Switching from autopilot to the driver

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Switching from autopilot to the driver: A transient performance analysis

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Abstract

One of the main safety concerns associated with semi-autonomous vehicles is the sharing of control between a human driver and an autonomous driving system. Even with an attentive driver, such switches in control may pose a threat to the safety of the driver and the surrounding vehicles. The aim of this study is to develop an indicator that can measure the level of safety during a driver take-over, using knowledge about the system known *a priori*. A model-based approach is used to analyse the system with special focus on the lateral dynamics of the vehicle. The driver and the vehicle are modelled as linear systems, and a path tracking controller is used to serve as an autonomous system. With this structure, shared control is studied as a switched system, in which the vehicle's lateral control switches between the autonomous system and the driver. A bound on the transient dynamics that arise due to a switch is derived, using the induced $\mathcal{L}_\infty$ norm. This bound is then used to formulate an indicator that checks if the states/outputs of interest are within acceptable limits. A comparison with simulation results has shown that the indicator successfully captures the effect of different system parameters on take-over safety, although in a slightly conservative manner. This indicator can be further developed as a tool to be used in the design and evaluation of shared-/multi-modal control systems in future vehicles.

Keywords

Switched control systems, transient dynamics, semi-autonomous driving systems, shared-/multi-modal control, human-machine systems, transition of control

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Introduction

Autonomous driving technology has the potential to become a game changer in the mobility industry of the 21st century.¹ People's perception of driving and transportation as a whole stand the chance of being transformed, given the far reaching opportunities for innovation that these systems have opened up. As of today, automotive manufacturers and suppliers have developed systems that are capable of achieving varying levels of driving autonomy, most of which fall under the SAE levels 2 and 3 of automation,¹ while some level 4 systems are also being experimented by ride hailing companies. Overall, there are two schools of thought within the industry, one that believes that semi-autonomous driving technology is crucial towards building people's trust in autonomous systems, while the other advocates for a direct shift to fully autonomous systems. It is, however, expected that semi-autonomous systems will have their place on the road until technology, infrastructure and regulation have matured enough to give way to fully autonomous systems.¹

The main challenge associated with the development of semi-autonomous systems is the sharing of control between the driver and the autonomous system. When the autonomous system reaches its boundaries, it has to alert the driver and hand over control safely,^{2,3} while the driver can also manually take or give control to the system. The driver's actions following a transfer of control are a major threat to safety, which has been proven by a number of studies in literature.^{2,4–6} Gold et al.⁵ studied the driver's behaviour during a transfer of control by observing his or her reaction time to a 'take-over' request from the vehicle. The results show that

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Figure 1. Classification of the different types of transitions, the dotted box highlights the focus of this research.

when the time given to resume control is reduced, decision-making and reactions are faster but are generally worse in quality. A similar study⁶ measured the frequency content of the steering angle and the lateral position of the vehicle after a transfer of control, which has shown that it takes around 35–40 s for the driver to stabilize the lateral control of the vehicle after resuming control. These studies help in understanding certain aspects of the driver's reactions from a human-factors perspective, but do not provide a systematic approach to study such scenarios. This research tries to bridge this gap by proposing a model-based method that can be used to assess the safety of a transition of control.

A classification of the different types of transitions of control, as presented in Figure 1, helps in developing a better understanding of the problem at hand. The classification is done in two levels, based on the initiation and the direction of transition, as shown in Figure 1. Based on initiation, the transition can be classified as 'driver initiated (DI)' or 'vehicle initiated (VI)' transitions and the direction of transition can be either from the driver to the vehicle (D2V) or from the vehicle to the driver (V2D).

This results in four classes of transitions, DI-D2V, DI-V2D, VI-V2D and VI-D2V transitions. Among these, the VI-V2D transitions have a higher potential for danger, compared to the other types of transitions. This is because in VI-V2D transitions, the driver is likely to have a much lower level of situational awareness when the transition is initiated. This is attributed to the low workload during the autonomous driving mode, in which, not much information related to the driving task has to be processed by the driver. This diminished situational awareness will not be immediately restored on switching to manual driving, posing a greater threat to a safe transition.⁴

However, in a DI type of transition, factors such as the driver's response to a take-over request from the vehicle and bringing him or her back in the driving loop, have a lower influence making the problem easier to analyse using mathematical models. Hence, this research tries to take a first step towards addressing the safety concerns during a transition of control by focusing on the DI-V2D type transitions, also termed in this article as a 'take-over'.

Research objectives

The objective of this research is to develop a 'switching performance indicator' that can be used to evaluate the safety of a switch from the autonomous system to the driver. The indicator should quantify the safety critical aspects of the transient dynamics that arise due to a switch. These safety critical aspects can be the amplitude, frequency content or settling time of the signal (state/output of the system) being studied. This study aims at deriving a performance indicator that can provide a bound on the amplitude of the transient dynamics, using knowledge about the system known *a priori*.

Methodology and assumptions

A take-over is studied in this study with special attention to the lateral dynamics of the vehicle, assuming that the driver takes over control of the vehicle only through the steering wheel. A model-based approach is followed, with the driver and the vehicle modelled as linear systems. A path tracking controller is used to emulate an autonomous driving system and shared control is studied as a switched system, in which the vehicle's lateral control switches between the driver and the autonomous system. The longitudinal speed of the vehicle is taken as a constant, as it is assumed to be controlled by a cruise control system with a bandwidth much lower than that of the lateral controller.

It is also assumed that after a switch from the autonomous system to the driver, the system stays in the manual mode for a sufficiently long period of time. This assumption is made because it is of interest in this research to focus on the transient dynamics that are present for a finite time interval following a switch. Having set the objectives and boundaries for this research, the next part of this section presents some supporting literature.

Supporting literature

Some studies in literature have developed tools that could help in achieving the objective of this research. An $\mathcal{L}_p$ string stability condition is proposed by Ploeg et al.⁷ for cascaded systems using its input–output properties. The $\mathcal{L}_2$ and $\mathcal{L}_\infty$ induced gains are used to arrive at corresponding string stability conditions. An analogy can be drawn between notions of string stability and that of a switching performance indicator, as both are measures of system performance and applicable over a finite time horizon. These induced gains could be used in the context of switched systems to derive bounds on the states/outputs based on the system dynamics, as it will be discussed in the subsequent sections.

It is to be noted here that notions of stability of the switched system are not investigated in this research, as it is assumed that the system stays in the manual mode for a sufficiently long duration after a switch. This

Figure 2. Block diagram – Driver-Vehicle switched control system.

assumption puts the system into a class of ‘slow-switched’ systems, for which stability can be guaranteed by defining constraints on the switching signal, provided the individual subsystems are stable. A lower bound on the average dwell time between consecutive switches is proposed by Hespanha and Morse,⁸ which guarantees stability for such systems. A specific application of this result to prove the stability of switches between the manual and automated driving is presented by Kaustubh et al.⁹

This article is organized as follows. The next section on ‘Modelling a take-over’ discusses the overall structure of the system and the models used for each of the system components. An analysis on the transient dynamics and the different methods to define a bound on it is presented in section ‘Analysis of the transient dynamics’. Following this, a comparison is done between simulation results and the obtained performance indicators in section ‘Simulation and validation’. The final section presents the conclusion and recommendations to improve on the results obtained in this study.

Modelling a take-over

As discussed in the previous section, shared control between the driver and the autonomous system can be modelled as a switched system, which is depicted in the block diagram shown in Figure 2. It can be seen that the vehicle is controlled either by the driver or the path tracking controller, and a transfer of control between the two can be studied as a switch between the two control modes. In the autonomous mode, the path tracking controller computes the set point steering angle (δ_{set}) which is realized by a steering actuator. In the manual mode, the driver’s perception and mental computation of the desired steering input (δ_{des}) can be seen as a controller, while the actuation is carried out by the human neuromuscular system, which is represented by the block ‘Driver dynamics’ in Figure 2. The models for

Figure 3. Single track vehicle model.

each of these components are discussed in the subsequent parts of this section.

Vehicle model

The lateral dynamics of the vehicle is modelled using a single track vehicle model,¹⁰ shown in Figure 3.

The notations used in Figure 3 along with the vehicle parameters that will be used in this section are listed in Table 1. The equations of motion corresponding to the lateral dynamics of the single track vehicle model are given by¹¹

$$\begin{aligned}\dot{v}_y &= -\frac{C_f + C_r}{mv_x} v_y - \left(\frac{l_f C_f - l_r C_r}{mv_x} + v_x \right) \omega_z + \frac{C_f}{m} \delta \\ \dot{\omega}_z &= -\frac{l_f C_f - l_r C_r}{I_{zz} v_x} v_y - \frac{l_f^2 C_f + l_r^2 C_r}{I_{zz} v_x} \omega_z + \frac{l_f C_f}{I_{zz}} \delta\end{aligned}\quad (1)$$

This vehicle model is intended to be used for the purpose of path tracking, which necessitates the definition of the path tracking errors that are depicted in Figure 4. The curve $\mathcal{C}$ in Figure 4 represents a reference path, with $\mathcal{O}^s$ being the point on the curve along the $\vec{e}_y^i$ direction. The set of vectors $\vec{e}^s = [\vec{e}_x^s \ \vec{e}_y^s]^T$ represents a reference frame centred at $\mathcal{O}^s$ with $\vec{e}_x^s$ tangent to the curve at $\mathcal{O}^s$. Furthermore, $\vec{e}^g = [\vec{e}_x^g \ \vec{e}_y^g]^T$ is a ground fixed reference frame centred at a stationary point $\mathcal{O}^g$.

Table 1. Vehicle parameters and variables (Toyota Prius test platform at TNO).¹²

Variable	Description	
v_y	Lateral speed (m/s)	
ω_z	Yaw rate (rad/s)	
ψ	Heading angle (rad)	
v_x	Vehicle forward speed (m/s)	
δ	Steering angle (rad)	
$F_{x,f}, F_{x,r}$	Front and rear longitudinal tyre forces (N)	
$F_{y,f}, F_{y,r}$	Front and rear lateral tyre forces (N)	
β	Body slip angle (rad)	
$\vec{v}_f, \vec{v}_r$	Front and rear wheel centre velocity vectors	
$\vec{v}$	Velocity vector of the centre of mass	
$\mathcal{O}^i$	Centre of mass of the vehicle	
$\vec{e}^i$	Vehicle fixed reference frame at $\mathcal{O}^i$	
α_f, α_r	Wheel slip angles (rad)	
Parameter	Description	Value
C_f	Front axle cornering stiffness (N/rad)	9.84×10^4
C_r	Rear axle cornering stiffness (N/rad)	1.98×10^5
l_f	Position of CG from front axle (m)	1.11
l_r	Position of CG from rear axle (m)	1.59
I_{zz}	Moment of inertia about the Z-axis (kg m ²)	2865.6
m	Mass of the vehicle (kg)	1625

CG: centre of gravity.

Figure 4. Geometric representation of the tracking errors $\vec{y}_e$ and ψ_e .

With this setting, the following two types of tracking errors are defined:¹³

- Angular error (ψ_e): Difference between the vehicle's heading angle (ψ) and θ_s , where θ_s is the orientation of $\vec{e}_x^s$ with respect to $\vec{e}_x^g$, that is, $\psi_e = \theta_s - \psi$.
- Distance error (y_e): The distance of the point $\mathcal{O}^s$ from the centre of mass of the vehicle along the $\vec{e}_y^i$ direction, that is, $y_e = \|\vec{r}_{\mathcal{O}^s/\mathcal{O}^i}\|$.

For small values of ψ_e and for highway driving ($v_x \gg y_e \omega_z$), the dynamics of the angular and distance error can be obtained as¹³

$$\dot{y}_e = -v_y + v_x \psi_e \quad (2)$$

$$\dot{\psi}_e = v_x \rho - \omega_z \quad (3)$$

where ρ is the curvature of the reference path.

The error dynamics given by equations (2) and (3) along with the vehicle dynamics (equation (1)) can be written in the state space form

$$\dot{X} = AX + B_1\delta + B_2\rho \quad (4)$$

where

$$A = \begin{bmatrix} -\frac{C_f + C_r}{mv_x} & -\frac{l_f C_f - l_r C_r}{mv_x} - v_x & 0 & 0 \\ -\frac{l_f C_f - l_r C_r}{I_{zz} v_x} & -\frac{l_f^2 C_f + l_r^2 C_r}{I_{zz} v_x} & 0 & 0 \\ -1 & 0 & 0 & v_x \\ 0 & -1 & 0 & 0 \end{bmatrix},$$

$$X = \begin{bmatrix} v_y \\ \omega_z \\ y_e \\ \psi_e \end{bmatrix}, \quad B_1 = \begin{bmatrix} \frac{C_f}{m} \\ \frac{l_f C_f}{I_{zz}} \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ v_x \end{bmatrix}.$$

Driver model

The human driver is modelled using two components: the 'Driver perception' and the 'Driver dynamics' as seen in Figure 2. The driver perception is modelled as a feedback controller, using cognitive models of the driver from literature. These models mimic human cognitive functions that are active during driving. Several models are proposed in literature which are summarized by Jürgensohn.¹⁴ The 'Pursuit-Compensatory' driver model proposed by McRuer et al.¹⁵ is found to be most suitable for manoeuvres that are involved in highway driving because it computes the driver's desired steering angle as the sum of a state feedback term that compensates for tracking errors and a feedforward term that aids in path following, as shown below

$$\delta_{des} = k_e(y_e + l_a \psi_e) + k_r \rho = K_d CX + k_r \rho \quad (5)$$

where $K_d = [k_e \quad k_e l_a]$ and $C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Here, l_a is the look ahead distance of the driver and gains k_e and k_r weigh the feedback and feedforward components, respectively.

Once the desired steering angle is known, the physical actuation by the driver (represented by the block 'Driver dynamics' in Figure 2) can be modelled using 'human operator models' from literature.¹⁶ These models replicate the driver's physical response to a cognitive reference input. The 'Quasi-linear model' developed in yet another work by McRuer and Krendel¹⁷ models the

human operator by means of a second-order transfer function from the desired steering angle (δ_{des}) to the actual steering angle achieved by the driver (δ_d) as given below

$$H(s) = \frac{\delta_d(s)}{\delta_{des}(s)} = \frac{k(T_L s + 1)}{(T_I s + 1)(T_N s + 1)} e^{-\tau_d s} \quad (6)$$

where difference in the task environment and in operator properties are described by means of five independent parameters: k , T_L , T_I , T_N and τ_d . τ_d is the reaction time of the driver in response to a change in the reference path. It can be considered as the time needed to perceive and process the necessary control input. T_N is the time constant of the neuromuscular dynamics, which is the time required for the muscles in the driver's arms to receive and execute the actions from the central nervous system. T_L and T_I are time constants of the lead lag filter which depend on how well the driver is accustomed to the vehicle or knows a certain route. k is the driver gain which is influenced by numerous factors like the driver's driving style, urgency and level of attention. This model is chosen because it provides the flexibility of modifying different properties of the driver, while fitting well in the linear control framework chosen for this study. The 'Quasi-linear model' is also widely accepted in literature and is used by a number of studies to model human driving behaviour in different driving conditions.¹⁸

A second-order Padé approximation¹⁹ is used in the place of the exact time delay in equation (6) because it was found that the frequency responses of the transfer function in equation (6) and its second-order Padé approximation are almost identical up to a frequency of 20 Hz, with their magnitude differing by a maximum of 10^{-5} dB and phase by 1° (for $\tau_d = 0.1$ s). Since equation (6) is to be used in this research only at low frequencies (< 1 Hz), this approximation is valid and simplifies the analysis by eliminating the exponential term.

This results in a fourth order transfer function for the operator model (equation (6)), which can be written in the state space form as

$$\begin{aligned} \dot{\Delta}_d &= A_d \Delta_d + B_d \delta_{des} \\ \delta_d &= C_d \Delta_d \end{aligned} \quad (7)$$

where $\Delta_d = [\delta_{d1} \delta_{d2} \delta_{d3} \delta_{d4}]^T$ is a set of dummy states for the minimal state space realization of equation (6), while A_d , B_d and C_d are the corresponding state space matrices. This results in the following closed-loop dynamics for the driver-vehicle system, obtained by combining equations (4), (5) and (7)

$$\dot{X}_d = A_{dc} X_d + B_{dc} \rho \quad (8)$$

$$\text{where } X_d = \begin{bmatrix} X \\ \Delta_d \end{bmatrix}, \quad A_{dc} = \begin{bmatrix} A & B_1 C_d \\ B_d K_d C & A_d \end{bmatrix} \quad \text{and} \\ B_{dc} = \begin{bmatrix} B_2 \\ B_d k_r \end{bmatrix}.$$

The next part of this section briefly discusses the path tracking controller used in this study as an autonomous system.

Path tracking controller

A path tracking controller with independent longitudinal and lateral control developed by Schmeitz et al.¹³ is used in this study. The longitudinal speed is assumed to be controlled by a cruise control or an equivalent system, and the forward speed is assumed to be constant for the time horizon under consideration.

The steering control input to the vehicle involves a feedback (δ_{fb}) and a feedforward term (δ_{ff}). The feedforward component (δ_{ff}) is derived from the steady state solution of the single track vehicle model (equation (1)) by substituting $\omega_z = v_x/R$, which is approximately the steady state yaw rate of a vehicle driven along a path of constant radius R ($= \rho^{-1}$) at a constant forward speed v_x . This results in an expression for the steady state steering angle, used here as the feedforward component (δ_{ff})

$$\delta_{ff} = (l + K_\delta v_x^2) \rho =: k_{ff} \rho \quad (9)$$

Here, $l = l_f + l_r$ and the understeer coefficient K_δ in equation (9) is given by

$$K_\delta = \frac{m}{l} \left(\frac{l_r}{C_f} - \frac{l_f}{C_r} \right)$$

The feedback component (δ_{fb}) is defined as a function of the tracking errors y_e and ψ_e , given by

$$\delta_{fb} = k_1 y_e + k_2 \psi_e = K_c C X \quad (10)$$

where $K_c = [k_1 \ k_2]$ is the feedback gain matrix.

Combining equations (9) and (10) the steering control input can be written as

$$\delta_{set} = K_c C X + k_{ff} \rho \quad (11)$$

The steering control input derived in equation (11) is realized through a steering actuator, which is modelled as a second-order transfer function from the set point (δ_{set}) steering angle (equation (11)) to the realized steering angle (δ_c), as given below

$$H_d(s) = \frac{\delta_c(s)}{\delta_{set}(s)} = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{2\beta_{str}}{\omega_0} s + 1} e^{-\tau_{str} s} \quad (12)$$

where ω_0 and β_{str} are the natural frequency and the damping constant, respectively, and τ_{str} is the delay in the actuator. This dynamics corresponds to the steering actuator system used in the Toyota Prius test platform at TNO.¹²

Similar to the human operator model in equation (6), a second-order Padé approximation is used for the delay in equation (12) and was found to be accurate up to ≈ 25 Hz (for $\tau_{str} = 0.1$ s) with maximum deviations similar to those discussed in the previous section for the human operator model. This results in a fourth-order

transfer function for the actuator dynamics, which can be written in the state space form as shown below

$$\begin{aligned}\dot{\Delta}_c &= A_c \Delta_c + B_c \delta_{set} \\ \delta_c &= C_c \Delta_c\end{aligned}\quad (13)$$

where $\Delta_c = [\delta_{c1} \delta_{c2} \delta_{c3} \delta_{c4}]^T$ is a set of dummy variables for the minimal state space realization of equation (12), while A_c , B_c and C_c are the corresponding state space matrices. This results in the following closed-loop dynamics for the controller–vehicle system, obtained by combining equations (1), (11) and (13)

$$\dot{X}_c = A_{cc} X_c + B_{cc} \rho \quad (14)$$

$$\text{where } X_c = \begin{bmatrix} X \\ \Delta_c \end{bmatrix}, \quad A_{cc} = \begin{bmatrix} A & B_1 C_c \\ B_c K_c C & A_c \end{bmatrix} \quad \text{and} \\ B_{cc} = \begin{bmatrix} B_2 \\ B_c K_{ff} \end{bmatrix}.$$

With the closed-loop dynamics for both control modes obtained in equations (8) and (14), the next part of this section discusses how a switch is made between the two systems, and some simulation results are presented to visualize the transient dynamics due to the switch.

Switching between the two modes

As discussed in section ‘Introduction’, a take-over is a switch from the controller–vehicle system (equation (14)) to the driver–vehicle system (equation (8)). The time instant of switch (τ_{to}) is an exogenous variable, as the switch is initiated by the driver out of his or her free will.

It is to be noted here that the states vectors X_c and X_d are not the same and for a switch from system (equation (14)) to (equation (8)), it is necessary to derive the initial conditions for system (equation (8)) from the state vector X_c at the time of switching (τ_{to}). It can be seen from equations (8) and (14) that the first set of states (X) are identical while the second set corresponding to the actuation dynamics (Δ_d and Δ_c) are different. The initial conditions for these states can be derived by imposing the conditions for continuity on the steering input and its derivatives, at the instant of switching, as shown in the equations below

$$\begin{aligned}\delta_c(\tau_{to}) &= \delta_d(\tau_{to}) \\ \dot{\delta}_c(\tau_{to}) &= \dot{\delta}_d(\tau_{to}) \\ \ddot{\delta}_c(\tau_{to}) &= \ddot{\delta}_d(\tau_{to}) \\ \dddot{\delta}_c(\tau_{to}) &= \dddot{\delta}_d(\tau_{to})\end{aligned}\quad (15)$$

Continuity up to the third derivative of the steering angle is imposed in equation (15), assuming that the driver takes over control in a theoretically smooth/continuous manner. In practice, such a smooth switch can be imposed by a suitably designed steering actuator system, a discussion on which is, however, out of the scope of this study.

From equation (7) we know

$$\delta_d = C_d \Delta_d = [0, 0, 0, 0, C_d] X_d =: C_{\Delta_d} X_d \quad (16)$$

Similarly, from equation (13), we know

$$\delta_c = C_c \Delta_c = [0, 0, 0, 0, C_c] X_c =: C_{\Delta_c} X_c \quad (17)$$

Taking derivatives of equations (16) and (17) and substituting them in equation (15), we obtain an expression for the initial conditions of the driver–vehicle system (equation (8)) as

$$X_d(\tau_{to}) = E^{-1}(F X_c(\tau_{to}) + G P(\tau_{to})) \quad (18)$$

where $P = [\rho \quad \dot{\rho} \quad \ddot{\rho}]^T$ while E , F and G are obtained as a result of solving the system of linear equations in equation (15).

The closed-loop dynamics for a take-over can now be written using equations (8) and (14) as

$$\dot{X}_c(t) = A_{cc} X_c(t) + B_{cc} \rho(t), \quad 0 \leq t \leq \tau_{to} \quad (19a)$$

$$\dot{X}_d(t) = A_{dc} X_d(t) + B_{dc} \rho(t), \quad \tau_{to} < t \leq t_{tr} \quad (19b)$$

where the initial condition $X_d(\tau_{to})$ of equation (19b) is given by equation (18) and t_{tr} is the end of the transient period that is chosen for the analysis. The outputs of interest are defined on the system using output matrices C_{cc} and C_{dc} as

$$y(t) = \begin{cases} C_{cc} X_c(t), & 0 \leq t \leq \tau_{to} \\ C_{dc} X_d(t), & \tau_{to} < t \leq t_{tr} \end{cases} \quad (20)$$

A comparison between a single lane change manoeuvre done in the autonomous mode, the manual mode and the one in which a take-over happens in between the manoeuvre is presented in Figure 5. In the simulations, the vehicle has a constant forward speed of 100 km/h and the take-over is initiated at 0.9 s after the start of manoeuvre. The width of the lanes is assumed to be 3.5 m, and the lane change is performed over a longitudinal distance of 105 m. The transient time duration (t_{tr}) is chosen as approximately 2.5 times the duration of the lane change. This was observed to be long enough to capture any peak transient response that occurs after a switch.

As highlighted in subsection ‘Research objectives and supporting literature’ of the ‘Introduction’, it can be seen from the plots for the lateral position (Y), lateral acceleration (a_y) and steering angle (δ) in Figure 5 that the peaks in the transients which arise due to the switch are more severe in comparison to the performance of the path tracking controller or the driver alone. The lateral position of the vehicle also crosses the road boundaries in the case of a take-over, which is clearly unsafe.

Note that the reference path for a lane change is defined as a fifth order polynomial function of the longitudinal position (x , measured along $\vec{e}_x^g$), as

$$y_{ref}(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 \quad (21)$$

Figure 5. Comparing a single lane change manoeuvre performed by a driver alone, path tracking controller alone and a take-over at 0.9 s, with $v_x = 100$ km/h and lane change length $L = 105$ m, lane width = 3.5 m. Vehicle parameters are given in Table 1, driver and controller parameters in Table 2 (section –‘Simulation and validation’).

where $y_{ref}(x)$ is the reference lateral position (along the direction $\vec{e}_y^g$) for the vehicle. A fifth order polynomial is chosen for the reference path because it provides six coefficients, a_0 up to a_5 , that can be used to set the initial and final conditions for the reference lateral position, speed and acceleration, respectively. The reference curvature, which is the input to systems (equations (8) and (14)), can now be derived as¹¹

$$\rho(x) = \frac{\frac{d^2 y}{dx^2}}{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}} \quad (22)$$

Discussions

Driver intention. The driver’s intention is important and quite detrimental to the dynamics of the vehicle after a take-over. During a lane change initiated by an autonomous system, it can be said that there are two logical intentions that the driver can have – to steer back into the initial lane or to continue the lane change (barring any other intentions as rare cases). The latter scenario is chosen in this study because it is relevant when the driver does not trust the autonomous system’s ability to perform the lane change safely and prefers to take-over and complete it manually. A number of studies

(summarized by Vlakveld⁴) have highlighted the deterioration in driving performance after a transition of control, applicable even when the driver is required to continue a simple lane keeping task after a take-over. This is taken as a motivation to study a take-over initiated during a more complex manoeuvre (a lane change) and to arrive at a performance measure, which can also be extended to more scenarios.

Moreover, the driver’s intentions after a take-over would be a choice of the reference trajectory to the driver model. Changing the reference trajectory after a switch between controllers would have a significant influence on the transients being studied here. Thus, it becomes difficult to isolate the effect of a switch in control from the combined effect on the transient dynamics.

Effect of parameter uncertainty. The damping of the lateral dynamics is significantly affected by the cornering stiffness of the tyres. This can be better understood from the expression for the damping coefficient¹¹ ζ of the second-order dynamics (equation (1))

$$\zeta \approx \frac{1}{\sqrt{\left(1 + \frac{K_s}{T} v_x^2\right)}} \quad (23)$$

Figure 6. Take-over @0.9 s, varying the front and rear cornering stiffness by $\pm 15\%$ (nominal values: $C_f = 1.131 \times 10^5$, $C_r = 1.683 \times 10^5$). Twenty randomly sampled cases are shown along with the worst case scenario ($C_f = 9.84 \times 10^4$, $C_r = 1.98 \times 10^5$).

A comparison is done in Figure 6, where the simulation presented in Figure 5 is performed with a $\pm 15\%$ uncertainty in the cornering stiffness (nominal values: $C_f = 1.131 \times 10^5$, $C_r = 1.683 \times 10^5$). The parameter combination leading to the worst case in the lateral dynamics is chosen for all simulations presented in this study.

Further analysis of the transients caused by a switch in control is presented in the next section, along with the derivation of a performance indicator to evaluate these dynamics.

Analysis of the transient dynamics

A first step in the analysis of the transient dynamics is to find a ‘switching performance indicator’ that can be used to verify if the output of interest satisfies a predefined bound. This can be useful especially for bounding states/outputs of the system that are safety critical for the application. The performance indicator is first derived for general switched linear systems, followed by a specific application to the take-over scenario. The resulting performance indicator is then simplified using two different approaches resulting in a total of three indicators of varying conservativeness and complexity, as it will be discussed in this section.

Bounding the transient dynamics of switched linear systems

The general form of the system (equation (19)) that is studied in this research can be written as

$$\dot{X}_1(t) = A_1 X_1(t) + B_1 u(t), \quad 0 \leq t \leq t_s \quad (24a)$$

$$\dot{X}_2(t) = A_2 X_2(t) + B_2 u(t), \quad t_s < t \leq T \quad (24b)$$

$$\text{given, } X_2(t_s) = f(X_1(t_s), u(t_s), \dot{u}(t_s), \ddot{u}(t_s) \dots) \quad (24c)$$

where $X_1(t) \in \mathbb{R}^n$ and $X_2(t) \in \mathbb{R}^n$ are the state vectors, and the pairs $(A_1 \in \mathbb{R}^{n \times n}, B_1 \in \mathbb{R}^n)$ and $(A_2 \in \mathbb{R}^{n \times n}, B_2 \in \mathbb{R}^n)$ are the corresponding state and input matrices of the first (equation (24a)) and second (equation (24b)) subsystems (of order n), hereafter referred to as ‘System 1’ and ‘System 2’, with $u(t) \in \mathbb{R}$ as their common input. The time instant of switch (t_s) is considered as an exogenous factor and the initial condition for System 2 is a given function (f) of the states of System 1, the input ($u(t)$) and its derivatives at the instant of switching. It is also to be noted here that the System 2 is assumed to be active at least till time ‘ T ’ after the switch, which has to be large enough to include possible peak transient responses of the system after the switch. A specific definition of T is, however, not proposed in this research and has to be decided based on the dynamics of System 2. Finally, the output ($y(t)$) is defined using output matrices $C_1^T \in \mathbb{R}^n$ and $C_2^T \in \mathbb{R}^n$, given by

$$y(t) = \begin{cases} C_1 X_1(t), & 0 \leq t \leq t_s \\ C_2 X_2(t), & t_s < t \leq T \end{cases} \quad (25)$$

The required boundedness condition can be expressed as shown below

$$\|y(t)\|_{\infty}^{(t_s, T]} \leq y_{\max} \quad (26)$$

where $\|\cdot\|_{\infty}^{(t_s, T]}$ denotes a norm that is computed over the finite time interval $(t_s, T]$ and $y_{\max}$ is the maximum allowable absolute value of the output defined in equation (25). The output signal before the switch is only a function of the dynamics of System 1, its initial condition and the input, which are given and assumed to be such that bound (equation (26)) is satisfied for $t \leq t_s$. A sufficient condition for satisfying (equation (26)) is presented in the following theorem.

Theorem 1. Let equation (24) represent the dynamics of a switched system, with each of its subsystems (equations (24a) and (24b)) being exponentially stable linear time invariant (LTI) systems. Then, the amplitude boundedness condition (equation (26)) is satisfied if

$$\frac{y_{t_s} + \|g_2(t - t_s)\|_{\mathcal{L}_1}^{(t_s, T]} \|u(t)\|_{\infty}^{(t_s, T]}}{y_{\max}} \leq 1 \quad (27)$$

where $g_2(t)$ is the impulse response corresponding to the output defined in equation (25) for $t \in (t_s, T]$ and y_{t_s} is given by

$$y_{t_s} = |C_1 e^{A_1 t_s} X_0 + C_1 \int_0^{t_s} e^{A_1(t_s - \tau)} B_1 u(\tau) d\tau|$$

where X_0 is the initial condition of system (24a).

Proof. The output $y(t)$, as defined in equation (25) for $t \in (t_s, T]$, can be written as the solution of the system (equation (24)), given by

$$y(t) = \underbrace{C_2 e^{A_2(t-t_s)} X_2(t_s)}_{y_\alpha(t)} + \underbrace{C_2 \int_0^{t-t_s} e^{A_2(t-t_s-\tau)} B_2 u(\tau + t_s) d\tau}_{y_\beta(t)}, \quad t_s < t \leq T \quad (28)$$

where $y_\alpha(t)$ represents the zero-input response and $y_\beta(t)$ represents the zero-state response of system (equation (24b)) after the switch. The finite time $\mathcal{L}_\infty$ norm of the output $y(t)$ can be written as

$$\|y(t)\|_{\infty}^{(t_s, T]} = \|y_\alpha(t) + y_\beta(t)\|_{\infty}^{(t_s, T]} \quad (29)$$

Using the triangle inequality for $\mathcal{L}_1$ norms

$$\|y(t)\|_{\infty}^{(t_s, T]} \leq \|y_\alpha(t)\|_{\infty}^{(t_s, T]} + \|y_\beta(t)\|_{\infty}^{(t_s, T]} \quad (30)$$

where

$$\|y_\alpha(t)\|_{\infty}^{(t_s, T]} = \max_{t_s < t \leq T} |C_2 e^{A_2(t-t_s)} X_2(t_s)| \quad (31)$$

Since the subsystems of equation (24) are exponentially stable, A_1 and A_2 are Hurwitz. Moreover, for A_2 having distinct eigenvalues, the argument of the max function in the RHS of equation (31) is a decaying function, and an upper bound for $\|y_\alpha(t)\|_{\infty}$ can be obtained at $t = t_s$ as

$$\|y_\alpha(t)\|_{\infty}^{(t_s, T]} < |C_2 X_2(t_s)| =: y_{t_s} \quad (32)$$

In order to evaluate $\|y_\beta(t)\|_{\infty}^{(t_s, T]}$ in equation (30), the following Lemma is first discussed:

Lemma 1. If $G(s)$ is the transfer function of a system with the input signal $u(t)$, output signal $y(t)$ and $g(t)$ the impulse response matrix of $G(s)$, then for zero initial conditions, the output $y(t)$ is bounded according to²⁰

$$\|y(t)\|_{\infty} \leq \|g(t)\|_{\mathcal{L}_1} \|u(t)\|_{\infty}$$

Since $y_\beta(t)$ is the zero-state output of equation (24b), Lemma 1 can be used to bound it as

$$\|y_\beta(t)\|_{\infty}^{(t_s, T]} \leq \|g_2(t - t_s)\|_{\mathcal{L}_1}^{(t_s, T]} \|u(t)\|_{\infty}^{(t_s, T]} \quad (33)$$

where $g_2(t)$ is the impulse response matrix of the transfer function from the input $u(t)$ to the output $y(t)$, which can be computed as

$$g_2(t) = C_2 e^{A_2 t} B_2, \quad t \geq 0 \quad (34)$$

The bound (equation (30)) can now be written using equations (32) and (33) as

$$\|y(t)\|_{\infty}^{(t_s, T]} \leq y_{t_s} + \|g_2(t - t_s)\|_{\mathcal{L}_1}^{(t_s, T]} \|u(t)\|_{\infty}^{(t_s, T]} \quad (35)$$

A sufficient condition for satisfying the upper bound on the output defined in equation (26) can, thus, be formulated using equation (35) to obtain

$$\frac{y_{t_s} + \|g_2(t - t_s)\|_{\mathcal{L}_1}^{(t_s, T]} \|u(t)\|_{\infty}^{(t_s, T]}}{y_{max}} \leq 1$$

Discussion

The sufficient condition derived in Theorem 1 can be seen as a switching performance indicator, for satisfying bounds on the output of the form (equation (26)). By definition, the indicator focuses on the transient time period after a switch and is applicable for a general input ($u(t)$). It is also to be noted that using the triangle inequality for the $\mathcal{L}_\infty$ norms in equation (30) and subsequently using Lemma 1 for bounding $\|y_\beta(t)\|_{\infty}^{(t_s, T]}$, a considerable level of conservativeness is introduced in the final bound that is obtained. The level of conservativeness is discussed in section – ‘Simulation and validation’, by comparing the performance indicator with simulation results. Theorem 1 will now be applied to the specific case of a take-over scenario using the models developed in section – ‘Modelling a take-over’.

Bounding the transient dynamics of a take-over

Theorem 1, when applied to system (equation (19)) for output (equation (20)), gives a sufficient condition for satisfying a bound of the form (equation (26)) as given below

$$G_1 := \frac{y_{\tau_{to}} + \|g_d(t - \tau_{to})\|_{\mathcal{L}_1}^{(\tau_{to}, t_{tr}]} \|\rho(t)\|_{\infty}^{(\tau_{to}, t_{tr}]}}{y_{max}} \leq 1 \quad (36)$$

where

$$y_{\tau_{to}} = |C_{cc} e^{A_{cc} \tau_{to}} X_{c0} + C_{cc} \int_0^{\tau_{to}} e^{A_{cc}(\tau_{to}-\tau)} B_{cc} \rho(\tau) d\tau|$$

with X_{c0} as the initial condition of system (equation (19a)) and $g_d(t)$ as the impulse response corresponding to the output (equation (20)) of the driver–vehicle system (equation (19b)).

A simpler representation of G_1 (equation (36)) can be obtained using some properties of system (equation (19)) and input $\rho(t)$. This is done in the following parts of this section, by first expressing the finite time $\mathcal{L}_1$ norm of the impulse response (that appears in equation (36)) using the eigenvalues of the system, followed by approximating the input signal as a sinusoid for a lane change manoeuvre.

A growth relationship for finite time $\mathcal{L}_1$ norms. The $\mathcal{L}_1$ norm of the impulse response $g_d(t)$ that appears in equation (36) is to be computed over a finite time period ($\tau_{to} < t \leq t_{tr}$) which can alternatively be represented in terms of the eigenvalues of the state matrix (A_{dc}). This is done by taking advantage of the fact that the state matrices of the closed-loop dynamics in equation (19) are Hurwitz (known from the models developed in section – ‘Modelling a take-over’).

The impulse response $g_d(t)$ can be computed from the state space matrices as

$$g_d(t) = C_{dc} e^{A_{dc}t} B_{dc} \quad (37)$$

As A_{dc} is Hurwitz, there exist positive constants c and λ such that the impulse response can be bounded as²¹

$$|g_d(t)| \leq c e^{-\lambda t} \quad (38)$$

Moreover, since all eigenvalues of A_{dc} have multiplicity equal to 1, λ can be chosen as the absolute value of the largest (least negative) real part of the eigenvalues.²¹ Finding c analytically for the upper bound (equation (38)) to be tight, is challenging, given the complex structure of $g_d(t)$ (due the high order (8th) of the state space). However, a work around is proposed here, using the absolute maximum of the impulse response ($\|g_d(t)\|_\infty$). First, the time (t_∞) at which the absolute maximum value is attained by the impulse response is defined as

$$t_\infty = \{t | |g_d(t)| = \|g_d(t)\|_\infty, \quad t \geq 0\} \quad (39)$$

Here, t_∞ is a finite quantity by virtue of A_{dc} being Hurwitz. Moreover, the transient time period t_{tr} should be such that $(t_{tr} - \tau_{to}) \geq t_\infty$.

An approximate lower bound on c can be formulated using equations (38) and (39) as

$$\begin{aligned} \|g_d(t)\|_\infty &= |g_d(t_\infty)| \leq c e^{-\lambda t_\infty} \\ c &\geq \|g_d(t)\|_\infty e^{\lambda t_\infty} \end{aligned} \quad (40)$$

A word of caution has to be made here regarding using the lower bound obtained in equation (40), as it is a necessary but not a sufficient condition for satisfying (equation (38)). It can, however, be seen as a reasonable initial estimate to obtain an acceptable value for c , that does not make the RHS of equation (38) a conservative bound on $|g_d(t)|$.

An illustration of the bound (equation (38)) for the impulse response of the lateral tracking error of the vehicle (y_e) is presented in Figure 7 with the value of c set at its minimum value obtained in equation (40). In this example, $g_d(t)$ is computed using the vehicle parameters (Table 1) and driver model parameters (Table 2), with the values for c and λ obtained as 341.24 and 0.0639, respectively.

The finite time $\mathcal{L}_1$ norm of $g_d(t)$ can be computed using the bound in equation (38), giving

Table 2. Driver model parameters^{15,17} and path tracking controller parameters.^{12,13}

Driver parameters		Controller parameters	
k	0.24	ω_0 (rad/s)	17.5
T_L (s)	16	β_{str} (N m s/rad)	0.7
τ_d (s)	0.099	τ_{str} (s)	0.1
T_I (s)	0.91	k_1	0.008
T_N (s)	0.47	k_2	0.339
k_e	0.0071	k_{ff}	0.274
l_a	14.08		
k_r	0.08		

$$\begin{aligned} \int_0^\tau |g_d(t)| dt &= \int_0^\tau |C_{dc} e^{A_{dc}t} B_{dc}| dt \\ &\leq c \int_0^\tau e^{-\lambda t} dt \\ &= \frac{c}{\lambda} (1 - e^{-\lambda \tau}) \end{aligned} \quad (41)$$

where τ is the time up to which the $\mathcal{L}_1$ norm is to be computed. The final expression obtained in equation (41) can be seen as a growth relationship that bounds the finite time $\mathcal{L}_1$ norm, as it grows with τ at an exponential rate governed by λ . A comparison between this relation and the actual value of the $\mathcal{L}_1$ norm is presented in Figure 8, for the output chosen as the lateral position error (y_e).

It can be seen that the bound is satisfied but gets increasingly conservative for larger values of τ . However, by virtue of the simplicity that the relation offers, it deserves to be used as a substitute for the actual norms in equation (36) for obtaining a more intuitive sufficient condition for bounding the output amplitude, given by

$$G_2 := \frac{y_{\tau_{to}} + \frac{c}{\lambda} (1 - e^{-\lambda(t_{tr} - \tau_{to})}) \| \rho(t) \|_{\infty}^{(\tau_{to}, t_{tr})}}{y_{max}} \leq 1 \quad (42)$$

Note that equation (42) is obtained by substituting τ as the transient time period ($t_{tr} - \tau_{to}$) in equation (41) and subsequently using it instead of $\|g_d(t - \tau_{to})\|_{\mathcal{L}_1}^{(\tau_{to}, t_{tr})}$ in equation (36). Although a simpler expression for the sufficient condition is obtained in equation (42), it is more conservative than equation (36) due to the fact that a bound on the finite time $\mathcal{L}_1$ norm is used instead of its actual value. The level of conservativeness introduced in equation (42) will be verified in section ‘Simulation and validation’.

The next part of this section discusses how the results obtained in this section can be used along with the properties of the input ($\rho(t)$) for a lane change manoeuvre to obtain a similar bound on the output.

Bounding the transients of a lane change manoeuvre. The input to the system represented by equation (19) is the

Figure 7. Comparing $|g_d(t)|$ with the bound (equation (38)) and c equal to the lower bound in equation (40), for output chosen as the lateral tracking error y_e . Here, $c = 341.24$ and $\lambda = 0.0639$.

Figure 8. Comparison of the growth relationship (equation (41)) with the actual value of the finite time $\mathcal{L}_1$ norm of $g_d(t)$, for output chosen as the lateral position error (y_e). Here, $c = 341.24$ and $\lambda = 0.0639$.

curvature of the reference trajectory, which can be approximated by a low-frequency sinusoid (0.2 – 0.4 Hz in this study) for a single lane change manoeuvre. The input can, thus, be written as

$$\rho(t) = \begin{cases} A_p \sin(\omega t) & 0 \leq t \leq t_{lc} \\ 0 & t > t_{lc} \end{cases} \quad (43)$$

where A_p and ω are the amplitude and frequency of the sinusoid. The time required to perform the lane change (t_{lc}) is equal to the time period of the sinusoid resulting in $\omega = 2\pi/t_{lc}$. A comparison between the sinusoidal approximation (equation (43)) with the actual

Figure 9. Comparison between the sinusoidal approximation (equation (46)) with the actual polynomial curvature (equation (23)), for a single lane change of $t_{lc} = 3.8$ s, $v_x = 100$ km/h and lane width = 3.5 m.

polynomial curvature (equation (22)) for a single lane change manoeuvre is given in Figure 9.

The output signal (equation (20)) after the take-over ($\tau_{to} < t \leq t_{tr}$) can be written as the sum of the zero-input and zero-state responses, of which the zero-state response can be written as a convolution integral of the input with the impulse response $g_d(t)$, as shown below

$$|y(t)| = \begin{cases} |C_{dc} e^{A_{dc}(t-\tau_{to})} X_d(\tau_{to}) + \int_{\tau_{to}}^{t-\tau_{to}} g_d(t-\tau_{to}-\tau) \rho(\tau+\tau_{to}) d\tau| & \tau_{to} < t \leq t_{lc} \\ |C_{dc} e^{A_{dc}(t-\tau_{to})} X_d(\tau_{to}) + \int_{t_{lc}-\tau_{to}}^{t-\tau_{to}} g_d(t-\tau_{to}-\tau) \rho(\tau+\tau_{to}) d\tau| & t_{lc} < t \leq t_{tr} \end{cases} \quad (44)$$

Here, the output is derived in two parts due to a similar definition of the input (equation (43)), separated at $t = t_{lc}$. It is to be noted that the only difference between the two separate parts of the output is in the upper limit of the convolution integral, being limited to $(t_{lc} - \tau_{to})$ for $t > t_{lc}$ as the input is 0 during this time period.

A bound on the output can now be derived by substituting the bound on the impulse response (equation (38)) for the actual impulse response $g_d(t)$ in equation (44) which gives

$$|y(t)| \leq \begin{cases} |C_{dc} e^{A_{dc}(t-\tau_{to})} X_d(\tau_{to}) + A_p c \{ \sin(\omega t - \theta) + e^{-\lambda(t-\tau_{to})} \sin(\theta - \phi) \}| & \tau_{to} < t \leq t_{lc} \\ |C_{dc} e^{A_{dc}(t-\tau_{to})} X_d(\tau_{to}) + A_p c \{ e^{-\lambda(t-\tau_{to})} \sin(\theta - \phi) - e^{-\lambda(t-t_{lc})} \sin \theta \}| & t_{lc} < t \leq t_{tr} \end{cases} \quad (45)$$

$$=: \gamma(t), \quad t \in (\tau_{to}, t_{tr}]$$

where $\theta = \arctan(\omega/\lambda)$, $\phi = \omega\tau_{to}$ and $\omega = 2\pi/t_{lc}$.

An absolute bound over the output can be found by evaluating the RHS of equation (45) ($\gamma(t)$) and taking the maximum over $t \in (\tau_{to}, t_{tr}]$. A sufficient condition for satisfying output bounds of the form equation (26) can then be defined as

$$G_3 := \max_{t \in (\tau_{to}, t_{tr}] y_{max}} \frac{\gamma(t)}{y_{max}} \leq 1 \quad (46)$$

An illustration of this absolute bound is presented in the next section by comparing it with the bounds obtained in equations (36), (42) and through simulations. An illustration of how these bounds (or performance indicators) can be used to obtain safe regions on a specific parameter space is also discussed in the next section.

Simulations and validation

Using the models developed in section – ‘Modelling a take-over’, simulations are performed to obtain the peak values of the transients in the states/outputs after a switch. The simulations are performed for a single lane change manoeuvre, similar to the ones presented in subsection – ‘Switching between the two modes’ of the previous section. The following two independent factors are identified that can be varied, to generate different take-over conditions:

- Length of the lane change (L) – a measure of the severity of the manoeuvre initiated by the autonomous system. A shorter lane change is more severe for the same forward speed (v_x) of the vehicle.
- Time of take-over (τ_{to}) – as already discussed, this is the parameter that defines the time instant at which the control switches from the autonomous system to the driver.

These two parameters are selected in order to study the effect of the input trajectory and the switching parameter on the transient lateral dynamics of the vehicle (for a given driver, vehicle and path tracking controller). Note that the input to the system for the lane change manoeuvre is the reference curvature defined in equation (22).

For the purpose of illustration, the lateral acceleration of the vehicle (a_y (m/s²)) is chosen as the output of interest and its maximum allowable absolute value ($a_{y_{max}}$) is set at 4 m/s². The parameter values of the driver model and the path tracking controller are summarized in Table 2.

The peak absolute value of the output (here a_y) is first obtained through simulations by numerically solving the dynamics of the system (equation (19)) which is then compared with the bounds G_1 (equation (36)), G_2 (equation (42)) and G_3 (equation (46)), as shown in Figure 10.

Figure 10. Comparison of G_1 , G_2 and G_3 with simulations for a constant $L = 105$ m, $\tau_{to} = [0.1 \dots 3.9]$ s and $a_{y_{max}} = 4$ m/s². The safe limit (corresponding to a value of 1, when $\|a_y\|_{\infty} = a_{y_{max}}$) is indicated by the red dashed line.

It can be seen that the bound G_1 is the least conservative followed by G_3 and G_2 . This is due to the fact that G_1 is derived using the actual impulse response function making it less conservative in comparison to G_3 and G_2 , which use the bound on the impulse response (equation (41)). Having a simpler representation (G_2 and G_3) in terms of the eigenvalues of the system has, thus, resulted in an increase in the conservativeness. Furthermore, using a more precise definition of the input signal has helped in reducing the conservativeness from G_2 to G_3 . Overall, it can be said that G_1 provides a bound that is least conservative and most generic at the same time. The bound G_1 is now compared with the simulations for a range of values for L and τ_{to} as shown in Figure 11.

It can be seen that G_1 bounds the absolute peak of the transients obtained through simulations for the entire range of parameters, although it is still conservative to some degree. The safe level for the bound is indicated by a plane at a value of 1 (indicated in blue, in Figure 11). The portions of the surfaces that have a value greater than 1 can be projected on to the L versus τ_{to} plane, resulting in a classification of the parameter space into safe and unsafe regions, as illustrated in Figure 12.

Although using the bound G_1 demarcates a larger area as unsafe in the parameter space, it follows the trend seen in simulations that a smaller value of L (faster lane change) means more severe take-overs and for a given value of L some values of τ_{to} are more critical.

The bounds developed in this research can, thus, be used as performance indicators for the transient

Figure 11. Illustration of the absolute bound on $a_y(t)$ obtained from G_1 (equation (39)) with respect to simulation results, for $L = [90 \dots 140]$ m and $\tau_{to} = [0.1 \dots 3.5]$ s.

Figure 12. Safe and unsafe regions in the parameter space L (m) versus τ_{to} (s), obtained by projecting the portions of the bounds in Figure 11 that have a value greater than 1, on to the L (m) versus τ_{to} (s) plane.

dynamics which in turn divides parameter spaces into safe/unsafe regions similar to Figure 12.

Discussion

The specific regions demarcated in Figure 12 are applicable to the path tracking controller and the driver model chosen for this study. The same criterion can be applied to any choice of controllers or driver models that fit the system description (equation (24)), to obtain a similar classification of a parameter space that is of interest to the reader.

Moreover, no specific safety standards are used here, given the fact that there are no readily available standards for such manoeuvres involving take-overs in a

semi-autonomous driving system. However, the idea here is to use the indicators developed to evaluate a system for meeting future safety standard requirements.

Conclusion and recommendations

Conclusion

This research has put forth a switching performance indicator that can be used to evaluate the level of safety during and shortly after a driver initiated take-over from an autonomous driving system. The driver and the vehicle are modelled as LTI systems, and a path tracking controller is used to serve as an autonomous driving system. The take-over is then studied as a switch from the path tracking controller to the driver. The safety of a take-over is evaluated based on whether an output of interest satisfies a predefined absolute bound, after the take-over. This is done by first deriving a sufficient condition that can guarantee an absolute bound on the output after a switch, followed by obtaining a representation of the sufficient condition in terms of the eigenvalues of the system and the properties of the input signal. These sufficient conditions or bounds can also be interpreted as switching performance indicators as they can be used to pre-determine the safety of a take-over based on the structure and settings of the system components (driver, vehicle and path tracking controller).

Finally, a comparison is made between the bounds obtained through simulations and the one obtained by evaluating the sufficient conditions. It has been found that the bounds given by the sufficient conditions capture the trend seen in the simulation results to some extent, as a function of two parameters that define the input signal and the switching time instant, respectively. Even though the level of conservativeness in these bounds make them difficult to be used directly in real applications, they are useful for an initial estimation of a bound on the transient dynamics while designing systems that involve shared or multi-modal control, without having to perform extensive simulations. It is also observed that reducing the conservativeness of such an approach is a challenge beyond what is achieved here.

Recommendations for future work

In this research, the performance or safety of a switch is evaluated only in terms of bounding the absolute peak value of the states/outputs, which is not sufficient to completely quantify the transient dynamics arising due to a switch. The frequency content of the transients can also be a major concern for safety, which can be studied, for example, using the induced $\mathcal{L}_2$ norms as done Ploeg et al.⁷ or the root-mean-square gains as proposed by Hespanha.²²

An investigation on developing a unified indicator that combines different properties of the transients and a quantitative measure of its conservativeness are some

steps that can be taken towards making such an approach useful in real-world applications. Furthermore, an investigation into applying such an approach to different driver intentions after a take-over can be an interesting extension of this work.

It can also be interesting to apply the indicator developed in this research to non-linear vehicle models, sophisticated driver models and lateral controllers. Further research is also necessary in determining how such performance indicators can be effectively used in the design of semi-/multi-modal autonomous driving systems.

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Appendix I

Notation

$a_0 \dots a_5$	reference path coefficients
a_y	lateral acceleration
A_p	amplitude of approx. sinusoidal input
C_f, C_r	front and rear tyre cornering stiffness
$C_{\Delta_d}, C_{\Delta_c}$	output matrices for δ_d, δ_c
f	function defining initial conditions for System 2
g	impulse response matrix
g_1, g_2	impulse responses, general Systems 1, 2
G_1, G_2, G_3	bound sounds on the transient dynamics
I_{zz}	moment of inertia about Z-axis
k	gain – operator model
k_e	feedback gain – driver
k_r	feedforward gain – driver
k_1, k_2	feedback gains – controller

k_{ff}	feedforward gain – controller
K_c	feedback gain matrix – controller
K_d	feedback gain matrix – driver
K_δ	understeer coefficient
l_a	look ahead distance
l	wheelbase
L	length of lane change
m	mass of the vehicle
$\mathcal{O}$	reference point
t_s	general system, switching time instant
T	general system, transient time period
T_L	lead time constant – human operator
T_I	lag time constant – human operator
T_N	neuromuscular time constant
t_{tr}	transient time period
t_{lc}	lane change time
t_s	switching time instant – general system
u	general input
v	velocity
X_1, X_2	state vectors – general Systems 1, 2
X	state vector
X_0	initial state vector
y	lateral position

Greek symbols

α	lateral slip angle of the tyre
β	heading angle
β_{str}	steering actuator damping coefficient
γ	analytic expression for the output signal
δ	steering angle
$\delta_{d_1} \dots \delta_{d_4}$	dummy variables – driver dynamics
$\delta_{c_1} \dots \delta_{c_4}$	dummy variables – steering actuator dynamics
Δ	vector of dummy variables
ζ	damping coefficient of the lateral dynamics
θ_s	orientation of the reference path at point s
λ	least negative eigenvalue
ρ	curvature of reference path
τ_{to}	take-over time instant
τ_d	delay – driver dynamics
τ_{str}	delay – steering actuator
ϕ	phase angle
ψ	heading angle
ω	frequency of the approx. sinusoidal input
ω_0	steering actuator natural frequency
ω_z	yaw velocity