \frac{\sum_{h \in H} \sum_{r \in R_h} \psi_{hra} f_{ihr}^{(n-1)}}{F_{ih}} \right) (B_{ia}^{(n)} + K_{ia} (B_{ia}^{(n)})) + \frac{\sum_{h \in H} \sum_{r \in R_h} \psi_{hra} f_{ihr}^{(n-1)}}{F_{ih}} B_{ia}^0 \right) \times y_l(x_l^{(n)}), \forall i \in I, h \in H, r \in R_h. \quad (20)$$

From Eq. (20), it is known that $U_{ihr}^{(n)}$ is continuous with path flows $\mathbf{f}^{(n-1)}$ and link flows $\mathbf{x}^{(n)}$ under Assumption 3. Hence, for any i, h and r , V_{ihrl} is continuous with the current path flows $\mathbf{f}$. $\square$

Proposition 1 also indicates that the vector of path net utilities is determined by the series previous day-to-day path flows. During the adjustment process, a critical point for an activity need is derived as follows. For any i, h and a , let $\bar{B}_{iha}^{(n)}$

make $V_{ihr}(\tilde{B}_{iha}^{(n)}) = V_{ihw}$, where $r \in R_h$ and $w \in \tilde{R}_{ih}^{(n)}$. It is noted that $V_{ihw} \geq 0$ because the path net utility of stay-at-home is zero. According to Eqs. (12), (13), (17) and Assumption 3, we have

$$V_{ihr}(\tilde{B}_{iha}^{(n)}) = V_{ihw} \Leftrightarrow \tilde{B}_{iha}^{(n)} = \frac{c_l^0 + g_l^0 t_l^0 z_l(x_l) + V_{ihw} - U_{ihr}^{-l} + C_{ihr}^{-l}}{\alpha_{la} \rho_{ira} y_l(x_l)}, \forall i \in I, h \in H, a \in A, l \in L_r \cap L_a, w \in \tilde{R}_{ih}^{(n)}, \quad (21)$$

where U_{ihr}^{-l} and C_{ihr}^{-l} are the sums of all activity link utilities and travel link disutilities on path r without activity link $l \in L_r \cap L_a$ obtained by traveler class i from h , namely $U_{ihr}^{-l} = U_{ihr} - u_{ihr,l}$ and $C_{ihr}^{-l} = C_{ihr} - c_l$, respectively. As $V_{ihr} \leq V_{ihw}$ always holds when $r \in R_h$ and $w \in \tilde{R}_{ih}^{(n)}$, $\tilde{B}_{iha}^{(n)}$ is the upper bound of $B_{iha}^{(n)}$, namely $B_{iha}^{(n)} \leq \tilde{B}_{iha}^{(n)}$. In case of $B_{iha}^{(n)} < \tilde{B}_{iha}^{(n)}$, according to Eq. (21), we have $V_{ihr}(B_{iha}^{(n)}) < V_{ihw}$ and $r \notin \tilde{R}_{ih}^{(n)}$. It signifies that when there is traveler class i who resides at h with the need for an activity lower than its critical point, path r is not in the set of best paths if a link corresponding to the activity is an element of path r . Then, some traveler class i using path r from h will be swapped out on the next day, i.e., $f_{ihr}^{(n+1)} < f_{ihr}^{(n)}$, according to the definition of the flow transfer vector $\Phi(\mathbf{f})$. It is noted that $B_{iha}^{(n)} = \tilde{B}_{iha}^{(n)}$ holds only if path r is in $\tilde{R}_{ih}^{(n)}$ and the flow of traveler class i on path r will not decrease on the next day.

In addition, for any i , hand a , there is break-even point for each activity need, denoted as $\tilde{B}_{iha}$ making $v_{ihr}(\tilde{B}_{iha}) = 0$ for $l \in L_r \cap L_a$. According to Eqs. (12), (13), (17), (18) and Assumption 3, we have

$$v_{ihr}(\tilde{B}_{iha}^{(n)}) = 0 \Leftrightarrow \tilde{B}_{iha}^{(n)} = \frac{c_l^0 + g_l^0 t_l^0 z_l(x_l)}{\alpha_a \rho_{ira} y_l(x_l)}, \forall i \in I, h \in H, a \in A, l \in L_r \cap L_a. \quad (22)$$

According to Eqs. (21) and (22), it is known that $\tilde{B}_{iha}^{(n)} > \tilde{B}_{iha}^{(n)}$ holds when $V_{ihw} - U_{ihr}^{-l} + C_{ihr}^{-l} > 0$. Note that if a is the only activity (except stay-at-home) on path r , $V_{ihw} - U_{ihr}^{-l} + C_{ihr}^{-l} > 0$ always holds due to $U_{ihr}^{-l} = 0$ and $C_{ihr}^{-l} > 0$. Otherwise, $\tilde{B}_{iha}^{(n)} \leq \tilde{B}_{iha}^{(n)}$ holds when $V_{ihw} - U_{ihr}^{-l} + C_{ihr}^{-l} \leq 0$. The following remark is made.

Remark 1. In case of $V_{ihw} - U_{ihr}^{-l} + C_{ihr}^{-l} > 0$:

$$\begin{aligned} r \in \tilde{R}_{ih}^{(n)} \text{ and } v_{ihr} > 0 \text{ hold if } \tilde{B}_{iha}^{(n)} < B_{iha}^{(n)} = \tilde{B}_{iha}^{(n)}, \\ r \notin \tilde{R}_{ih}^{(n)} \text{ and } v_{ihr} > 0 \text{ hold if } \tilde{B}_{iha}^{(n)} < B_{iha}^{(n)} < \tilde{B}_{iha}^{(n)}, \\ r \notin \tilde{R}_{ih}^{(n)} \text{ and } v_{ihr} \leq 0 \text{ hold if } B_{iha}^{(n)} \leq \tilde{B}_{iha}^{(n)} < \tilde{B}_{iha}^{(n)}, \end{aligned}$$

otherwise, in case of $V_{ihw} - U_{ihr}^{-l} + C_{ihr}^{-l} \leq 0$:

$$\begin{aligned} r \in \tilde{R}_{ih}^{(n)} \text{ and } v_{ihr} = 0 \text{ hold if } B_{iha}^{(n)} = \tilde{B}_{iha}^{(n)} = \tilde{B}_{iha}^{(n)}, \\ r \in \tilde{R}_{ih}^{(n)} \text{ and } v_{ihr} < 0 \text{ hold if } B_{iha}^{(n)} = \tilde{B}_{iha}^{(n)} < \tilde{B}_{iha}^{(n)}, \\ r \notin \tilde{R}_{ih}^{(n)} \text{ and } v_{ihr} < 0 \text{ hold if } B_{iha}^{(n)} < \tilde{B}_{iha}^{(n)} < \tilde{B}_{iha}^{(n)}. \end{aligned}$$

It is worth noting that $v_{ihr} > 0$ is not a necessary condition of path r being in $\tilde{R}_{ih}^{(n)}$. A typical example is that when there is a subsequent activity in r that is complementary with activity a (such as entertainment after work), r could be in the set of paths with highest net utility even though the net utility of conducting a in r is negative. Conversely, when there is a subsequent activity in r that is mutually-exclusive with activity a , r maybe not in the set of paths with highest net utility even though the net utility of conducting activity a in r is positive.

Furthermore, from Eqs. (3), (12) and (13), the path marginal net utility for traveler class i with respect to link flow on path r , i.e., $\partial V_{ihr} / \partial x_{ihr,l}$, can be derived as

$$\frac{\partial V_{ihr}}{\partial x_{ihr,l}} = \frac{\partial \sum_{l \in L} \delta_{ihr,l} v_{ihr,l}}{\partial x_{ihr,l}} = \frac{\partial v_{ihr}}{\partial x_{ihr,l}} = \alpha_{la} \rho_{ira} B_{iha} \frac{dy_l}{dx_l} - g_l^0 t_l^0 \frac{dz_l}{dx_l}, \forall i \in I, h \in H, l \in L, \quad (23)$$

where $\alpha_{la} \rho_{ira} B_{iha} dy_l/dx_l$ and $g_l^0 t_l^0 dz_l/dx_l$ respectively denote the marginal positive and negative network effects. As mentioned above, both items are non-negative under Assumptions 3 and 4. Furthermore, for any non-activity link l , α_{la} is equal to zero and thus $\partial V_{ihr} / \partial x_{ihr,l} < 0$ always holds. For an activity link l , we have

$$\alpha_{la} \rho_{ira} B_{iha} \frac{dy_l}{dx_l} > g_l^0 t_l^0 \frac{dz_l}{dx_l} \Leftrightarrow \frac{\partial V_{ihr}}{\partial x_{ihr,l}} > 0, \forall i \in I, h \in H, l \in L, a \in A, \quad (24)$$

$$\alpha_{la} \rho_{ira} B_{iha} \frac{dy_l}{dx_l} < g_l^0 t_l^0 \frac{dz_l}{dx_l} \Leftrightarrow \frac{\partial V_{ihr}}{\partial x_{ihr,l}} < 0, \forall i \in I, h \in H, l \in L, a \in A. \quad (25)$$

Remark 2. When condition (24) holds, the marginal positive network effect for traveler class i is dominant on the activity link, which causes a positive feedback dominance in AT-RAP for traveler class i . Conversely, when condition (25) holds, the marginal negative network effect for traveler class i is dominant on the activity link, which causes negative feedback dominance in AT-RAP for traveler class i .

5. Properties of the dynamic system

5.1. Needs-based activity-travel equilibria

At the macroscopic level, the activity-travel flow assignment can be considered as a $\sum_{i \in I} \sum_{h \in H} F_{ih}$ -player non-cooperative game. Specifically, the travelers represent players, the path sets are action sets, the path net utilities are pay-offs depending on the strategies of the other travelers, and each traveler tries to maximize his/her path net utility. As travelers make a mixed path choice $\mathbf{P}_{ih}$, the activity-travel flow assignment is a mixed-strategy game. Under Assumption 4, the Wardropian user equilibrium, in this $\sum_{i \in I} \sum_{h \in H} F_{ih}$ -player non-cooperative mixed-strategy game, is defined as a state, in which no traveler wants to change his/her mixed path choice unilaterally. It can be expressed by finding a vector $\mathbf{f} \in \Omega_R$ such that the following condition holds:

$$V_{ihr}(\mathbf{B}, \mathbf{f}) \begin{cases} = \max_{w \in R_h} V_{ihw}(\mathbf{B}, \mathbf{f}), & \text{if } f_{ihr} > 0, \\ \leq \max_{w \in R_h} V_{ihw}(\mathbf{B}, \mathbf{f}), & \text{if } f_{ihr} = 0, \end{cases} \quad \forall i \in I, h \in H, r \in R_h, \quad (26)$$

Note that Eq. (26) establishes an integrated model to formulate the flow assignment problem and obtain the user equilibria across multiple dimensions of activity-travel choices. For Eq. (26), it is well known that, at an equilibrium state, each traveler from h and i only chooses those paths that have the maximum net utility, and the paths that are not used should have net utilities lower than or equal to the maximum net utility.

Proposition 2. *On each day, there is at least one equilibrium path/link flow vector.*

Proof. On each day, according to Eq. (6), it is known that the activity need vector $\mathbf{B}$ is determined by path flows on the previous day and independent of the path flows on the current day. From Eq. (1), we know the set of feasible path flows is a compact closed convex set with $\Omega_R \subset \mathbb{R}_+^{||I|| \sum_{h \in H} ||R_h||}$. In addition, the path net utilities are continuous with the path flows (Proposition 1). Therefore, a user equilibrium satisfying Eq. (26) exists. Furthermore, due to the inclusion of 0–1 integer variables δ_{hrl} and ψ_{hra} , $\mathbf{V}(\mathbf{B}, \mathbf{f})$ is non-convex on Ω_R . Consequently, Eq. (26) may have multiple local solutions. On the other hand, the set of feasible link flows is a compact closed convex set with $\Omega_L \subset \mathbb{R}_+^{||I|| \sum_{h \in H} \sum_{r \in R_h} ||L_r||}$. From Proposition 1, the link net utilities are continuous with the link flows and thus the equilibrium link flows exist. A necessary condition of the uniqueness of equilibrium link flows is the Jacobian matrix of $\mathbf{v}(\mathbf{B}, \mathbf{x})$ being positive definite. According to $x_l = \sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} x_{ihrl}$, for any i, h, r, l and i', h', r', l' , it is known that $\partial v_{ihrl} / \partial x_{i'h'r'l'} = \partial v_{ihrl} / \partial x_{ihrl}$ if $l = l'$ and 0 otherwise. By expanding the link net utility Jacobian matrix, it is known that the link net utility Jacobian matrix is not positive definite and thus $\mathbf{v}(\mathbf{B}, \mathbf{x})$ is not strictly monotonic on Ω_L . Therefore, there may be multiple equilibrium solutions of link flow vector. $\square$

The equivalency between the Wardropian user equilibrium and a mixed-strategy Nash equilibrium in the traffic network has been proven by Haurie and Marcotte (1985) and Bell and Cassir (2002). Here, we provide a proof for mixed activity-travel choice in the framework of multi-state supernetworks.

Proposition 3. *On each day, a user equilibrium is equivalent to a $\sum_{i \in I} \sum_{h \in H} F_{ih}$ -player non-cooperative mixed-strategy Nash equilibrium.*

Proof. At the macroscopic level, under Assumption 4, $f_{ihr} \cong p_{ihr} F_{ih}$ holds for any i, h and r according to the Weak Law of Large Number, and thus $f_{ihr} = 0$ holds if and only if $p_{ihr} = 0$. According to Eq. (26), $p_{ihr} = 0$ when $r \notin \tilde{R}_{ih}$. At the microscopic level, any traveler m of class i from home h chooses a mixed strategy of path choice $\mathbf{P}_{ih}^m$ against all other travelers' strategies $\mathbf{P}^{-m}$. This mixed strategy $\mathbf{P}_{ih}^m$ is considered as a probability vector of all feasible paths for i from home h , i.e., $\mathbf{P}_{ih}^m = (p_{ihr}^m, r \in R_h)^T$, where p_{ihr}^m is the probability that traveler m of class i from home h choosing path r . Furthermore, the path net utility of traveler m from home h can be formulated in the following linear additive form

$$V_{ih}^m(\mathbf{P}) = \sum_{r \in R_h} p_{ihr}^m V_{ih}^m(\mathbf{P}^{-m}, r), \quad \forall i \in I, h \in H, m \in F_{ih}, \quad (27)$$

where $V_{ih}^m(\mathbf{P}^{-m}, r)$ is the path net utility of traveler i choosing pure path strategy r while all other travelers follow a mixed strategy $\mathbf{P}^{-m}$. For any home h , since traveler class i is assumed to be homogeneous, it has the same pure strategy set of path choice and thus they are replaceable. Therefore, for any home h , both $p_{ihr}^m = p_{ihr}^j = p_{ihr}$ and $V_{ih}^m(\mathbf{P}^{-m}, r) = V_{ih}^j(\mathbf{P}^{-j}, r)$ hold for any class i traveler m, j and path r . Then, we have

$$p_{ihr} > 0 \Leftrightarrow p_{ihr}^m > 0, p_{ihr} = 0 \Leftrightarrow p_{ihr}^m = 0, \quad \forall i \in I, h \in H, m \in F_{ih}, r \in R_h, \quad (28)$$

$$V_{ih}^m(\mathbf{P}^{-m}, r) \begin{cases} = \max_{w \in R_h} V_{ih}^m(\mathbf{P}^{-m}, w) \Rightarrow p_{ihr}^m > 0, \\ < \max_{w \in R_h} V_{ih}^m(\mathbf{P}^{-m}, w) \Rightarrow p_{ihr}^m = 0, \end{cases} \quad \forall i \in I, h \in H, m \in F_{ih}, r \in R_h. \quad (29)$$

Furthermore, due to $f_{ihr} \cong p_{ihr} F_{ih}$, we have

$$V_{ih}^m(\mathbf{P}^{-m}, r) \cong V_{ihr}(\mathbf{f}), \quad \forall i \in I, h \in H, m \in F_{ih}, r \in R_h. \quad (30)$$

Eq. (30) indicates that the net utility of class i traveler m using pure path strategy r is approximately equal to the net utility of path r corresponding to flow pattern $\mathbf{f}$. Under Assumption 2, synthesizing Eqs. (28)–(30), we have

$$V_{ihr}(\mathbf{f}) = \max_{w \in R_h} V_{ihw}(\mathbf{f}) \Rightarrow p_{ihr} > 0 \Leftrightarrow p_{ihr}^m > 0 \Leftarrow V_{ih}^m(\mathbf{P}^{-m}, r) = \max_{w \in R_h} V_{ih}^m(\mathbf{P}^{-m}, w), \forall i \in I, h \in H, r \in R_h, \quad (31)$$

$$V_{ihr}(\mathbf{f}) < \max_{w \in R_h} V_{ihw}(\mathbf{f}) \Rightarrow p_{ihr} = 0 \Leftrightarrow p_{ihr}^m = 0 \Leftarrow V_{ih}^m(\mathbf{P}^{-m}, r) < \max_{w \in R_h} V_{ih}^m(\mathbf{P}^{-m}, w), \forall i \in I, h \in H, r \in R_h. \quad (32)$$

It follows that an activity-travel user equilibrium is equivalent to a mixed-strategy Nash equilibrium. $\square$

It is noted that according to Nash (1951), there is at least one mixed-strategy Nash equilibrium to the above $\sum_{i \in I} \sum_{h \in H} F_{ih}$ -player non-cooperative game, which is consistent with Proposition 2. In addition, on each day, the activity need vector is the major determinant of the set of user equilibria.

5.2. Convergence and stability analysis

In order to investigate the convergence of the proposed dynamic needs-based activity-travel system, it is necessary to construct two indicators to measure the convergence of activity needs and path flows, respectively. Particularly, each convergence indicator is considered as a gap function measuring how close the dynamic system is to a stationary or equilibrium point. However, it is impractical to get the gap of path flows and activity needs between the current point and equilibrium because the stationary point is unknown in advance or there may not even be an equilibrium. Therefore, we try to construct gap functions using the historical and current activity needs and path flows. Logically, the more travelers on those paths that have lower net utilities than the maximum one, the larger the gap obtained, which can be formulated as $f_{ihr}^{(n)} (\max_{w \in R_h} V_{ihw}(\mathbf{f}^{(n)}) - V_{ihr}(\mathbf{f}^{(n)}))$. Similarly, the gap function of activity needs can be formulated as $\psi_{hra} f_{ihr}^{(n)} |B_{iha}^{(n)} - B_{iha}^{(n-1)}|$. Summing up the relative gaps across all paths of all travel classes, the need and flow convergence indicators can be respectively expressed as

$$O_B(\mathbf{B}^{(n)}) = \frac{\sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} \psi_{hra} f_{ihr}^{(n)} |B_{iha}^{(n)} - B_{iha}^{(n-1)}|}{\sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} \psi_{hra} f_{ihr}^{(n)} B_{iha}^{(n)}}, \quad (33)$$

$$O_F(\mathbf{f}^{(n)}) = \frac{\sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} f_{ihr}^{(n)} (\max_{w \in R_h} V_{ihw}(\mathbf{f}^{(n)}) - V_{ihr}(\mathbf{f}^{(n)}))}{\sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} f_{ihr}^{(n)} |V_{ihr}(\mathbf{f}^{(n)})|}. \quad (34)$$

Proposition 4. AT-RAP converges to some stationary point coinciding with needs-based activity-travel equilibria.

Proof. Substituting Eqs. (6) and (9) into respectively Eqs. (33) and (34), it is known that both the need convergence indicator $O_B(\mathbf{B}^{(n)})$ and the flow convergence indicator $O_F(\mathbf{f}^{(n)})$ are equal to zero if and only if the flow transfer vector $\Phi(\mathbf{f}) = 0$. From Eq. (8), it is found that both $\sum_{n=1}^{\infty} \lambda^{(n)} = \infty$ and $\sum_{n=1}^{\infty} (\lambda^{(n)})^2 < \infty$ hold. Then, the AT-RAP dynamic system Eq. (7) can be seen as a Euler method in structure. As a result, the day-to-day $\mathbf{f}^{(n)}$ converges to some stationary point even if the monotonicity of path flows is not guaranteed (Huang and Lam, 2002). Furthermore, since Eq. (7) is a Euler method, any flow pattern $\mathbf{f} \in \Omega_R$ satisfies the activity-travel equilibrium condition expressed in Eq. (26) if and only if it is a stationary point for the dynamic system (see Theorem 2 in Zhang and Nagurney, 1996). $\square$

Based on Propositions 3, 4 and 5, the dynamic needs-based activity-travel system converges to a steady state, which coincides with a mixed-strategy Nash equilibrium and the equivalent Wardropian user equilibrium. In addition, following Nagurney and Zhang (1997), AT-RAP is stable if the Euclidean distance between the path flow pattern sequence $\mathbf{f}^{(n)}$ initialized with any $\mathbf{f}^0$ and any equilibrium flow pattern $\mathbf{f}$, namely, $\|\mathbf{f}^{(n)} - \mathbf{f}\|$ is monotonically non-increasing from day to day. In this regard, let B_{iha}^* make $\partial v_{ihrl}(B_{iha}^*) / \partial x_{ihrl} = 0$. From Eqs. (12), (13), (17), (18) and Assumption 3, we then have

$$\frac{\partial v_{ihrl}}{\partial x_{ihrl}} = \alpha_{la} \rho_{ira} B_{iha}^* \frac{\partial y_l}{\partial x_l} - c_l^0 - g_l^0 t_l^0 \frac{\partial z_l}{\partial x_l} = 0 \Leftrightarrow B_{iha}^* = \frac{g_l^0 t_l^0}{\alpha_a \rho_{ira}} \frac{dz_l}{dx_l} / \frac{dy_l}{dx_l}, \forall i \in I, h \in H, a \in A. \quad (35)$$

Proposition 5. If $B_{iha} \leq B_{iha}^*$ holds for any i, h and a on each day, AT-RAP is stable.

Proof. As mentioned above, the dynamic AT-RAP system Eq. (7) is a Euler method, the criterion of the stability, such that link net utility $\mathbf{v}(\mathbf{x})$ is monotonically decreasing with link flows $\mathbf{x}$ (see Theorem 5 in Nagurney and Zhang, 1997). For non-activity links, under Assumption 3, the link marginal net utility with respect to link flow can be derived as

$$\frac{\partial v_{ihrl}}{\partial x_{ihrl}} = -g_l^0 t_l^0 \frac{dz_l}{dx_l} < 0, \forall i \in I, h \in H, r \in R_h, l \in L. \quad (36)$$

For activity links, under Assumption 4, we have

$$\frac{\partial v_{ihrl}}{\partial x_{ihrl}} = \alpha_a \rho_{ira} B_{iha} \frac{dy_l}{dx_l} - g_l^0 t_l^0 \frac{dz_l}{dx_l} \begin{cases} > 0, & \text{if } B_{iha} > B_{iha}^*, \\ \leq 0, & \text{if } B_{iha} \leq B_{iha}^*, \end{cases} \forall l \in L_A. \quad (37)$$

Fig. 3. (a) Study area in Beijing and (b) the extracted traffic networks.

From Eqs. (36) and (37), it is known that all link net utilities are monotonically decreasing with link flows when condition $B_{iha} \leq B_{iha}^*$ holds for any i, h and a on each day, and thus the AT-RAP is stable. $\square$

Remark 3. On a day, when the positive network effect is very small on each activity link, B_{iha}^* is very large which easily guarantees $B_{iha} \leq B_{iha}^*$ for any i, h and a . The flow on this activity tends to decrease according to Remark 2. As a result, the need may build-up from day to day, and there likely exist a combination of i, h and a making $B_{iha} > B_{iha}^*$ on a certain day. Therefore, the sufficient conditions of stability of AT-RAP are often difficult to satisfy. However, if the positive network effect does not exist in SNK, AT-RAP should be stable.

6. Numerical example

6.1. Basic setting

This section illustrates the proposed AT-RAP in a simplified transportation network of Beijing. Based on Beijing Transport Annual Report (2017), Tongzhou and Dingfuzhuang-Guanzhuang districts are the major residential areas in the east of Beijing, and the major work place for the two residential districts is the CBD (Central Business District) of Beijing. As shown in Fig. 3(a), the two residential districts are connected to the CBD by Jingdong Expressway and Batong Subway Line; in addition, two major shopping locations are Guomao (near the CBD) and Xidan-Wangfujing Districts. By extracting and simplifying the network of the above districts, as shown in Fig. 3(b), travelers live at location h_1 (Tongzhou) and h_2 (Dingfuzhuang-Guanzhuang), and two activities a_1 (work) and a_2 (shopping) are conducted in the center, where activity a_1 is conducted at location d_1 (CBD) and activity a_2 can be conducted at location d_2 (Guomao) or d_3 (Xidan-Wangfujing). Locations h_1 and h_2 are connected with location d_1 by directed private vehicle links 1, 2, 5, 6 (Jingdong Expressway) and directed public transit links 3, 4, 7, 8 (Batong Subway Line). Activity location d_1 is connected with d_2 via directed walking links 11 and 12; d_1 is connected with d_3 via directed private vehicle links 9 and 10. There are four parking places located at h_1, h_2, d_1 and d_3 , respectively.

We obtain the corresponding SNK as shown in Fig. 4. Besides link 0 (stay-at-home) and traffic links 1–12, other links numbered 13–23 are defined as follows. Links 13, 14 and 15 represent conducting activity a_1 at d_1 , conducting activity a_2 at d_2 and conducting activity a_2 at d_3 , respectively. Directed links 16–23 represent picking up or parking car at h_1, h_2, d_1 and d_3 , respectively. Detailed parameter settings for all activity-travel links are shown in Table 1. The symbol “/” means “not relevant”. For simplicity, the need build-up rate for activities a_1 and a_2 are assumed to be constant, i.e., $K_{i1}(\cdot) = k_{i1}$ and $K_{i2}(\cdot) = k_{i2}$. The functions for modeling positive and negative network effect multipliers are specified as follows

$$y_l(x_l) = 1 + \beta_l x_l^{\gamma_l}, \forall l \in L, \quad (38)$$

$$z_l(x_l) = 1 + \eta_l \left(\frac{x_l}{s_l} \right)^{\mu_l}, \forall l \in L. \quad (39)$$

where parameters are assigned different values for different types of links. For example, $\beta_l = 0$ and $\eta_l = 0$ imply there is no positive nor negative network effect on link l and thus travelers incur the free flow travel utility or cost. When γ_l and μ_l are equal to 1, the positive and negative network effect multipliers are linear. Finally, according to Eqs. (17), (19), (20), (38) and (39), the path net utility obtained by class i travelers traversing r from h on day n is derived as

Fig. 4. Multi-state supernetworks for travelers living at (a) h_1 and (b) h_2 .

Table 1
Parameter settings of links.

Link number	Link attribute								
	c_l^0	g_l^0	s_l	t_l^0	α_{l1}, α_{l2}	β_l	γ_l	μ_l	η_l
1, 2	1	0.1	7000	20	0, 0	0.15	0.1	4	0.15
3, 4	3	0.2	9000	25	0, 0	0.15	0.1	2	0.15
5, 6	1	0.1	8000	20	0, 0	0.15	0.1	4	0.15
7, 8	3	0.2	12,000	30	0, 0	0.15	0.1	2	0.15
9, 10	1	0.1	6000	15	0, 0	0.15	0.1	4	0.15
11,12	0	0.2	15,000	20	0, 0	0.15	0.1	1	0
13	0	0.2	15,000	480	1, 0	0.2	0.1	2	0.2
14	15	0.4	6000	60	0, 1	0.2	0.1	2	0.2
15	15	0.4	6000	60	0, 1	0.2	0.1	2	0.2
16,17,18,19	1	0.1	6000	1	0, 0	0.1	0.1	1	0.1
20,21	3	0.1	10,000	1	0, 0	0.1	0.1	1	0.1
22,23	1	0.1	4000	1	0, 0	0.1	0.1	1	0.1

$$\begin{aligned}
 V_{ihr}^{(n)} = & \sum_{l \in L} \delta_{hrl} \alpha_{la} \rho_{ira} \left(\left(1 - \frac{\sum_{h \in H} \sum_{r \in R_h} \psi_{hra} f_{ihr}^{(n-1)}}{F_{ih}} \right) (B_{ia}^{(n)} + k_{ia}) + \frac{\sum_{h \in H} \sum_{r \in R_h} \psi_{hra} f_{ihr}^{(n-1)}}{F_{ih}} B_{ia}^{(n-1)} \right) \\
 & \times \left(1 + \beta_l \left(\sum_{i \in I} \sum_{r \in R_h} \sum_{r \in R_h} \delta_{hrl} f_{ihr}^{(n)} \right)^{\gamma_l} \right) \\
 & - \sum_{l \in L} \delta_{hrl} \left(c_l^0 + g_l^0 t_l^0 \left(1 + \eta_l \left(\sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} \delta_{hrl} \frac{f_{ihr}^{(n)}}{s_l} \right)^{\mu_l} \right) \right), \forall i \in I, h \in H, r \in R_h.
 \end{aligned} \quad (40)$$

It is noted that there is a one-to-one correspondence between a traveler and a private vehicle. All travelers and private vehicles should depart from their homes and return to there the end of day. For example, as shown in Fig. 4(b), a path denoted by the ordered link set {18, 5, 21, 13, 11, 14, 12, 20, 6, 19} shows that a traveler first picks up the car at h_2 and drives to d_1 for parking and conducting a_1 , after that, walks to d_2 for conducting a_2 , and finally goes back to pick up the car at d_1 and drives home. In fact, any path is a one to one mapping with a combined choice of traffic route, mode, activity location, and sequence. As listed in Table 2, paths 0 and 1–15 are feasible for travelers living at h_1 ; while paths 0 and 16–26 are feasible for travelers living at h_2 . Suppose there are two traveler classes, 6000 travelers of class 1 and 3000 of class 2 at h_1 ; and there are 4000 travelers of class 1 and 2000 of class 2 in total at h_2 . Finally, set activity reference needs as $B_{11}^0 = 100, B_{12}^0 = 10, B_{21}^0 = 0.1, B_{22}^0 = 30$; set need build-up rates $k_{11} = 20, k_{12} = 5, k_{21} = 2, k_{22} = 10$; set parameters $\lambda^0 = 0.08$ and $\omega = 30$.

6.2. Computational results

Initially, suppose all travelers of each class are evenly distributed on all feasible paths except stay-at-home. For comparison, we first show the day-to-day needs-based activity-travel dynamics and equilibrium. First of all, from Figs. 5–8, it is observed that the adjustment processes and convergence of activity need, activity location choice, path flow, and path net

Table 2
Feasible paths.

Path number	Activity sequence	$\rho_{1r1},$ ρ_{1r2}	$\rho_{2r1},$ ρ_{2r2}	Link order													
				1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th
0	stay-at-home	/, /	/, /	0	/	/	/	/	/	/	/	/	/	/	/	/	/
1	a_1	1, /	1, /	3	7	13	8	4	/	/	/	/	/	/	/	/	/
2	a_1	1, /	1, /	16	1	5	21	13	20	6	2	17	/	/	/	/	/
3	a_1	1, /	1, /	16	1	19	7	13	8	18	2	17	/	/	/	/	/
4	a_2	/, 1	/, 1	3	7	11	14	12	8	4	/	/	/	/	/	/	/
5	a_2	/, 1	/, 1	16	1	5	21	11	14	12	20	6	2	17	/	/	/
6	a_2	/, 1	/, 1	16	1	19	7	11	14	12	8	18	2	17	/	/	/
7	a_2	/, 1	/, 1	16	1	5	9	23	15	22	10	6	2	17	/	/	/
8	a_1, a_2	1, 1.1	1, 1	3	7	13	11	14	12	8	4	/	/	/	/	/	/
9	a_1, a_2	1, 1.1	1, 1	16	1	5	21	13	11	14	12	20	6	2	17	/	/
10	a_1, a_2	1, 1.1	1, 1	16	1	19	7	13	11	14	12	8	18	2	17	/	/
11	a_2, a_1	1, 0.9	1, 1	3	7	11	14	12	13	8	4	/	/	/	/	/	/
12	a_2, a_1	1, 0.9	1, 1	16	1	5	21	11	14	12	13	20	6	2	17	/	/
13	a_2, a_1	1, 0.9	1, 1	16	1	19	7	11	14	12	13	8	18	2	17	/	/
14	a_1, a_2	1, 1.1	1, 1	16	1	5	21	13	20	9	23	15	22	10	6	2	17
15	a_2, a_1	1, 0.9	1, 1	16	1	5	9	23	15	22	10	21	13	20	6	2	17
16	a_1	1, /	1, /	7	13	8	/	/	/	/	/	/	/	/	/	/	/
17	a_1	1, /	1, /	18	5	21	13	20	6	19	/	/	/	/	/	/	/
18	a_2	/, 1	/, 1	7	11	14	12	8	/	/	/	/	/	/	/	/	/
19	a_2	/, 1	/, 1	18	5	21	11	14	12	20	6	19	/	/	/	/	/
20	a_2	/, 1	/, 1	18	5	9	23	15	22	10	6	19	/	/	/	/	/
21	a_1, a_2	1, 1.1	1, 1	7	13	11	14	12	8	/	/	/	/	/	/	/	/
22	a_1, a_2	1, 1.1	1, 1	18	5	21	13	11	14	12	20	6	19	/	/	/	/
23	a_2, a_1	1, 0.9	1, 1	7	11	14	12	13	8	/	/	/	/	/	/	/	/
24	a_2, a_1	1, 0.9	1, 1	18	5	21	11	14	12	13	20	6	19	/	/	/	/
25	a_1, a_2	1, 1.1	1, 1	18	5	21	13	20	9	23	15	22	10	6	19	/	/
26	a_2, a_1	1, 0.9	1, 1	18	5	9	23	15	22	10	21	13	20	6	19	/	/

Fig. 5. Day-to-day changes in activity needs for the travelers from (a) h_1 and (b) h_2 .

utility exhibit very different patterns for different traveler classes even when they live in the same district. All these differences are derived from the different settings of activity needs. It implies that the chain effects from need via activity-travel to flow dynamics do exist. As shown in Figs. 5 and 6, the adjustment processes and convergence of activity need and location choice exhibit similar patterns for the same traveler class even for different residential districts. Activity needs and the numbers of travelers at the activity locations change at first and then stabilize. Both the activity needs and location choices of class 1 travelers converge earlier than those of class 2 travelers. To be more specific, as shown in Figs. 7(a) and (c), for class 1 travelers, as the day progresses, the flows on paths 9, 14, 21, 22 and 25 fluctuate at first and then stabilize while other paths cannot attract travelers after sixty days. When the flow adjustment converges, there are 2328 and 3672 travelers from h_1 choosing paths 9 and 14 respectively; there are respectively 1086, 609 and 2305 travelers from h_2 choosing paths 21, 22 and 25. As given above, reference needs of the two activities for class 1 travelers are relatively high and the utilities can compensate the respective travel costs. In addition, conducting a_1 prior to a_2 is favored more by class 1 travelers. In particular, all used paths have activity sequence $\{a_1, a_2\}$.

As shown in Fig. 8(a), during the process, class 1 travelers from h_1 realize that choosing paths 9 and 14 generates a higher net utility and thus some continuously switch to them from other paths until a final equilibrium state is achieved,

Fig. 6. Day-to-day changes in activity location choices for the travelers from (a) h_1 and (b) h_2 .

Fig. 7. Day-to-day path flow adjustment processes for travelers (a) class 1 at h_1 , (b) class 2 at h_1 , (c) class 1 at h_2 and (d) class 2 at h_2 .

in which used paths 9 and 14 have the maximum and equal net utility, which is 133.9. As shown in Fig. 8(c), during the process, class 1 travelers from h_2 realize that choosing paths 21, 22 and 25 generates a higher net utility and thus some continuously switch to them from other paths until a final equilibrium state is achieved, in which used paths 21, 22 and 25 have the maximum and equal net utility, which is 141.0. These results are consistent with Propositions 3 and 5.

As shown in Figs. 7(b) and (d), for class 2 travelers, the flows on paths 7, 14, 20 and 25 fluctuate at first and then stabilize while other paths barely attract traveler after thirty days. When the adjustment converges, there are respectively 2112 and 884 class 2 travelers from h_1 choosing paths 7 and 14; there are respectively 1391 and 597 class 2 travelers from h_2 choosing paths 20 and 25. Therefore, the need of activity a_1 may be lower than the critical need on some days. Then, the utility of conducting activity a_1 is very low on these days and it may not compensate the corresponding travel costs. On

Fig. 8. Day-to-day changes of path net utilities for travelers (a) class 1 at h_1 , (b) class 2 at h_1 , (c) class 1 at h_2 and (d) class 2 at h_2 .

Fig. 9. The influence of tiny (a) β_1 on day-to-day activity location choices of the travelers from (a) h_1 and (b) h_2 .

the path level, in Figs. 8(b) and (d), class 2 travelers from h_1 realize that the net utility of path 14 changes up and down opposed to that of path 7, and class 2 travelers from h_2 realize that the net utility of path 25 changes up and down opposed to that of path 20. Thus, some class 2 travelers continuously adjust their activities and path choices until a final equilibrium state is achieved.

6.3. Influence of key parameters on the performance of the dynamic system

As mentioned in Eq. (39), β_l and η_l are the positive and negative network effects, respectively. By setting $\beta_l = 10^{-3}$ and $\eta_l = 10^{-3}$ for all links and keeping other parameters the same as in Section 6.1, the corresponding activity location choices

Fig. 10. The influences of tiny η_l on day-to-day activity location choices of the travelers from (a) h_1 and (b) h_2 .

Fig. 11. Path flow adjustment processes for travelers (a) class 1 at h_1 , (b) class 2 at h_1 , (c) class 1 at h_2 and (d) class 2 at h_2 under $\omega = +\infty$.

are illustrated in Figs. 9 and 10, respectively. Specifically, as shown in Fig. 9, when $\beta_l = 10^{-3}$, the positive network effect is very small, which causes the dominance of negative feedback in AT-RAP. It further leads to both activity locations d_2 and d_3 being used by class 1 travelers. Besides, it is observed that the flow fluctuates for the travelers from h_2 . However, as shown in Fig. 10, when $\eta_l = 10^{-3}$, the negative network effect is very small, which causes the dominance of positive feedback in AT-RAP. It leads to the result for both traveler classes that activity location d_3 attracts increasingly larger flows while the flows at activity location d_2 keep diminishing to zero. These results confirm Remarks 2 and 3. Furthermore, it can also be observed that in either case of $\beta_l = 10^{-3}$ and $\eta_l = 10^{-3}$, the day-to-day changes of activity location choices are flattened out compared with Fig. 6 because the weak positive or negative network effect reduces the complexity of AT-RAP.

Furthermore, by setting $\omega = +\infty$ and keeping other parameters the same as in Section 6.1, the day-to-day path flow adjustment processes are illustrated in Fig. 11. It is observed that the day-to-day flow adjustment process does not converge.

This is because when $\omega = +\infty$, $\lambda^{(n)} = \lambda^0$ holds, which means that travelers never change, and then the proposed AT-RAP cannot be seen as a Euler method in structure. It is worth noting that the path flows of class 2 travelers from h_1 and h_2 are both almost periodic due to periodic changes in the need of activity 1.

7. Conclusion

Understanding people's activity-travel behavior is critical for effective policy-making in strategic and operational travel demand forecasting and traffic management. Over the past years, activity-based models have gradually replaced trip-based models in travel demand analysis, certainly in academic research, due a more valid representation of daily activity-travel behavior. However, the three core components of activity-based models have largely been treated in isolation due to the lack of an integrative and cohesive modeling framework. In view of the advantages of the concept of needs, this study moves the state of the art a step forward by elaborating the multi-state supernetwork. It formulates the chain relationship between dynamic needs, activities, travel demand and traffic flows, combining dynamic activity generation, activity-travel scheduling, and traffic flow evolution. Both positive and negative network effects are captured, and integrated into the dynamic needs-based activity-travel system. As a result, it is found that through the day-to-day adjustment process, the proposed system converges to a steady state, which coincides with a mixed-strategy Nash equilibrium and equivalent Wardropian user equilibrium. The proposed propositions are explored by simulated numerical examples.

Nevertheless, several facets of this basic framework need further investigation. First, to do fully justice to needs-based theory, the mapping from multiple activities to a set of underlying needs, should be further developed. Second, the needs of individuals should be modelled in the larger context of interpersonal interactions, e.g., at the household level. Third, the mapping from dynamic needs to activity-travel utilities/disutilities needs further empirical analyses. Fourth, it is also meaningful to integrate within-day dynamics and time-dependent (dis)utility functions in the model system. Depending on the time-dependent traffic flow propagation principles, travel times and disutilities other than formulated as BPR functions have a significant impact on the choice of ATPs and day-to-day traffic flows. For that matter, the dynamic activity-travel assignment model proposed in Liu et al. (2015) and the space-time supernetwork representation in Liao (2016) provide a solid knowledge base. Fifth, given the ubiquity of travel information (e.g., Liu et al., 2017; Li et al., 2018) and travel time uncertainty (e.g., Liu and Liu., 2018), incorporating travel information into travelers' (dis)utility functions and choice mechanisms possibly causes different network effects. Sixth, this study has the limitation of relying on an explicit enumeration of ATPs. Combining path generation methods (e.g., Leventhal et al., 1973; Ouyang et al., 2011; Ramadurai and Ukkusuri, 2011; Wang et al., 2019) seems promising to enhance model applicability. Seventh, a more fundamental elaboration would acknowledge uncertainty in travel times the supernetwork representation (e.g., Liao et al., 2014) and different behavioral mechanisms that capture decision making under uncertainty. Last but not least, the present study is based on simple linear utility functions that are not in line with empirical findings. It may be that different dynamics will be observed if these simple functions are replaced with nonlinear functions. We leave these developments for future research.

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