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Stochastic Modelling of Protection Systems: Comparison of Four Mathematical Techniques

by
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Stochastic Modelling of Protection Systems: Comparison of Four Mathematical Techniques

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Abstract

Power system protection systems are designed to automatically take countermeasures against situations in power systems that are undesired. If these countermeasures are not taken the extent of the undesired situation will grow. This is an important aspect of the stochastic behaviour of power systems.

A review is given of stochastic models of protection systems as described in literature. It is concluded that a standard is needed for the treatment of stochastic aspects of protection systems. This standard can be used as a basis for models and for collecting failure statistics. The proposed standard is based on a division of all protection system actions into correct operations, mal-trips and fail-to-trips. The mal-trips are divided into instantaneous mal trips and potential mal trips. Suggestions about the mathematical treatment of the stochastic protection system actions have been made.

A small part of a power system, including one or more protection systems, is modelled by using four mathematical techniques: Markov theory, renewal theory, Petri nets and simulation. In these models the protection system can work correctly or fail to operate when short circuits occur. When the cost of maintenance and the cost of a failure to operate are defined, the optimal maintenance frequency can be determined. When using simulation this is the most difficult because of its statistical nature. Using the analytical techniques, this is much easier.

When the model becomes more complex, the usefulness of the techniques changes. Markov models are still useful and relatively easy to handle, but only exponential distributions can be used. Renewal theory cannot be used for modelling large systems. Petri nets can be used, but they are difficult to design. The advantage over Markov models is that deterministic transition times between states can be incorporated into the model. Simulation is very suitable for modelling large systems. The main concerns are the large computational demands and the interpretation and processing of the statistical output.

Keywords: power system protection / power system reliability / optimal maintenance / probability theory / Markov processes / renewal theory / petri nets / Monte Carlo simulation

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Chapter 1
Introduction

In the Electrical Energy Systems Section of Eindhoven University of Technology, for some years stochastic phenomena in power systems have been studied with support of the Econometrics Section of the Catholic University of Brabant. Its main goal was to be able to analyse the reliability of the supply of electrical energy. As a result of this co-operation a computer program was designed that can evaluate the reliability of industrial networks taking into account, among other things, short circuits, relay malfunctions and maintenance [10].

Because protection systems are designed to take countermeasures against situations that are dangerous for the continuity of the supply, their reliability has an enormous influence on the reliability of the supply. If the design of a protection system is adequate and it works properly, the effects of faults in power system components will be minor because the fault will be cleared fast. The negative effects of the faults increase if design or operation of protection systems fails. One of the causes of this is the extra time the fault is present in the power system. The reaction of protection systems to a fault in the power system is not completely predictable. Because of this, an investigation has started on the stochastic behaviour of protection systems.

The first action that is undertaken in this investigation is to make an inventory of the existing stochastic models and mathematical techniques that can be used in modelling. Both aspects are reported in this work.

Chapter 2 treats the descriptive part of this work. It is intended as the start of what should lead to an overall model of the stochastic behaviour of the protection. Section 2.1 is an elementary introduction to power systems. It is meant for non power system engineers to get an understanding of the total system protection systems are part of. In section 2.2 the protection system itself is discussed in three steps, its function, the principles of protection and the realisation. This part may be omitted by the reader who has detailed knowledge of protection systems. Section 2.3 contains a review of the most important stochastic models of protection systems, described in literature. Section 2.4 is a description of, according to the writers, the most important stochastic aspects of protection systems.

Chapter 3 is an application of four stochastic theories to one protection phenomenon: the failure to operate. After an introduction in section 3.1, the next four sections, 3.2 to 3.5, respectively deal with Markov theory, renewal theory, Petri nets and simulation. Every section contains a summary of the most important axioms of the technique and the mathematical treatment of the models. Section 3.6 holds the results of calculations performed on the models of the previous four sections.

Finally chapter four contains the conclusions and directions for future research.
Chapter 2
Stochastic aspects of the protection

2.1 Introduction to power systems

It is the function of powers supply to produce electrical energy of high and constant quality for a price as low as possible. These constraints are contradictory. In practice a balance has to be found between quality and price of the supply. The structure of power system, used to reach this goal, is briefly described in this section.

The electrical energy is generated by a conversion process in power stations. The energy source can be of a chemical, atomic, kinetic or other nature. After conversion the energy is transported through metal conductors. If these conductors are shielded by a solid insulating material they are called cables. If they are hanging from towers with only a solid insulating material where they are hanging from the towers, they are called overhead lines. A large number of lines, cables, power stations and users are connected to each other, forming an electrical network. Often this network is designed so that single components can be removed without negative effects. The flow of energy to all the users then can be sustained by other components. This is called redundancy.

With respect to the height of the voltage used in power systems there are two opposing phenomena. If a large amount of energy has to be transported, it is most economic to use a high voltage in order to decrease the loss of energy due to the flow of electrical current. When small amounts of energy are transported, e.g. close to the user, high voltage is expensive because of the high investments in insulation. From this it can be concluded that there is a need to transport the same energy under different voltages. This problem is solved by using voltages that are sinusoidal in time. The power system then is an alternating current or AC system. Normally 50Hz or 60Hz is used for the frequency. The biggest advantage of AC systems is the fact that power transformers can be used to transform the voltage level under which the energy is transported. It was found that the use of three conductors with three voltages, which have the same frequency and amplitude but different in phase by 120°, was advantageous. These systems are called three phase systems.

If one of the components of the power system mentioned above has to be connected or disconnected, switches are used. The most important kind of switch in this report is the circuit breaker. It has the ability to switch under all conditions that can occur in the power system even during short circuits when the currents are one or two orders of magnitude higher than normal.

The part of the power system discussed up to now is called the primary part of the power system. It contains the parts that conduct the electrical current and maintain the voltage directly related to the generation, transport and distribution of energy. Beside this there is equipment for protection and control of the power system. This equipment is used for
transport and processing of data. It is called the secondary part of the power system. The interfaces that convert the signals in the primary part to signals that can be used in the secondary part of the power system are called measurement transformers. There are two types, namely voltage transformers for transformation of the voltage and current transformers for transformation of the current. These measurement transformers are placed at strategic places in the power system, e.g. at both sides of a transformer or line. The signals from the measurement transformers then are offered to the equipment for protection and monitoring.

It is the task of the protection system to identify a specified set of undesired situations on the basis of the signals from the measurement transformers. Because of their destructive character and because of the danger to the continuity of power supply, short circuits are the most important situations the protection system has to identify. After identification of a short circuit in the object under consideration the protection will give the command to a specified set of circuit breakers to disconnect the faulted component from the rest of the power system. The detection of the short circuit and the opening of circuit breakers have to be as fast as possible.

The place where conductors join or split, possibly of different voltage levels, are called substations. Power transformers, switches, measurement transformers and the protection apparatus are also located here. The different conductors that belong to the same phase are all connected to the same busbar, which is nothing more than a metal bar isolated at its mounting points. Every substation has at least one set of three busbars (one per phase).
2.2 Description of protection systems

2.2.1 Function of protection systems

In section 2.1, some basic elements of power systems are discussed. In case everything in the power system was predictable there would be no need for a protection system. However, there are events that cannot be predicted. Some of these events can harm people, machinery or the continuity of supply. The most important example is the short circuit. In substations, where these events may require operating of circuit breakers, protection equipment will be installed. The protection then has to decide, from the signal of the measurement transformers, when the circuit breaker needs to open. In figure 2.1, the single line diagram of this concept is shown. In reality there are three conductors and three pairs of measurement transformers. These are all connected to one protection apparatus that commands sets of three circuit breakers to open or close.

![Figure 2.1 Basic scheme of protection equipment.]

If there is a short circuit in a component, e.g. transformer or cable, very high currents will flow, which can cause damage to the power system. Also the continuity of supply is endangered because of the collapse of the voltage close to the short circuit. Because of this, the component has to be disconnected from the power system as fast as possible. In order to influence the supply of energy as little as possible, it is best that only the faulted component, is disconnected. Thus, the function of short circuit protection can be described as:

a. remove a short circuited component from the system as fast as possible,
b. do not remove any unnecessary components from the system.

The division into two functions will show to be important for the stochastic behaviour of the protection.

Other events for which protection systems are designed to interfere are power frequency deviations, the overload of a component and gas production in transformers. In this section however, only the short circuit protection will be discussed.

The function of protection systems can be defined as efficient automatic countermeasures against specified undesired situations. The principles, which are used to achieve this in short circuit protection, are discussed in paragraph 2.2.2. The realisation of the equipment then is described in paragraph 2.2.3.
2.2.2 Principles of short circuit protection

Many different principles are used in short circuit protection to determine if there is a short circuit. Mostly the phase voltage and current signals in one or more places of a network are available after they have been transformed by voltage and current transformers. Besides the actual values of the signals, several functions, which are derived from them, can be used when investigating the presence of a short circuit. Some of these functions are:

- the average of the signal over a certain time in history
- the root mean square (RMS) of the signal over a certain time in history
- the average or RMS of certain harmonics in the signal
- the presence of certain transient patterns in the signal in an interval in history
- the phase angle between current and voltage fundamental frequency signals.

The criterion for detection of short circuits is based on the comparison of one or more functions based on the above-mentioned functions with threshold values. The apparatus that performs this comparison is called the protective relay. The principles, used most in short circuit relays, are discussed here.

For reasons of economy and simplicity, only current or only voltage transformers are used in some situations. Relays using only current for fault detection are called current relays. Besides short circuit detection ($I > I$), these relays are used to detect overloads ($> I$). Because the currents during a short circuit are mostly an order of magnitude higher than under normal conditions, the detection is based on the comparison of the actual value or the rate of change of the current with a threshold. With this kind of relay the possibilities for selectivity, i.e. the reaction to short circuits in a certain zone, are limited. The threshold current level and reaction time can be coordinated, but often this is not enough.

A relay that applies a criterion only to voltage signals is called a voltage relay. If a short circuit is present, the voltages of the phases involved will drop. The threshold RMS-voltage under which the relay will give a switching command can be set. Besides phase-to-earth voltages, neutral point displacement voltages can be used for protection purposes. The selectivity of this relay is even more limited than that of the current relay.

If both current and voltage signals are available, among others the following three functions can be derived.

1) **Active and reactive power flow** can be obtained as follows:

\[
\begin{align*}
P &= I \cdot V \cdot \cos(\varphi) \\
Q &= I \cdot V \cdot \sin(\varphi) \\
S &= P + jQ
\end{align*}
\]  

(2.1)

With $P$ the active power flow, $Q$ the reactive power flow, $S$ complex power, $I$ and $V$ the RMS values of current and voltage of the same phase and $\varphi$ the phase angle between them. This offers the possibility to distinguish between a short circuit in one direction and in the other. If a certain circuit breaker is meant to open only if the short circuit is in a certain direction, this is possible by using a current relay for the detection of the short circuit current and a power direction relay for the detection of the power flow direction. In this case, both relays together decide on the desired action of the protection.

2) If only the angle $\varphi$ is known, given (2.1), the ratio between active and reactive
power is known.

$$\tan(\varphi) = \frac{\cos(\varphi)}{\sin(\varphi)} = \frac{P}{Q}$$  \hspace{1cm} (2.2)

Beside this, the following is known. When $\varphi$ is between $-90^\circ$ and $90^\circ$ the power is flowing in the direction defined positive for the current. The power is flowing in the opposite direction when $\varphi$ is between $90^\circ$ and $270^\circ$. If $\varphi$ is close to $90^\circ$, there is reactive power flow only and a short circuit is present. This way to detect a short circuit ahead of the measuring point is easier to implement than the method mentioned under 1 and therefore more used.

3) If a short circuit is present between one phase and earth, the impedance of this phase ($Z$) seen from the measuring point of the protection is equal to:

$$Z = \frac{V}{I} \cos(\varphi) + j \frac{V}{I} \sin(\varphi)$$  \hspace{1cm} (2.3)

This impedance is equal to the sum of the line impedance and the short circuit impedance. The impedance of the line is mainly reactive and depends on the distance between the protection apparatus and the fault location. The nonlinear impedance of the short circuit is mainly resistive and mostly within a certain range. This leads to the conclusion that if the impedance is within an area of the complex plain a short circuit is present within a certain distance. In figure 2.2 this area is marked 1st. By increasing this area the protected zone can be enlarged. In figure 2.2 one bigger area marked 2nd is drawn. Relays, which use this technique, are called distance relays, because they can monitor a line up to a certain distance from the protection system, or impedance relays.

![Impedance Plane](image)

**Figure 2.2** The impedance plane showing an example of areas the distance protection system uses as a criterion for short circuit detection.

By using these three functions, the selectivity is much higher then with current and voltage relays. All three methods enable determination of the direction of the short circuit. The distance relay even enables the user to define a border of the protected zone at another place then were the measurement transformers are stationed.

When using the information of measurement transformers at different locations in a substation or in different substations, the so-called differential protection or zone protection is possible. An example of this is one direction relay at both ends of a circuit of an overhead line. When both relays detect a short circuit in the direction of the line and they know that the other does too, they can immediately give a switching command for the short circuit has to be on the protected line.
2.2.3 Realisation of protection systems

The function of the protection system described in paragraph 2.2.1 is realised by the protection systems that are located in substations. With information from the measurement transformers and on the base of a principle as described in section 2.2.2 the decision is made whether or not action has to be undertaken. This paragraph is an introduction to the way the protection equipment is realised. At first, the different parts of which a protection system normally consists, and their dependencies will be discussed. Then, the different techniques, used for realisation, are described.

The sensing part of the protection consists of current and voltage transformers. In the simplest case, there is only one transformer connected to a protection system that measures the voltage of the neutral point of a transformer or the current between this point and earth. But, often there are three current transformers, one for each phase, three voltage transformers or even three current and three voltage transformers. If voltages or currents in more than one place in the power system are used in the protection principle, even more measurement transformers can be part of one protection system. The function of the measurement transformers is to transform the high voltages and currents to voltages and currents with nominal values of for instance 100V and 5A with given accuracy. Because a voltage or current at one place in the power system can be used for different protection purposes the signals of the measurement transformers are often offered to more than one protection system. The typical way the protection systems are connected to the measurement transformers is shown in figure 2.3. They are parallel connected to the voltage transformers and series connected to the current transformers. In case the risk of the dependence on one transformer for all protection systems is too high, two transformers, which transform the same voltage or current, can be used. The protection systems then are divided into two groups, one for each transformer.

![Figure 2.3 Typical connection scheme of the signal transformers.](image)

The execution part of the protection consists of circuit breakers. If the protection system has decided on a switching operation a signal is sent to the circuit breaker. It is the function of the circuit breaker to interrupt all possible currents on command in a specified time. A circuit breaker consists of two conducting electrodes that contact each other when it is closed. The opening of the circuit breaker is performed using pneumatic or spring energy. The contacts open in a fluid or gas that is designed to extinguish the arc that forms a conducting path between the separated electrodes. Circuit breakers do not belong to one protection system only. Normally they perform the switching actions in command of several protection systems.

The energy needed by the protection system is stored in a battery that is constantly being refilled. If there is a disturbance in the supply of the battery it can still provide the
protection system with energy for several hours. One battery normally provides several protection systems with energy.

The most important part of the protection system is the protection relay. Note that often the total protection system is also called relay. It is the apparatus that processes the signals from the measurement transformers and when needed, gives commands to the circuit breakers. The power that is used in this process is provided by the battery. The protection principle and the interconnections between measurement transformers, the circuit breakers and the relay are fixed. There is however a possibility of setting some parameters of the protection method, e.g. threshold currents, voltages and times.

If the protection principle involves information that is not directly available in the substation where the relay is based communication with other substations is used to transport the data. If the distance is not too big, signals from measurement transformers can be directly transported through electrical conductors. Otherwise, the signals have to be transported through telephone lines or optical data communication lines. In this case, the data of several protection systems can be transported through one communication channel.

Through history three techniques have been used for making relays, namely:
• electromechanical
• static (electronic)
• digital or numerical
The electromechanical relays are used even since the first power systems (early 20th century). The simplest form is a coil that attracts an iron rod if the current is higher than a certain threshold. The movement of the rod closes a switch that is the command for the circuit breaker to open. If this threshold current is lower than the short circuit current but higher than the normal current this system is a short circuit relay. In almost all relays, regardless of the technique that is used, the command for the circuit breaker to open is generated by an electromechanical relay.

When the transistor was developed in the sixties the first static relays were designed. However it took 10 years before they were reliable enough to be used. They are very much the same as electromechanical relays but without moving parts, which explains the name, static relay. Because of this, static relays are faster than electromechanical relays. However, the functions that can be performed are mainly the same.

The development of the microprocessor in the eighties lead to a new kind of relay, the digital relay. Its architecture is completely different from the other two types of relays. The signals from the measurement transformers are sampled and converted from analog to digital. This digital information is processed using digital filters and algorithms that are designed using programming languages. Digital communication between (parts of) protection systems is used. The renewed architecture enables more complicated protection principles to be used. Also, one digital protection system can perform protection functions traditionally performed by more than one protection system. Communication with the control centre can be used to change the setting of the relay and to perform remote inspection.
2.3 Existing stochastic models

In this section an overview of protection models used in reliability analysis and a few stochastic models of the protection used in other areas will be presented. The models are not discussed in full detail, only the main characteristics are presented. For more information, references are included.

Of the qualities required of the protection [e.g. 44, 50] the two of main interest to us here are:

**Selectivity or discrimination**
The protection’s effectiveness in isolating only the faulty part of the system.

**Stability**
The property of remaining in operation with faults occurring outside the protected zone.

In other words, the power system protection should isolate the fault and refrain from action for the rest. These two aspects of the protection lead to two aspects of the reliability of the protection as defined by the IEC [26]:

- **Reliability of protection**
  the probability that a protection can perform a required function under given conditions for a given time interval.

- **Dependability**
  the probability for a protection of not having a failure to operate under given conditions for a given time interval.

- **Security**
  the probability for a protection of not having an unwanted operation under given conditions for a given time interval.

Here the two principle failure modes of the protection appear, namely failure to operate and unwanted operation. These two terms will reappear further on, but there is more to failure of the protection than this. For example, the unwanted operation can be spontaneous, or due to an event in the power system (often a fault outside the zone to be protected, i.e. an external fault). The failure can be due to a relay failure, a circuit breaker failure, a current transformer failure or even due to an error in the calculation of the setting.

The first aspect of the protection to be modelled in a stochastic way was the fault clearance time [24]. Once the probability density function of the fault clearance time is known, the required time-grading can be calculated for any given value of the acceptable chance of unwanted operation. A similar concept is used for stochastic assessment of transient stability [3, 34]. The protection is here considered to consist of 4 parts:

- Main protection
- Backup protection
- Breaker failure protection
- Circuit breaker

All four parts have a failure probability. The first three parts have a conditional probability density function (pdf) for the fault-clearing time. From this the pdf of the fault-clearing time is calculated.

Most authors agree that mal-trips (unwanted operations) are easy to model. The mal-trip event occurs, and part of the system is isolated immediately. With the failure to operate some modelling difficulties arise. The protection can fail such that it will no longer trip in case of a fault. But, nothing remarkable will happen in the system until such a fault occurs. The protection is said to be in a dormant fail-to-trip-state. Several authors have tried to calculate the probability that a relay is unavailable during a fault from pdf’s for time to failure and time to repair. The earliest publications on this subject [13, 20] are mainly of
conceptual value. The model proposed by Singh and Patton [46] has been a reference for several years. Their model includes faults normally cleared by the protection, a dormant fail-to-trip-state, faults occurring while the relay is in the dormant fail-to-trip-state and preventive maintenance. A more detailed model has recently been proposed by Anderson and Agarwal [6]. They further take into account: inspection of the relay while the component to be protected is still in operation, maintenance on component and relay at the same time, switching after a fault normally cleared, switching after a fault cleared by the backup protection, failure of the protection while the component is in repair. All this is placed into a Markov-model which is analytically solved. The few peculiarities in the model are probably due to their need to obtain an analytical solution. Attempts to further improve the model are made by Kumm et al. [33], and by Meeuwsen et al. [40]. Meeuwsen et al. include mal-trips in the model. Kumm et al. include unrevealed failure, detectable by self-test, and unrevealed failure, not detectable by self-test.

Many authors include some protection model in their reliability analysis [e.g. 2, 4, 5, 7, 12, 15, 18, 19, 21, 22, 23, 28, 29, 31, 32, 35, 39, 45, 47]¹. The protection is either explicitly modelled or included in the circuit breaker model [15, 21, 22, 23, 29, 45]. Most authors distinguish between mal-trip and fail-to-trip. In most cases mal-trip is modeled through a failure rate and fail-to-trip by means of a probability. Also fairly generally taken into account are short circuits in circuit breakers [7, 17, 18, 21, 22, 23, 45]. Such a fault must be cleared by opening of backup breakers. Especially in redundant systems (as most transmission systems are) these failures can contribute significantly to the number of supply interruptions. Endrenyi [17] proposed a model for switching after circuit breaker faults. The fault is cleared by the minimum set of circuit breakers. After the switching time some components are taken back into operation. The part that remains isolated contains the faulted component and is limited by the minimum set of switching elements (which can include circuit breakers). This is further implemented by Grover and Billinton [21] and by Guertin and Lamarre [23]. In their modelling, dynamic components (circuit breakers, reclosers, disconnect switches etc.) can be in any of the following 5 states:
- normally operating and healthy
- faulted
- out of operation for repair or preventive maintenance
- stuck when called upon to open or not closing when called upon to do so
- unwanted operation
A fail-to-trip of a circuit breaker (or of the relay behind it) will also lead to the opening of backup breakers. The switching after a primary fault can be modelled in the same way as after a circuit breaker fault [23, 32]. So can switching after a mal-trip due to an external fault. Rai et al. [45] give an interesting list of possible circuit breaker failures.
- active or short failure, insulation failure:
  - a short circuit in the circuit breaker
  - a short circuit in the current transformer
- active or short failure, stuck breaker failure:
  - measurement error in current transformer

¹This list is admittedly incomplete, but thought to be a representative cross-section containing the most important publications.
- incorrect relay setting
- mechanical error in circuit breaker
- electrical error in circuit breaker
- passive or open failure
  - measurement error in current transformer
  - electrical error in circuit breaker
  - mechanical error in circuit breaker
  - operator error
  - preventive maintenance on circuit breaker

Anderson [4] distinguishes between fail-to-operate due to electronic circuit element failure and due to improper setting and mal-trip due to spurious input data and incorrect calibration.

Kula et al. [32] use in their model probabilities for the following.
- failure of relay
- failure of communication between relay and circuit breaker
- fail-to-operate of circuit breaker
- failure of the circuit breaker failure protection

A further step in understanding failures of the protection is distinguishing between "spurious mal-trips" and "unrevealed mal-trips" [2, 7, 8, 10, 11, 19, 31]. Spurious mal-trips or instantaneous mal-trips occur at random and can be described through a pdf for the time to failure. Unrevealed mal-trips or potential mal-trips should be treated in the same way as dormant fail-to-trip. The protection gets in a potential mal-trip state and nothing will happen until an external fault occurs. Alternatively, this phenomenon is described by means of a constant probability for mal-trip during an external fault [10, 11]. The main modelling problem is that all external faults are different. A relay might trip for some external faults, but certainly not for all. There should thus be different states or different probabilities for different external faults. Dobraca et al. [14] solved this problem by defining a "set of vulnerable relays" for each fault. Only relays within this set have a non-zero probability of mal-trip during the fault. Bollen [10, 11] introduced a protection tree, defined for each fault, in which probabilities of mal-trips are given for relays during operation of the main protection, as well as during operation of the backup protection. We will come back to this protection tree further on.

The IEEE Gold Book [29] gives guidance for performing reliability calculations in industrial commercial power systems. The following protection failures are taken into account:

- Failure of a continuously required function leading to:
  - component short circuit resulting in operation of the backup protection
  - switching device opening without the proper command
  - switching device closing without the proper command

- Failure of the response function leading to:
  - switching device failure to open on command
  - switching device failure to close on command
  - protection trips incorrectly during a fault outside of the protected zone

Interesting here is that failure of the protection is not restricted to failure of a protection relay. It is simply assumed that there are possible short circuits which automatically lead to operation of the backup protection. This is especially interesting as recent data indicates that most failures of the protection are not due to relay failures [27], but e.g. due to incorrect setting.
Bollen and Massee [9] go one step further and divide protection failure in four categories.

- **Failure of the protective concept**: those "failures" which were present in the protection concept and known to the designer.
- **Failure of the model**: those failures which were present in the protection concept but not known to the designer, due to the limitations of the model used.
- **Failure of the setting**: failure due to a discrepancy between the design setting and actual setting.
- **Failure of the device**: failure due to any mechanical, electrical or hardware error in the relay, the circuit breaker, the current transformer, communication channels, etc.

There are a few more interesting protection models that need to be mentioned here.

Bitzer et al. [8] consider mal-function of the protection as a specific type of "common-mode failure". The authors give, among others, the following types of protection malfunctions:

- Mal-trip during normal system conditions
- Mal-trip during an earth fault
- Mal-trip during switching-off of another component
- A fault in the dead zone

For a study in a fairly overall redundant 110kV system, with system data from surveys in the same system, the authors find that about 30% of the interruptions due to common-mode failures is caused by protection malfunction. The other "common-mode" failures are failures of a circuit while the parallel circuit is out of operation (even if there is no causal relation to them whatsoever).

Nordin and Bubenko [43] use a model in which the protection of a component C consists of primary protection, local backup, remote backup level 1 and remote backup level 2. For every component C, there is a set $I_e$ containing the primary protection devices. For every device $PC_i, C_i$, there is a set $J_{C_i}$ containing the local backup protection devices, etc. In other words, all primary protection devices have to operate for the fault to be cleared. If one of them fails to operate, the backup protection of the failed device has to operate. Every device has a probability of fail-to-operate. These probabilities are zero for the remote backup level 2 devices.

Allen and Adraktas [1] use a model that looks further than the circuit breaker. In other models stuck-breaker probabilities are assumed to be independent. However, protection schemes sometimes have common components, in which case this assumption is no longer valid. For every protective system they set up an event tree, for which the protective system is split into functional parts (like fault detector, relay, trip signal generator and circuit breaker). Each of them is given a probability of failure upon request and of operation without request. For each of the events in the event tree, the result is determined (in opening or non-opening of the breaker) plus the event probability. The influence of shared components can be taken into account by setting an event tree for the whole terminal station.

Bollen [9, 11] models the correct and incorrect behaviour of the protection by introduction of protection trees. For every possible short circuit position, a protection tree is defined. It gives primary relays and as many levels of backup relays as deemed necessary. To each relay in the tree a probability to operate and a fault-clearing time are allocated. In this way, failure to operate due to a short circuit as well as mal-trip due to a short circuit are taken
In this case relay #5 has a 100%-98% = 2% chance of fail-to-trip; relays #1, #2 and #6 have a 2% chance of mal-trip during short circuit no. #7. If relay #5 fails to operate (either due to the 2% chance of fail-to-trip or because it is in a dormant fail-to-trip state), relay #15 will certainly trip, relay #20 with a probability of 75%, and relay #1 and #2 with a probability of 3%, and so on.

2Apart from that, spontaneous mal trips of relays and the relay getting into a dormant-fail-to-trip state are taken into account.
2.4 A standard stochastic model of the protection

In section 2.3 a review is given of the most important stochastic models of protection systems that are discussed in literature. From this section it is clear that an enormous variety exists in terms and approaches. If a standard would exist for the base of models of protection systems, this would have several advantages.

• It is easier to understand the models
• Comparisons between models would be possible
• It could be a base for collecting service statistics, the results of which can be used in the models

As an attempt to create such a standard model, in this section, those phenomena that make up the stochastic behaviour of the protection system, including measurement transformers and circuit breakers are described. Depending on the application, part of this model can be used while other parts are neglected or deliberately excluded. The model consists of definitions and names of several types of failures of protection systems with suggestions for mathematical treatment. An assumption of the model is that the user of it has a moderately detailed knowledge of the power system under consideration including its protection systems. The topology of the equipment and the main parameters that influence its functions should be known, e.g. the impedance of a cable, the setting of a protection system, the state of a switch (open or close), etc. The model presented does not give numbers for probabilities and other parameters. They depend strongly on the kind of equipment used and are to be obtained by collecting and processing statistics.

All actions of the protection system are divided into three classes.

1. correct-trip
2. mal-trip
3. fail-to-trip

The rest of this section deals with these classes in the same order.

ad.1 The correct-trip is the easiest to model of all three. A correct-trip means that for a certain fault in the power system the right switches open within a specified maximum reaction time. The time to opening of the circuit breakers will be a probability density function (pdf) which is zero for times bigger than the maximum reaction time.

ad.2 The second class of stochastic actions is the mal-trip that is defined as a circuit breaker opening or closing when this is not needed. This class can be divided in two subclasses: the instantaneous and the potential mal-trip.

The instantaneous mal-trip is the opening or closing of a circuit breaker because of a failure in the protection system, independent of the state of the power system. The failure is in all cases an equipment failure. When dividing the protection system into functional parts as in paragraph 2.2.3 (measurement transformers, communication equipment, computer algorithms, circuit breakers, etc.) the failure can be classified with respect to the part of the protection system where it occurred. With this it is possible to determine the effect of the failure, i.e. it can be determined which circuit breakers will open or close. Instantaneous mal-trips can be modelled by defining a pdf for the time to failure of all parts of the protection system.

The potential mal-trip is the erroneous opening or closing of one or more circuit breakers directly related to an event in the power system. It can be due to a fault or a systematic imperfection of the protection system. These can be defined as follows.

• Potential mal-trips due to systematic imperfections are mal-trips on which maintenance has no influence. They can be divided into two groups:
  • Failure of the concept. This are systematic imperfections that are known by
the designer.

- Failure of the model. Those systematic imperfections that are not known by the designer.

- Potential mal-trips due to faults of protection systems are those mal trips on which maintenance can have influence.

Because the effect of the failure depends on it, for both types of potential mal-trips it is important to distinguish between failures in different parts of the protection system (paragraph 2.2.3).

Because of systematic imperfections, each protection system has a set of events in the power system that will always lead to a potential mal-trip. Then, the pdf of the time the first potential mal-trip is equal to the pdf of the first occurrence of such an even. For potential mal-trips, which require a fault in the protection system followed by an event in the power system, it can be said that every fault has a set of events that will result in a mal-trip. In case the protection system is faulted but there has not been an event to trigger the mal-trip, by means of preventive maintenance the fault can be identified and repaired. According to this, the potential mal-trip can be modelled by defining, for every protection system:

1) The events that will always result in a mal-trip
2) The pdfs of all faults in the protection system and for all these faults which power system events will lead to a mal-trip.
3) The influence of preventive maintenance on it.

ad.3 The third class of actions of the protection system is the fail-to-trip. A fail-to-trip is the not opening or closing of one or more circuit breakers when required. Its stochastic behaviour is very similar to that of the potential mal-trip. A fail-to-trip can also be divided into those that are caused by a systematic imperfection of the protection system and those caused by a fault of the protection system. In both cases an event in the power system has to occur for a failure to happen. In case of a systematic imperfection of the protection system there is a set of events that will always result in a fail-to-trip. In case of a fault in the protection system there is a set of events that will result in a fail-to-trip when they occur while the fault is present in the protection system. However, if maintenance is performed before this, the fault in the protection system may be neutralised. The three points that are mentioned at the end of the description of the potential mal-trip, after exchanging "mal-trip" for "fail-to-trip", also apply when modelling the fail-to-trip.

Although the similarity of potential mal-trips and fail-to-trips have been stressed, there is an important difference between them. The power system events that are important for fail-to-trips are those to which the protection system should react, e.g. all short circuits in a transformer. The power system events that are important for potential-mal trips are those events to which the protection system possibly could react. This last group of events is much more difficult to define and thus harder to model than the first group.
Chapter 3
Stochastic modelling techniques

In this chapter four stochastic techniques, namely Markov theory, renewal theory, Petri nets and simulation, are used for modelling systems consisting of part of a power system including one or more protection systems. In section 3.1 the kind of system that will be modelled is describes. The subsequent four sections each deal with one stochastic technique. These sections contain an overview of the most important mathematical tools and a detailed description of the treatment of the models. Section 3.6 contains the results of calculations performed on these models.

3.1 Introduction

3.1.1 The dormant fail-to-trip

Of the protection system actions discussed in section 2.4, besides the correct trip only the fail-to-trip will be considered in the models presented here. It is caused by an unrevealed fault, called the dormant fail-to-trip state, in the relay. This type of failure is considered here because it is the one on which maintenance has a dominating impact. The main goal of the models presented in this chapter is to find the optimal maintenance strategy that leads to the lowest operational cost of the system. To find this strategy, a cost structure has to be defined. The cost structure used in this chapter consists of two parts:

\[ C_m \]  This is the cost of performing maintenance on one relay. It is assumed to be independent of the state in which the relay is found (dormant or healthy).

\[ C_f \]  This is the cost of one failure to operate (fail-to-trip).

Since these costs can differ a lot in different situations, no values are assigned to these variables. Instead the calculations are performed for a wide range of the ratio \( C_f/C_m \). The cost of the system is expressed in a per unit system with \( C_m \) as a base. With this structure, it is possible to apply the models in various situations where one or more protection systems and an object, which requires protection, are present. Three examples (a, b and c) of situations where the models are applicable are shown in figure 3.1.

In the first example, a transformer and its protection are shown. The cost of failure to

![Figure 3.1](image)

Figure 3.1  Examples of situations where the models presented in this chapter can be applied.
operate is the additional damage caused by the extra time the short circuit current flows. Another cost of failure to operate in this situation could be caused by the voltage dip that lasts longer when the protection system fails to operate. This voltage dip can for instance cause computers to lose data stored in electronic memory.

Example b shows a radial power supply. If the protection system works correctly, a user normally does not experience an interruption of a short circuit that is located downstream. However, if all downstream protection systems fail, an upstream protection system will operate leading to an interruption of more users than necessary. In this situation the cost of failure to operate is equal to the cost of this interruption.

A totally different situation is shown in example c. Here the object is a person (for instance, a president) and the protection systems are bodyguards who protect the president against assassination attempts. If we assume that body guards are losing their concentration and motivation if nothing exciting happens for a long time and that regular training or briefings raise their concentration the models presented can be applied to this system. The cost of failure of the protection consists of all costs associated to the killing of the president. The cost of maintenance is equal to whatever it costs to restore the concentration of one bodyguard.

If essentially different phenomena are to be considered, the beforementioned cost structure can be substituted by a more complex structure. An example of this is the cost of investment needed to install the protection systems that cannot be modelled with the cost structure presented here.

3.1.2 Stochastic techniques

A variety of stochastic techniques can be used to calculate the properties of systems with stochastic components. Four of these theories will be discussed in this chapter. The following four sections each deal with one of them. Their applicabilities differ a lot. The two main characteristics, in this perspective, are the number of states that can be defined in the models and the set of distribution functions that can be used. Some examples of distribution functions are described in appendix 1. An overview of the theories and their applicability is presented in figure 3.2. In this figure every model is represented by a square with its name. The graph shows for every theory which distributions can be applied and how many states can be defined.

The overall superiority of the simulation theory over the other theories is not as big as one might conclude from figure 3.2. The simulation theory is not an analytical method like all others. 

![Figure 3.2 Applicability of stochastic techniques.](image_url)
the others. It is based on random number generating techniques and statistical analysis. This leads to a number of disadvantages over analytic methods.

- More computational time required.
- Less insight in the results.
- Less accuracy.

This is why simulation is only used when other methods are inadequate.

### 3.1.3 Assumptions and situation description

The following assumptions are used in modelling the system:

- After a failure to operate, which is followed by a repair, and after maintenance the relay is working properly.
- The times to failure of a relay are independent and exponentially distributed with parameter $\rho$. This means that if a relay is last seen healthy at time $t_n$, then the chance the relay is dormant at time $t$ is equal to:

$$P(T \leq t) = 1 - e^{-\rho(t-t_n)} \quad \text{for } t \geq t_n$$  \hspace{1cm} (3.1)

Here $T$ is the moment at which the relay becomes dormant.

- Times between short circuits are independent and exponentially distributed with parameter $\lambda$. Since short circuits are cleared immediately in the model, the chance that a short circuit will happen in the next time interval $t$ is equal to:

$$P(T_{sh} \leq t) = 1 - e^{-\lambda t}$$  \hspace{1cm} (3.2)

Here $T_{sh}$ is the time to the next short circuit.

- The times between two successive maintenances (TBM) are independent. Four different maintenance strategies are used in this chapter.

#### #1

In a system with $r$ relays, the TBM is exponentially distributed with parameter $\mu r$. All $r$ relays are maintained at the same time. The average number of relays maintained in one unit of time then equals $\mu$.

#### #2

This strategy can only be applied in a system where all protection systems consist of two identical relays. If one of these relays trips, the protection function will be performed and no failure to operate will occur. The TBM is exponentially distributed with parameter $2\mu r$. At every maintenance one relay of every pair is maintained. At the following maintenance, the other relays will be maintained.

#### #3

As strategy #1 but with TBM constant instead of exponentially distributed. The TBM is always equal to $\frac{r}{\mu}$.

---

**Figure 3.3** Example of the two situations that are modelled.
As strategy #2, the TBM however is constant and equal to $r/2\mu$. In this strategy the maintenance is again performed in turns on pairs of relays. In all strategies the average number of relays maintained per unit of time is equal to $\mu$.

The duration of short circuits, maintenance and repairs is neglected.

With these assumptions the model can be described as follows. Occurrence rates of short circuits are constant with rate $\lambda$. Every relay can be either healthy or dormant. Transition from the first state to the latter, which we will call relay failure, has a constant occurrence rate $\rho$. Transition in the opposite direction can only be caused by maintenance or by a failure to operate (short circuit when a specified number of relays is dormant). Both maintenance and failures to operate cost money. The aim of the models presented in the following four sections is to find the maintenance parameter $\mu$ that results in the lowest total cost in a given situation with a given maintenance strategy.

The three analytical techniques, Markov theory, renewal theory and Petri nets are used to model two situations. The first situation contains one relay. If this relay is dormant a short circuit will lead to a failure to operate (FtO). The second situation contains two relays. They both have to be dormant for a short circuit to cause a FtO. The second example of figure 3.1 is shown in figure 3.3 for both situations.

From figure 3.2 it can be concluded that not all maintenance types can be modelled by all theories. Since Markov theory does not include deterministic times between events it can not be used to model maintenance strategies #3 and #4. This is why strategy #1 and #2 are introduced as an equivalent to the other strategies. In table 3.1 the models are given with a specification of their properties.

Table 3.1 Overview of the Markov, renewal and Petri net models.

<table>
<thead>
<tr>
<th>Name</th>
<th>Theory</th>
<th>Situation</th>
<th>Maintenance Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Markov</td>
<td>1</td>
<td>#1</td>
</tr>
<tr>
<td>M2</td>
<td>Markov</td>
<td>2</td>
<td>#1</td>
</tr>
<tr>
<td>M3</td>
<td>Markov</td>
<td>2</td>
<td>#2</td>
</tr>
<tr>
<td>R1</td>
<td>renewal</td>
<td>1</td>
<td>#3</td>
</tr>
<tr>
<td>R2</td>
<td>renewal</td>
<td>2</td>
<td>#3</td>
</tr>
<tr>
<td>R3</td>
<td>renewal</td>
<td>2</td>
<td>#4</td>
</tr>
<tr>
<td>PN1</td>
<td>Petri net</td>
<td>1</td>
<td>#3</td>
</tr>
<tr>
<td>PN2</td>
<td>Petri net</td>
<td>2</td>
<td>#3</td>
</tr>
<tr>
<td>PN3</td>
<td>Petri net</td>
<td>2</td>
<td>#4</td>
</tr>
</tbody>
</table>

The simulation models are introduced in section 3.4.
3.2 Markov theory

3.2.1 Theory

In this section Markov theory will be introduced and applied in three models, M1, M2 and M3. Paragraph 3.2.1 contains definitions and the most important rules for calculation of properties of Markov models. The reader who is interested in the backgrounds of this is referred to [49]. The three models are described in detail in this paragraph 3.2.2. The results of calculations will be presented in section 3.6 along with the results of the other techniques.

Markov theory is used for modelling systems in which the transition rates between states only depend on the state the system is in, not on the history of the system. Different types of Markov models exist. In this chapter stationary continuous time Markov models with a finite number of states will be discussed. From here, this type of Markov model is meant when it is not explicitly mentioned otherwise.

The memoryless property of Markov models can be defined as follows.

\[
P(X(t_1) = x_1 \mid X(t_2) = x_2, X(t_3) = x_3, \ldots, X(t_n) = x_n) = P(X(t_1) = x_1 \mid X(t_2) = x_2)
\]

with \( t_1 > t_2 > t_3 > \ldots > t_n \) (3.3)

In which the stochastic variable \( X(t) \) indicates the state of the system. If \( X(t) = x \) then we say the system is in state \( x \) at time \( t \). In words the equation means that the future state of the system only depends on the present state, not on the history of the system.

The property that the model is stationary can be illustrated by the following equation.

\[
P(X(u + t) = x_i \mid X(u) = x_i) = P(X(t) = x_i \mid X(0) = x_i) = P_i(t)
\]

(3.4)

This means that the chance of the transitions from one state to the other depends only on the length of the interval in which the transition may take place.

We now define the infinitesimal transition rates \( q_{ij} \).

\[
q_{ij} = \begin{cases} 
\lim_{\Delta t \to 0} \frac{P(X(u + \Delta t) = x_j \mid X(u) = x_i)}{\Delta t} & \text{for } j \neq i \\
\sum_{k \neq j} -q_{ij} & \text{for } j = i 
\end{cases}
\]

for \( j \neq i \) (3.5)

It can be shown that the sojourn time in every state \( x_i \) is exponentially distributed with parameter \(-q_{ii}\). This means that if the system is in state \( x_i \) at time \( t \), the time to the next transition \( T(t) \) has the following distribution function.

\[
P(T(t) \leq s) = 1 - e^{-q_i s}
\]

(3.6)

When leaving state \( x_i \), the chance of entering state \( x_j \) is equal to \( q_{ij}/q_i \).

The rowvector \( p(t) \) contains the distribution at time \( t \). The initial distribution \( p(0) \) holds the chances the system is in the individual states at \( t=0 \).
In order to calculate the dynamic state probabilities of the Markov model, the Chapman-Kolmogorov equation has to be solved. This is a set of linear first order differential equations.

\[
\frac{\partial \rho(t)}{\partial t} = \rho(t) \cdot Q
\]

(3.7)

Here \(Q\) denotes the matrix with transition rates \(q_e\) called the infinitesimal generator. The analytical solution of (3.7) is given by

\[
\rho(t) = \rho(0) \cdot e^{Qt}
\]

(3.8)

The matrix exponential in this formula is defined as

\[
e^{Qt} = \sum_{k=0}^{\infty} \frac{t^k}{k!} Q^k
\]

(3.9)

where \(Q^k\) denotes the \(k^{th}\) matrix product of matrix \(Q\). This infinite series can be solved analytically only in very special cases. Otherwise, it has to be approximated numerically which is difficult because of numerical instabilities of the calculation methods.

Two states in a Markov model are said to be communicating if either of them can be reached in the future when the system is currently in the other. When all pairs of states in a Markov model are communicating the limit of \(\rho(t)\) for \(t\) to infinity converges to the rowvector \(\pi\), called the equilibrium distribution, independent of the initial distribution. This means that if we are interested in the probabilities after the transient phenomena are extinct the initial distribution is insignificant.

\[
\pi = \lim_{t \to \infty} \rho(t)
\]

(3.10)

The equilibrium distribution is also equal to the fractions of time the system spends in the individual states in the long run. This vector can be calculated by solving the following set of linear equations.

\[
\pi \cdot Q = 0
\]

(3.11)

Here \(Q\) is a rowvector of the same dimension as \(\rho_\infty\) containing only zeros. The singularity in the matrix \(Q\) causes the liberty to scale the solution at will. However because \(\pi\) is a probability distribution the additional constraint applies that the components of the vector should add up to 1.

\[
1 = \sum_{i=1}^{n} \pi_i
\]

(3.12)

Where \(\pi_i\) is the \(i^{th}\) component of vector \(\pi\).

At the end of this paragraph the most important advantages and disadvantages of Markov theory are given. The advantages are:

- The method is analytical. This means that results can be obtained fast and that the effect of individual parameters on the performance of the system can be evaluated.
- Because of its simplicity the method is easy to understand and easy to handle.
- Relatively easy calculation of the equilibrium.
- The possibility to model systems with many states.
The disadvantages are:
- Only modelling of exponential and Erlang distributions is possible.
- The dynamic transition chances have to be approximated using unstable numerical techniques.

3.2.2 Protection models

In figure 3.4 the standard graphical representation of the three Markov models is shown. Here the circles or vertices represent states. The directed edges represent the transitions that are possible. The transition rates are shown next to the edges. They can be ordered in the matrix $Q$. The graph and the matrix are equivalent descriptions of the model. If one of them is given the other can be derived.

![Graphical representation of models M1, M2 and M3.]

Figure 3.4  Graphical representation of models M1, M2 and M3.

The three models will be discussed individually.

Model M1:
In this model there is one protection system or relay. The times between maintenance are exponentially distributed with parameter $\mu$.

The two states of model M1 are defined as follows:

state 1: The relay is healthy. Short circuits and maintenance do not cause a change of state in the model. Only failure of the relay (rate $\rho$) will change the state of the model.

state 2: This state represents the dormant relay. Once the relay is dormant two things can happen. Either a maintenance operation (rate $\mu$) or a short circuit (rate $\lambda$) will occur. If a short circuit occurs while the system is in state 2, a failure to operate is the result. This is noticed and the relay is repaired. Because of this, both events will lead to the same result that is represented by a transition to state 1.

The infinitesimal generator of model M1 is given by.

$$Q = \begin{bmatrix} -\rho & \rho \\ \mu+\lambda & -\mu-\lambda \end{bmatrix} \quad (3.13)$$

The rate at which failures to operate occur is equal to $\lambda \pi_2$. This leads to a total average cost per unit of time $C$ that is equal to

$$C = C_{\pi} + C_{\lambda} \pi_2 \quad (3.14)$$

Using equations (3.11) and (3.12) the equilibrium distribution $\pi$ is found.
Substitution of equation (3.15) into (3.14) leads to:

\[ C = C_m \mu + \frac{C \Delta \mu}{\mu + \lambda + \rho} \quad (3.16) \]

Using the derivative with respect to \( \mu \) the minimal cost is found to be obtained at:

\[ \mu = \frac{C \Delta \mu}{C_m} - \lambda - \rho \quad (3.17) \]

**Model M2:**

This model contains two relays. Short circuits lead to a fail-to-trip if both relays are dormant at the time of occurrence of the short circuit. Maintenance is performed simultaneously at both relays. The times between maintenance are exponentially distributed with parameter \( \mu/2 \).

The three states of model M2 are defined as follows.

- **state 1:** Both relays are healthy. As in model M1, short circuits and maintenance do not cause a transition when the system is in this state. Both relays can fail with rate \( \rho \). This causes a total transition rate out of state one, equal to \( 2\rho \).

- **state 2:** One of the relays is dormant. In this state short circuits do not have any influence on the state of the model because the relay that is healthy will work properly. Maintenance with rate \( \mu/2 \) will cause a transition to state 1 while the failure of the healthy relay (rate \( \rho \)) will cause a transition to state 3.

- **state 3:** In this state both relays are dormant. Maintenance (rate \( \mu/2 \)) will cause a transition to state 1. Short circuits (rate \( \lambda \)) while the system is in state 3 will cause a failure to operate and the repair of both the relays that is represented by a transition to state 1.

The infinitesimal generator of this model is given by:

\[
Q = \begin{bmatrix}
-2\rho & 2\rho & 0 \\
\mu/2 & -\mu/2 - \rho & \rho \\
\mu/2 + \lambda & 0 & -\mu/2 - \lambda
\end{bmatrix}
\]

The equilibrium distribution is equal to:

\[
\pi = \frac{1}{12\lambda \rho + 8\rho^2 + 2\lambda \mu + 6\rho \mu + \mu^2} \begin{bmatrix} (2\lambda + \mu)(2\rho + \mu), 4\rho(2\lambda + \mu), 8\rho^2 \end{bmatrix}
\]

The total average cost per year \( C \) then is given by:
\[ C = 2C_m \mu + C_r \lambda \eta_3 \]
\[ = C_n \mu + \frac{C_r \lambda \rho^2}{12 \lambda \rho + 8 \rho^2 + 2 \lambda \mu + 6 \rho \mu + \mu^2} \]  

(3.20)

Determination of the minimum of this function implies the evaluation of a third order polynomial equation.

**Model M3:**

This model is very similar to model M2. The only difference is that maintenance is performed in turns and the times between maintenance are exponentially distributed with parameter \( \mu \) instead of \( \mu/2 \).

The four states of model M3 are defined as follows.

**state 1** Both relays are healthy. In this model the two relays are not maintained at the same time. This is why two states are created for one relay dormant, state 2 and state 3. Both states can be reached by a transition from state 1 with rate \( \rho \).

**state 2** In this state the relay, which will be maintained next, is healthy while the other is dormant. There are two ways out of this state. If the relay, which is healthy, fails (rate \( \rho \)), state 4 will be reached. If the relay that is healthy will be maintained (rate \( \mu \)), state 3 will be reached.

**state 3** In this state the relay, which will be maintained next, is dormant and the other is healthy. Maintenance (rate \( \mu \)) will bring the system back in state 1 while by the failing of the relay, which is healthy, (rate \( \rho \)) state 4 will be reached.

**state 4** Here both relays are dormant. A short circuit (rate \( \lambda \)) will cause a failure to operate leading to the repair of both relays. Maintenance of one of the relays (rate \( \mu \)) will causes a transition to state 3.

The cost function is given by.

\[ C = C_n \mu + C_r \lambda \rho \eta_2 \]
\[ = C_n \mu + \frac{C_r \lambda \rho^2(2 \rho + 3 \mu)}{3 \lambda \rho^2 + 2 \rho^3 + 5 \lambda \rho \mu + 5 \rho^2 \mu + \lambda \mu^2 + 4 \rho \mu^2 + \mu^3} \]  

(3.21)

The minimum of this function has to be obtained numerically.
3.3 Renewal theory

3.3.1 Theory

Renewal theory can be used to model a certain set of problems. It is mostly used in the following subjects:

- Queuing problems
- Storage management
- Reliability/availability analysis.

A detailed description of renewal theory is given in [49]. In all problems the same base is used. A series of non-negative independent random variables \( X_1, X_2, X_3, \ldots \) with a common distribution function \( F(t) \) is defined.

\[
(X_{\nu})_{\nu=1}^{\infty} \sim F(t) \quad \Rightarrow \quad \forall \nu \in \{1,2,3,\ldots\} \quad P(X_\nu \leq t) = F(t) \tag{3.22}
\]

In case of the three applications mentioned above the variables would typically be representing times between arrivals of customers at a queue, sizes of orders in a store or the times between failures of components in a system. The cumulative stochastic variable \( S_n \) is the sum of the first \( n \) variables defined above.

\[
S_n = \sum_{i=1}^{n} X_i \tag{3.23}
\]

They represent the arrival time of the \( n^{th} \) customer in a queue, the total size of orders up to the \( n^{th} \) order or the time of the \( n^{th} \) failure.

For positive \( t \), we define the stochastic function \( N(t) \) as:

\[
N(t) = \text{the largest integer } n \text{ for which } S_n \leq t \tag{3.24}
\]

This means \( N(t) \) can represent the number of customers having entered a queue at time \( t \), the amount of orders minus one when a total demand of \( t \) is reached or the number of failures having occurred at time \( t \). The renewal function \( M(t) \) is defined as the expected value of the counting function \( N(t) \).

\[
M(t) = E[N(t)] \tag{3.25}
\]

The renewal function can be determined in different ways, four of which will be discussed here.

1) By solving equation (3.26).

\[
M(t) = F(t) + \int_0^t M(t-x) dF(x) \tag{3.26}
\]

Analytically, this is only possible in exceptional cases.

2) By using relation (3.27).

\[
\mathcal{L}[M(t)] = \frac{\mathcal{L}[F(t)]}{1 - \mathcal{L}[F(t)]} \tag{3.27}
\]
where

\[ \mathcal{L}[G(t)] = \int_0^\infty e^{-st} dG(t) \quad (3.28) \]

for any function \( G \) of bounded variation. Here, \( \mathcal{L} \) denotes the Laplace-Stieltjes transform.

3) By using (3.29).

\[ M(t) = \sum_{n=1}^{\infty} F_n(t) \quad (3.29) \]

with

\[ F_n(t) = P(S_n \leq t) \quad (3.30) \]

4) By using the fact that the renewal function satisfies the following asymptotic behaviour:

\[ \lim_{t \to \infty} \left( M(t) - t \frac{1}{\mu_1} - \frac{\mu_2}{2\mu_1} + 1 \right) = 0 \quad (3.31) \]

Here \( \mu_1 \) and \( \mu_2 \) are the first and second moment of the distribution function \( F(t) \). For relatively large values of \( t \) the asymptote is often used as an approximation of the renewal function. In the models defined in this chapter the renewal function typically has to be known for small values of \( t \).

All methods have their own restrictions. Depending on the distribution function \( F(t) \) and the application, the most useful technique should be chosen. In this chapter the second method is used. This is possible because the distribution function under consideration is the distribution function of the sum of exponentially distributed stochastic variables. Relation (3.27) can be evaluated analytically using mathematical computer packages.

One class of stochastic systems has got a property that makes it easy to analyse. These systems are called regenerative systems. The property is that at an infinite series of moments \( \{T_1, T_2, T_3, \ldots\} \) the system is in the same state and starts anew, independent of the past. These moments are called the regenerative moments. All probabilities and distributions in the system are the same whatever regenerative moment is taken as the origin. The moments can be stochastic or deterministic. Equation (3.32) is a very useful result for regenerative processes.

\[ \lim_{t \to \infty} \frac{R(t)}{t} = \frac{E(R_n)}{E(T_n - T_{n-1})} = \frac{E(R_1)}{E(T_1)} \quad (3.32) \]

\[ \text{The regular Laplace transform of } G(t) \text{ is equal to } \int_0^\infty G(t)e^{-st}dt. \]
Where $R_n$ are independent stochastic variables representing the total reward earned in the $n^{th}$ regenerative cycle and $R(t)$ is the total reward earned up to time $t$. The statement can be used in order to obtain the average value of some variable in the long run by analysing only one cycle.

Just like in the previous section we will sum up the advantages and disadvantages of the theory presented in this section. The advantages are:

- The method is analytical.
- Very general results can be obtained that are independent of the precise shape of the distribution function.

The disadvantages are:

- Every problem requires new analytical attention. There is not one method that can be automated.
- Only systems with a few states can be modelled.
- Numerical approximations become necessary when modelling other than the most simple systems.

3.3.2 Protection models

Model R1:
In this model there is one relay and the times between maintenance are equal to $1/\mu$. At $t \in \{0, 1/\mu, 2/\mu, \ldots \}$ the system is in the same state. Thus, these moments are regenerative moments. At these moments, the relay is healthy because maintenance has just been performed. The time to failure ($TTF$) of the relay is exponentially distributed with parameter $\rho$, its distribution function is called $G(t)$. The time to short circuit ($TTSH$) is exponentially distributed with parameter $\lambda$, its distribution function is called $H(t)$. The time to the next maintenance operation is deterministic and equal to $1/\mu$.

$$G(t) = 1 - e^{-\rho t}$$
$$H(t) = 1 - e^{-\lambda t}$$

Let $N(t)$ denote the number of FToS occurring in the interval $[0, t)$. When defining the reward function $R(t) = N(t)$ equation (3.32) become,

$$\lim_{t \to \infty} \frac{N(t)}{t} = \frac{E[N\left(\frac{n+1}{\mu}\right) - N\left(\frac{n}{\mu}\right)]}{1/\mu} = \frac{\mu M(1/\mu)}{1/\mu} = \mu M(1/\mu)$$

To be able to determine the renewal function first the distribution function $F(t)$ of the times between FToS has to be found. Because of the memoryless property of the exponential distribution, function the $TTSH$ at the time the relay fails is still distributed according to $H(t)$. Because of this, all times between FToS of the FToS that occur before $t = 1/\mu$ consist of the sum of one $TTF$ and one $TTSH$. With this we can write for $F(t)$:

$$F(t) = G(t) \ast H(t)$$
$$\mathcal{L}[F(t)] = \mathcal{L}[G(t)] \ast \mathcal{L}[H(t)]$$

Here the $\ast$ stands for the convolution operator for distribution functions that is defined by:

$$G(t) \ast H(t) = \int_0^t G(t-x)dH(x) = \int_0^t H(t-x)dG(x)$$
Substitution of (3.33) and (3.35) in (3.27) and using Laplace inversion leads to the following equation.

$$M_{n1}(t) = \frac{\lambda \rho t}{\lambda + \rho} + \frac{\lambda \rho e^{-\lambda t} - \lambda t}{(\lambda + \rho)^2}$$  \hspace{1cm} (3.37)

The average cost per year $C$ for this model is equal to:

$$C = C_n \mu + C_t \lim_{t \to \infty} \frac{N(t)}{t}$$  \hspace{1cm} (3.38)

Substitution of (3.34) and (3.37) in this equation leads to:

$$C = C_n \mu + C_t \left[ \frac{\lambda \rho}{\lambda + \rho} + \mu \lambda t e^{-\lambda t} - \lambda t \frac{\lambda + \rho}{(\lambda + \rho)^2} \right]$$  \hspace{1cm} (3.39)

Using numerical techniques, the value of $\mu$ can be found for which this cost is minimal.

**Model R2:**

This model is very similar to model R1. But, because there are two relays instead of one the models differ in two aspects.

1) Maintenance is performed every $2/\mu$ instead of every $1/\mu$.

2) Two relays must be dormant before a FtO can occur. Because of this, the distribution function $G(t)$ of the second relay becoming dormant is equal to:

$$G(t) = (1 - e^{-\mu t})^2$$  \hspace{1cm} (3.40)

The calculation of $M_{n2}(t)$ is similar to that of $M_{n1}(t)$, but instead of $G(t)$ from equation (3.33) $G(t)$ from equation (3.40) has to be used. The result can be substituted in the following equation for the average cost per year $C$.

$$C = C_n \mu + C_t \frac{\mu}{2} M_{n2} \left( \frac{2}{\mu} \right)$$  \hspace{1cm} (3.41)

**Model R3:**

In this model there are two relays that are maintained in turns. If $t \in \{ ..., -2/\mu, 0, 2/\mu, \ldots \}$ one of the relays is maintained, and if $t \in \{ ..., -1/\mu, 1/\mu, 3/\mu, \ldots \}$ the other relay is maintained.

An interval with length $1/\mu$, starting with the maintenance of one of the relays, will be studied. After the maintenance at $t = 0$ one relay will be healthy and $G_s(t)$ is the distribution function of its TTF.

$$G_s(t) = 1 - e^{-\mu t}$$  \hspace{1cm} (3.42)

The other relay has got a non-zero probability of being dormant already at $t=0$. This probability depends on the time that has elapsed since the last maintenance ($1/\mu$ ago) or repair (due to a FtO). In this model the assumption is made that failures to operate are rare and did not occur in the last regenerative interval. Then $G_b(t)$ is the distribution function of the TTF of this relay.
\[ G_b(t) = G_a \left( t + \frac{1}{\mu} \right) = 1 - e^{-\lambda t} e^{-\mu t} \quad (3.43) \]

Then, the time to the first F\text{tO} is distributed according to \( F_1(t) \).

\[ F_1(t) = (G_a(t) G_b(t)) \cdot H(t) \quad (3.44) \]

Since after the first F\text{tO} both relays are repaired, the time between the first and second F\text{tO} and after that between all other F\text{tOs} are distributed like in model R2. Now, the following relation applies.

\[ M_{R3}(t) = \sum_{n=1}^{\infty} n P(N(t) = n) = \sum_{n=1}^{\infty} P(N(t) \geq n) = \sum_{n=1}^{\infty} P(S_n \leq t) \quad (3.45) \]

\[ = F_1(t) + F_1(t) \cdot \sum_{n=2}^{\infty} P(S_n - S_1 \leq t) = F_1(t) + F_1(t) \cdot M_{R2}(t) \]

By evaluation of this expression and substitution of the result in equation (3.46) for the average cost per year \( C \), the optimal maintenance parameter \( \mu \) can be found.

\[ C = C_n \mu + C_1 \mu M_{R3}(1/\mu) \quad (3.46) \]
3.4 Petri nets

3.4.1 General description

In 1962, in his PhD. dissertation, C.A. Petri defined a technique for modelling systems with dependent parallel processes. In these models actions are possible if a finite number of preconditions are satisfied. A digraph, in which the vertices can be split up into two disjoint classes, is used to represent the system. These graphs are referred to as Petri nets (PNs).

Figure 3.5 shows an example of a Petri net. The vertices represented by a circle stand for places or states. In our example, they are given a name consisting of a number preceded by a capital P. The vertices represented by a line stand for transitions that can occur in a system. Their name consists of a number preceded by a capital T. The states connected to transitions by an arc directed to the transition are called the incoming states of the transition. The states connected to a transition by an arc directed to the state are called the outgoing states of the transition. There are no arcs connecting two places or two transitions. A transition can only fire when all incoming states are active. The firing of a transition deactivates all incoming states and activates all outgoing states.

PNs are used to describe the causal logic of systems. For instance, the PN in figure 3.5 can be used to describe the chemical relations between six chemical products, P1 to P6.

The presence of products P1 and P2 is needed to produce product P3. Product P3 can decompose spontaneously producing P4, P5 and P6, and so on.

In order to keep track of the active states, marks or tokens are introduced. In the graphical representation a mark is drawn in all states that are active. In mathematical notation a rowvector is defined. The k'th element of this vector will be 0 if state Pk is not active. Otherwise, it will be 1. The mathematical or graphical specification of the places that are marked is called a marking. Which markings can be reached depends on the initial set of states that is active, the initial marking. For the example given in figure 3.5 the initial marking (1, 1, 0, 0, 0, 0) is chosen. In figure 3.6 the marking before and after firing of transition T1 are shown.
Figure 3.7 Reachability graph of the example PN.

Given an initial marking the set of situations that can be reached can be determined. From this, a reachability graph can be made in which the vertices are all different markings that can occur in the PN with a certain initial situation. The directed arcs of the reachability graph show the possible transitions between two markings. Figure 3.7 shows the reachability graph of the PN from figure 3.5 with initial marking \((1,1,0,0,0,0)\).

Up to this point the basics of PN theory have been discussed. All applications show the issues mentioned above. Apart from that, depending on the application, different extensions have been given.

- It is possible to allow more than one mark in a state. The vector of the marking then shows the number of marks in each state.
- It is possible that in one place more than one mark is deleted or created when firing one transition. If so, a transition is only possible if the incoming states have at least as much marks as required for this transition.
- A special kind of arc can be used from ingoing places to transitions. These arcs, called inhibitor arcs, have the property that the transition will be disabled if a mark is present in the ingoing state.
- A priority of firing may be defined if there is a conflict. If two or more transitions have one or more incoming places in common, marks in these places are necessary to enable both transitions. Firing of one transition will disable firing of the other. A priority list can be used to define which transition will fire.
- There are two ways to introduce time in PN's. The first uses firing durations. This means there is a delay between removal of marks in incoming places and creation of marks in outgoing places. The second one uses firing delays. Here the transitions fire only after the incoming states have been continuously marked for a certain time. For both ways a special case called an immediate transition can be defined as a timed transition with delay and duration equal to zero.

Up to here no stochastics have been mentioned. The way stochastics can be introduced depends on the application. A summary of the most important possibilities is presented.

- The introduction of exponentially distributed firing delays by Molloy [41], Natkin [42] and Symons [48] was the first attempt to include stochastics in PNs. Molloy proved the result is isomorphic to continuous time Markov chains described in section 3.2 and named this modelling tool stochastic Petri nets (SPN). Being isomorphic means that for every state of one model exactly one state of the other model can be specified so that the models have the same properties.
- Marsan, Balbo and Conte [37] introduced generalised stochastic Petri nets (GSPN) in which two types of transitions are defined namely, transitions with exponentially distributed firing delays and immediate transitions. If two conflicting immediate transitions are enabled, the decision of which transition will fire is taken by chance. This model is still isomorphic to continuous time Markov chains.
- Marsan and Chiola [38] introduced deterministic stochastic Petri nets (DSPN) as a further extension of SPNs. In DSPNs a transition may be associated with an
exponentially distributed or a deterministic firing delay. An analytical technique for the calculation of steady state possibilities has been developed under the assumption that in no possible marking of the DSPN two or more deterministic transitions are concurrently enabled.

- Extended stochastic Petri nets (ESPN) in which arbitrary distributions can be associated with firing delays have been introduced by Dugan, Trivedi, Geist and Nicola [16]. ESPNs can be investigated using simulation.
- Timed Petri nets (TPN) use stochastic transition durations instead of stochastic transition delays. This means the marks of incoming places are removed immediately when transition are enabled and are put in the outgoing places only after a time associated to an exponential distribution. Reference [51] is a review on this subject.
- An extension of this technique is called generalised timed Petri nets (GTPN) [25] in which arbitrary distributions can be associated with firing durations. Because it has more possibilities than Markov theory but still is an analytical technique, in this section DSPNs will be used to model the stochastic aspects of protection systems.

3.4.2 Deterministic stochastic Petri net theory

Before describing a method for calculating the steady-state distribution of DSPNs, some general terms and properties will be introduced. In the next paragraph three models will be presented that are based on DSPNs.

Deterministic transitions fire if they are enabled without interruption for a given time $\tau_d$. This is called a race with enabling memory.

To calculate the steady state distribution we define a continuous time Markov chain in which the states $S_i$ are related to the PN markings $M_i$. It is called the embedded Markov chain or EMC. The transition rates are chosen in a way that the steady state probability of the corresponding state in the two models are related, but they are not equal in every case. For this, we are interested in the mean sojourn times of the system in certain (groups of) markings and transition chances between them. With respect to this we can distinguish three kinds of markings:

a. Markings in which one deterministic transition and no exponential transitions are enabled.

b. Markings in which only exponential transitions are enabled.

c. Markings in which one deterministic transition and at least one exponential transition are enabled.

For these three categories of markings we will discuss here how they are related to the EMC.

ad a: A marking $M_i$ with just one deterministic transition corresponds to one state $S_i$ of the EMC. Although in this marking $M_i$ there is only one transition enabled, because of the presence of immediate transitions more than one marking $M_j$ can be the next nonvanishing marking with, $P(M_i \rightarrow M_j) > 0$. Markings in which immediate transitions are enabled are called vanishing markings and are not represented in the EMC because it is impossible to explicitly model immediate transitions in Markov theory. All markings $M_j$ that can become the next nonvanishing marking after firing of the deterministic transition in $M_i$, are each represented by a state $S_j$ in the EMC. The transition rates $\lambda(S_i \rightarrow S_j)$ are given by the following equation.
A marking $M_j$ with only exponential distributions is also represented by one state $S_j$ in the EMC. The markings $M_j$ that can become the next nonvanishing marking after $M$, each are represented by one state $S_j$ in the EMC. The mean sojourn time in state $S_j$ is $1/\lambda_j$. Where $\lambda_j$ is the sum of all transition rates $\lambda(S_i \rightarrow S_j)$. These rates are equal to the corresponding rates in the Petri net.

When both deterministic and exponential transitions are enabled the representation in the EMC is much more complicated. The nonvanishing marking $M$, which is reached first, in which the deterministic marking $Td$ with transition delay $\tau_d$ is enabled is represented by state $S$ in the EMC. From here on, not all transitions in the Petri net are represented by a transition in the EMC. Exponential Petri net transitions, which do not disable the deterministic transition, are not represented by one transition in the EMC. In the EMC two kinds of transitions can occur, preemptable transitions, which disable the deterministic transition, and the deterministic transition itself. In both kinds of transitions it is taken into account that in the Petri net any number of exponential transitions could have fired before the corresponding transition fires.

With respect to the deterministic transition $Td$ the markings can be divided in two groups. Those in which the deterministic transition is enabled belong to the group MD. Those who disable the deterministic transition belong to group MP. When the deterministic transition fires, the resulting marking can be in either group. When placing the states of group MD first the infinitesimal generator $Q$ of the transitions between these markings is as follows.

$$Q = \begin{bmatrix} D & P \\ Q_{21} & Q_{22} \end{bmatrix}$$

Here $D$ contains the transition rates of exponential transitions between markings in MD and $P$ contains transition rates of exponential transitions from markings of group MD to group MP. When all members of group MP are defined as absorbing states from which no exit is possible, the following infinitesimal generator $Q'$ applies.

$$Q' = \begin{bmatrix} D & P \\ 0 & 0 \end{bmatrix}$$

In the EMC all markings $M_j$ that can be reached by firing of $Td$ or by the firing of a preemptable transition are represented by a state $S_j$. This is illustrated in figure 3.8. The chance of reaching these markings the next time $Td$ fires or is preempted is equal to.

$$P(S_i \rightarrow S_j) = \psi_i \exp^{\psi_i \tau_d} \Delta_d \psi_j^T$$

Here $\psi_i$ is a rowvector of appropriate length of all zeros but the $i^{th}$ entry that is a one. The distribution matrix $\Delta_d$ is defined as follows.

$$\Delta_d = \begin{bmatrix} \Delta_d,d & \Delta_d,p \\ 0 & 1 \end{bmatrix}$$

The upper part holds the effects of the firing of transition $Td$. The bottom part represents the situation in which $Td$ is preempted.
Now we can manipulate the transition rates of the EMC so that the expected sojourn time in state $S_i$ is equal to the mean sojourn time in class MD. This mean sojourn time $E(t_{MD})$ is given by equation.

$$E(t_{MD}) = \sum_{i \in S_{EMD}} u_i \begin{bmatrix} D^{-1}(e^{\alpha u} - I) & 0 \\ 0 & 0 \end{bmatrix} u_j$$

(3.52)

The transition rate between states $S_i$ and $S_j$ in this situation is equal to.

$$\lambda(S_i \rightarrow S_j) = \frac{P(S_j \rightarrow S_i)}{E(t_{MD})}$$

(3.53)

From the steady state distribution of the EMC it is possible to compute the steady state distribution of the DSPN. The steady state probability $P(M)$ of a marking $M$ that enables only exponential transitions or only one deterministic transition is equal to the steady state probability $P(S)$ of the corresponding state $S$ of the EMC.

The steady state probability $P(S)$ of a state $S$ of the EMC that represents the presence in a class of markings MD contributes to the steady state probability $P(M)$ of marking $M \in MD$ as follows.

$$\text{contribution of } P(S) \text{ to } P(M) = \frac{P(S)}{E(t_{MD})} \begin{bmatrix} D^{-1}(e^{\alpha u} - I) & 0 \\ 0 & 0 \end{bmatrix} u_j$$

(3.54)

Taking the sum of the contributions of all states $S_i$ related to $M$ leads to the steady state probability $P(M)$. Using this technique, the whole distribution of the DSPN can be determined.

Since the matrixexponential can mostly not be evaluated analytically, it has to be approximated numerically. Computer programs for evaluating DSPNs are available [36], but because the models presented in this section could be evaluated using a regular mathematical program they were not used.
The advantages of DSPN are:
• Both exponential and deterministic transition delays can be modelled.
• An analytical method exists for finding the equilibrium distribution.
• Parallel processes can be modelled.

The disadvantages are:
• DSPNs, as other PNs, are difficult to design.
• When the model is complicated large computational efforts are to be expected to calculate the equilibrium distribution.

3.4.3 Protection models

Model PN1:
This model is identical to model R1. There is one relay and maintenance is performed when \( t \in \{0, 1/\mu, 2/\mu, \ldots\} \). The Petri net of this model and its initial marking are shown in figure 3.9. The mark in P1 represents the relay that is healthy. The mark in P3 is part of the maintenance structure of the model. The three kinds of transitions present in the model are defined as follows.

- Hollow bar: Exponential transition with a rate printed next to it.
- Solid bar: Deterministic transition with a transition delay printed next to it.
- Thin bar: Immediate transition with transition delay equal to zero.

![Figure 3.9 Petri net of model PN1.](image)

The transition causing the mark to move from P1 to P2 with rate \( \rho \) represents the failure of the relay. The transition in the opposite direction with rate \( \lambda \) represents a failure to operate (FtO). At \( t = 1/\mu \) the deterministic transition will cause the mark in P3 to move to P4. After this, one of the immediate transitions will fire causing the initial marking to reappear. The arc that ends with a circle is a prohibiting arc. This means that the transition involved can only fire if P2 is unmarked.

![Figure 3.10 Reachability graph of model PN1.](image)

The reachability graph of this model is shown in figure 3.10. In this graph two different types of transitions can be distinguished. Solid arcs represent exponential transitions while dashed arcs represent deterministic transitions. Immediate transitions are represented by
the deterministic or exponential transition it succeeds.
Since model PN1 is identical to model R1 no mathematical evaluation has been performed
for model PN1.

Model PN2:

Model PN3:
This model contains two relays. Maintenance is performed in turns at \( t \in \{0, 1/\mu, 2/\mu, \ldots\} \).
Because in model R3, the assumption is made that in the previous regenerative interval no
FtOs occurred PN3 and R3 are not identical. That is why a mathematical evaluation of model PN3 will follow here.

The Petri net of model PN3 will be presented in two stages. Figure 3.13 contains the part of the PN that is similar to model M3. If the places P1 to P4 are marked, they represent:

- **P1**: Both relays are healthy.
- **P2**: The relay, which will not be maintained next, is dormant while the other is healthy.
- **P3**: The relay, which will be maintained next, is dormant while the other is healthy.
- **P4**: Both relays are dormant.

In this part of the Petri net all the exponential transitions of the total Petri net are included. The infinitesimal generator $X$ is defined by:

$$
X = \begin{bmatrix}
-2\rho & \rho & \rho & 0 \\
0 & -\rho & 0 & \rho \\
0 & 0 & -\rho & \rho \\
\lambda & 0 & 0 & -\lambda
\end{bmatrix}
$$

Figure 3.13  Part of the Petri net of model PN3.

Figure 3.14  Petri net of model PN3.
The total PN is shown in figure 3.14. Here places P5 and P6 again are part of the maintenance mechanism. At \( t = 1/\mu \) maintenance will be performed. The mark in P6 causes one of the immediate transitions to fire by which P5 will be marked again and one of the following actions will occur.

If P1 is marked, no other marks will move.
If P2 is marked, its mark will be moved to P3.
If P3 is marked, its mark will be moved to P1.
If P4 is marked, its mark will be moved to P3.

The reachability graph is shown in figure 3.15. Note the similarity between the graphs of the Markov models M1, M2 and M3 in figure 3.4 and the reachability graphs of models PN1, PN2 and PN3.

\[ \begin{align*}
1 & = (1, 0, 0, 0, 0) \\
2 & = (0, 1, 0, 0) \\
3 & = (0, 0, 1, 0) \\
4 & = (0, 0, 0, 1) \\
\end{align*} \]

Figure 3.15 Reachability graph of model PN3.

Two distributions \( \pi_{a,k} \) and \( \pi_{b,k} \) are defined as the distribution of the reachability graph before and after the \( k \)-th maintenance respectively. The following recursive relation applies.

\[
\begin{align*}
\pi_{b,k} &= (\pi_{a,k} + m_{a,k} + \pi_{a,k}), 0, 0, 0, 0, 0) \\
\pi_{a,k+1} &= \pi_{b,k} e^{Xt} \\
\end{align*} \quad k \in \mathbb{N} \tag{3.56}
\]

Here \( \pi_{a,k} \) is the \( k \)-th element of \( \pi_{a,k} \).

For \( k \) to infinity these distributions will converge to \( \pi_a \) and \( \pi_b \) respectively.

\[
\begin{align*}
\pi_a &= (\pi_{a1}, \pi_{a2}, \pi_{a3}, \pi_{a4}) = \lim_{k \to \infty} \pi_{a,k} \\
\pi_b &= (\pi_{b1}, \pi_{b2}, \pi_{b3}, \pi_{b4}) = \lim_{k \to \infty} \pi_{b,k} \tag{3.57}
\end{align*}
\]

These equilibrium distributions can be calculated by solving the following set of linear equations that evolves from (3.56).

\[
\begin{bmatrix}
\pi_{a1} \\
\pi_{a2} \\
\pi_{a3} \\
\pi_{a4} \\
\pi_{b1} \\
\pi_{b2} \\
\pi_{b3} \\
\end{bmatrix} = 
\begin{bmatrix}
0 & e^{x_{11}} & e^{x_{12}} & e^{x_{13}} & e^{x_{14}} & 1 & 0 & 1 & 0 \\
0 & e^{x_{21}} & e^{x_{22}} & e^{x_{23}} & e^{x_{24}} & 0 & 0 & 1 & 0 \\
0 & e^{x_{31}} & e^{x_{32}} & e^{x_{33}} & e^{x_{34}} & 0 & 0 & 0 & 1 \\
\end{bmatrix} 
\begin{bmatrix}
\pi_{a1} \\
\pi_{a2} \\
\pi_{a3} \\
\pi_{a4} \\
\pi_{b1} \\
\pi_{b2} \\
\pi_{b3} \\
\end{bmatrix} \tag{3.58}
\]

Where \( e^{x_{ij}} \) is the element in the \( i \)-th row and \( j \)-th column of the matrixexponential \( e^{Xt} \) for \( t = 1/\mu \). The following additional constraint applies.
$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \quad (3.59)$$

The average probability that the system is in state 4 of the reachability graph $\rho_{-4}$ is defined by:

$$\rho_{-4} = \lim_{t \to \infty} \frac{1}{t} \int_0^t P(\text{system in state 4 of the reachability graph at time } x) dx \quad (3.60)$$

It is equal to:

$$\rho_{-4} = \mu \int_0^{\mu} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} dt \quad (3.61)$$

Next $\rho_{-4}$ will be substituted in the following cost function.

$$C = C_n + C_l \rho_{-4} \quad (3.62)$$

This is the desired result. The numerical results of this model and model R3 are compared in section 3.6.
3.5 Monte Carlo simulation

3.5.1 Theory

The Monte Carlo simulation technique differs from those of sections 3.2, 3.3 and 3.4 by the fact that it is not analytical but uses statistics. For detailed information on simulation the reader is referred to [30]. Instead of analysing the stochastic system it is simulated over and over again until the desired accuracy is reached. The simulation of stochastic events involves a random or pseudo-random generator that generates numbers $X_i$ on the interval $[0,1)$ with a uniform distribution.

$$\forall i \in \mathbb{N} \quad P(X_i \leq x) = \begin{cases} 0 & \text{for } x \in (-\infty,0) \\ x & \text{for } x \in (0,1) \\ 1 & \text{for } x \in [1,\infty) \end{cases} \quad (3.63)$$

The advantage of a pseudo-random generator over a random generator is property that results can be reproduced. One possibility these numbers can be realised in a way that they can be reproduced is by use of the following recursive formula.

$$X_{i+1} = \frac{(X_i \cdot a \cdot n) \mod n}{n} \quad (3.64)$$

The values of $a$ and $n$ must be carefully chosen. In the simulation examples of section 3.6 the following values used, $a = 950706376$ and $n = 2^{31}-1$. The value of $X_0$ (in the examples equal to 1) is called the seed and has to be specified by the user.

A set of stochastic variables $Y_i$ with distribution function $F(y)$ can be realised by using the realisations of $X_i$ in the following way:

$$Y_i = F^{-1}(X_i) \quad (3.65)$$

With $F^{-1}(x)$ the inverse of the distribution function. In case of an exponential distribution with parameter $\lambda$ this becomes:

$$Y_i = \frac{\ln(1-X_i)}{-\lambda} \quad (3.66)$$

The advantages and disadvantages will be discussed here. The advantages are:

- In theory, all systems, that can be thought of, can also be simulated.
- Little mathematical knowledge is required.
- When the goal is to obtain the index numbers in one single situation, even for simple systems that can be evaluated analytically, simulation is a competitive technique.

The disadvantages are:

- The results of simulation are statistical results.
- Simulation is time consuming if the system is complex, the desired precision is high or the index numbers have to be known for many values of the system parameters. A combination of all three factors leads to unacceptable computational efforts.
- When evaluating an performance measures as a function of a system parameter, the same time-consuming simulation has to be performed for different values of this
41

Figure 3.16 Situations on which the simulation models are based.

system parameter. After that a best fit technique has to be performed to the statistical results of the simulation to get a functional dependency.

3.5.2 Protection models

Three models S1, S2 and S3 will be simulated. Their structure is similar to that of the models of the previous sections. However, different situations are modelled. Figure 3.16 shows the two situations that are modelled. In both situations a double circuit line is shown. The electrical energy is transported from the left to the right.

- Short circuits can occur in both circuits (rate \( \lambda \) per circuit) after which the protection will normally disconnect the circuit involved. The other circuit can transport enough energy so no interruption will be experienced by the users at the end of the line.
- Since the repair time is neglected there can be no short circuit in one circuit while the other is being repaired.
- All relays can fail. The TTF is exponentially distributed with parameter \( p \).
- If a short circuit occurs in a circuit where at one end all relays are dormant, a FtO will occur. The user will experience an interruption of supply and all relays in the circuit are repaired.
- After maintenance is performed on a group of relays, these relays will be healthy. The maintenance strategies used, are the same as defined in section 3.1.

Table 3.2 shows an overview of the models and its properties. Model S1 has been evaluated using both simulation and Markov theory. Model S2 has been evaluated using both simulation and renewal theory. Because of its complexity, model S3 was only evaluated using simulation.

<table>
<thead>
<tr>
<th>name</th>
<th>theory</th>
<th>situation</th>
<th>maintenance strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>simulation/Markov</td>
<td>3</td>
<td>#1</td>
</tr>
<tr>
<td>S2</td>
<td>simulation/renewal</td>
<td>3</td>
<td>#3</td>
</tr>
<tr>
<td>S3</td>
<td>simulation</td>
<td>4</td>
<td>#3</td>
</tr>
</tbody>
</table>

With the assumptions mentioned, the two circuits are stochastically independent and can be modelled separately. The total number of FtOs is thus double the number of FtOs due to short circuits in one circuit. In figure 3.17 a flow chart is shown by which the simulation
of the three models is performed. Because the models are all regenerative, with the moments at which maintenance is performed as regeneration moments, only one regenerative interval was simulated a large number of times.

By $TTU$ is meant the time to the moment where the circuit is unprotected. It is equal to the time to the moment where all relays at one end of the circuit are dormant.

By $\min\{\text{Exp}_2+p, \text{Exp}_2+p\}$ is meant the minimum of two realisations of a variable with distribution function $\text{Exp}_2+p$. A realisation of a variable that is distributed $\text{Exp}_2+p$ is equal to the sum of a realisation of a variable that is exponentially distributed with parameter $2p$ and a realisation of a variable that is exponentially distributed with parameter $p$.

When determining the time to short circuit ($TTSH$), the memoryless property of the exponential distribution is used. At the moment the circuit becomes unprotected the $TTSH$ is still exponentially distributed with parameter $\lambda$.

In model $S1$, the time between two successive regenerative moments is exponentially distributed with parameter $\mu/4$ which means that for the simulation of every regenerative interval a value with this distribution function has to be realised. In the other models the length of the regenerative interval is constant and equal to $4/\mu$ (model $S2$) or $8/\mu$ (model $S3$).

The average cost per unit of time for all models is given by:

$$C = C_0 \mu + \frac{2 \cdot C\text{FtOs}}{\text{total simulation length}}$$

Here the total simulation length is the sum of the simulation lengths of all simulations. An estimation of the expected cost per time unit, with certain values of $\mu, \lambda$ and $p$, then can be obtained by simulation of this system.
3.6 Results

3.6.1 Results of analytical techniques

In the previous sections of this chapter four theories and twelve models, three for every theory, were presented. In this chapter the results of calculations with these models are presented. Because the models R1 and PN1 are identical, only calculations have been performed using model R1. The same is true for models R2 and PN2. The models R3 and PN3 differ by the presence (R3) or absence (PN3) of the assumption that there has been no FtO in the previous regenerative period.

All models have four variables namely the transition rates $\lambda$ and $\rho$, the maintenance parameter $\mu$ and the ratio $C_r/C_m$. In real systems, all variables but $\mu$ are difficult to influence. Because of this, first the average cost per year will be calculated as a function of $\mu$ (figure 3.18). The other parameters were initially chosen to be equal to:

- $\lambda = 1/10$ years
  This is approximately equal to the failure rate of 10 km of cable or line. Different values will be used later on.

- $\rho = 1/20$ years
  This is an arbitrary value. It will be the same for all simulations.

- $C_r/C_m = 100$
  This is an arbitrary value. Later on this value will be varied over four orders of magnitude.

In this figure the solid lines are the results for the Markov models, the course dashed lines are the results for the renewal models and the fine dashed line is the result for model PN3. The lines that correspond to models R3 and PN3 are hard to tell apart in the calculations performed. So, for these conditions the extra assumption of model R3 that no FtO occurred in the previous renewal interval, appears to be justified. The straight line through the origin represents the average cost of maintenance per year that is proportional to the maintenance parameter $\mu$. The difference between the straight line and each curve is proportional to the expected number of FtOs. The lower the curve, the more effective the system design and the maintenance strategy at the given maintenance frequency.

![Figure 3.18 The average cost per year as a function of the maintenance parameter $\mu$.](image)

The minimum corresponds to the optimal frequency. The sum of maintenance costs and interruption costs is at its lowest possible value. A reduction of the maintenance frequency would lead to reduction of maintenance costs, but this would be more than compensated by an increase in interruption costs.

The optimal value of the maintenance parameter $\mu$ is found by determination of the
Figure 3.19 The optimal cost and maintenance frequency $\mu$ as a function of the ratio $C_i/C_m$ with $\lambda = 1/250$ years.

Figure 3.20 The optimal cost and maintenance frequency $\mu$ as a function of the ratio $C_i/C_m$ with $\lambda = 1/10$ years.

Figure 3.21 The optimal cost and maintenance frequency $\mu$ as a function of the ratio $C_i/C_m$ with $\lambda = 1/1$ years.
minimum in the corresponding line in figure 3.18. The minimal cost is different for different models. For Markov models it is essentially bigger than for the corresponding renewal models. So, if in practice a constant maintenance interval is applied, this cannot realistically be modelled by an exponentially distributed maintenance interval. If this is still done, the optimal \( \mu \) found in the model, will not really be the optimal \( \mu \) in reality. The minimal cost will be overestimated.

In the situation modelled in figure 3.18 the minimum of the models with two relays is smaller than the minimum of the models with one relay. And, maintenance in turns is better than performing maintenance to all relays at the same time. In practice however, the maintenance per relays might be more expensive if maintenance is performed in turns. Which of these two effects is the largest can be calculated when all system parameters are known.

When varying the ratio \( C_i/C_m \) the minimal cost and optimal \( \mu \) become functions. For \( C_i/C_m \in [10, 10^5] \) these are shown in figures 3.19, 3.20 and 3.21 in log-log graphs for \( \lambda = 1/250 \) years (transformer or busbar), \( \lambda = 1/10 \) years (10 km of cable or line) and \( \lambda = 1/1 \) year (100 km of line) respectively. In every figure two graphs are shown. The left graph shows the minimal cost and the right graph shows the optimal frequency for all analytical models.

From these figures it can be concluded that:

- Markov models lead to higher minimum cost per year than the corresponding renewal models or Petri net models. It seems that regular times between maintenance have to be preferred over irregular maintenance intervals with equal expected value.
- The optimal cost for a system with two relays is lower than for a system with one relay. This is not surprising since two relays fail to operate less than one relay. Note that the relays is are not taken into account. The difference between the two systems (e.g. M1 and M2) should account for the additional costs of the second relay.
- The minimal cost of models with maintenance in turns is lower than the model based on the same theory with maintenance at the same time for both relays.

It is remarkable that, for models with two relays, multiplication of the cost of a FtO by 100 (shifting 2 to the right in the log-log graph) only increases the cost per year by a factor 3. At the right of the graphs, \( C \) is proportional to \( (C_i/C_m)^{1/2} \) if there is one relay in the model and proportional to \( (C_i/C_m)^{1/3} \) if the model has two relays.

---

**Figure 3.22** The average cost per year of models S1, S2 and S3 as a function of maintenance parameter \( \mu \) with \( C_i/C_m = 100 \).
3.6.2 Results of simulation

The calculations performed on the simulation models are shown in figures 3.22 ($C_r/C_m = 100$) and 3.23 ($C_r/C_m = 10^5$). Models S1 and S2 have been evaluated both analytically and by use of simulation. Model S3 was only evaluated using simulation. The graphs are similar to the one in figure 3.18.

![Graph](image)

Figure 3.23 The average cost per year of models S1, S2 and S3 as a function of maintenance parameter $\mu$ with $C_r/C_m = 10000$.

For different values of the maintenance parameter $\mu$ the average cost per year was calculated. The simulation results were obtained by simulating the regenerative period 100,000 times for 50 different values of $\mu$. Because of the noisy shape of the simulation results, the minimum is hard to determine. A best fit method should be used for it. The relative positions of the lines in figures 3.22 and 3.23 are similar to those in figure 3.18. The models with maintenance strategy #1 are more expensive then the ones with maintenance strategy #3 and two redundant relays lead to a lower minimal cost then no redundancy in the protection system.
Chapter 4
Conclusions and directions for future research

The model presented in section 2.4 can be used to start a discussion on standardising the treatment of stochastic phenomena of protection systems. The large variety of terms and descriptions in the review, presented in section 2.3, shows that there is a need for this. Such a standardised model can then be taken as a basis for applications, neglecting some of the phenomena or adding new phenomena to it. Also, the standardised model can be a basis for the collection of service statistics.

In chapter three, four techniques (Markov Theory, renewal theory, Petri nets and simulation), are described and used for modelling systems, consisting of a part of a power system including one or more protection systems. The phenomenon that was modelled was the failure to operate of the protection system. From this chapter it may be concluded that, taking into account their restrictions, all methods can be used to study the stochastic behaviour of such a simple power system. Using analytical techniques, Markov theory, renewal theory and Petri nets, it is more easy to obtain functional dependencies than by using the statistical simulation. However, when more complicated systems are concerned, this advantage is outweighed by the disadvantage that only reasonable simple systems can be modelled. The simulation technique, however, can be used for modelling highly complicated systems. The biggest disadvantage of the analytical techniques is that they are limited to relatively simple systems. The biggest disadvantage of simulation is that it might be very time and memory consuming to come to useful statements. For a more detailed overview of this, the reader is referred to figure 3.2.

Under the conditions as described in sections 3.1 and 3.5, the following conclusions can be drawn from the results presented in section 3.6.

- By using stochastic analysis, it is possible to obtain the economic optimum of a system in which maintenance cost and failure cost occur.
- When taking into account the higher cost of investment of two protection systems over one, with respect to the cost of a failure to operate, there is a point above which it is beneficial to install a second protection system with the same function.
- The difference between systems with exponentially distributed maintenance intervals and systems with constant maintenance intervals is significant. This means that simplification of the latter by application of the former will not lead to accurate results.
- Systems with exponentially distributed maintenance intervals are more expensive than systems with constant maintenance intervals.
- In systems with two protection systems, which perform the same function, it is beneficial to maintain both systems alternatingly instead of simultaneously.

Stochastic techniques can be used to model bigger systems. For this, the simulation technique can always be used. The biggest concerns then are the computation time, memory constraints and the processing of data. If Markov theory is used the modelling
abilities are restricted to exponential distributions. It is however the easiest method to handle and to use for calculations. Renewal theory is probably not useful for modelling bigger systems. Finally, in theory Petri nets can be used, but they are difficult to design. Apart from the abilities of Markov theory deterministic transition times can be modelled. A special computer program is needed to calculate the equilibrium distribution [36, 38]. When modelling larger systems, there are some interesting research questions that can be investigated:

- The protection systems in a substation can be split up into parts with a certain function as described in paragraph 2.2.3. If the times between failures of all parts and the effects of the failures are known, by studying different configurations, the best one can be found, like finding out whether application of one or two measurement transformers for one signal is optimal.
- The use of digital relays makes it possible to perform several different protection functions in one relay. What is the effect of this on the reliability of the supply?
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Appendix 1

Frequently used distribution functions

The distribution function of a positive stochastic variable, e.g. a time to failure of a light bulb TTF, is defined by:

\[ F(t) = P(TTF \leq t) \quad \text{for } t \geq 0 \]  

(A.1)

The distribution function is monotonously not falling, can adopt values in the range [0,1] and is not necessarily continuous and differentiable.

If it exists, the derivative of the distribution function is called the probability density function \( f(t) \).

\[ f(t) = \frac{dF(t)}{dt} = \lim_{\Delta t \to 0} \frac{P(t < TTF \leq t + \Delta t)}{\Delta t} \]  

(A.2)

The third important function that we will define here is the failure rate \( \lambda(t) \) that is equal to the probability density function given the fact that the bulb is still burning.

\[ \lambda(t) = \lim_{\Delta t \to 0} \frac{P(t < TTF \leq t + \Delta t | TTF > t)}{\Delta t} \]  

(A.3)

\[ = \lim_{\Delta t \to 0} \frac{P(t < TTF \leq t + \Delta t \cap TTF > t)}{\Delta t P(t < TTF)} \]  

(A.4)

\[ = \frac{f(t)}{1 - F(t)} \]

Distribution functions can be characterised by its moments \( \mu_1, \mu_2, \ldots \). These are defined by the following Riemann-Stieltjes integral.

\[ \mu_n = \int_0^\infty t^n dF(t) \]  

(A.5)

The parameter \( \mu_n \) is called the \( n^{th} \) moment of the distribution function. If the distribution function is continuous and differentiable, this can be written as:

\[ \mu_n = \int_0^\infty t^n f(t) \, dt \]  

(A.6)

The first moment is also called the expected value of the random variable \( X \).

\[ \mu_1 = E\{X\} = \int_0^\infty t \, dF(t) \quad \text{if } X - F(t) \]  

(A.7)

Where the operator ' \( - \) ' means that the stochastic variable \( X \) is distributed according to the function \( F(t) \). The standard deviation \( \sigma \) of random variable \( X \) is equal to:

\[ \sigma(X) = \sqrt{\mu_2 - \mu_1^2} \]  

(A.8)

If we know the distribution function of two stochastic variables, then what is the
distribution function of the sum of the two variables? Let \( X \sim F(t) \) and \( Y \sim G(t) \) then, if they are mutually independent:

\[
X + Y \sim F(t) * G(t) = F(X + Y \leq t) = \int F(t-x) \, dG(x)
\]

\[
= \int G(t-x) \, dF(t)
\]

\( (F(t) * G(t)) \) is called the convolution of \( F(t) \) and \( G(t) \). If \( X \) and \( Y \) are identically distributed, the distribution function of the sum is called the autoconvolution of \( F(t) \). In general we have the recurrent relation.

\[
X_1 + X_2 + \ldots + X_n \sim F^{[n]}(t) = F^{[n-1]}(t) * F(t)
\]

Here \( F^{[n]}(t) \) is the \( n \text{th} \) autoconvolution of \( F(t) \).

Some important distributions, which are used frequently for modelling purposes, will be discussed here.

From theoretical point of view, the most important one is the exponential distribution. It has the property of memorylessness. This is probably explained most directly by the fact that the failure rate function is constant in time. If the stochastic variable represents the TTF, of for instance a new light bulb, the exponential distribution means that the bulb does not age or rejuvenate. Or, the bulb does not know how long it burned. The distribution function of the TTF of an old bulb that did not fail is the same as that of a new one. The exponential function is the only one with this property. Because of this, the function is very useful for modelling purposes. In case of the exponential distribution, the three functions, defined before, look like this.

\[
\text{Exp}(\lambda) : \quad F(t) = 1 - e^{-\lambda t} \quad f(t) = \lambda e^{-\lambda t} \quad \lambda(t) = \lambda \quad \text{for } t \geq 0 \quad (A.11)
\]

If there are two variables, \( X_1 \) and \( X_2 \), which are independent and exponentially distributed with parameters \( \lambda_1 \) and \( \lambda_2 \) respectively, then the minor of both realisations is exponentially distributed with transition rate equal to \( \lambda_1 + \lambda_2 \).

\[
\text{if} \quad X_1 \sim \text{Exp}(\lambda_1) \quad \text{and} \quad X_2 \sim \text{Exp}(\lambda_2)
\]

\[
\text{then} \quad \min\{X_1, \, X_2\} \sim \text{Exp}(\lambda_1 + \lambda_2) \quad (A.12)
\]

The last property of the exponential distribution to be discussed here can be formulated as follows. If a certain event takes place with interoccurrence times that are exponentially distributed with parameter \( \lambda \) but is only selected with a probability \( p \), then the interoccurrence times of selected events are \( \text{Exp}(p\lambda) \) distributed.

The next distribution to be discussed is the gamma distribution \( \Gamma(\alpha, \lambda) \). The density function \( f(t) \) and distribution function \( F(t) \) are given by.

\[
\Gamma(\alpha, \lambda) : \quad F(t) = \frac{1}{\Gamma(\alpha)} \int_0^t u^{\alpha-1} e^{-\lambda u} \, du \quad f(t) = \frac{\lambda^{\alpha} t^{\alpha-1} e^{-\lambda t}}{\Gamma(\alpha)} \quad \text{for } t \geq 0 \quad (A.13)
\]

Here the shape parameter \( \alpha \) and scale parameter \( \lambda \) both are positive. The \( \Gamma(\alpha) \) function is defined by:

\[
\Gamma(\alpha) = \int_0^\infty e^{-t^\alpha} \, dt \quad (A.14)
\]
For all $a$ it has the property $\Gamma(a+1) = \Gamma(a)a$. For a positive integer $k$ the gamma function is equal to $(k-1)!$. In this case the distribution is sometimes called the Erlang-$k$ distribution of which the distribution function and density function are equal to:

$$
\text{Erlang-}k: \quad F(t) = 1 - \sum_{j=0}^{k-1} \frac{e^{-\lambda t}}{j!} \quad f(t) = \frac{\lambda^k t^{k-1}}{(k-1)!} e^{-\lambda t} \quad \text{for } t \geq 0 \quad (A.15)
$$

This distribution is equal to the $k^{th}$ autoconvolution of the $\text{Exp}(\lambda)$ distribution. Thus, a variable that is Erlang-$k$ distributed is similar to the distribution of the sum of $k$ variables that are each $\text{Exp}(\lambda)$ distributed.

The lognormal distribution is characterised by the functions:

$$
\text{Logn}(\alpha, \lambda): \quad F(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\ln(t - \lambda)} e^{-\frac{u^2}{2\sigma^2}} du \quad f(t) = \frac{1}{\alpha t \sqrt{2\pi}} e^{-\frac{-(\ln(t - \lambda))^2}{2\sigma^2}} \quad \text{for } t > 0 \quad (A.16)
$$

Here the shape parameter $\alpha$ is positive and the scale parameter $\lambda$ can be any real value.

The Weibull distribution is characterised by the functions:

$$
\text{Weibull}(\alpha, \lambda): \quad F(t) = 1 - e^{-\lambda t^\alpha} \quad f(t) = \alpha \lambda t^{\alpha-1} e^{-\lambda t^\alpha} \quad \lambda(t) = \alpha \lambda t^{\alpha-1} \quad \text{for } t > 0 \quad (A.17)
$$

Here both shape and scale parameter, $\alpha$ and $\lambda$ respectively, are positive constants.

The four distributions mentioned above are lined up in table A.1 together with some of their properties.

### Table A.1 Properties of distribution functions.

<table>
<thead>
<tr>
<th>distr.</th>
<th>$\mu_1$</th>
<th>$c^2_X$</th>
<th>$\lambda(t)$ rising</th>
<th>$\lambda(t)$ falling</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>from</td>
<td>to</td>
<td>from</td>
<td>to</td>
</tr>
<tr>
<td>Exp($\lambda$)</td>
<td>$\frac{1}{\lambda}$</td>
<td>1</td>
<td>$c^2_X &lt; 1$</td>
<td>0</td>
</tr>
<tr>
<td>$\Gamma(\alpha, \lambda)$</td>
<td>$\frac{\alpha}{\lambda}$</td>
<td>$\frac{1}{\alpha}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Logn($\alpha, \lambda$)</td>
<td>$e^{\frac{\ln\lambda^\alpha}{\sigma^2}}$</td>
<td>$e^{\alpha'} - 1$</td>
<td>small $t$</td>
<td>$t \to \infty$</td>
</tr>
<tr>
<td>Weibull($\alpha, \lambda$)</td>
<td>$\Gamma\left(\frac{1+\frac{1}{\alpha}}{\lambda}\right)$</td>
<td>$\Gamma\left(\frac{1+\frac{1}{\alpha}}{\lambda}\right)$</td>
<td>$c^2_X &lt; 1$</td>
<td>0</td>
</tr>
</tbody>
</table>
Appendix 2
Mathematica input of figure 3.18

In[1] := <<Calculus' LaplaceTransform'

In[2]:=

\[ l = 1/10 \]
\[ r = 1/20 \]
\[ C_m = 1 \]
\[ C_f = 100 \]

(* Markov models *)

In[3]:=

\[ m a t r = \{\{2r, 0, -r, -l\}, \{-r, mu + r, 0, 0\}, \{-r, -mu, mu + r, -mu\}, \{0, -r, -r, mu + l\}\} \]

In[4]:= matr[[1]]

In[5]:=

\[ p i M 3[\mu] = LinearSolve[m a t r, \{0, 0, 0, 1\}]\]

\[ c p y M 1[\mu] = mu C_m + 1 C_f r / (mu + l + r) \]  (* cpy = cost per year *)

\[ c p y M 2[\mu] = mu C_m + 1 C_f 8 r^2 / (mu^2 + 2mu l + 6mu r + 12l r + 8 r^2) \]

\[ c p y M 3[\mu] = mu C_m + 1 C_f p i M 3[\mu] \]

(* renewal models *)

\[ G[t_] = 1 - Exp[-r t] \]
\[ H[t_] = 1 - Exp[-l t] \]

\[ L G[s_] = s LaplaceTransform[ G[t] , t , s] \]
\[ L G 2[s_] = s LaplaceTransform[ G[t]^2 , t , s] \]
\[ L H[s_] = s LaplaceTransform[ H[t] , t , s] \]
\[ L F[s_] = LG[s] L H[s] \]
\[ L F 2[s_] = LG 2[s] L H[s] \]
\[ L G R 3[s_] = LaplaceTransform[ G[t] G[t + 1/mu] , t , s] \]

\[ L M R 1[s_] = L F[s] / (1 - L F[s]) \]
\[ L M R 2[s_] = L F 2[s] / (1 - L F 2[s]) \]

\[ M R 1[t_] = InverseLaplaceTransform[ L M R 1[s] / s , s , t] \]
\[ M R 2[t_] = InverseLaplaceTransform[ L M R 2[s] / s , s , t] \]
\[ M R 3[t_] = InverseLaplaceTransform[ L G R 3[s] L H[s] (1 + L M R 2[s]) / s , s , t] \]

\[ c p y R 1[\mu] = mu (C_m + C_f M R 1[1/mu]) \]
\[ c p y R 2[\mu] = mu (C_m + C_f M R 2[2/mu])/2 \]
\[ c p y R 3[\mu] = mu (C_m + C_f M R 3[1/mu]) \]

(* Petri net model P3 *)

\[ Q = \{\{-2r , r , r , 0\}, \{0 , -r , 0 , r\}, \{0 , 0 , -r , r\}, \{l , 0 , 0 , -l\}\} \]

\[ e Q[t_] = Simplify[ ComplexExpand[ MatrixExp[ Q t ] ] ] \]
zero={0,0,0,0}
X=Transpose[{Join[ zero , {1,0} ],
    Join[ zero , {1,0} ],
    Join[ zero , {0,1} ],
    Join[ zero , {0,1} ],
    {a15,a25,a35,a45,0,0},
    {a16,a26,a36,a46,0,0}}

X[[1]]=[0,0,0,0,1,1]

pid[mu_]=Simplify[
    Take[ LinearSolve[ X , {1,0,0,0,0,0} ] , -2] 
    /.{ a25 -> eQ[1/mu][[1]][[2]],
         a35 -> eQ[1/mu][[1]][[3]],
         a45 -> eQ[1/mu][[1]][[4]],
         a26 -> eQ[1/mu][[2]][[2]],
         a36 -> eQ[1/mu][[2]][[3]],
         a46 -> eQ[1/mu][[2]][[4]] } ]

f[mu_,t_]=Simplify[ pid[mu] . Take[ Transpose[ eQ[ t ] ][{ 4 }],2] ]

cpyP3[mu_]:= mu + Cf 1 mu NIntegrate[ f[mu,t] , {t,0,1/mu} ]

(* output *)

MatrixForm[
    Table[ N[{ mu,
                cpyM1[mu],
                cpyM2[mu],
                cpyM3[mu],
                cpyR1[mu],
                cpyR2[mu],
                cpyR3[mu],
                cpyP3[mu] } , 4 ],
    {mu,0.06,3,0.06} ], TableSpacing -> {0,2} ] >>res1.asc
Appendix 3
Mathematica input of figures 3.19, 3.20 and 3.21

<<Calculus' LaplaceTransform'

\( l = 1/1 \) (* value of figure 3.21 *)
\( r = 1/20 \)
\( Cm = 1 \)

(* Markov models *)

\[
\text{matr} = \{(2r, 0, -\mu, -1), \\
(-r, \mu + r, 0, 0), \\
(-r, -\mu, \mu + r, -r), \\
(0, -r, -r, \mu + 1)\}
\]

\( \text{matr}[[4]] = \{1, 1, 1, 1\} \)

\( \text{piM3}[\mu_1] = \text{LinearSolve}[\text{matr}, \{0, 0, 0, 1\}] \)

\( \text{cpyM1}[\mu, \text{Cf}_1] = \mu \cdot Cm + 1 \cdot \text{Cf}_1 r / (\mu + 1 + r) \) (* cpy = cost per year *)

\( \text{cpyM2}[\mu, \text{Cf}_2] = \mu \cdot Cm + 1 \cdot \text{Cf}_2 8 r^2 / (\mu^2 + 2 \mu l + 6 \mu r + 12 l r + 8 r^2) \)

\( \text{cpyM3}[\mu, \text{Cf}_3] = \mu \cdot Cm + 1 \cdot \text{Cf} \cdot \text{piM3}[\mu] \)

(* renewal models *)

\( \text{LG}[s] = \text{LaplaceTransform}[G[t] , t, s] \)
\( \text{LG2}[s] = \text{LaplaceTransform}[G[t]^2 , t, s] \)
\( \text{LH}[s] = \text{LaplaceTransform}[H[t] , t, s] \)
\( \text{LF}[s] = \text{LaplaceTransform}[\text{LG}[s] \cdot \text{LH}[s] \]
\( \text{LF2}[s] = \text{LaplaceTransform}[\text{LG2}[s] \cdot \text{LH}[s] \)
\( \text{LGR3}[s] = \text{LaplaceTransform}[G[t] \cdot G[t+1/mu] , t, s] \)

\( \text{LMR1}[s] = \text{LaplaceTransform}[L[t] , t, s] \)
\( \text{LMR2}[s] = \text{LaplaceTransform}[L[t]^2 , t, s] \)
\( \text{MR1}[t] = \text{InverseLaplaceTransform}[\text{LMR1}[s] , s, t] \)
\( \text{MR2}[t] = \text{InverseLaplaceTransform}[\text{LMR2}[s] , s, t] \)
\( \text{MR3}[t] = \text{InverseLaplaceTransform}[\text{LGR3}[s] \cdot \text{LH}[s] \cdot (1 + \text{LMR2}[s]) / s , s, t] \)

\( \text{cpyR1}[\mu, \text{Cf}] = \mu \cdot (Cm + \text{Cf} \cdot \text{MR1}[1/mu]) \)
\( \text{cpyR2}[\mu, \text{Cf}] = \mu \cdot (Cm + \text{Cf} \cdot \text{MR2}[2/mu] / 2) \)
\( \text{cpyR3}[\mu, \text{Cf}] = \mu \cdot (Cm + \text{Cf} \cdot \text{MR3}[1/mu]) \)

(* Petri net model P3 *)

\( \text{Q} = \{-2r, r, r, 0\}, \)
\( \{0, -r, 0, r\}, \)
\( \{0, 0, -r, r\}, \)
\( \{1, 0, 0, -1\} \)

\( \text{eq}[t] = \text{Simplify}[\text{ComplexExpand}[\text{MatrixExp}[\text{Q} \cdot t]]] \)

\( \text{zero} = \{0, 0, 0, 0\} \)
\begin{verbatim}
X = Transpose[{Join[zero, {1, 0}], 
Join[zero, {1, 0}], 
Join[zero, {0, 1}], 
Join[zero, {0, 1}], 
{a15, a25, a35, a45, 0, 0}, 
{a16, a26, a36, a46, 0, 0}]
- IdentityMatrix[6]

X[[1]] = {0, 0, 0, 0, 1, 1}

pid[\mu_,_] = Simplify[
  Take[LinearSolve[X, {1, 0, 0, 0, 0, 0}], -2]
  /. (a25 -> eQ[1/\mu][1][2],
a35 -> eQ[1/\mu][1][3],
a45 -> eQ[1/\mu][1][4],
a26 -> eQ[1/\mu][2][2],
a36 -> eQ[1/\mu][2][3],
a46 -> eQ[1/\mu][2][4])
]

f[\mu_, t_] = Simplify[pid[\mu] . Take[Transpose[eQ[t]][{4}], 2]]

cpyP3[\mu_, Cf_] := mu + Cf \mu \text{NIntegrate}[f[\mu, t], (t, 0, 1/\mu)]

(* output *)

tabel = Table[0, {33}]
Do[Cf = 10^(-x/8); Print[40 - x];
tabel[[x - 7]] = N[
  Do[
    If[tabel[[i, j, 1]] < 0, 
      tabel[[i, j, 2, 1, 2]] < 0, 
      tabel[[i, j]] = {10000, {\mu -> 10000}},
      {i, 33}, {j, 2, 8}]
  ];
  OutputForm[
    MatrixForm[
      Table[N[
        Log[10, tabel[[x]][[1]]],
        Log[10, tabel[[x]][[2]]][[1]],
        Log[10, tabel[[x]][[3]]][[1]],
        Log[10, tabel[[x]][[4]]][[1]],
        Log[10, tabel[[x]][[5]]][[1]],
        Log[10, tabel[[x]][[6]]][[1]],
        Log[10, tabel[[x]][[7]]][[1]],
        Log[10, tabel[[x]][[8]]][[1]],
      ]], {x, 33}], TableSpacing->{0, 2}
    ]
  ]
] >> res6.asc
\end{verbatim}
\begin{verbatim}
OutputForm[ MatrixForm[
Table[ N[
  { Log[10, tabel[[x]][[1]]],
    Log[10, mu] /. tabel[[x]][[2]][[2]],
    Log[10, mu] /. tabel[[x]][[3]][[2]],
    Log[10, mu] /. tabel[[x]][[4]][[2]],
    Log[10, mu] /. tabel[[x]][[5]][[2]],
    Log[10, mu] /. tabel[[x]][[6]][[2]],
    Log[10, mu] /. tabel[[x]][[7]][[2]],
    Log[10, mu] /. tabel[[x]][[8]][[2]]
  }, 4], {x, 33}], TableSpacing -> {0, 2} ] ] >> res7.asc
\end{verbatim}
Appendix 4
Mathematica input of the analytical curves in figures 3.22 and 3.23

<<Calculus\'LaplaceTransform'

l = 1/10
r = 1/20
Cm = 1
Cf = 10000 (* value corresponding with figure 3.23 *)

(* Markov model of S1 *)

matrS1={{4r , -1-mu/4 , -1-mu/4 , -mu/4 },
{-2r , 2r+1+mu/4 , 0 , -1 },
{-2r , 0 , 2r+1+mu/4 , -1 },
{0 , -2r , -2r , 2l+mu/4 }}

matrS1[[4]]={1,1,1,1}

pi2S1[mu_] = Simplify[ LinearSolve[matrS1,{0,0,0,1}][[2]] ]
pi3S1[mu_] = Simplify[ LinearSolve[matrS1,{0,0,0,1}][[3]] ]
pi4S1[mu_] = Simplify[ LinearSolve[matrS1,{0,0,0,1}][[4]] ]

cpyS1[mu_] = mu Cm + 1 Cf (pi2S1[mu] +pi3S1[mu] +2 pi4S1[mu])

(* renewal model of S2 *)

G[t_] = 1-Exp[-2r t]
H[t_] = 1-Exp[-1 t]

LG[s_] = s LaplaceTransform[ G[t], t , s]
LH[s_] = s LaplaceTransform[ H[t], t , s]
LF[s_] = LG[s] LH[s]
LM[s_] = LF[s]/(1-LF[s])
M[t_] = InverseLaplaceTransform[ LM[s]/s , s , t]

cpyS2[mu_] = mu (Cm + 2Cf M[4/mu]/4 )

(* output *)

OutputForm[ MatrixForm[ uittabel=Table[N[{x,cpyS1[x],cpyS2[x]}],{x,S,SO,l}],TableSpacing->{0,5} ] ] >>S1.asc
Appendix 5
Pascal source file of the simulations

PROGRAM SIMULATION_S1_4_RELAIS_EXP_MAINT;
{$N+}
{$E-}
{$B-}
{$V-}
USES CRT;

CONST
    SimNumber = 3; {1 if S1, 2 if S2, 3 If S3}
    NumberPeriods = 100000;
    rho = 1/20;
    lambda = 1/10;
    CostRatio = 100.;

VAR
    TTSh, TTU, TSim, mu, FtOsPerYear, 
    TTU1, TTU2, SimLength : DOUBLE;
    FtOs, i, j, 
    RNDLast : LONGINT;
    DF : TEXT;

(*------------------------------------*)
PROCEDURE GetInvExp(Lambda : DOUBLE;
VAR Out : DOUBLE;
VAR RNDLast : LONGINT);
{gets time when an event should occur}
VAR RNDGen : DOUBLE;
BigConst1, BigConst2, 
BigReal : EXTENDED; {10 byte REAL}
BEGIN
    BigConst1 := 950706376.0;
    BigConst2 := 2147483647.0;
    {in the books they told that this was a good random }
    {generator for computers with an 32 Bit Integer. }
    {what they forgot to tell was the fact that }
    {950706376*RNDLast > then the highest machinenumber }
    {so thats why I use 10byte REALS, so it's still a good}
    {random generator. (Better then the TurboPascal }
    {generator that gives different results with 80x87 }
    {and with no 80x87.)}
    BigReal := BigConst1 * RNDLast;
    RNDLast := ROUND(BigReal - TRUNC(BigReal/BigConst2) * BigConst2);
    RNDGen := (RNDLast/2)/1073741824;
    Out := -LN(RNDGen)/Lambda;
{end 5.3.7}

BEGIN {main}
Assign(df,'c:\s'+chr(SimNumber+48)+'.uit');
Rewrite(df);
CLRSCR;
RNDLast := 1;
For i := 1 to 50 do
BEGIN
mu:=0.06*i;
Case SimNumber of
  2: SimLength:=4/mu;
  3: SimLength:=8/mu;
end;
FtOs:=0;
FOR j:=1 to NumberPeriods do
BEGIN
  case SimNumber of
    1: GetInvExp(rho/4, SimLength, RNDLast);
    end;
    case SimNumber of
      1,2: GetInvExp(2*rho, TTU, RNDLast);
      3: begin
        GetInvExp(2*rho, TTU1, RNDLast);
        GetInvExp(2*rho, TTU2, RNDLast);
        TTU:=TTU1+TTU2;
        GetInvExp(2*rho, TTU1, RNDLast);
        GetInvExp(2*rho, TTU2, RNDLast);
        If TTU1+TTU2<TTU then TTU:=TTU1+TTU2;
      end;
    end;
  TSim:=TTU;
  WHILE SimLength>TSim DO
  BEGIN
    GetInvExp(lambda, TTSH, RNDLast);
    TSim:=TSim+TTSH;
    IF SimLength>TSim THEN
    BEGIN
      case SimNumber of
        1,2: GetInvExp(2*rho, TTU, RNDLast);
        3: begin
          GetInvExp(2*rho, TTU1, RNDLast);
          GetInvExp(2*rho, TTU2, RNDLast);
          TTU:=TTU1+TTU2;
          GetInvExp(2*rho, TTU1, RNDLast);
          GetInvExp(2*rho, TTU2, RNDLast);
          If TTU1+TTU2<TTU then TTU:=TTU1+TTU2;
        end;
      end;
      TSim:=TSim+TTU;
      FtOs:=FtOs+1;
    END
  END; {while}
END; {for j}
case SimNumber of
  1,2: FtOsPerYear:=mu*FtOs/(NumberPeriods*4);
  3: FtOsPerYear:=mu*FtOs/(NumberPeriods*8);
end;
writeln(df,mu:13,' ',2.*CostRatio*FtOsPerYear+mu:13);
writeln(i);
END;
Close(df);
END.
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