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LETTER TO THE EDITOR

NOTES ON THE PAPER: A THERMODYNAMIC
FRAMEWORK OF FRACTURE MECHANICS

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RECENTLY, Zhang and Karihaloo [1] published a paper about a thermodynamic framework for fracture mechanics. Their aim was "... to provide a rigorous framework of non-linear fracture mechanics, consistent with the laws of thermodynamics and to derive the necessary balance laws on the newly created surfaces." As such they expanded upon an approach used by Eftis and Liebowitz [2] in 1976, where all relevant variables appear twice, once as a property of the bulk of a body and once as a property of the surface. For example, bulk and surface density, bulk and surface entropy, etc. Physically such an approach makes sense, particularly in fracture mechanics where the actual amount of surface may change significantly as a crack propagates through a body.

The formulation of global balance laws is relatively simple. However, problems arise if one wants to obtain local balance laws. Eftis and Liebowitz [2] concluded that the local balance laws cannot be obtained as derived laws, but must be imposed as separate postulates. In their recent paper Zhang and Karihaloo [1], however, claim that the local balance laws can be derived from the global ones and the most important part of their paper is devoted to this derivation. Unfortunately their derivation contains a serious mistake. After correcting this mistake one finds that their basic result does not follow. Therefore one is forced to conclude that matters are still as they were after the appearance of the Eftis and Liebowitz paper.

In the remaining part of this note we will first show that the Zhang and Karihaloo results with respect to the balance law of mass lead to either a violation of their own starting position or to a physically highly unlikely result. Secondly, by tracing their derivation, we will show which error has occurred and that, after correction of the mistake, the basic results cannot be obtained. Thirdly and finally, a few remarks will be made concerning a thermodynamic framework in which both bulk and surface quantities appear on an equal footing. Reference to equation numbers or figure numbers in the paper of Zhang and Karihaloo is made by prefixing them with ZK.

Conservation of mass for the original and newly created surfaces is given by ZK(22):

$$\frac{D}{Dt} \int_{S_t} \rho^* da = 0 \quad (1)$$

where ρ^* is the mass density associated with the surface, S_t the material surface at time t and D/Dt the material derivative. Integrating eq. (1) over time one finds that the total mass $m^*(t)$ associated with the surface is:

$$m^*(t) = m^*(0) = \text{constant}, \quad (2)$$

provided we start our time count the moment a crack begins to propagate. Because no assumptions concerning material behaviour, loading, etc. were made to derive eq. ZK(22) it is a general one and our eq. (2) has the same generality. So, the equation surely applies to the thin strip shown in Fig. 1 where at $t = 0$ a crack starts and propagates through the strip in the period $0 \leq t \leq t_0$. If we take the dimensions of the strip such that $w/l \ll 1$ and $h/l \ll 1$ we find that the surface $S_t(t)$ of the system almost doubles in this period:

$$S_t(t_0) \approx 2S_t(0) \quad (3)$$

whereas the total mass remains constant. Now there are two possibilities. The first is that no mass is associated with the newly created surfaces, which introduces a strong inhomogeneity in ρ^* . This violates the basic starting point of the theory which is, after all, that mass should be associated with every surface. The second is that a relatively strong mass flow along the surface occurred. Moreover, the average surface mass density becomes dependent on S_t . This we find very unlikely from a physical point of view, particularly for solids.

Next, let us turn to the derivation given by Zhang and Karihaloo. The crucial step in their reasoning is the derivation of ZK(8) and ZK(9) from ZK(5). Since ZK(5) is basically a rewritten form of their starting postulate we accept this equation also. It reads:

$$\int_{V-A_F} [\dot{\Phi} + \Phi \operatorname{div} \mathbf{v} - \mathbf{Y} \cdot \nabla - \mathbf{g}] dV + \frac{D}{Dt} \int_{s(t)} \Phi^* da + \int_{s_f(t)} \mathbf{Y} \cdot \mathbf{n} da = 0, \quad (4)$$

in which Φ is any tensor field, Φ^* its corresponding surface quantity, $\mathbf{Y}$ the applied flow, $\mathbf{v}$ the velocity field, $\mathbf{g}$ the applied body force, $\mathbf{n}$ the outward normal, $s(t)$ the total surface at time t in the current configuration, $s_f(t)$ the newly created surfaces and $V - A_F$ the volume of the body minus the impending crack surface (see Fig. ZK1). According to Zhang and Karihaloo, eq. ZK(5) (our eq. 4) applies to any body containing a crack and they apply the equation to two regions $\mathcal{P}$ and $\mathcal{P}'$ (Fig. 2) chosen in such a way that $V = \mathcal{P} \cup \mathcal{P}'$ (see also Fig. ZK2). The crack is assumed to be in $\mathcal{P}'$ and $\partial \mathcal{P}$ is the boundary of

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Fig. 1. Crack in thin strip.

Fig. 2. Division of the body in two disjoint regions.

$\mathcal{P}$ and also part of the boundary of $\mathcal{P}'$. Equation (4) applied to $\mathcal{P}$ and to $\mathcal{P}'$ they obtain ZK(6) and ZK(7) which read respectively:

$$\int_{\mathcal{P}} [\dot{\Phi} + \Phi \operatorname{div} \mathbf{v} - \mathbf{Y} \cdot \nabla - \mathbf{g}] dv + \frac{D}{Dt} \int_{\partial \mathcal{P}} \Phi^* da = 0 \quad (5)$$

and

$$\int_{\mathcal{P}' - A_F} [\dot{\Phi} + \Phi \operatorname{div} \mathbf{v} - \mathbf{Y} \cdot \nabla - \mathbf{g}] dv + \frac{D}{Dt} \int_{\partial \mathcal{P}'} \Phi^* da + \frac{D}{Dt} \int_{s_F(t)} \Phi^* da + \int_{s_F(t)} \mathbf{Y} \cdot \mathbf{n} da = 0. \quad (6)$$

Adding eqs ZK(6) and ZK(7) (our eqs 5 and 6), using ZK(5) (our eq. 4) and the fact that V is the union of $\mathcal{P}$ and $\mathcal{P}'$ gives eq. ZK(8). This equation is:

$$\frac{D}{Dt} \int_{\partial \mathcal{P}} \Phi^* da = 0. \quad (7)$$

Using this equation they obtain finally eq. ZK(9) which is:

$$\int_{\mathcal{P}} [\dot{\Phi} + \Phi \operatorname{div} \mathbf{v} - \mathbf{Y} \cdot \nabla - \mathbf{g}] dv = 0. \quad (8)$$

Since $\mathcal{P}$ is an arbitrary part it is concluded that the integrand of eq. (8) should vanish, the result being that the local balance eqns ZK(10) and ZK(11) are obtained.

Now, the error in their derivation reproduced above is that the regions $\mathcal{P}$ and $\mathcal{P}'$ are considered as physical bodies. This is not the case because the boundary $\partial \mathcal{P}$ is not an actual, physical surface. Assigning surface values Φ^* to $\partial \mathcal{P}$ is nonsense for two reasons. The first is that, were this allowed, it is the same as assigning surface values to each and every part of the body because $\partial \mathcal{P}$ can be chosen at will. The second reason is that surface quantities are introduced precisely to discriminate between real (physical) surfaces and volumes. The reason for that is that one wants to account for the fact that atoms lying at the actual surface are differently influenced by all other atoms than those in the bulk of the material. Eftis and Liebowitz [2] discuss this matter quite extensively. Points on the boundary of some arbitrary region such as $\mathcal{P}$, only singled out in the course of an analysis and not lying on an actual surface, have to be considered as volume points because that is what they basically are.

If one tries to correct Zhang and Karihaloo's derivation, still aiming at "derived" local laws, it is found that this is not possible. Indeed, if one singles out the same regions as above one is faced with the problem of how to treat the boundary points if they are not part of an actual surface. If one postulates a reasonable balance law for regions completely within the actual body the local balance laws are in fact also postulated because the choice of these regions is arbitrary. Therefore we come to the conclusion that the result of Eftis and Liebowitz still stands to this day.

To conclude, let us make some remarks with respect to a thermodynamic framework of fracture mechanics where bulk and surface quantities occur on an equal footing. Since with an actual surface a certain thickness can be associated, the bulk and the surface can be viewed as two systems that interact with each other. Because the surface always surrounds the bulk, this interaction must be such that this condition is always met. Moreover, the bulk part can only exchange mass and energy with the surface. Mass exchange is expected to peak at the crack tip. An important parameter for this is basically the crack tip speed. The best manner to bring this into the theory is, in our opinion, by using the procedure followed by Gurtin [3–5], who surrounds the crack tip with a small region that moves with the crack tip at the same speed. The transition of bulk mass to surface mass takes place in this region. Although slightly different in meaning, such a zone in which energy is exchanged between parts of the system is in material science frequently called the "process zone". Setting up a full thermodynamic theory using this procedure is however, to our knowledge, a task which has yet to be performed.

REFERENCES

- [1] Chunyuan Zhang and B. L. Karihaloo, A thermodynamic framework of fracture mechanics. *Engng Fracture Mech.* **46**, 1023–1030 (1993).
- [2] J. Eftis and H. Liebowitz, On surface energy and the continuum thermodynamics of brittle fracture. *Engng Fracture Mech.* **8**, 459–485 (1976).
- [3] M. E. Gurtin, On the energy release rate in quasi-static elastic crack propagation. *J. Elasticity* **9**, 187–195 (1979).
- [4] M. E. Gurtin, Thermodynamics and the Griffith criterion for brittle fracture. *Int. J. Solids Structures* **15**, 553–560 (1979).
- [5] M. E. Gurtin and C. Yatomi, On the energy release rate in elastodynamic crack propagation. *Arch. rati. Mech. Anal.* **74**, 231–247 (1980).

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