Inheritance of workflows: an approach to tackling problems related to change

Citation for published version (APA):

Document status and date:
Published: 01/01/1999

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain.
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.

Download date: 20. Sep. 2019
Inheritance of Workflows
An approach to tackling problems related to change
by
W.M.P. van der Aalst and T. Basten

ISSN 0926-4515
All rights reserved
editors: prof.dr. R.C. Backhouse
         prof.dr. J.C.M. Baeten

Reports are available at:
http://www.win.tue.nl/win/cs

Computing Science Reports 99/06
Eindhoven, August 1999
Inheritance of Workflows
An approach to tackling problems related to change

W.M.P. van der Aalst and T. Basten

1 Dept. of Computing Science, Eindhoven University of Technology, The Netherlands
wsinwa@win.tue.nl
2 Dept. of Electrical Engineering, Eindhoven University of Technology, The Netherlands
tbasten@ics.ele.tue.nl

Abstract

Inheritance is one of the key issues of object-orientation. The inheritance mechanism allows for the definition of a subclass which inherits the features of a specific superclass. When adapting a workflow process definition to specific needs (ad-hoc change) or changing the structure of the workflow process as a result of reengineering efforts (evolutionary change), inheritance concepts are useful to check whether the new workflow process inherits some desirable properties of the old workflow process. Today’s workflow management systems have problems dealing with both ad-hoc changes and evolutionary changes. As a result, a workflow management system is not used to support dynamically changing workflow processes or the workflow processes are supported in a rigid manner, i.e., changes are not allowed or handled outside of the workflow management system. In this paper, we propose inheritance-preserving transformation rules for workflow processes and show that these rules can be used to avoid problems such as the “dynamic-change bug.” The dynamic-change bug refers to errors introduced by migrating a case (i.e., a process instance) from an old process definition to a new one. A transfer from an old process to a new process can lead to duplication of work, skipping of tasks, deadlocks, and livelocks. Restricting change to the inheritance-preserving transformation rules guarantees transfers without any of these problems. Moreover, the transformation rules can also be used to extract aggregate management information in case more than one version of a workflow process cannot be avoided.

Key words: Workflow management, Petri nets, inheritance, adaptive workflow, dynamic change, management information

Contents

1 Introduction 2
2 Preliminaries 6
  2.1 Notations for bags ................................................. 6
  2.2 Labeled Place/Transition nets .................................. 7
  2.3 Branching bisimilarity ...................................... 10
  2.4 WF-nets ......................................................... 12
  2.5 Soundness ..................................................... 14
3 Inheritance 16
  3.1 Inheritance relations ....................................... 17
  3.2 Inheritance-preserving transformation rules .......... 21

*Part of this work was done while the author was at sabbatical leave at the Large Scale Distributed Information Systems (LSDIS) laboratory of the University of Georgia.
**Part of this work was done while the author was employed at the Department of Computing Science of the Eindhoven University of Technology.
1 Introduction

Workflow-management technology aims at the automated support and coordination of business processes to reduce costs and flow times, and to increase quality of service and productivity [26, 38, 39]. A critical challenge for workflow management systems is their ability to respond effectively to process changes [37, 58]. Changes may range from ad-hoc modifications of the process for a single customer to a complete restructuring of the workflow process to improve efficiency [5]. Today’s workflow management systems are ill suited to dealing with change. They typically support a more or less idealized version of the preferred process. However, the real run-time process is often much more variable than the process specified at design-time. The only way to handle changes is to go behind the system’s back. If users are forced to bypass the workflow management system quite frequently, the system is more a liability than an asset.

Adaptive workflow aims at providing process support similar to contemporary workflow systems, but in such a way that the workflow system is able to deal with process changes. Recent papers and workshops show that the problems related to workflow change are difficult to solve [5, 7, 9, 18, 22, 23, 32, 33, 37, 47, 49, 55, 58]. Therefore, we take up the challenge to find techniques to add flexibility without losing the support provided by today’s systems.

Typically, there are two types of process changes: (1) ad-hoc changes and (2) evolutionary changes. Ad-hoc changes are handled on a case-by-case basis and affect only one case (i.e., process instance) or a selected group of cases. The change is the result of an error, a rare event, or special demands of the customer. Exceptions often result in ad-hoc changes. A typical example of an ad-hoc change is the need to skip a task in case of an emergency. A workflow process definition resulting from an ad-hoc change is called a variant of the workflow process. Ad-hoc change typically leads to many variants of a given workflow process running in parallel. Evolutionary change is of a structural nature: From a certain moment in time, the workflow changes for all new cases to arrive at the system. The change
is the result of a new business strategy, reengineering efforts, or a permanent alteration of external conditions (e.g., a change of law). Evolutionary change is typically initiated by the management to improve efficiency or responsiveness or is forced by legislature or changing market demands. A workflow process definition resulting from an evolutionary change is called a version. Evolutionary change results in new versions of a workflow process. New cases are handled according to the most recent version. Existing cases (i.e., work-in-progress) may also be influenced by an evolutionary change. Sometimes it is acceptable to handle running cases the old way. However, in many situations, cases need to be transferred from the old version to the new version.

Both ad-hoc and evolutionary change inevitably lead to one of the following two situations: Either there are multiple variants and/or versions which are active at the same time or cases need to be migrated from one variant/version to another. Today's workflow management systems have problems dealing with both situations. We use the term dynamic-change problem (cf. [22]) to refer to the anomalies caused by transferring cases from one process to another. The term management-information problem is used to refer to the problem of providing an aggregate overview of the work-in-progress in case of multiple versions and/or variants. The trend is towards an increasingly dynamic situation where both ad-hoc and evolutionary changes are needed to improve customer service and reduce costs continuously. Therefore, these problems are relevant for the next generation of workflow management systems.

In this paper, we use Petri nets to illustrate process-related concepts. In fact, we use a restricted class of Petri nets, namely the so-called WF-nets [1, 3]. In a WF-net, there is one source place and one sink place and all other nodes are on a path from source to sink. Readers not familiar with Petri nets and workflow modeling are referred to Section 2.

![Figure 1.1: The dynamic-change bug.](image)

Figure 1.1 shows two workflow process definitions illustrating the dynamic-change problem. If the sequential workflow process (left) is changed into the workflow process where tasks *send_goods* and *send_bill* can be executed in parallel (right), there are no problems, i.e., it is always possible to transfer a case from the left to the right. The sequential process has five possible states and each of these states corresponds to a state in the parallel process. For example, the state with a token in *p2* is mapped onto the state with a token in *s2* and *s3*. In both cases, tasks *prepare_shipment* and *send_goods* have been executed and *send_bill* and *record_shipment* still need to be executed. Now consider the situation
where the parallel process is changed into the sequential one, which means that cases need to be moved from the right-hand-side process to the left-hand-side process. For most of the states of the right-hand-side process, this is no problem, e.g., a token in $s_1$ and a token in $s_2$ are mapped onto one token in $P_1$, and a token in $s_2$ and a token in $s_3$ are mapped onto one token in $P_2$. However, the state with a token in both $s_1$ and $s_4$ (i.e., prepare.shipment and send.bill have been executed) causes problems because there is no corresponding state in the sequential process (where it is not possible to execute send.bill before send.goods). The example in Figure 1.1 shows that it is not straightforward to migrate old cases to the new process after a change.

The problem illustrated in Figure 1.1 is a result of reducing the degree of parallelism by making the process sequential. Similar problems occur when the ordering of tasks is changed, e.g., two sequential tasks are swapped. Extending the workflow with new tasks, removing parts, or aggregating a group of tasks into a single task may result in similar problems. When changing a workflow on-the-fly, i.e., running cases are transferred to the new process definition, the dynamic-change bug is likely to occur. Therefore, the problem is very relevant for workflow management systems truly supporting adaptive workflow. Today’s workflow management systems are not able to handle this problem. These systems typically use a versioning mechanism, i.e., every change leads to a new version and cases refer to the appropriate version. If a case starts using a version of the process, it will continue to use this version. The versioning mechanism may be suitable in some situations. An administrative process with a short flow time is a good candidate for a versioning mechanism. However, there are many situations where such a mechanism is not appropriate. If a case has a long flow time, then it is often not acceptable to handle existing cases the old way. Consider for example a process for handling mortgage loans. Mortgages typically have a duration of 20 to 30 years. If the mortgage process changes several times per year, this could lead to dozens of different versions running in parallel. To reduce costs and to keep the processes manageable, the number of active versions (i.e., versions still used by cases) should be kept to a minimum. Also for processes with a shorter flow time, it may be undesirable to have many versions running simultaneously. In fact, there may be legal reasons (i.e., starting from 1-1-2000 a new step in the process is mandatory) forcing the transfer of cases to the new process. Unfortunately, problems such as the one illustrated by Figure 1.1 make a direct transfer hazardous. Note that the dynamic-change problem is relevant for both ad-hoc change and evolutionary change. However, the problem is most prominent for evolutionary change where potentially many cases need to be transferred.

Another problem related to change is the problem that it may lead to multiple active versions/variants of the same process which makes it difficult to provide aggregate management information. Consider again Figure 1.1. Assume that the two workflow process definitions are versions of the same workflow process. At some point in time, the left-hand process may contain six running cases, two in state $p_1$, three in state $p_2$, and one in state $p_3$, whereas the right-hand process may contain four running cases, two in the state with tokens in $s_1$ and $s_2$ and two in the state with tokens in $s_1$ and $s_4$. To provide aggregate management information, these numbers must be combined in such a way that the result provides a meaningful representation of the amount of work-in-progress. In the example, the solution is not very difficult because each state in the left-hand process definition of Figure 1.1 has a corresponding state in the right-hand process definition. As a result, aggregate management information can be collected by projecting the states of all cases onto the right-hand process definition. Doing so yields that, for four of the total of ten cases, tasks send.goods, send.bill, and record.shipment still need to be executed; for two cases, send.goods and record.shipment still need to be performed; three cases are in a state that send.bill and record.shipment still need to be done, whereas for one case only task record.shipment still needs to be performed. It is possible to summarize this information by
counting the number of tokens resulting in each place of the right-hand process definition of Figure 1.1 when projecting the ten cases onto this process definition: Places $i$ and $o$ do not contain any tokens, place $s_1$ contains six tokens, place $s_2$ contains seven tokens, place $s_3$ contains four tokens, and, finally, $s_4$ contains three tokens. Although this example is not very complicated, in general, it is not straightforward to obtain aggregate management information when the different process definitions are more complex or their number is larger.

The management-information problem explained above occurs if multiple versions and/or variants of the workflow process cannot be avoided. For evolutionary change, the number of versions is often limited. In fact, if all cases are transferred, then there is just one active version (i.e., all running cases use the same version). However, in some situations, it is not possible nor desirable to transfer cases to the most recent process. There can be legal, managerial, or practical reasons that prevent the transfer of cases. In such a situation, there are multiple active versions of the same process. Ad-hoc change may lead to the situation where the number of variants may be of the same order of magnitude as the number of cases. The variants are customized to accommodate specific needs. To manage a workflow process with different versions/variants, it is desirable to have an aggregated view of the work-in-progress. Note that in a manufacturing process the manager can get a good impression of the work-in-progress by walking through the factory. For a workflow process handling digitized information, this is not possible. Therefore, it is of the utmost importance to supply the manager with tools to obtain a condensed but accurate view of the workflow processes. Although the problem of extracting aggregate management information is relevant for both ad-hoc and evolutionary change, it is most prominent for ad-hoc change.

To tackle the dynamic-change problem and the management-information problem, we propose an approach based on the inheritance-preserving transformation rules introduced in [13, 4]. Inheritance is one of the key concepts of object-orientation. Classes and objects in object-oriented design correspond to workflow process definitions and cases in a workflow management context. In object-oriented design, inheritance is typically restricted to the static aspects (e.g., data and methods) of an object class. For workflow management, the dynamic behavior of cases is of prime importance. The inheritance-preserving transformation rules used in this paper focus on workflow process definitions in a Petri-net-based setting. The four inheritance relations presented in this paper use branching bisimilarity (to compare processes) in combination with the notions of encapsulation and abstraction. Encapsulation corresponds to blocking tasks, whereas abstraction corresponds to hiding tasks. Restricting process changes to the inheritance-preserving transformation rules presented in this paper makes a direct transfer possible in any state while avoiding problems such as the one illustrated by Figure 1.1. Note that the inheritance rules can only be used to avoid the dynamic-change bug, i.e., it is a preventive treatment of the problem. If changes such as the one shown in Figure 1.1 are allowed, the only cure is to postpone the transfer in case of problems. As a result, in such a case, there may be several active versions of the same workflow process. There may be other reasons for having multiple active versions, e.g., by law, cases are forced to be handled the old way. In case of ad-hoc workflow, there are also multiple active versions of the same process (called variants). The presence of multiple active versions and/or variants of the same process can obscure the status of the whole workflow. Fortunately, the inheritance-preserving transformation rules can also be used to construct aggregate management information. The inheritance notions allow for the definition of concepts such as the Greatest Common Divisor (GCD) and Least Common Multiple (LCM) of a set of vari-

---

1The notions of encapsulation and abstraction in this paper are inspired by process-algebraic concepts (see [11]). In process algebra, the terms “encapsulation” and “abstraction” have a different meaning than the same terms in object-oriented design.
These concepts can be used to create a condensed overview of the work-in-progress. Clearly, the dynamic-change problem and the management-information problem are related. By solving the dynamic-change problem (i.e., instantly migrating all cases to a single version of the process), there is no need to construct aggregate management information because there is just one active version. However, ad-hoc changes inevitably lead to multiple variants and, as illustrated by Figure 1.1, multiple active versions of a workflow process are sometimes unavoidable.

The remainder of this paper is organized as follows. In Section 2, we introduce the basic concepts and the techniques we are going to use. The approach presented in this paper is based on a special subclass of Petri nets (WF-nets) and a notion of correctness named soundness [1, 3]. Section 3 introduces the inheritance notions and the inheritance-preserving transformation rules used in this paper. In Section 4, the use of inheritance in a workflow management context is discussed. Section 5 tackles the problems related to dynamic change using the inheritance-preserving transformation rules. In Section 6, it is shown that the results can also be used to create aggregate management information. In Section 7, we consider the use of tools to support the notions presented in this paper. Finally, Section 8 summarizes the results.

2 Preliminaries

This section introduces the techniques used in the remainder. Standard definitions for bags and Petri nets are given. Moreover, more advanced concepts such as branching bisimilarity, workflow nets, and soundness are presented. These preliminaries are required to define the inheritance concepts in an unambiguous way.

2.1 Notations for bags

In this paper, bags are defined as finite multi-sets of elements from some alphabet \( A \). A bag over alphabet \( A \) can be considered as a function from \( A \) to the natural numbers \( \mathbb{N} \) such that only a finite number of elements from \( A \) is assigned a non-zero function value. For some bag \( X \) over alphabet \( A \) and \( a \in A \), \( X(a) \) denotes the number of occurrences of \( a \) in \( X \), often called the cardinality of \( a \) in \( X \). The set of all bags over \( A \) is denoted \( \mathcal{B}(A) \). The empty bag, which is the function yielding 0 for any element in \( A \), is denoted \( \emptyset \). For the explicit enumeration of a bag, a notation similar to the notation for sets is used, but using square brackets instead of curly brackets and using superscripts to denote the cardinality of the elements. For example, \([a^2, b, c^3]\) denotes the bag with two elements \( a \), one \( b \), and three elements \( c \); the bag \([a^2 \mid P(a)]\) contains two elements \( a \) for every \( a \) such that \( P(a) \) holds, where \( P \) is some predicate on symbols of the alphabet under consideration. To denote individual elements of a bag, the same symbol “\( a \)” is used as for sets: For any bag \( X \) over alphabet \( A \) and element \( a \in A, a \in X \) if and only if \( X(a) > 0 \). The sum of two bags \( X \) and \( Y \), denoted \( X + Y \), is defined as \([a^n \mid a \in A \land n = X(a) + Y(a)]\). The difference of \( X \) and \( Y \), denoted \( X - Y \), is defined as \([a^n \mid a \in A \land n = (X(a) - Y(a)) \max 0]\). The binding of sum and difference is left-associative. The restriction of \( X \) to some domain \( D \subseteq A \), denoted \( X \mid D \), is defined as \([a^{X(a)} \mid a \in D]\). Restriction binds stronger than sum and difference. The notion of subbags is defined as expected: Bag \( X \) is a subbag of \( Y \), denoted \( X \subseteq Y \), if and only if, for all \( a \in A \), \( X(a) \leq Y(a) \). Note that any finite set of elements from \( A \) also denotes a unique bag over \( A \), namely the function yielding 1 for every element in the set and 0 otherwise. Therefore, finite sets can also be used as bags. If \( X \) is a bag over \( A \) and \( Y \) is a finite subset of \( A \), then \( X - Y, X + Y, Y - X \), and \( Y + X \) yield bags over \( A \). Moreover, \( X \leq Y \) and \( Y \leq X \) are defined in a straightforward manner.
2.2 Labeled Place/Transition nets

In this section, we define a variant of the classic Petri-net model, namely labeled Place/Transition nets. For a more elaborate introduction to Petri nets, the reader is referred to [21, 41, 48]. Let \( U \) be some universe of identifiers; let \( L \) be some set of action labels.

**Definition 2.1. (Labeled P/T-net)** An \( L \)-labeled Place/Transition net, or simply labeled P/T-net, is a tuple \((P, T, F, \ell)\) where:

1. \( P \subseteq U \) is a finite set of places,
2. \( T \subseteq U \) is a finite set of transitions such that \( P \cap T = \emptyset \),
3. \( F \subseteq (P \times T) \cup (T \times P) \) is a set of directed arcs, called the flow relation, and
4. \( \ell : T \rightarrow L \) is a labeling function.

In the Petri-net literature, the class of Petri nets introduced in Definition 2.1 is sometimes referred to as the class of (labeled) ordinary P/T-nets to distinguish it from the class of Petri nets that allows more than one arc between a place and a transition.

Let \((P, T, F, \ell)\) be a labeled P/T-net. Elements of \( P \cup T \) are referred to as nodes. A node \( x \in P \cup T \) is called an input node of another node \( y \in P \cup T \) if and only if there exists a directed arc from \( x \) to \( y \); that is, if and only if \( x \preceq y \). Node \( x \) is called an output node of \( y \) if and only if there exists a directed arc from \( y \) to \( x \). If \( x \) is a place in \( P \), it is called an input place or an output place; if it is a transition, it is called an input or an output transition. The set of all input nodes of some node \( x \) is called the preset of \( x \); its set of output nodes is called the postset. Two auxiliary functions \( \bullet \cdot \cdot : (P \cup T) \rightarrow \mathcal{P}(P \cup T) \) are defined that assign to each node its preset and postset, respectively. For any node \( x \in P \cup T \),

\[
\bullet x = \{ y \mid y \preceq x \} \quad \text{and} \quad x \bullet = \{ y \mid x \preceq y \}.
\]

Note that the preset and postset functions depend on the context, i.e., the P/T-net the function applies to. If a node is used in several nets, it is not always clear to which P/T-net the preset/postset functions refer. Therefore, we augment the preset and postset notation with the name of the net whenever confusion is possible: \( x \cdot \bullet \) is the preset of node \( x \) in net \( N \); and \( x \bullet \cdot \) is the postset of node \( x \) in net \( N \).

A labeled P/T-net as defined above is a static structure. Figure 2.2 shows the graphical representation of a P/T-net. Places are represented by circles; transitions are represented by rectangles. Attached to each place is its identifier. Attached to each transition is its label. Transition identifiers are only included if it is necessary to distinguish different transitions with the same label.

Labeled P/T-nets have a dynamic behavior. The behavior of a net is determined by its structure and its state. To express the state of a net, its places may contain tokens. In labeled P/T-nets, tokens are nothing more than simple markers (see Figure 2.2). The distribution of tokens over the places is often called the marking of the net.

**Definition 2.3. (Marked, labeled P/T-net)** A marked, \( L \)-labeled P/T-net is a pair \((N, s)\), where \( N = (P, T, F, \ell) \) is an \( L \)-labeled P/T-net and where \( s \) is a bag over \( P \) denoting the marking of the net. The set of all marked, \( L \)-labeled P/T-nets is denoted \( \mathcal{N} \).

The dynamic behavior of marked, labeled P/T-nets is defined by a so-called firing rule, which is simply a transition relation defining the change in the state of a marked net when executing an action. To define the firing rule, it is necessary to formalize when a net is allowed to execute a certain action.
Definition 2.4. (Transition enabling) Let \((N, s)\) be a marked, labeled P/T-net in \(N\), where \(N = (P, T, F, \ell)\). A transition \(t \in T\) is enabled, denoted \((N, s)[t]\), if and only if each of its input places \(p\) contains a token. That is, \((N, s)[t] \Leftrightarrow \bullet t \leq s\).

When a transition \(t\) of a labeled P/T-net is enabled, the net can fire this transition. Upon firing, \(t\) removes a token from each of its input places and adds a token to each of its output places. This means that upon firing \(t\), the marked net \((N, s)\) changes into another marked net \((N, s - \bullet t + \bullet t)\).

Definition 2.5. (Firing rule) The firing rule \(_t \) \in \mathcal{L} \subseteq N \times L \times N\) is the smallest relation satisfying for any \((N, s)\) in \(N\), with \(N = (P, T, F, \ell)\), and any \(t \in T\),
\[
(N, s)[t] \Rightarrow (N, s)[\ell(t)](N, s - \bullet t + \bullet t).
\]

The labeled P/T-net shown in Figure 2.2 is used to illustrate the firing rule. The net models the processing of complaints by the complaints desk of a fictitious Company X. The complaints desk handles complaints of customers about the products produced by Company X. Each complaint is registered before it is classified. Depending on the classification of the complaint, a letter is sent to the customer or an inquiry is started. The inquiry starts with a consultation of the department involved, followed by a discussion with the customer. Based on this inquiry, the necessary actions are taken. Finally, the dossier is filed. Figure 2.2 shows the process definition which is used to configure the workflow management system used by the employees of the complaints desk. The marking shown in Figure 2.2 is \([i]\), i.e., the state with one token in place \(i\). Transition \(register\) is the only transition enabled in this marking. Firing \(register\) results in the state \([pending\_complaint, registered]\), i.e., two tokens are produced. Then, \(classify\_complaint\) will fire followed by either \(send\_letter\) or \(contact\_department\), \(contact\_customer\), and \(take\_action\). Finally, \(file\_dossier\) will fire. Note that \(file\_dossier\) consumes two tokens and produces one token.

The firing rule determines the set of so-called reachable markings of a marked P/T-net. A marking \(s\) is reachable from the initial marking \(s_0\) of a marked net \((N, s_0)\) if and only if there exists a sequence...
of enabled transitions whose execution leads from \( s_0 \) to \( s \). This paper uses the following notations for sequences. Let \( A \) be some alphabet of identifiers. A sequence of length \( n \), for some natural number \( n \in \mathbb{N} \), over alphabet \( A \) is a function \( \sigma : \{0, \ldots, n - 1\} \to A \). The sequence of length zero is called the empty sequence and written \( e \). For the sake of readability, a sequence of positive length is usually written by juxtaposing the function values: For example, a sequence \( \sigma = [(0, a), (1, a), (2, b)] \), for \( a, b \in A \), is written \( aab \). The set of all sequences of arbitrary length over alphabet \( A \) is written \( A^* \).

**Definition 2.6. (Firing sequence)** Let \( (N, s_0) \) with \( N = (P, T, F, \ell) \) be a marked, labeled P/T-net in \( \mathcal{N} \). A sequence \( \sigma \in T^* \) is called a firing sequence of \( (N, s_0) \) if and only if \( \sigma = e \) or, for some positive natural number \( n \in \mathbb{N} \), there exist markings \( s_1, \ldots, s_n \in B(P) \) and transitions \( t_1, \ldots, t_n \in T \) such that \( \sigma = t_1 \ldots t_n \) and, for all \( i \) with \( 0 \leq i < n \), \((N, s_i)(t_{i+1})\) and \( s_{i+1} = s_i - \bullet t_{i+1} + t_{i+1} \cdot \). Sequence \( \sigma \) is said to be enabled in marking \( s_0 \), denoted \((N, s_0)(\sigma)\). Firing the sequence \( \sigma \) results in the unique marking \( s \), denoted \((N, s_0)(\sigma)(N, s)\), where \( s = s_0 \) if \( \sigma = e \) and \( s = s_n \) otherwise.

The marked, labeled P/T-net \((N, [i])\) shown in Figure 2.2 has many enabled firing sequences. For example, firing sequence register classify_complaint contact_department is enabled. Executing this sequence results in marking [pending_complaint, contact_cust].

As mentioned, a marking of a labeled P/T-net is reachable if and only if there is a firing sequence leading from the initial marking to that marking.

**Definition 2.7. (Reachable markings)** The set of reachable markings of a marked, labeled P/T-net \((N, s) \in \mathcal{N} \) with \( N = (P, T, F, \ell) \), denoted \([N, s]\), is defined as the set \([s' \in B(P) | (\exists \sigma : \sigma \in T^*: (N, s) \{\sigma\}(N, s'))]\).

Consider for example the marked, labeled P/T-net \((N, [i])\) shown in Figure 2.2. There are two firing sequences leading to marking \([0]\). Therefore, \([0]\) is reachable. In total, there are seven markings reachable from \([i]\).

For the purpose of analyzing processes defined by P/T-nets, many properties have been defined and studied. Some properties refer to the net structure, while others refer to the dynamic behavior of a marked P/T-net. The following two definitions refer to structural properties. The first definition uses the standard notations for the inverse of a relation \( R (R^{-1}) \) and the reflexive and transitive closure of \( R (R^*) \).

**Definition 2.8. (Connectedness)** A labeled P/T-net \( N = (P, T, F, \ell) \) is weakly connected, or simply connected, if and only if, for every two nodes \( x \) and \( y \) in \( P \cup T \), \( x(F \cup F^{-1})^* y \). Net \( N \) is strongly connected if and only if, for every two nodes \( x \) and \( y \) in \( P \cup T \), \( xF^* y \).

In the remainder of this paper, we assume all nets to be weakly connected. Moreover, we assume all nets to have at least two nodes. Nets without places or transitions do not make any sense.

Another structural property is the so-called free-choice property:

**Definition 2.9. (Free-choice P/T-net)** A free-choice P/T-net is a (labeled) P/T-net \((P, T, F, \ell) \) as in Definition 2.1 such that, for all transitions \( t, u \in T \), either \( t \cap u = \emptyset \) or \( t = u \).

Free-choice P/T-nets are characterized by the fact that two transitions sharing an input place always share all their input places. From a pragmatic point of view, the class of free-choice P/T-nets is of particular interest; many workflow management systems use a diagramming technique which corresponds to free-choice nets. The class of free-choice P/T-nets combines a reasonable expressive power.
with strong analysis techniques. Consequently, free-choice P/T-nets have been extensively studied in
the literature. The most important results on free-choice P/T-nets have been brought together in [21].

An example of a property which refers to the dynamics of a marked P/T net is boundedness.

**Definition 2.10. (Boundedness)** A marked, labeled P/T-net $(N, s) \in \mathcal{N}$ is bounded if and only if the
set of reachable markings $[N, s]$ is finite.

In a bounded net, the number of tokens in any place is bounded. If the maximum number of tokens in
each place is one, then the net is safe.

**Definition 2.11. (Safeness)** A marked, labeled P/T-net $(N, s) \in \mathcal{N}$ with $N = (P, T, F, \ell)$ is safe if
and only if, for any reachable marking $s' \in [N, s]$ and any place $p \in P$, $s'(p) \leq 1$.

Note that safeness implies boundedness.

A transition is dead if and only if there is no reachable marking enabling that transition.

**Definition 2.12. (Dead transition)** Let $(N, s)$ be a marked, labeled P/T-net in $\mathcal{N}$. A transition $t \in T$
is dead in $(N, s)$ if and only if there is no reachable marking $s' \in [N, s]$ such that $(N, s')[t]$.

A property stronger than the absence of dead transitions is liveness. A P/T-net is live if and only if,
no matter what marking has been reached, it is always possible to enable an arbitrary transition of the
net by firing a number of other transitions.

**Definition 2.13. (Liveness)** A marked, labeled P/T-net $(N, s) \in \mathcal{N}$ with $N = (P, T, F, \ell)$ is live if
and only if, for every reachable marking $s' \in [N, s]$ and transition $t \in T$, there is a reachable marking
$s'' \in [N, s')$ such that $(N, s'')[t]$.

2.3 Branching bisimilarity

To formalize the inheritance concepts mentioned in the introduction, we need to formalize a notion
of equivalence. Labeled P/T-nets are equipped with an equivalence relation that specifies when two
different marked, labeled P/T-nets have the same (observable) behavior. By choosing different equiv­
alence relations different semantics are obtained. For more information on the different semantics for
concurrent systems the reader is referred to [27, 28, 45]. In this paper, we use branching bisimilarity
[30] as the standard equivalence relation on different marked, labeled P/T-nets in $\mathcal{N}$.

The notion of a silent action is pivotal to the definition of branching bisimilarity. Silent actions are
actions (i.e., transition firings) that cannot be observed. Silent actions are denoted with the label $\tau$, i.e.,
only transitions in a P/T-net with a label different from $\tau$ are observable. Note that we assume that $\tau$
is an element of $L$. The $\tau$-labeled transitions are used to distinguish between external, or observable,
and internal, or silent, behavior. A single label is sufficient, since all internal actions are equal in the
sense that they do not have any visible effects.

As explained in the next subsection, in the context of workflow management, we want to distin­
guish successful termination from deadlock. A termination predicate defines in what states a marked
P/T-net can terminate successfully. If a marked, labeled P/T-net is in a state where it cannot perform
any actions or terminate successfully, then it is said to be in a deadlock. Assume that $\downarrow \subseteq \mathcal{N}$ is some
arbitrary termination predicate.

To define branching bisimilarity, two auxiliary definitions are needed: (1) a relation expressing that
a marked, labeled P/T-net can evolve into another marked, labeled P/T-net by executing a sequence
of zero or more $\tau$ actions; (2) a predicate expressing that a marked, labeled P/T-net can terminate by
performing zero or more $\tau$ actions.
Definition 2.14. The relation $\rightarrow \subseteq \mathcal{N} \times \mathcal{N}$ is defined as the smallest relation satisfying, for any $p, p', p'' \in \mathcal{N}$, $p \rightarrow p$ and $(p \rightarrow p' \land p' \{\tau\} p'') \Rightarrow p \rightarrow p''$.

Definition 2.15. The predicate $\downarrow \subseteq \mathcal{N}$ is defined as the smallest set of marked, labeled P/T-nets satisfying, for any $p, p', p'' \in \mathcal{N}$, $\downarrow p \Rightarrow \downarrow p$ and $(\downarrow p \land p' \{\tau\} p) \Rightarrow \downarrow p'$.

Let, for any two marked, labeled P/T-nets $p, p' \in \mathcal{N}$ and action $\alpha \in L$, $p \{\alpha\} p'$ be an abbreviation of the predicate $(\alpha = \tau \land p = p') \lor p \{\alpha\} p'$. Thus, $p \{\tau\} p'$ means that zero $\tau$ actions are performed, when the first disjunct of the predicate is satisfied, or that one $\tau$ action is performed, when the second disjunct is satisfied. For any observable action $a \in L \setminus \{\tau\}$, the first disjunct of the predicate can never be satisfied. Hence, $p \{\{a\}\} p'$ is simply equal to $p \{a\} p'$, meaning that a single $a$ action is performed.

Definition 2.16. (Branching bisimilarity) A binary relation $\mathcal{R} \subseteq \mathcal{N} \times \mathcal{N}$ is called a branching bisimulation if and only if, for any $p, p', q, q' \in \mathcal{N}$ and $\alpha \in L$.

1. $p\mathcal{R}q \land p \{\alpha\} p' \Rightarrow (\exists q', q'': q', q'' \in \mathcal{N} : q \rightarrow q'' \land q'' \{\{\alpha\}\} q' \land p\mathcal{R}q'' \land p'\mathcal{R}q)$,

2. $p\mathcal{R}q \land q \{\alpha\} q' \Rightarrow (\exists p', p'' : p', p'' \in \mathcal{N} : p \rightarrow p'' \land p'' \{\{\alpha\}\} p' \land p''\mathcal{R}q \land p'\mathcal{R}q)$, and

3. $p\mathcal{R}q \Rightarrow (\downarrow p \Rightarrow \downarrow q \land \downarrow q \Rightarrow \downarrow p)$.

Two marked, labeled P/T-nets are called branching bisimilar, denoted $p \sim_b q$, if and only if there exists a branching bisimulation $\mathcal{R}$ such that $p\mathcal{R}q$.

Figure 2.17: The essence of a branching bisimulation.

Figure 2.17 shows the essence of a branching bisimulation. The firing rule is depicted by arrows. The dashed lines represent a branching bisimulation. A marked, labeled P/T-net must be able to simulate any action of an equivalent marked, labeled P/T-net after performing any number of silent actions, except for a silent action which it may or may not simulate. The third property in Definition 2.16 guarantees that related marked, labeled P/T-nets always have the same termination options.

Branching bisimilarity is an equivalence relation on $\mathcal{N}$, i.e., $\sim_b$ is reflexive, symmetric, and transitive.

Property 2.18. Branching bisimilarity, $\sim_b$, is an equivalence relation.

Branching bisimilarity was first introduced in [29]. The definition given in this subsection is slightly different from the original definition. In fact, it is the definition of semi-branching bisimilarity, which was first defined in [57]. It can be shown that the two notions are equivalent in the sense that they define the same equivalence relation on marked, labeled P/T-nets [30, 12]. The reason for using the alternative definition is that it is more concise and more intuitive than the original definition. A comparison of the two definitions can be found in [12].

2.4 WF-nets

The use of Petri nets for workflow modeling has been suggested by many authors (e.g., [2, 7, 8, 24, 36, 42]) and several workflow management systems use Petri nets as a design language, e.g., COSA [51], INCOME [46], and BaanWorkflow [10]. In fact, most commercial workflow management systems use a modeling language which corresponds to a subset of Petri nets (typically free-choice P/T-nets [3]).

Before we present the class of nets we use in the remainder of this paper, we introduce the basic concepts and terminology used in the workflow-management domain. These are the concepts supported by today's workflow management systems and also recognized by standardization bodies such as the Workflow Management Coalition [39].

Workflows are case-based, i.e., every piece of work is executed for a specific case. Examples of cases are a mortgage, an insurance claim, a complaint, a tax declaration, an order, or a request for information. Cases are often generated by an external customer. However, it is also possible that a case is generated by another department within the same organization (internal customer). The goal of workflow management is to handle cases as efficiently and effectively as possible. A workflow process is designed to handle similar cases. Cases are handled by executing tasks in a specific order. The workflow process definition specifies which tasks need to be executed for a case and in what order (i.e., the life cycle of one case in isolation). Alternative terms for workflow process definition are: “procedure,” “flow diagram,” and “routing definition.” Since tasks are executed in a specific order, it is useful to identify conditions which correspond to causal dependencies between tasks. A condition holds or does not hold (true or false). Each task has pre- and postconditions: The preconditions should hold before the task is executed and the postconditions should hold after execution of the task. Many cases can be handled by following the same workflow process definition. As a result, the same task has to be executed for many cases. A task which needs to be executed for a specific case is called a work item. An example of a work item is the order to execute task “send refund form to customer” for case “complaint sent by customer Baker.” Most work items are executed by a resource. A resource is either a machine (e.g., a printer or a fax) or a person (participant, worker, employee). In most offices, the resources are mainly human. However, because workflow management is not restricted to offices, we prefer the term resource. Resources are allowed to deal with specific work items. To facilitate the allocation of work items to resources, resources are grouped into classes. A resource class is a group of resources with similar characteristics. There may be many resources in the same class and a resource may be a member of multiple resource classes. If a resource class is based on the capabilities (i.e., functional requirements) of its members, it is called a role. If the classification is based on the structure of the organization, such a resource class is called an organizational unit (e.g., team, branch, or department). A work item which is being executed by a specific resource is called an activity. If we take a photograph of the state of a workflow, we see cases, work items, and activities. Work items link cases and tasks. Activities link cases, tasks, and resources.

In this paper, we abstract from the resources and focus on the process aspect. In fact, we only consider the life cycle of one case in isolation. Cases only interact with each other via competition.
for resources. The problems introduced in Section 1 are not related to the allocation of resources to tasks or the interaction between cases. Therefore, given the topic of this paper, it is reasonable to abstract from resources and to consider just one case at a time. We also abstract from workflow attributes. A workflow attribute is a specific piece of information used for the routing of a case. One can think of a workflow attribute as a control variable or a logistic parameter. A workflow attribute may be the age of a customer, the department responsible, or the registration date, and is used to make routing decisions. We abstract from these workflow attributes for the following reasons. In reality, the routing decisions (i.e., OR-splits) are based on workflow attributes whose values depend on application data and/or the behavior of the persons and applications involved. Since workflow attributes are typically set by external entities (i.e., resources, applications, or electronic messages), they cannot be modeled accurately. Therefore, we consider each choice to be a non-deterministic one. We also abstract from workflow attributes because it allows us to use P/T-nets rather than high-level Petri nets. From an analysis point of view, the class of P/T-nets is preferable because of the availability of efficient algorithms and powerful analysis tools.

In the process dimension, it is specified which tasks need to be executed and in what order. Modeling a workflow process definition in terms of a P/T-net is rather straightforward: Tasks are modeled by transitions, conditions are modeled by places, and cases are modeled by tokens. Consider for example Figure 2.2. The P/T-net shown specifies the processing of complaints; each case corresponds to one complaint. There are seven tasks. Each task is modeled by a transition. Place $i$ models the condition that a new case has been created. The token in place $i$ refers to a newly created case for which no tasks have been executed yet.

A marked, labeled P/T-net which models a workflow process definition is called a WorkFlow net (WF-net). A WF-net satisfies two requirements. First, a WF-net has one place $i$ without any input transitions and one place $a$ without output transitions. A token in $i$ corresponds to a case which needs to be handled; a token in $a$ corresponds to a case which has been completed. Second, in a WF-net there are no dangling tasks (transitions) and/or conditions (places). Every task and condition should contribute to the processing of cases. Therefore, every node of a WF-net should be located on a path from place $i$ to place $a$. The latter requirement corresponds to strongly connectedness if place $a$ is connected to $i$ via an additional transition $i$.

Definition 2.19. (WF-net) Let $N = (P, T, F, \ell)$ be an $L$-labeled P/T-net. Net $N$ is a workflow net (WF-net) if and only if the following conditions are satisfied:

1. case creation: $P$ contains an input place $i \in U$ such that $\bullet i = \emptyset$,
2. case completion: $P$ contains an output place $a \in U$ such that $a \bullet = \emptyset$, and
3. connectedness: $\bar{N} = (P, T \cup \{i\}, F \cup \{(a, i), (i, i), (i, a)\}, \ell \cup \{(i, \tau)\})$ is strongly connected.

For any WF-net $N$, the extended net $\bar{N}$ used to formulate the connectedness constraint is called the short-circuited net. The label of the new transition in the short-circuited net is not important. For the sake of convenience, the label is set to $\tau$.

The P/T-net shown in Figure 2.2 is a WF-net satisfying the requirements given in Definition 2.19. The reader is referred to [3] for more information on modeling workflow process definitions in terms of WF-nets.

The input place $i$ corresponds to the initial state and the output place $a$ corresponds to the final state, i.e., a case starts in marking $[i]$ and completes in marking $[a]$. In the previous subsection, we introduced branching bisimilarity which distinguishes successful termination and deadlock. A
workflow can only terminate successfully in marking \([o]\). Therefore, for WF-nets, the termination predicate is defined as follows.

**Definition 2.20.** The class of marked, labeled P/T-nets \(N\) is equipped with the following termination predicate: 

\[ \downarrow = \{(N, [o]) \mid N \text{ is a WF-net} \}. \]

Note that the fact that a WF-net contains the output place \(o\) does not necessarily mean that, in the initial marking, it has the option to terminate successfully. In the next subsection, we address among other things successful termination of workflow processes.

### 2.5 Soundness

As mentioned, a workflow process definition specifies the life cycle of one case in isolation. This means that we are interested in the behavior of WF-nets that have initially a single token in the special place \(i\). The requirements given in Definition 2.19 only refer to the structure of the P/T-net modeling a workflow process definition. Despite these structural requirements, the behavior of a WF-net can contain problems such as deadlocks, livelocks, dangling references upon completion of a case, and tasks that can never be executed. Consider for example the WF-net shown in Figure 2.21. The WF-net describes the procedure for handling complaints. It is an extension of the WF-net shown in Figure 2.2. However, while extending this WF-net an error has been introduced. If the task `send_letter` is executed, a deadlock occurs; the system gets stuck in the marking `pending_complaint, inform_man`. The source of this problem is place `error`. This place is depicted in bold and named `error` to highlight the crux of the problem. Another problem occurs if `ignore_complaint` is executed; a token gets stuck in place `error` and the case completes (i.e., a token is put in place \([o]\)) without removing this token. The token in `error` can be seen as a dangling reference to the already completed case. Since most workflow management systems have no garbage collection, such a completion is undesirable. Moreover, after completing a case, it should be guaranteed that no tasks are executed for this case. To avoid these, and other, problems we formulate additional requirements.

**Definition 2.22. (Soundness)** A WF-net \(N\) is said to be **sound** if and only if the following conditions are satisfied:

1. **safeness**: \((N, [i])\) is safe,
2. **proper completion**: for any reachable marking \(s \in [N, [i]], o \in s\) implies \(s = [o]\),
3. **completion option**: for any reachable marking \(s \in [N, [i]], [o] \in (N, s)\), and
4. **dead transitions**: \((N, [i])\) contains no dead transitions.

Soundness is the minimal requirement any workflow process definition should satisfy. The first requirement in Definition 2.22 states that a sound WF-net is safe. This is a reasonable assumption since places in a WF-net correspond to conditions which are either true (marked by a token) or false (empty). The second requirement states that the moment a token is put in place \([o]\) all the other places should be empty, which corresponds to the completion of a case without dangling references. The third requirement states that starting from the initial marking \([i]\) it is always possible to reach the marking with one token in place \(o\), which means that it is always feasible to complete a case successfully. The last requirement, which states that there are no dead transitions, corresponds to the requirement that for each task there is an execution of the workflow in which the task is performed.

The notion of soundness used in this paper is slightly stronger than the notion of soundness used in previous publications, i.e., the first requirement is not present in [1, 3]. The safeness requirement
has been added to stress the fact that places correspond to conditions which either hold (one token) or do not hold (no tokens). In addition, the requirement allows for the simplification of the inheritance rules pivotal to this paper.

The WF-net shown in Figure 2.2 is sound. This can easily be verified by inspecting the seven reachable states. The WF-net shown in Figure 2.21 is not sound because the second requirement (proper completion: marking \([o, \text{error}]\) is reachable), the third requirement (completion option: deadlock in marking \([\text{pending\_complaint, inform\_man}]\)), and the fourth requirement (dead transitions: \(\text{in­form\_management}\) will never fire) of Definition 2.22 (Soundness) are not guaranteed.

The notion of soundness coincides with liveness and safeness of the short-circuited net.

**Theorem 2.23. (Characterization of soundness)** A WF-net \(N\) is sound if and only if \((\bar{N}, [i])\) is live and safe.

**Proof.** The proof is similar to the proof of Theorem 11 in [1]. The only difference is that in this paper a stronger notion of soundness is used, which implies safeness rather than boundedness of the short-circuited net.

This theorem shows that standard Petri-net-based analysis techniques can be used to verify soundness. Consider for example the WF-net shown in Figure 2.2. We can use one of the many standard Petri-net-based analysis tools (cf. [19]) to verify that the short-circuited net is live and safe. The exact complexity of deciding whether a WF-net is sound is not known. A very straightforward approach to deciding soundness is the construction of a coverability graph (see, for example, [48]) of the short-circuited net in its initial marking. This approach requires, in the worst case, non-primitive recursive space. Other existing results suggest that deciding soundness for arbitrary WF-nets requires at least exponential space in the number of nodes of the net. However, most workflow management systems use a modeling language which corresponds to free-choice P/T-nets ([3]). For free-choice WF-nets, soundness can be decided in polynomial time ([1]). Practical experience with workflow management
systems that allow for the design of non-free-choice WF-nets (e.g., COSA) shows that even the more complex workflows have less than 100,000 states and can be checked using tools that are based on the coverability-graph algorithm such as Woflan [6]. Woflan is briefly described in Section 7; for more information, the interested reader is referred to [53, 54].

**Definition 2.24. (Workflow process definition)** A workflow process definition is a sound WF-net. The set of all workflow process definitions is denoted $\mathcal{W}$.

Class $\mathcal{W}$ is the class of labeled P/T-nets that is interesting in the context of workflow management. Members of this class are called workflow process definitions and are guaranteed to be correct with respect to the criteria mentioned in Definitions 2.19 (WF-net) and 2.22 (Soundness). Note that, formally, a workflow process definition is defined as a P/T net without an initial marking. However, in the remainder, we typically consider markings reachable from the initial marking $[i]$. Therefore, if no initial marking is given explicitly, the initial marking of a workflow process definition is assumed to be $[i]$.

In this paper, branching bisimilarity is used as a behavioral equivalence relation. Therefore, in the remainder, we assume that branching bisimilarity is the standard equivalence relation for comparing workflow process definitions.

**Definition 2.25. (Behavioral equivalence of workflow process definitions)** For any two workflow process definitions $N_0$ and $N_1$ in $\mathcal{W}$, $N_0 \sim_b N_1$ if and only if $(N_0, [i]) \sim_b (N_1, [i])$.

### 3 Inheritance

Inheritance is one of the cornerstones of object-oriented programming and object-oriented design. The basic idea of inheritance is to provide mechanisms which allow for constructing subclasses that inherit certain properties of a given superclass. In our case, a class corresponds to a workflow process definition (i.e., a sound WF-net; see Definition 2.24) and objects (i.e., instances of the class) correspond to cases. In most object-oriented design methods, a class is characterized by a set of attributes and a set of methods. Attributes are used to describe properties of an object. Methods specify operations on objects (e.g., create, destroy, and change attribute). Note that attributes and methods only describe the static aspects of an object. The dynamic behavior of an object is either hidden inside the methods or modeled explicitly. (In UML [17], the behavior of an object is modeled in terms of a state machine.) Although the dynamic behavior of objects is an intrinsic part of the class description (either explicit or implicit), inheritance of dynamic behavior is not well-understood. (See [13] for an elaborate discussion on this topic and pointers to related work.) Since every object-oriented programming language supports inheritance with respect to the static structure of a class (i.e., the interface consisting of attributes and methods), this is remarkable. Since workflow management aims at supporting business processes, results on inheritance of static aspects are not very useful in this context. However, we can use the work presented in [13, 4, 14] where inheritance of dynamic behavior is dealt with in a comprehensive manner. Other approaches either focus on very specific inheritance relations or abstract from the causal relations between tasks/methods. Consider for example the work by Malone et al. [40] where inheritance is defined for tasks and processes. They also provide tool support for navigating through a space of processes using specialization and generalization links. Unfortunately, the control or routing structure is not taken into account, i.e., causal relations between tasks are not considered. Some of the workflow management systems available claim to be object-oriented and
thus provide some support for inheritance. For example, the workflow management system InCon­
cert [35] allows for building workflow class hierarchies. Unfortunately, inheritance is restricted to
attributes and the structure of a process is not taken into account. Many workflow management sys­
tems have been implemented using object-oriented programming languages. However, these systems
do not offer object-oriented mechanisms such as inheritance to the workflow designer or the designer
has to program code to benefit from the object-oriented features provided by the host language. Nev­
evertheless, we think that inheritance is a very useful concept for workflow management. Therefore, we
advocate the use of the inheritance notions presented in [13, 4, 14] and illustrate the usefulness by
tackling the problems related to change.

3.1 Inheritance relations

In this subsection, we define four inheritance relations for workflow processes. Consider two work­
flow process definitions $x$ and $y$ in $W$. When is $x$ a subclass of $y$? Process definition $x$ is a subclass
of superclass $y$ if $x$ inherits certain features of $y$. Intuitively, one could say that $x$ is a subclass of
$y$ if and only if $x$ can do what $y$ can do. Clearly, all tasks present in $y$ should also be present in $x$.
Moreover, $x$ will typically add new tasks. Therefore, it is reasonable to demand that $x$ can do what $y$
can do with respect to the tasks present in $y$. With respect to new tasks (i.e., tasks present in $x$ but not
in $y$), there are basically two mechanisms which can be used. The first mechanism simply disallows
the execution of any new tasks and then compares the resulting behavior of $x$ with the behavior of $y$.
This mechanism leads to the following notion of inheritance.

If it is not possible to distinguish the behaviors of $x$ and $y$ when only tasks of $x$ that are
also present in $y$ are executed, then $x$ is a subclass of $y$.

Intuitively, this definition conforms to blocking tasks new in $x$. The resulting inheritance concept is
called \textit{protocol inheritance}; $x$ inherits the protocol of $y$.

Another mechanism would be to allow for the execution of new tasks but to consider only the
effects of old ones.

If it is not possible to distinguish the behaviors of $x$ and $y$ when arbitrary tasks of $x$ are
executed, but when only the effects of tasks that are also present in $y$ are considered, then
$x$ is a subclass of $y$.

This inheritance notion is called \textit{projection inheritance}; $x$ inherits the projection of workflow process
definition $y$ onto the old tasks. Projection inheritance conforms to hiding or abstracting from tasks
new in $x$.

Recall from Section 2.3 that branching bisimilarity is the equivalence used to compare the behav­
iors of marked P/T-nets and, thus, the behaviors of workflow process definitions. Also recall that the
action label $\tau$ is used to denote internal or unobservable actions. As a consequence, hiding tasks in
a workflow process definition can be achieved by renaming these tasks to $\tau$. In the remainder of this
paper, we assume that the set of action or task labels $L$ used in WF-nets (see Definition 2.19) is equal
to the set $O$ of observable tasks extended with $\tau$, i.e., $L = \{\tau\} \cup O$.

Although the distinction between the two inheritance mechanisms presented above may seem sub­
tle, the corresponding inheritance notions are quite different. To illustrate this difference, we use the
eight workflow process definitions shown in Figure 3.1. Process definition $N_0$ corresponds to a sequen­
tial workflow process which consists of three tasks: \textit{register}, \textit{handle}, and \textit{archive}. Each of the other
workflow process definitions (i.e., $N_1$, $N_2$, $N_3$, and $N_4$) extends $N_0$ with an additional task: \textit{check}. In
process definition $N_1$, task check can be executed arbitrarily many times between register and handle. Process $N_1$ is a subclass of $N_0$ with respect to protocol inheritance; if check is blocked, then $N_1$ is identical to $N_0$. Process $N_1$ is also a subclass of $N_0$ with respect to projection inheritance; if every execution of check is hidden, then $(N_1, \{i\})$ is equivalent (branching bisimilar) to $(N_0, \{i\})$. In $N_2$, task check can be executed instead of task handle. Process $N_2$ is a subclass of $N_0$ with respect to protocol inheritance; if check is blocked, then $N_2$ is equivalent to $N_0$. Process definition $N_2$ is not a subclass of $N_0$ with respect to projection inheritance, because it is possible to skip task handle by executing the (hidden) task check. In process definition $N_3$, task check is executed in parallel with task handle. Process $N_3$ is not a subclass of $N_0$ with respect to protocol inheritance; if check is blocked, then task archive cannot be executed. However, $N_3$ is a subclass of $N_0$ with respect to projection inheritance. If one abstracts from the newly added parallel task check, one cannot distinguish $N_3$ and $N_0$. Task check is inserted between handle and archive in the remaining workflow process definition $N_4$ shown in Figure 3.1. Process $N_4$ is not a subclass of $N_0$ with respect to protocol inheritance; if check is blocked, then the process deadlocks after executing task handle. However, $N_4$ is a subclass of $N_0$ with respect to projection inheritance. If one abstracts from check, one cannot observe any differences between the behaviors of $N_4$ and $N_0$. 

Figure 3.1: Five workflow process definitions.
The two mechanisms (i.e., blocking and hiding) result in two orthogonal inheritance notions. Therefore, we also consider combinations of the two mechanisms. A workflow process definition is a subclass of another workflow process definition under protocol/projection inheritance if and only if both by hiding the new methods and by blocking the new methods one cannot detect any differences, i.e., it is a subclass under both protocol and projection inheritance. In Figure 3.1, \( N_1 \) is a subclass of \( N_0 \) with respect to protocol/projection inheritance. The two mechanisms can also be used to obtain a weaker form of inheritance. A workflow process definition is a subclass of another workflow process definition under life-cycle inheritance if and only if by blocking some newly added tasks and by hiding some others one cannot distinguish between them. Life-cycle inheritance is more general than the other three inheritance relations. All workflow process definitions shown in Figure 3.1 are subclasses of \( N_0 \) with respect to life-cycle inheritance. A detailed study of the four inheritance relations can be found in [13]. For the purpose of this paper, it suffices to formalize the relations. We do not go into much detail about the properties of the inheritance relations.

To formalize the four forms of inheritance, we introduce two operators on P/T-nets, namely encapsulation and abstraction. Encapsulation is used to block tasks; abstraction is used to hide tasks. The two operators are inspired by the encapsulation and abstraction operators known in process algebra [11]. The operators can be defined on labeled P/T-nets as follows.

**Definition 3.2. (Encapsulation)** Let \( N = (P, T_0, F_0, \ell_0) \) be an \( L \)-labeled P/T-net. For any \( H \subseteq O \), the encapsulation operator \( \partial_H \) is a function that removes from a given P/T-net all transitions with a label in \( H \). Formally, \( \partial_H(N) = (P, T_1, F_1, \ell_1) \) such that \( T_1 = \{ t \in T_0 \mid \ell_0(t) \not\in H \} \), \( F_1 = F_0 \cap ((P \times T_1) \cup (T_1 \times P)) \), and \( \ell_1 = \ell_0 \cap (T_1 \times L) \).

Note that removing transitions from a WF-net as defined in Definition 2.19 might yield a result that is no longer a WF-net.

**Definition 3.3. (Abstraction)** Let \( N = (P, T, F, \ell_0) \) be an \( L \)-labeled P/T-net. For any \( I \subseteq O \), the abstraction operator \( \tau_I \) is a function that renames all transition labels in \( I \) to the silent action \( \tau \). Formally, \( \tau_I(N) = (P, T, F, \ell_1) \) such that, for any \( t \in T \), \( \ell_0(t) \in I \) implies \( \ell_1(t) = \tau \) and \( \ell_0(t) \not\in I \) implies \( \ell_1(t) = \ell_0(t) \).

Given these two operators, the four notions of inheritance can be defined as follows:

**Definition 3.4. (Inheritance relations)**

1. **Protocol inheritance:**
   For any workflow process definitions \( N_0 \) and \( N_1 \) in \( W \), workflow process definition \( N_1 \) is a subclass of \( N_0 \) under protocol inheritance, denoted \( N_1 \preceq_{pr} N_0 \), if and only if there is an \( H \subseteq O \) such that \( (\partial_H(N_1), [i]) \sim_b (N_0, [i]) \).

2. **Projection inheritance:**
   For any workflow process definitions \( N_0 \) and \( N_1 \) in \( W \), workflow process definition \( N_1 \) is a subclass of \( N_0 \) under projection inheritance, denoted \( N_1 \preceq_{pj} N_0 \), if and only if there is an \( I \subseteq O \) such that \( (\tau_I(N_1), [i]) \sim_b (N_0, [i]) \).

3. **Protocol/projection inheritance:**
   For any workflow process definitions \( N_0 \) and \( N_1 \) in \( W \), workflow process definition \( N_1 \) is a subclass of \( N_0 \) under protocol/projection inheritance, denoted \( N_1 \preceq_{pp} N_0 \), if and only if there is an \( H \subseteq O \) such that \( (\partial_H(N_1), [i]) \sim_b (N_0, [i]) \) and an \( I \subseteq O \) such that \( (\tau_I(N_1), [i]) \sim_b (N_0, [i]) \).
4. Life-cycle inheritance:

For any workflow process definitions \( N_0 \) and \( N_1 \) in \( \mathcal{W} \), workflow process definition \( N_1 \) is a subclass of \( N_0 \) under life-cycle inheritance, denoted \( N_1 \leq_{lc} N_0 \), if and only if there are an \( I \subseteq O \) and an \( H \subseteq O \) such that \( I \cap H = \emptyset \) and \( \tau_I \circ \delta_H(N_1)([i]) \sim_k (N_0, [i]) \).

The four inheritance relations are based on the equivalence notion (branching bisimilarity) introduced in Definition 2.16. Note that for life-cycle inheritance the new tasks are partitioned into two sets \( H \) and \( I \): Tasks that are blocked by means of the operator \( \delta_H \) and tasks that are hidden by means of \( \tau_I \). It is easy to see that protocol/projection inheritance implies both protocol and projection inheritance. Moreover, protocol inheritance implies life-cycle inheritance and projection inheritance also implies life-cycle inheritance. However, life-cycle inheritance does not imply protocol or projection inheritance. Consider for example the workflow process definition shown in Figure 3.5. This workflow process definition extends the process definition shown in Figure 2.2 with four new tasks: \textit{inform\_customer}, \textit{contact\_management}, \textit{inform\_management}, and \textit{ignore\_complaint}. It corresponds to the WF-net of Figure 2.21 without the place \textit{error}. (Note that in contrast to the WF-net of Figure 2.21 the soundness property is satisfied.) The question is whether the extended workflow process definition shown in Figure 3.5 is a subclass of the workflow process definition shown in Figure 2.2. It is not a subclass under protocol inheritance; blocking \textit{contact\_management} results in a potential deadlock. It is also not a subclass under projection inheritance; by executing \textit{ignore\_complaint}, the original task \textit{send\_letter} is skipped. Since protocol/projection inheritance requires both protocol inheritance and projection inheritance, the extended workflow process definition is clearly not a subclass under protocol/projection inheritance. However, the extended workflow process definition shown in Figure 3.5 is a subclass of the workflow process definition of Figure 2.2 under life-cycle inheritance; by hiding \textit{contact\_management} and \textit{inform\_management}, blocking \textit{ignore\_complaint}, and hiding or blocking \textit{inform\_customer}, one obtains a workflow process definition that is branching bisimilar to the original one.

![Figure 3.5: An extended workflow process definition.](image-url)
The four inheritance relations introduced in this subsection have a number of desirable properties. For example, the relations are preorders (i.e., they are reflexive and transitive; see Property 4.3.21 in [13]). Furthermore, if one workflow process definition is a subclass of another workflow process definition under any of the four inheritance relations and vice versa, then the two workflow process definitions are equivalent as defined in Definition 2.25 (i.e., the two workflow process definitions are branching bisimilar; see Property 4.3.23 in [13]). In other words, the four inheritance relations are anti-symmetric. A relation that is reflexive, anti-symmetric, and transitive is a partial order. Thus, the following property is given without further proof.

**Property 3.6.** Assuming $\cong$, as defined in Definition 2.25, as the equivalence on workflow process definitions, $\leq_b$, $\leq_{pt}$, $\leq_{pj}$, and $\leq_{pp}$ are partial orders.

Another observation is that the definition of life-cycle inheritance does not allow that some executions of a task are blocked while other executions of the same task are hidden or left untouched in determining a subclass relationship between two workflow process definitions. To illustrate this restriction, consider a workflow process definition $N_5$. Process definition $N_5$ is an extension of workflow process definition $N_0$ of Figure 3.1 that combines the two extensions of process definitions $N_2$ and $N_3$ in the same figure. Process definition $N_5$ is not a subclass under life-cycle inheritance of $N_1$, whereas the workflow process definitions $N_2$ and $N_3$ are. The reason is that life-cycle inheritance does not allow the encapsulation of task check when it is executed as an alternative to task handle and the abstraction of task check when it is executed in parallel to handle. However, it is not difficult to generalize the definition of life-cycle inheritance, or any of the other three inheritance relations for that matter, in such a way that it is allowed to treat different executions of the same task in a different way. It simply requires the use of temporary task names to distinguish the different executions of a single task. For example, such a variant of life-cycle inheritance could be defined as follows. If a workflow process definition is a subclass under the current definition of life-cycle inheritance of another workflow process definition, then any renaming of the tasks new in the subclass yields a subclass under the variant of life-cycle inheritance. Consider, for example, the variant $N_6$ of the workflow process definition $N_5$ introduced above in which the two checks have names check$^2_2$ and check$^3_3$, respectively. It is not difficult to see that blocking one of these tasks and hiding the other one proves that $N_0$ is a subclass of $N_6$ under the current definition of life-cycle inheritance. Renaming the two tasks check$^2_2$ and check$^3_3$ to check proves that $N_5$ is a subclass of $N_6$ under the proposed variant of life-cycle inheritance. However, in this paper, we do not formalize the generalizations of the four inheritance relations along the lines discussed in this paragraph. The goal is to focus on the important concepts that play a role when applying inheritance notions in the context of workflow management. Although the above generalizations might be useful in some occasions, they distract from the essential concepts.

Finally, the question remains which inheritance relation is appropriate. The answer to this question depends on the context. For some applications, a very liberal notion of inheritance is suitable (i.e., life-cycle inheritance). For other applications, a more restrictive notion is desirable. In Section 4, we discuss the usefulness of the inheritance relations in different application domains.

### 3.2 Inheritance-preserving transformation rules

The inheritance relations of the previous subsection by themselves are not always immediately useful. The workflow designer can only benefit from the inheritance relations if there is a method or a tool to support workflow changes which preserve inheritance. For this purpose, we present four inheritance-preserving transformation rules. Each of these transformation rules can be used to construct a subclass
of a given workflow process definition by extending it. The rules are local and relatively easy to check (from a computational point of view). Furthermore, they correspond to typical design constructs used by a workflow designer to extend or change a workflow.

The rules presented in this paper are slightly different versions of the rules presented in [13, 4, 14]. The main distinction is the requirement that workflow process definitions (called life cycles in [13, 4, 14]) have to be safe. Therefore, the rules are named $PPS$, $PTS$, $PJS$, and $PJ3S$ rather than $PP$, $PT$, $PJ$, and $PJ3$. The safeness requirement simplifies the formulation of the rules and allows for generalizations with respect to the free-choice requirements stated in [13].

Two auxiliary definitions are needed for the definition of the transformation rules.

**Definition 3.7. (Alphabet)** The alphabet operator is a function $\alpha : N \rightarrow P(O)$. Let $(N, s)$ be a marked, $L$-labeled PfP-net in $N$, with $N = (P, T, F, \ell)$. The alphabet of $(N, s)$ is defined as the set of visible labels of all transitions of the net that are not dead: $\alpha(N, s) = \{ \ell(t) \mid t \in T \land \ell(t) \neq \tau \land t \text{ is not dead} \}$.

Since workflow process definitions do not contain dead transitions, the alphabet of a workflow process definition equals the set of its observable transition labels.

**Property 3.8. (Alphabet of a workflow process definition)** For any $N = (P, T, F, \ell) \in W$, the alphabet $\alpha(N, [\ell])$ equals $\{ \ell(t) \mid t \in T \land \ell(t) \neq \tau \}$.

**Proof.** It follows immediately from Definitions 2.24 (Workflow process definition), 2.22 (Soundness), and 3.7 (Alphabet). \hfill \Box

For the sake of simplicity, the alphabet of a workflow process definition $N \in W$ is denoted $\alpha(N)$.

**Definition 3.9. (Union of labeled P/T-nets)** Let $N_0 = (P_0, T_0, F_0, \ell_0)$ and $N_1 = (P_1, T_1, F_1, \ell_1)$ be two $L$-labeled P/T-nets such that $(P_0 \cup P_1) \cap (T_0 \cup T_1) = \emptyset$ and such that, for all $t \in T_0 \cap T_1$, $\ell_0(t) = \ell_1(t)$. The union $N_0 \cup N_1$ of $N_0$ and $N_1$ is the labeled P/T-net $(P_0 \cup P_1, T_0 \cup T_1, F_0 \cup F_1, \ell_0 \cup \ell_1)$. If two P/T-nets satisfy the abovementioned two conditions, their union is said to be well defined.

The rule that is the easiest one to understand is presented first. It is named $PPS$ and preserves both protocol and projection inheritance. Transformation rule $PPS$ is illustrated in Figure 3.10. Let $N_0$ be a workflow process definition. Let $N$ be a (connected) P/T-net such that the union $N_1 = N_0 \cup N$ is well defined. The workflow process definition $N_1$ is a subclass of process definition $N_0$ under protocol/projection inheritance if the following four conditions are satisfied: (1) $N_0$ and $N$ only share a single place $p$, (2) all transitions of $N$ have a label which does not appear in the alphabet of $N_0$, (3) each transition of $N$ with $p$ as one of its input places has a visible label, and (4) $(N, [p])$ is live and safe. Transformation rule $PPS$ shows that under protocol/projection inheritance, it is allowed to postpone behavior. When $(N_1, [\ell])$ reaches a state in which place $p$ is marked, it is possible to iterate the behavior defined by $N$ an arbitrary number of times before continuing with the original behavior. The requirement that $(N, [p])$ is live and safe guarantees that every token consumed from place $p$ by a transition of $N$ can always be returned to $p$. This property of $N$ is crucial for the correctness of rule $PPS$.

**Theorem 3.11. (Protocol/projection-inheritance-preserving transformation rule $PPS$)** Let $N_0 = (P_0, T_0, F_0, \ell_0)$ be a workflow process definition in $W$. If $N = (P, T, F, \ell)$ is a labeled P/T-net with place $p \in P$ such that
Figure 3.10: A protocol/projection-inheritance-preserving transformation rule.

1. \( p \not\in \{i, o\} \), \( P_0 \cap P = \{p\} \), \( T_0 \cap T = \emptyset \),
2. \( (\forall t : t \in T : \ell(t) \not\in \alpha(N_0)) \),
3. \( (\forall t : t \in T \land p \in \bullet t : \ell(t) \neq \tau) \),
4. \( (N, \{p\}) \) is live and safe, and
5. \( N_1 = N_0 \cup N \) is well defined,

then \( N_1 \) is a workflow process definition in \( \mathcal{W} \) such that \( N_1 \preceq_{pp} N_0 \).

**Proof.** Transformation rule \( PPS \) is a special case of two rules that are presented in the remainder, namely the rules \( PTS \) of Theorem 3.13 and \( PJS \) of Theorem 3.15. Since \( PTS \) preserves protocol inheritance and \( PJS \) preserves projection inheritance, it is shown that rule \( PPS \) preserves both protocol and projection inheritance and, thus, protocol/projection inheritance. The proof is a simplification of the proof of Theorem 4.3.30 in [13]. (The free-choice requirement in [13] is replaced by the condition that both \( N_0 \) and \( N \) are safe.)

In Figure 3.1, \( N_1 \) can be constructed from \( N_0 \) using transformation rule \( PPS \): place \( p_1 \) is the place shared by \( N_0 \) and the extension containing transition \( \text{check} \).

The remaining three transformation rules of this subsection are all based on the same principles as rule \( PPS \). The second transformation rule of this subsection, named \( PTS \), preserves protocol inheritance. It is illustrated in Figure 3.12. Let \( N_0 \) be a workflow process definition and \( N \) a labeled P/T-net such that the union \( N_1 = N_0 \cup N \) is well defined. Let \( P^I \) be the set of places in \( N_0 \) which are in the preset of some transition in \( N \) and let \( P^O \) be the set of places in \( N_0 \) which are in the postset of some transition in \( N \). Net \( N_1 \) is a workflow process definition and a subclass of \( N_0 \) under protocol inheritance if the following conditions are satisfied: (1) both \( P^I \) and \( P^O \) are not empty; (2) \( N_0 \) and \( N \) do not share any other nodes than the places in \( P^I \) and \( P^O \); (3) transitions in \( N \) consuming tokens from places in \( N_0 \) have preset \( P^I \) and transitions in \( N \) producing tokens for places in \( N_0 \) have postset \( P^O \); (4) all transitions of \( N \) with input places in \( N_0 \) have a visible label which does not appear in the alphabet of \( N_0 \); (5) \( N_0 \) extended with a transition \( u \) consuming tokens from \( P^I \) and producing tokens for \( P^O \) is a workflow process definition and (6) \( N \) extended with a transition \( v \) consuming tokens from \( P^O \) and producing tokens for \( P^I \) is live and safe if in the initial state all places in \( P^I \) are marked. Transformation rule \( PTS \) corresponds to the choice construct. Workflow process definition \( N_0 \) is extended with one or more alternative branches of behavior defined by extension \( N \). To make sure that
$N$ returns tokens to the original workflow process definition, $N$ extended with $v$ and with initial marking $P^I$ has to be live (enabling $v$ corresponds to marking all places in $P^O$). Net $N$ extended with $v$ is required to be safe to make sure that once the tokens are returned nothing is left in the other places of $N$. To make sure that marking all places in $P^O$ does not violate the soundness property, $N_0$ extended with $u$ is required to be a workflow process definition; transition $u$ emulates extension $N$. Note that both $u$ and $v$ are only introduced to formulate the requirements of transformation rule $PTS$; they are not present in the original workflow net $N_0$, the extension $N$, or the subclass $N_1$. The transitions of $N$ with input places in $N_0$ act as so-called guards. It is not difficult to see that encapsulating the guards leads to a net whose behavior is identical to the behavior of the original workflow process definition.

\begin{equation}
\begin{align*}
N_1 &= N_0 \cup N \\
\text{then } N_1 \text{ is a workflow process definition in } \mathcal{W} \text{ such that } N_1 \leq_{PT} N_0.
\end{align*}
\end{equation}
Proof. The proof consists of two parts. In the first part, it is shown that $N_1$ is a workflow process definition (see Definition 2.24). In the second part, a branching bisimulation is given showing that $N_0$ and $N_1$ satisfy the requirement stated in Definition 3.4-1 (Protocol inheritance).

To prove that $N_1$ is a workflow process definition, we first prove that $N_1$ is a WF-net (see Definition 2.19); second, we prove that $N_1$ is sound (see Definition 2.22).

Net $N_1$ must have one input place $i$ with an empty preset and one output place $o$ with an empty postset. Therefore, we have to show that, for all new transitions $t$ of $N$, $t \not\in o^i$ and $t \not\in o^o$. Based on requirements 1 and 3, this is equivalent to showing that $o \not\in P^I$ and $i \not\in P^O$. Since $N_0^N$ is a WF-net (requirement 5), this is guaranteed. The connectedness requirement of a WF-net follows from the fact that $(N^N, P^I)$ is live and safe (requirement 6). Hence, we conclude that $N_1$ is a WF-net.

To prove that $N_1$ is sound, we show that the short-circuited net $(\tilde{N}_1, [i])$ is live and safe (Theorem 2.23).

To prove safeness, assume that there is a marking $s \in [\tilde{N}_1, [i]]$ and a place $p \in P_1$ such that $s(p) > 1$. Let $T^I = \{ t \in T \mid o_t = P^I \}$ and $T^O = \{ t \in T \mid o_t = P^O \}$. Observe that $(N^N, P^I)$ is safe (requirement 6) and, thus, places in $P \setminus (P^I \cup P^O)$ are empty after firing a transition $t \in T^O$. Two cases can be distinguished. First, assume that $p \in P_0$. Based on the above observation, it is only possible that $s(p) > 1$ if two transitions in $T^I \setminus T^O$ are fired without firing a transition in $T^O$ in between. This means that, in $(N_0^N, [i])$, it is possible to fire the special transition $u$ twice in a row. However, recall that $N_0^N$ is a workflow process definition (requirement 5), which means that $(N_0^N, [i])$ is safe (Theorem 2.23). Clearly, this yields a contradiction. Second, assume that $p \in P_0$. Observe that firing a transition in $T^I \setminus T^O$, followed by firing (zero or more) transitions in $T^O \setminus T^O$, followed by firing a transition in $T^O$ is equivalent to firing transition $u$ in $(N_0^N, [i])$. Thus, since $(N_0^N, [i])$ is safe, it is only possible that $s(p) > 1$ if two transitions in $T^O \setminus T^O$ are fired without firing a transition in $T^I$ in between. (Only in this way, it is possible to create in $(\tilde{N}_1, [i])$ markings of places in $P_1$ that are not reachable in $(N_0^N, [i])$.) However, this contradicts again the above observation that places in $P \setminus (P^I \cup P^O)$ are empty after firing a transition $t \in T^O$. Consequently, $(\tilde{N}_1, [i])$ is safe.

To prove liveness of $(\tilde{N}_1, [i])$, observe that $(N^N, P^I)$ is live (requirement 6) and, thus, firing a transition in $T^I \setminus T^O$ can always be followed by firing a transition in $T^O$. Furthermore, recall that $N_0^N$ is a workflow process definition (requirement 5) which means that $(\tilde{N}_0^N, [i])$ is live (Theorem 2.23). Thus, transitions in $T_0$ are live in $(\tilde{N}_1, [i])$, because they are live in $(N_0^N, [i])$ and the subnet $N$ can only postpone enabling, and transitions in $T$ are live in $(\tilde{N}_1, [i])$, because $(N^N, P^I)$ is live and a state which marks all places in $P^I$ is reproducible in $(\tilde{N}_1, [i])$.

Combining the results obtained so far proves that $(\tilde{N}_1, [i])$ is live and safe and thus $N_1$ is sound (Theorem 2.23).

It remains to be proven that $N_1$ is a subclass under protocol inheritance of $N_0$. According to Definition 3.4-1 (Protocol inheritance), it must be shown that there exists an $H \subseteq O$ such that $(\partial_H(N_1), [i]) \sim_H (N_0, [i])$. Set $H$ is chosen to be $\alpha(N_1) \setminus \alpha(N_0)$. Let $R$ be the relation $\{((\partial_H(N_1), s), (N_0, s)) \mid s \in B(P_0)\}$. Using requirement 4, it is straightforward to prove that this relation is a branching bisimulation between $(\partial_H(N_1), [i])$ and $(N_0, [i])$, as defined in Definition 2.16.

To illustrate transformation rule $PTS$, we use the workflow process definitions shown in Figure 3.1. Process definition $N_1$ can be constructed from $N_0$ using transformation rule $PTS$; net $N$ is the net containing place $p_1$ and transition $check$. Note that, in this particular case, $P^I$ and $P^O$ coincide. Net $N_2$ can also be constructed from $N_0$ using transformation rule $PTS$. Since the remaining workflow process definitions (i.e., $N_3$ and $N_4$) are no subclasses of $N_0$ with respect to protocol inheritance, it makes no sense to try and apply $PTS$ to obtain either of these workflow process definitions.

It is easy to see that transformation rule $PPS$ of Theorem 3.11 is a special case of rule $PTS$. This
can be proven by assuming that the sets $P^I$ and $P^O$ of Theorem 3.13 are both equal to the singleton set containing place $p$ of Theorem 3.11.

The readers familiar with the inheritance transformation rule $PT$ presented in [13], will note some differences. The additional requirements stated in Theorem 3.13, in conjunction with the requirement that workflow process definitions are safe, allow for the local verification of the soundness properties. There is no need to verify that the whole WF-net $N_I$ is sound, as is required in [13]. It suffices to check that $N_0^u$ is sound and that $(N_0^u, P^I)$ is live and safe.

The next transformation rule of this subsection, $PJS$, preserves projection inheritance. Theorem 3.15 given below formalizes transformation rule $PJS$. Figure 3.14 illustrates the rule. It shows that rule $PJS$ corresponds to a sequential composition. New behavior may be inserted between sequential parts of a workflow process definition, yielding a subclass under projection inheritance. In contrast to the previous two transformation rules, the original workflow process definition is modified. Basically, transformation rule $PJS$ says that it is allowed to replace an arc in the original workflow process definition by an entire P/T-net. The original workflow process definition $N_0$ contains a place $p$ which has a transition $t_p$ as one of its input transitions. The modification of $N_0$ is based upon a P/T-net $N$ sharing place $p$ and transition $t_p$ with $N_0$. Place $p$ is the only input place of $t_p$ in $N$. The result of the transformation rule is the P/T-net $N_I$ obtained by taking the union of $N_0$ and $N$ after removing both the arc between $t_p$ and $p$ from $N_0$ and the arc between $p$ and $t_p$ from $N$. The requirement that $(N, \{p\})$ is live and safe guarantees that $N_I$ always has the option to move every token that transition $t_p$ would normally have put into place $p$ to place $p$ by only firing transitions of $N$. The requirement that all transitions of $N$ other than $t_p$ are labeled with task identifiers not appearing in the alphabet of $N_0$ guarantees that hiding these tasks does not influence the behavior of the original workflow process definition.

To formulate transformation rules $PJS$ and $PJS$, we use $\bullet_{\cdot}$ and $\bullet_\cdot$ to denote the preset and postset functions of $N$. Without this notation it is not possible to distinguish the preset and postset functions of the extension $N$ from those of the original workflow process definition $N_0$ and the resulting subclass $N_I$.

**Figure 3.14:** A projection-inheritance-preserving transformation rule.

**Theorem 3.15.** (Projection-inheritance-preserving transformation rule $PJS$) Let $N_0 = (P_0, T_0, F_0, \ell_0)$ be a workflow process definition in $W$. If $N = (P, T, F, \ell)$ is a labeled P/T-net with place $p \in P$ and transition $t_p \in T$ such that

1. $p \notin \{i, o\}$, $P_0 \cap P = \{p\}$, $T_0 \cap T = \{t_p\}$, $(t_p, p) \in F_0$, and $\bullet_{t_p} = \{p\}$,
2. \((\forall t : t \in T \setminus T_0 : \ell(t) \not\in \alpha(N_0))\),

3. \((N, [p])\) is live and safe, and

4. \(N_1 = (P_0, T_0, F_0 \setminus \{(t_p, p)\}, \ell_0) \cup (P, T, F \setminus \{(p, t_p)\}, \ell)\) is well defined, then \(N_1\) is a workflow process definition in \(\mathcal{W}\) such that \(N_1 \leq_{p_0} N_0\).

**Proof.** The proof is similar to the proof of Theorem 4.3.40 in [13]. (The free-choice requirement in [13] is replaced by the condition that both \(N_0\) and \(N\) are safe.) \(\Box\)

In Figure 3.1, \(N_4\) can be constructed from \(N_0\) using transformation rule \(PJS\); the arc between transition handle and place \(p_2\) is replaced by the net containing place \(p_3\) and transition check.

As mentioned in the proof of Theorem 3.11, transformation rule \(PPS\) of Theorem 3.11 is a special case of transformation rule \(PJS\) of Theorem 3.15. For more details, the reader is referred to [13]. (It is an interesting exercise to prove this claim.)

To formulate the last transformation rule of this subsection, the following auxiliary definition is needed. A place of a marked P/T-net is said to be redundant or implicit if and only if it does not depend on the number of tokens in the place whether any of its output transitions is enabled by some reachable marking.

**Definition 3.16. (Implicit place)** Let \((N, s)\) with \(N = (P, T, F, \ell)\) be a marked, labeled P/T-net. A place \(p \in P\) is called implicit in \((N, s)\) if and only if, for any reachable markings \(s' \in (N, s)\) and any transition \(t \in p \bullet, s' \geq \bullet t \setminus \{p\} \Rightarrow s' \geq \bullet t\).

Implicit places and their properties have been studied in [15, 20].

Transformation rule \(PJS\) is formalized in Theorem 3.18 given below. It shows under what restrictions it is allowed to extend a workflow process definition with a parallel branch of behavior. The result of rule \(PJS\) is a subclass of the original workflow process definition under projection inheritance. It is illustrated in Figure 3.17. As before, \(N_0\) is the original workflow process definition. Again, the modification of \(N_0\) is based on a P/T-net \(N\) containing a place \(p\) such that \((N, [p])\) is live and safe. The two net structures \(N_0\) and \(N\) share two transitions \(t_i\) and \(t_o\). In \(N\), place \(p\) is the only input place of \(t_i\) and the only output place of \(t_o\). Furthermore, \(p\) has no other input or output transitions. The net structure \(N_1\) resulting from transformation rule \(PJS\) is defined as the union of \(N_0\) and \(N\) after the removal of place \(p\). These assumptions mean that transitions \(t_i\) and \(t_o\) function as the input and output transition of the extra parallel branch modeled by \(N\). The basic idea is that the P/T-net \((N_1, [i])\) satisfies the property that every firing of transition \(t_i\) is eventually followed by a firing of transition \(t_o\). The requirement that \((N, [p])\) is live and safe guarantees that each time transition \(t_i\) fires the resulting tokens in places of \(N\) can be moved to the input places of transition \(t_o\) in \(N\) by only firing transitions of \(N\) other than \(t_i\) and \(t_o\). In addition, to guarantee that \((N_1, [i])\) satisfies the desired property, also \((N_0, [i])\) must be such that every firing of \(t_i\) is followed by exactly one firing of \(t_o\). To achieve this goal, assume that \(N_0\) is extended with a place \(q\) with \(t_i\) as its only input transition and \(t_o\) as its only output transition. Requiring that place \(q\) is implicit in this extension guarantees that a firing of transition \(t_o\) is always preceded by a firing of \(t_i\). It is not difficult to see that the number of tokens in \(q\) (zero or one) corresponds to the number of firings of transition \(t_i\) which have not yet been followed by a firing of \(t_o\). To guarantee that \((N_0, [i])\) cannot terminate without firing \(t_o\) as many times as \(t_i\), the extension of \(N_0\) with place \(q\) must be such that it cannot put a token in place \(q\) while leaving tokens in \(q\). Clearly, this is achieved when the extension of \(N_0\) with \(q\) yields another workflow process definition. The combination of the requirements on \(N_0\) and \(N\) implies that \(N_1\) is a workflow process definition satisfying the property that every firing of \(t_i\) is eventually followed by a firing of \(t_o\).
Theorem 3.18. (Projection-inheritance-preserving transformation rule PJ3S) Let \( N_0 = (P_0, T_0, F_0, \ell_0) \) be a workflow process definition in \( W \). Let \( N = (P, T, F, \ell) \) be a labeled P/T-net. Assume that \( q \in U \) is a fresh identifier not appearing in \( P_0 \cup T_0 \cup P \cup T \). If \( N \) contains a place \( p \in P \) and transitions \( t_i, t_o \in T \) such that

1. \( p \in P_0 \), \( p^N = \{ t_i \} \),
2. \( P_0 \cap P = \emptyset, T_0 \cap T = \{ t_i, t_o \} \),
3. \( (\forall t : t \in T \setminus T_0 : \ell(t) \notin \alpha(N_0)) \),
4. \( (N, [p]) \) is live and safe,
5. \( N_1 = N_0 \cup (P \setminus \{p\}, T, F \setminus \{(p, t_i), (t_o, p)\}, \ell) \) is well defined,
6. \( q \) is implicit in \( (N_0^q, [i]) \) with \( N_0^q = (P_0 \cup \{q\}, T_0, F_0 \cup \{(t_i, q), (q, t_o)\}, \ell_0) \), and
7. \( N_0^q \) is a workflow process definition,

then \( N_1 \) is a workflow process definition in \( W \) such that \( N_1 \preceq_{pj} N_0 \).

Proof. The proof is similar to the proof of Theorem 4.3.46 in [13]. (The free-choice requirement in [13] is replaced by the condition that both \( N_0 \) and \( N \) are safe.)

The transformation rule defined in Theorem 3.18 is named PJ3S for historical reasons; PJ3S corresponds to one of the axioms given in [13].

It is easy to see that workflow net \( N_3 \) in Figure 3.1 can be constructed from \( N_0 \) using transformation rule PJ3S.

In Section 3.1, we concluded that the workflow process definition shown in Figure 3.5 is a subclass of the workflow process definition shown in Figure 2.2 with respect to life-cycle inheritance. It is not difficult to see that the transformation rules presented in this section can be used to construct the subclass of Figure 3.5 from the superclass of Figure 2.2. Transformation rule PTS can be used to add the alternative task ignore_complaint. Rule PJ3S can be used to add task inform_management in-between send_letter and ready. The parallel task contact_management can be added using transformation rule PJ3S. Task inform_customer can be added using either PPS, PTS, or PJ3S.

In this subsection, four transformation rules have been presented to construct subclasses of workflow process definitions under different forms of inheritance. The rules correspond to design constructs that are often used in practice, namely choice, iteration, sequential composition, and parallel
composition. If a designer sticks to these rules, inheritance is guaranteed. In the remainder, we show that the transformation rules can be used to avoid the problems discussed in the introduction. The rules are also interesting from a computational point of view. By using the inheritance-preserving transformation rules rather than making arbitrary changes, the complexity of checking whether the extended workflow process definition is a subclass of the original process definition is reduced considerably. Note that all requirements specified for the transformation rules can be verified locally, i.e., each requirement which involves the evaluation of dynamic behavior is a constraint on either the original workflow process definition $N_0$ or the extension $N$. For none of the rules, it is required to verify the dynamic behavior (e.g., liveness, safeness, and reachability) of the combined net $N_1$. Soundness of the subclass $N_1$ follows from local requirements. Nevertheless, the complexity of many of the requirements in the transformation rules appears to be at least EXPSPACE hard. From a practical point of view, this is not an unconquerable problem. The requirements are of the same complexity as checking soundness. As argued in Section 2.5, existing tools such as Woflan can already verify the soundness property for complex workflows encountered in practice. Moreover, if only free-choice WF-nets are allowed, as is the case in most of the workflow management systems, all requirements can be verified in polynomial time. See [13] and [3] for more details.

4 Inheritance in the workflow-management domain

In this section, we discuss the usefulness of inheritance concepts in the context of workflow management. To address this issue, it is worthwhile to consider the following two trends:

- The shift from a "Sellers' Market" to a "Buyers' Market" in the last 30 years has resulted in an increase in the number of products and services offered to the customer. Consider for example mortgage loans; today, most financial institutions offer various types of mortgage loans. Moreover, the customer expects flexibility, i.e., the standard product or service has to be customized.
- Today's enterprises have a complex and rapidly changing structure. Communication mechanisms such as Electronic Data Interchange (EDI) and the Internet have enabled Electronic commerce (E-commerce) and extended/virtual enterprises. As a result, many business processes have become interorganizational or intraorganizational.

The impact of these two trends on workflow management is significant. As a result of the first trend, the number of workflow processes and variants for these processes has increased considerably. The second trend has resulted in inter/intraorganizational workflows distributed over several sites and involving heterogeneous resources. Workflow processes are moving from long-lasting, well-defined, centralized business processes to dynamically changing, distributed business processes with many variants. Given these developments and the associated problems, inheritance concepts are of particular relevance for the next generation of workflow management systems. To illustrate this, we give a number of situations where the inheritance concept can be used to tackle certain problems.

4.1 Ad-hoc change

It is widely recognized that workflow management systems should provide flexibility. However, as indicated in the introduction, today's workflow management systems have problems dealing with change. A particular kind of change is ad-hoc change. Ad-hoc change affects individual cases, i.e., it refers to changes on a case-by-case basis rather than structural modifications of the workflow process definition. An ad-hoc change is typically the result of an error, a rare event, or special demands of the
customer. Exceptions often result in ad-hoc changes. A typical example of ad-hoc change is skipping a task in case of an emergency. This kind of change is often initiated by some external factor. A typical dilemma related to ad-hoc change is the problem to decide what kinds of changes are allowed and the fact that it is impossible to foresee all possible changes.

The inheritance concepts presented in this paper can offer some support for ad-hoc change. The predefined workflow process definition is the superclass. The modified workflow process definition resulting from an ad-hoc change should be a subclass of this superclass under one of the four inheritance relations. By enforcing this requirement, certain properties are preserved. In a process resulting from an ad-hoc change that is a subclass of the predefined workflow process under protocol inheritance, new alternatives are offered but every sequence of tasks possible in the superclass is also possible in the subclass. For example, under protocol inheritance, it is possible to skip existing tasks by introducing “bypass” tasks. When projection inheritance is used, it is not allowed to skip existing tasks. However, it is possible to add new tasks in-between or in parallel. Which notion of inheritance is most appropriate depends on the situation. It is also possible to use different inheritance notions for different parts of the workflow process, e.g., subflows without interaction outside the company may be changed under life-cycle inheritance and subflows which communicate with external actors can only be changed under projection inheritance. Note that in general it is not possible to foresee all potential changes. The inheritance relations allow for formulating rules with respect to change rather than enumerating all possible exceptions.

Ad-hoc change typically leads to many variants of a given workflow process. Since such changes often correspond to exceptions, it is not desirable to combine all these variants in a single complex workflow. By using inheritance rather than creating a copy and modifying it each time a change is needed, only changes need to be stored. Moreover, as shown in Section 6, it is possible to provide aggregate management information.

4.2 Evolutionary change

New technology, new laws, and new market requirements lead to modifications of the workflow process definitions at hand. Evolutionary change refers to changes of a structural nature: From a certain moment in time, the process changes for all new cases to arrive at the system. This type of change is the result of a new business strategy, reengineering efforts, or a permanent alteration of external conditions (e.g., a change of law). Evolutionary change is initiated by the management to improve efficiency or responsiveness, or is forced by legislature or changing market demands. Evolutionary change always affects new cases but it may also influence old cases. Basically, there are four ways to deal with existing cases:

- **Restart**
  All existing cases are aborted and restarted in the new process. At any time, all cases use the same routing definition. For most workflow applications, it is not acceptable to restart cases because it is not possible to rollback work or it is too expensive to flush cases.

- **Abort**
  All existing cases are stopped and are not processed any further, i.e., all pending cases are aborted and considered to be ready. This approach is used if all existing cases are completed by hand. In general, this solution is not acceptable.

- **Proceed**
  Each case refers to a specific version of the workflow process. Newer versions do not affect old
cases. Most workflow management systems support such a versioning mechanism. A drawback of this approach is that old cases cannot benefit from an improved routing definition. In addition, it might be a complex task to manage to many versions.

- **Transfer**
  Existing cases are transferred to the new process, i.e., they can directly benefit from evolutionary changes. Often, the transfer of cases is the preferred solution. The term dynamic change, introduced in the introduction, refers to the problem of transferring cases to a consistent state in the new process.

Evolutionary change can cause problems internal to the company such as the dynamic-change problem illustrated in Figure 1.1. In addition, it can also cause confusion for the outside world. If a customer is used to receiving goods before the bill and now starts receiving the bill before the goods, this may result in irritation. Therefore, change needs to be restricted and the designer needs to be aware of the fact that the environment can be affected by certain changes. The inheritance rules presented in the previous section can be used to restrict changes. Since observable behavior is one of the cornerstones of the defined inheritance notions (see Sections 3.1 and 2.3), the degree of change as observed by the environment can be quantified.

If external business partners are involved in a change which does not satisfy certain inheritance requirements, they need to agree on such a change because the change might have externally noticeable effects. As a consequence, business partners will be able to notice the differences and need to act accordingly.

In Section 5, we show that by restricting evolutionary change to the four inheritance rules presented in the previous section the dynamic change problem can be avoided.

### 4.3 Workflow templates

Although workflow processes within different enterprises have common elements, they are typically designed from scratch. Also within large companies, it is often not possible to specify a workflow process definition once and replicate it across all parts of the company that are in need of such a process. Local differences have to be taken into account and prohibit the use of one uniform solution. As a result, workflow processes are typically designed from scratch and the “wheel” is re-invented every day. To avoid re-inventing the wheel, one can use workflow templates. A workflow template is a standard design of a common workflow process. An enterprise or a department within the enterprise can use such a workflow template as the starting point for the design of a new workflow process. The standard solution provided by the template is changed to accommodate specific needs. The use of templates allows the designers to reflect local differences (resulting from specific regulations, organizational structures, and other particularities) and still re-use the common parts.

The idea of using templates for workflow processes is not new. Today’s Enterprise Resource Planning (ERP) systems such as SAP R/3 (SAP AG, Walldorf, Germany, [34]) and BaanERP (Baan Company, Barneveld, The Netherlands, [44]) offer hundreds of ready-made workflow templates (often named business models or reference models) that can be used as a starting point for configuring the system. These workflow templates are often based on “best business practices” and reflect the experiences of leading enterprises. Although the set of workflow templates offered by today’s workflow management systems is still limited, it is clear that the use of templates will increase to avoid starting from scratch every time a new workflow has to be designed.

Current ERP and workflow management systems provide limited support for templates. A designer has to make a copy of a template and customize it to accommodate specific needs. This “copy and modify” approach is not very sophisticated; any change is allowed and changes to the template
do not affect the workflow processes designed using the template. Instead of the “copy and modify”
approach, one could also use an approach based on inheritance. By establishing an inheritance rela-
tionship between the customized workflow process and the corresponding template, it is possible to
restrict change and certain changes of the template can be transferred to the customized process.

4.4 E-commerce

Traditional Electronic commerce (E-commerce) mainly using Electronic Data Interchange (EDI) is
rapidly moving to the Internet. Moreover, E-commerce is moving from long-lasting well-defined
business relationships to a more dynamic situation, where parties having no prior trading relationship
engage in a common business process. Consequently, the operational boundaries between organiza-
tions have become fluid. As a result, it is difficult to separate interorganizational business processes
from the intraorganizational ones. E-commerce has complicated the management of business pro-
cesses. The processes are scattered over multiple organizations and are subject to frequent changes.
There are many problems which need to be solved to enable the enactment of workflows crossing
organizational borders. These problems are of a conceptual (e.g., how to design interorganizational
workflows), a technical (e.g., how to exchange data over the internet), a financial (e.g., how to dis-
tribute the benefits), and/or a managerial (e.g., who is responsible) nature. Two interesting conceptual
problems that may benefit from the inheritance concepts presented in this paper are the following.

- How to agree on a common workflow without having to know all the details of each others
  business processes?

- How to allow for local changes (e.g., one of the business partners involved optimizes its internal
  process) without the need for global coordination?

To solve the first problem one could design a simple common workflow where only the tasks which
are relevant for all partners are specified. Then, the common workflow is partitioned over the business
partners involved, i.e., the global workflow process is split into local parts. Each business partner ex-
tends/refines the local workflow process until it can be made operational. However, changing the local
workflow can cause problems, e.g., swapping two tasks can lead to deadlocks of the shared workflow.
To avoid such problems, three of the four inheritance-preserving transformation rules can be used
(PPS, P1S, and P13S). If the changes of the local workflow preserve projection inheritance, the other
business partners cannot detect any differences and therefore problems such as deadlocks and live-
locks can be avoided. The notion of projection inheritance is appropriate because it only allows for
changes having internal effects. Note that protocol inheritance and life-cycle inheritance are not suit-
able for this application. An alternative route in one of the local workflows may lead to a deadlock of
the overall workflow process. The second problem can be solved by using the same mechanism (i.e.,
projection inheritance); if local change is restricted to the transformation rules preserving projection
inheritance, then the other business partners cannot detect any differences and there is no need for a
new agreement on the global protocols.

The examples given in this section illustrate that inheritance concepts can be used to support ad-hoc
change, evolutionary change, workflow templates, and E-commerce. Clearly, this paper is just a start-
ing point for augmenting workflow management systems with inheritance concepts. For example, all
the examples given in this paper focus on the process perspective, i.e., only the control flow and rout-
ing aspects are considered. Inheritance is equally important for the other perspectives dealing with the
organization, data, applications, and operations. However, since the process perspective is dominant
in workflow management applications, we restrict ourselves to this perspective. In the remainder, we show that inheritance can truly assist in dealing with the problems identified in Section 1.

5 Dynamic change

The problem of dynamic change was introduced using Figure 1.1. If a sequential process is changed to a parallel one, there are no problems. However, if the degree of parallelism is reduced, there are states in the old process which do not correspond to states in the new process. The state with a token in both $s_1$ and $s_4$ (right-hand side of Figure 1.1) cannot be mapped onto a state in the sequential process (left-hand side). Putting a token in $i$, $p_1$, or $p_2$ will result in the double execution of task send_bill. Putting a token in $p_2$, $p_3$, or $o$ will result in the skipping of (at least) task send_goods. The problem identified does not only apply to the situation where the degree of parallelism is changed. For example, swapping or removing tasks may lead to similar problems. This is the reason most workflow management systems do not allow dynamic change, i.e., if a workflow process is changed, then all existing cases are handled the old way and the new process only applies to new cases. Every case has a pointer to a version of the workflow and each version is maintained as long as there are cases pointing to it. For some applications, this solution will do. However, if the flow time of a case is long, it may be unacceptable to process running cases the old way. Consider for example the change of a 4-year curriculum at a university to a 5-year one. It is too expensive to offer both curricula for a long time. Sooner or later, cases (i.e., students) need to be transferred. Other examples are mortgage loans and insurance policy's with a typical flow time of decades. Maintaining old versions of a process is often too expensive and may cause managerial problems. It is also possible that there are regulations (e.g., new laws) enforcing a dynamic change.

The inheritance-preserving transformation rules do not solve the problem indicated in Figure 1.1. The only way to avoid the incorrect execution of cases is to postpone the transfer of running cases in state $[s_1, s_4]$. The inheritance-preserving transformation rules can be used to avoid such problems by restricting change to those changes where a correct transfer is always possible.

The remainder of this section is organized as follows. First, we introduce the notion of a transfer rule, i.e., a rule to map cases from one workflow process definition to another. Then, we give concrete, generic transfer rules to map cases from a superclass to a subclass (i.e., specialization). Finally, we provide generic rules to support generalization, i.e., mapping cases from a subclass to a superclass.

5.1 Valid transfer rules

In this subsection, we assume the presence of two workflow process definitions: the old one and the new one. Cases present in the old process at the moment of change need to be transferred from the old workflow process definition to the new one, i.e., each case in the old process definition has to be removed and mapped onto a new case in the new process definition. Since the state of the case in the new process definition depends on the state of the case in the old process definition, we need to define a transfer rule.

Definition 5.1. (Transfer rule) Let $N_0 = (P_0, T_0, F_0, \ell_0)$ and $N_1 = (P_1, T_1, F_1, \ell_1)$ be two workflow process definitions in $W$. A transfer rule $r$ from $N_0$ to $N_1$ (notation $N_0 \xrightarrow{r} N_1$) is a partial function mapping markings of $N_0$ onto markings of $N_1$, i.e., $r : B(P_0) \rightarrow B(P_1)$.

A transfer rule maps states of an old workflow process definition onto states of a new workflow process definition, i.e., a transfer rule is a partial function mapping markings of the old process definition onto
markings of the new process definition. Note that the function can be partial \((\text{dom}(r) \subseteq B(P_0))\), i.e., if the state of the case is not in the domain of the transfer rule, then the case is not transferred. Clearly, not every transfer rule is acceptable. The transfer of a case according to a transfer rule should not result in deadlocks or other anomalies. In this paper, we consider a transfer rule to be acceptable if and only if every transfer of a case results in a state in the new workflow process definition which is also reachable by newly initiated cases. A transfer rule satisfying this property is called valid.

**Definition 5.2. (Valid transfer rule)** Let \(N_0\) and \(N_1\) be two workflow process definitions in \(W\) and \(r\) a transfer rule \(N_0 \rightarrow N_1\). Transfer rule \(r\) is valid if and only if, for all \(s \in \text{dom}(r) \cap [N_0, [i]]\), \(r(s) \in [N_1, [i]]\).

Basically, a valid transfer rule makes sure that the soundness property of the target workflow net is guaranteed for cases that are transferred from another workflow process definition.

**Property 5.3.** Let \(N_0 = (P_0, T_0, F_0, \ell_0)\) and \(N_1 = (P_1, T_1, F_1, \ell_1)\) be two workflow process definitions in \(W\) and \(r\) a valid transfer rule \(N_0 \rightarrow N_1\). For any \(s \in \text{dom}(r) \cap [N_0, [i]]\), the following conditions are satisfied:

1. safeness: for any place \(p \in P_1\), \(r(s)(p) \leq 1\),
2. proper completion: if \(o \in r(s)\), then \(r(s) = [o]\), and
3. completion option: \([o] \in [N_1, r(s)]\).

**Proof.** It follows directly from Definitions 2.24 (Workflow process definition), 2.22 (Soundness), and 5.2 (Valid transfer rule). \(\square\)

There are three generic transfer rules which are guaranteed to be valid.

**Definition 5.4. \((r_i, r_o, \text{ and } r_\emptyset)\)** Let \(N_0\) and \(N_1\) be two workflow process definitions in \(W\). Transfer rules \(N_0 \rightarrow N_1\), \(N_0 \rightarrow N_1\), and \(N_0 \rightarrow N_1\) are defined as follows:

- \(r_i\) is the transfer rule which maps every possible marking onto \([i]\), i.e., for any \(s \in [N_0, [i]]\), \(r_i(s) = [i]\).
- \(r_o\) is the transfer rule which maps every possible marking onto \([o]\), i.e., for any \(s \in [N_0, [i]]\), \(r_o(s) = [o]\).
- \(r_\emptyset\) is the transfer rule with the empty domain, i.e., \(\text{dom}(r_\emptyset) = \emptyset\).

**Property 5.5.** \(r_i, r_o, \text{ and } r_\emptyset\) are valid.

**Proof.** It follows directly from Definitions 2.24 (Workflow process definition), 2.22 (Soundness), 5.2 (Valid transfer rule), and 5.4; states \([i]\) and \([o]\) are reachable in any sound workflow net and the transfer rule with the empty domain is trivially valid. \(\square\)

Transfer rules \(r_i\), \(r_o\), and \(r_\emptyset\) correspond to three of the four policies described in Section 4.2. Rule \(r_i\) corresponds to restarting all existing cases, \(r_o\) corresponds to aborting all existing cases, and \(r_\emptyset\) corresponds to completing all running cases according to the old process definition (i.e., the use of a versioning mechanism). In the remainder of this section, we do not consider these trivial transfer rules; we focus on transfer rules which lead to a direct transfer (i.e., no postponement such as in \(r_\emptyset\)) to a meaningful state (i.e., not by default to \([i]\) or \([o]\)). Each of the rules presented corresponds to one of the inheritance-preserving transformation rules presented in Section 3.2.
5.2 Transfer of cases from superclass to subclass

In this paper, we consider two types of transfer rules, namely from a class to a subclass and from a class to a superclass. In this subsection, we present transfer rules mapping states of a class onto states of a subclass. Each of the transfer rules is based on one of the inheritance-preserving transformation rules of Section 3.2.

Before we introduce the transfer rules corresponding to respectively \(PPS\), \(PTS\), \(PJS\), and \(PJ3S\), we introduce the identity function as a generic transfer rule.

Definition 5.6. \((rid)\) Let \(N_0\) and \(N_1\) be two workflow process definitions in \(\mathcal{W}\). \(N_0 \xrightarrow{rid} N_1\) is the transfer rule which corresponds to the identity function, i.e., for any \(s \in [N_0, [i]]\), \(rid(s) = s\).

Transfer rule \(rid\) is not necessarily valid. A state in the first workflow process definition does not have to exist in the second process definition. However, \(rid\) turns out to be a suitable transfer rule for \(PPS\), \(PTS\), and \(PJS\).

The first transfer rule is based on the inheritance-preserving-transformation rule illustrated by Figure 3.10 (rule \(PPS\), see Theorem 3.11). Since \(PPS\) only adds new alternative behavior and does not restrict the behavior of the part corresponding to the "old" workflow process definition in any way, the case can simply be transferred without changing its state.

Theorem 5.7. (Transfer rule \(r_{PPS}\)) Let \(N_0\) and \(N_1\) be two workflow process definitions satisfying the requirements stated in Theorem 3.11. Mapping \(r_{PPS} = rid\) is a transfer rule \(N_0 \xrightarrow{r_{PPS}} N_1\) that is valid.

Proof. Recall that \(PPS\) is a special case of \(PTS\). Therefore, the validity of \(r_{PPS}\) follows directly from the validity of the transfer rule \(r_{PTS}\) which is presented next (see Theorem 5.9). However, it is easy to see that \(r_{PPS}\) is valid: The extension \(N\) only adds behavior, i.e., \([N_0, [i]] \subseteq [N_1, [i]]\). Therefore, the identity function is valid.

Figure 5.8 illustrates transfer rule \(r_{PPS}\). Inheritance-preserving transformation rule \(PPS\) has been used to extend the old workflow process definition on the left-hand-side with task \(inform\_customer\) which can be executed at any point between registration and filing. Transfer rule \(r_{PPS}\) transfers each case from the left-hand-side process definition to the right-hand-side process definition without changing the state. Figure 5.8 also illustrates transfer rule \(r_{1PPS}\) which is defined in Section 5.3.

The second transfer rule is based on inheritance-preserving transformation rule \(PTS\) and is identical to \(r_{PPS}\).
Theorem 5.9. (Transfer rule \( r_{PTS} \)) Let \( N_0 \) and \( N_1 \) be two workflow process definitions satisfying the requirements stated in Theorem 3.13. Mapping \( r_{PTS} = r_{id} \) is a transfer rule \( N_0 \rightarrow N_1 \) that is valid.

Proof. Let \( N_0 = (P_0, T_0, F_0, \ell_0) \) and \( N_1 = (P_1, T_1, F_1, \ell_1) \) be two workflow process definitions satisfying the requirements stated in Theorem 3.13. Since \( P_0 \subseteq P_1 \) and the extension \( N \) only enables new behavior (rather than restricting the existing behavior), it follows that \([N_0, [i]] \subseteq [N_1, [i]]\). This observation implies that the identity function \( r_{PTS} \) is valid.

Transfer rule \( r_{PTS} \) is illustrated in Figure 5.10. Using inheritance-preserving transformation rule \( PTS \), the old workflow process definition on the left-hand-side is extended with an alternative branch containing tasks \( ignore\_complaint \) and \( update\_statistics \). Every state in the left-hand-side process definition is also reachable in the right-hand-side process definition. Therefore, it is easy to see that the transfer rule \( r_{PTS} \) is valid in this particular situation. (Note that \( update\_statistics \) is not present in Figure 3.5. The task has been added to the WF-net shown in Figure 5.10 to introduce a subclass which has states not present in the superclass, i.e., the states marking place \( ignored \).)

The third transfer rule can be used when new tasks are inserted between existing sequential tasks as defined in Theorem 3.15 (i.e., transformation rule \( PJS \)).

Theorem 5.11. (Transfer rule \( r_{PJS} \)) Let \( N_0 \) and \( N_1 \) be two workflow process definitions satisfying the requirements stated in Theorem 3.15. Mapping \( r_{PJS} = r_{id} \) is a transfer rule \( N_0 \rightarrow N_1 \) that is valid.

Proof. The proof is similar to the proofs of Theorems 5.7 and 5.9. Since \( P_0 \subseteq P_1 \) and the added part \( N \) only inserts new behavior and does not restrict the existing behavior, it follows that \([N_0, [i]] \subseteq [N_1, [i]]\). This implies that the identity function \( r_{PJS} \) is valid.

Figure 5.12 illustrates transfer rule \( r_{PJS} \). Inheritance-preserving transformation rule \( PJS \) has been used to insert task \( inform\_management \) between \( send\_letter \) and \( ready \). The addition of this task only introduces new states. Therefore, transfer rule \( r_{PJS} \) transfers each case without changing the state. At a glance, an alternative transfer rule might be a mapping that transfers the token in place \( ready \) of the left-hand process definition to place \( inform\_man \) of the right-hand process definition. Such a transfer rule would imply that the newly added task \( inform\_management \) must be executed for the transferred case. However, such a transfer is only meaningful if the token in place \( ready \) of the left-hand process definition is the result of executing task \( send\_letter \). Clearly, this is not necessarily the case.
Transfer rules $r_{PJS}$, $r_{PPS}$, and $r_{PJ5}$ are rather trivial because additional behavior (i.e., alternative branches or parts inserted in-between existing parts) is introduced without eliminating existing states. The transfer rule corresponding to transformation rule $PJ3S$ is more complex because $PJ3S$ adds parallel behavior rather than additional behavior. When adding parallel behavior, it may be necessary to mark places in the newly added parts. Consider for example the two workflow process definitions shown in Figure 5.13. The left-hand-side process definition has been transformed into the right-hand-side process definition using transformation rule $PJ3S$; task contact management has been added such that it can be executed in parallel with contact.customer. Clearly, the state with a token in both pending_complaint and registered in the left-hand-side process definition should be mapped onto the identical state in the right-hand-side process definition. Also tokens marking i, classified, ready, and o should be transferred to the same place in the right-hand-side process definition. However, if one of the places contact_cust or cust_contacted is marked, then transferring the case to the identical state will result in a deadlock. Consider for example a case with tokens in both pending_complaint and registered in the left-hand-side process definition. If this state is transferred without modifications to the right-hand-side process definition, task take_action can never be executed, because place man_contacted will never get marked. The only way to solve this problem is to add an additional token to either place contact_man or man_contacted. In a conservative approach, contact_man is marked and the new task contact_management is required to be executed before task take_action is performed. In a progressive approach, man_contacted is marked and task take_action can be executed without executing the new task contact_management.
Figure 5.13 illustrates two complicating issues when transferring cases under the inheritance-preserving transformation rule $P_{J3S}$: (1) Sometimes (but not always) additional tokens need to be added, and (2) when adding tokens, there are sometimes multiple ways to add these tokens (e.g., conservative or progressive approach). Closely observing the requirements stated in Theorem 3.18 provides a solution for the first issue; the implicit place $q$ acts as some kind of counter indicating whether the newly added part should be marked with additional tokens. The second issue is dealt with by providing two transfer rules: a conservative or pessimistic one ($r_{PJS,C}$) and a progressive or optimistic one ($r_{PJS,P}$). To define these two rules and prove their validity, we use the following lemma.

**Lemma 5.14.** Let $N = (P, T, F, \ell)$ and $N^q = (P_q, T_q, F_q, \ell_q)$ be two workflow process definitions in $W$ and $q$ a place in $U$ such that $q \in P_q$, $P = P_q \setminus \{q\}$, $T = T_q$, $F = F_q \cap ((P \times T) \cup (T \times P))$, $\ell = \ell_q$, and $q$ is implicit in $(N^q, [i])$. For any reachable marking $s \in [N, [i]]$ and firing sequences $\sigma_1, \sigma_2 \in T^*$ such that $(N, [i]) [\sigma_1] (N, s)$ and $(N, [i]) [\sigma_2] (N, s)$, there is a unique marking $s' \in [N^q, [i]]$ such that, for all $p \in P$, $s'(p) = s(p)$ and $(N^q, [i]) [\sigma_1] (N^q, s')$ and $(N^q, [i]) [\sigma_2] (N^q, s')$.

**Proof.** It is well known that any firing sequence $\sigma \in T^*$ enabled in $(N, [i])$ and resulting in $s$ (i.e., $(N, [i]) [\sigma] (N, s)$ is enabled in $(N^q, [i])$ (i.e., $(N^q, [i]) [\sigma]$) (see Definition 3.16 (Implicit place) and [15, 20]). Assume $s_1$ and $s_2$ are the two markings in $[N^q, [i]]$ such that $(N^q, [i]) [\sigma_1] (N^q, s_1)$ and $(N^q, [i]) [\sigma_2] (N^q, s_2)$. It follows from Definition 3.16 that, for all $p \in P$, $s_1(p) = s_2(p) = s(p)$. Thus, it remains to be proven that $s_1(q) = s_2(q)$. Without loss of generality, we can assume that $s_1(q) > s_2(q)$. Since $N$ is a workflow process definition, there is an $s_3$ such that $(N, s_3) (N, [s])$ (see Definition 2.24). Since $q$ is implicit in $(N^q, [i])$, $s_3$ is also enabled in $(N^q, s_1)$ and $(N^q, s_2)$. Let $s_1'$ and $s_2'$ be two markings such that $(N^q, s_1) [\sigma_3] (N^q, s_1')$ and $(N^q, s_2) [\sigma_3] (N^q, s_2')$. It follows from the fact that $s_1 > s_2 \geq s$ that $s_1' > s_2' \geq [s]$. However, the fact that $s_1' > [s]$ contradicts the fact that $N^q$ is a workflow process definition (no proper completion; see Definition 2.22 (Soundness)). This contradiction shows that $s_1 = s_2$, which completes the proof.

For any two workflow process definitions $N$ and $N^q$ satisfying the requirements of Lemma 5.14, the lemma states that any two firing sequences leading to the same marking in $N$ also lead to the same marking in $N^q$ and that, in addition, these two markings are identical with respect to places of $N$. Lemma 5.14 implies that the function in the following definition is well defined.

**Definition 5.15.** Let $N = (P, T, F, \ell)$, $N^q = (P_q, T_q, F_q, \ell_q)$, and $q$ be defined as in Lemma 5.14. Function $id_q : [N, [i]] \rightarrow [N^q, [i]]$ is defined as follows. For any reachable marking $s \in [N, [i]]$ and firing sequence $\sigma \in T^*$ such that $(N, [i]) [\sigma] (N, s)$, $id_q(s) = s'$ where $s'$ is the unique marking in $[N^q, [i]]$ defined by $(N^q, [i]) [\sigma] (N^q, s')$.

Consider the projection-inheritance-preserving transformation rule $P_{J3S}$ of Theorem 3.18. To define the requirements of this transformation rule, the implicit place $q$ was added to the original workflow $N_0$. Place $q$ is a virtual place (i.e., it is not really present in one of the workflow process definitions $N_0$ or $N_1$) and has been added to make sure that every activation of the extension defined by $N$ (i.e., a firing of transition $t_o$) is followed by a deactivation (i.e., a firing of transition $t_o$). Note that workflow process definitions $N_0$ and $N_0^q$ of Theorem 3.18 satisfy the requirements of Lemma 5.14 and Definition 5.15. Function $id_q$ as defined in Definition 5.15 counts the number of tokens in $q$. Since workflow process definitions are safe, for any reachable marking $s$ of $N_0$, either $id_q(s) = s$ or $id_q(s) = s + [q]$. Recall the two complications concerning the transfer of cases under the inheritance-preserving transformation rule $P_{J3S}$ identified above. If $id_q$ indicates that $q$ is not marked, then
it suffices to transfer a case from the superclass \( N_0 \) to the subclass \( N_1 \) without changing its state. However, if \( id_q \) indicates that \( q \) is marked (i.e., \( q \) contains one token), then the newly added parallel branch of behavior needs to be marked when transferring a case. There are at least two ways to mark the parallel part: (1) The output places of \( t_i \) are marked (the conservative/pessimistic approach) or (2) the input places of \( t_o \) are marked (the progressive/optimistic approach). Therefore, we define two transfer rules based on P13S: \( T_{P13S,C} \) and \( T_{P13S,P} \).

**Theorem 5.16.** (Transfer rules \( T_{P13S,C} \) and \( T_{P13S,P} \)) Let \( N_0 = (P_0, T_0, F_0, t_0) \) and \( N_1 = (P_1, T_1, F_1, \ell_1) \) be two workflow process definitions satisfying the requirements stated in Theorem 3.18. Moreover, let \( q \) and \( N^q_0 \) be as defined in Theorem 3.18 and \( id_q : [N_0, [i]] \rightarrow [N^q_0, [i]] \) as defined in Definition 5.15. Finally, assume that \( S_q = \{ s \in [N_0, [i]] \mid id_q(s)(q) > 0 \} \).

If \( T_{P13S,C} \) is a transfer rule \( N_0 \) \( T_{P13S,C} \) \( N_1 \) such that, for all \( s \in B(P_0) \setminus S_q, r_{P13S,C}(s) = s \) and, for all \( s \in S_q, r_{P13S,C}(s) = s + t_i \setminus P_0 \), then \( T_{P13S,C} \) is valid.

If \( T_{P13S,P} \) is a transfer rule \( N_0 \) \( T_{P13S,P} \) \( N_1 \) such that, for all \( s \in B(P_0) \setminus S_q, r_{P13S,P}(s) = s \) and, for all \( s \in S_q, r_{P13S,P}(s) = s + t_o \setminus P_0 \), then \( T_{P13S,P} \) is valid.

**Proof.** Let \( s \in [N_0, [i]] \) and \( \sigma \in T_0^* \) be such that \((N_0, [i]) [\sigma] (N_0, s)\). We have to prove that \( r_{P13S,C}(s) \in [N_1, [i]] \) and \( r_{P13S,P}(s) \in [N_1, [i]] \).

Let \( t_i, t_o, N \) be as defined in Theorem 3.18. In the firing sequence \( \sigma, t_i \) and \( t_o \) occur alternatingly, i.e., at any point in the sequence the number of times \( t_i \) has occurred is equal to the number of times \( t_o \) has occurred or \( t_o \) has occurred one time extra. This property is a direct result of the fact that \( q \) is implicit in \((N^q_0, [i])\) (which means that \( \sigma \) is enabled in \((N^q_0, [i])\) and that \( N^q_0 \) is a workflow process definition (which means that \( q \) is safe). Because \( t_i \) and \( t_o \) occur alternatingly in \( \sigma \), we distinguish two possibilities:

1. The number of times \( t_i \) occurs in \( \sigma \) (possibly zero) is equal to the number of times \( t_o \) occurs in \( \sigma \), which means that \( s \not\in S_q \). Note that \( \sigma \) is not necessarily enabled in \((N_1, [i])\) when \( t_i \) occurs in \( \sigma \). Let \( \sigma_* \in (T \setminus \{t_i, t_o\})^* \) be such that \((N, t_i \bullet \cdot \cdot \cdot) [\sigma_*] (N, \bullet t_o) \cdot \cdot \cdot \cdot \). Such a firing sequence exists, because \((N, [p])\) is live and safe (see Theorem 3.18). Let \( \sigma' \) be a modification of the firing sequence \( \sigma \) where immediately after every occurrence of \( t_i \) the sequence \( \sigma_* \) is inserted. Sequence \( \sigma' \) is enabled in \((N_1, [i])\) and results in state \( s \). Since \((N_1, [i]) [\sigma'] (N_1, s) \) and \( r_{P13S,C}(s) = r_{P13S,P}(s) = s \) \((s \not\in S_q)\), both \( r_{P13S,C}(s) \) and \( r_{P13S,P}(s) \) are elements of \([N_1, [i]]\).

2. Transition \( t_i \) occurs precisely once more in sequence \( \sigma \) than transition \( t_o \), which means that \( s \in S_q \). Also in this case, \( \sigma \) is not necessarily enabled in \((N_1, [i])\) and in both states \( r_{P13S,C}(s) \) and \( r_{P13S,P}(s) \) some of the places in \( P_1 \setminus P_0 \) are marked. Construct sequence \( \sigma_* \) as before \((i.e.,(N, t_i \bullet) [\sigma_*] (N, \bullet t_o)) \). Sequence \( \sigma' \) is the modification of the firing sequence \( \sigma \) where immediately before every occurrence of \( t_o \) the sequence \( \sigma_* \) is inserted. Sequence \( \sigma' \) is enabled in \((N_1, [i])\) and results in state \( r_{P13S,C}(s) = s + t_i \setminus P_0 \) (note that \( s \in S_q \)). Hence, \( r_{P13S,C}(s) \in [N_1, [i]] \). Sequence \( \sigma'' \) is the sequence obtained by concatenating \( \sigma' \) and \( \sigma_* \). Also \( \sigma'' \) is enabled in \((N_1, [i])\) and results in state \( r_{P13S,P}(s) = s + t_o \setminus P_0 \). Hence, also \( r_{P13S,P}(s) \in [N_1, [i]] \).

In Figure 5.13, the left-hand-side process definition has been transformed into the right-hand-side process definition by adding task `contact_management` using transformation rule P13S. Consider a case in the left-hand process definition with a token in both `pending_complaint` and `contact_cust`. If transfer rule \( T_{P13S,C} \) is used, this case is transferred to the state in the right-hand process definition.
where pending_complaint, contact_cust, and contact_man are marked. Transfer rule $r_{PIJS, P}$ marks man_contacted instead of contact_man.

The transfer rules presented thus far map states of a superclass onto states of a subclass. If a series of inheritance-preserving transformation rules is applied to a workflow process definition, then it is possible to construct a composite transfer rule which is valid and maps any state of the original workflow process definition (superclass) onto the new process definition (subclass). The transfer rules presented in this subsection imply that for dynamically changing workflow process definitions following the rules PTS, PPS, PIJS, and PI3S problems such as deadlocks, livelocks, and dangling references can be avoided. At the end of the next subsection, the construction of composite transfer rules is illustrated by means of our running example.

5.3 Transfer of cases from subclass to superclass

The transfer rules of the previous subsection assume that cases are transferred from a superclass to a subclass. However, one can think of applications where the inheritance-preserving transformation rules presented in Section 3.2 are applied in the reverse direction. Note that none of the rules of Section 3.2 assumes a direction. Therefore, the inheritance-preserving transformation rules can also be applied to create a superclass based on a subclass. This means that a workflow process definition is not extended but reduced (i.e., parts of the workflow process definition are removed). For example, a parallel branch can be removed by applying PI3S in the reverse direction. If the inheritance-preserving transformation rules are applied in the reverse direction, we also need transfer rules which map states of the subclass onto states of the superclass. These transfer rules are presented in the remainder of this section.

First, let us consider a superclass workflow process definition and a subclass workflow process definition constructed by applying rule PPS of Theorem 3.11 in the reverse direction. Transfer rule $r_{PPS}^{-1}$ is used to map states from the subclass to the superclass.

**Theorem 5.17.** (Transfer rule $r_{PPS}^{-1}$) Let $N_0 = (P_0, T_0, F_0, \epsilon_0)$ and $N_1 = (P_1, T_1, F_1, \epsilon_1)$ be two workflow process definitions satisfying the requirements stated in Theorem 3.11. Moreover, let $p$ be the place as defined in Theorem 3.11 and let $r_{PPS}^{-1}$ be a transfer rule $N_1 \xrightarrow{r_{PPS}^{-1}} N_0$ such that, for all $s \in B(P_0)$, $r_{PPS}^{-1}(s) = s$, and, for all $s \in B(P_1) \setminus B(P_0)$, $r_{PPS}^{-1}(s) = s \oplus P_0 \oplus [p]$. Transfer rule $r_{PPS}^{-1}$ is valid.

**Proof.** Transformation rule PPS is a special case of rule PTS of Theorem 3.13. This can be proven by assuming that the sets $P^T$ and $P^O$ of Theorem 3.13 are both equal to the singleton set containing place $p$. Under this assumption, transfer rule $r_{PPS}^{-1}$ equals the transfer rules corresponding to PTS applied in the reverse direction which are presented next. Both transfer rules $r_{PIJS, C}^{-1}$ and $r_{PIJS, P}^{-1}$ stated in Theorem 5.20 correspond to $r_{PPS}^{-1}$. Thus, the validity of $r_{PPS}^{-1}$ follows immediately from the validity of $r_{PIJS, C}^{-1}$ and $r_{PIJS, P}^{-1}$.

Figure 5.8 illustrates transfer rule $r_{PPS}^{-1}$. Inheritance-preserving transformation rule PPS has been used to reduce the old workflow process definition on the right-hand side into the new workflow process definition on the left-hand side. Since the removed task inform_customer did not add any new states $r_{PPS}^{-1}$ corresponds (in this particular situation) to the identity function. If the removed part would have been a network of tasks, then all tokens in the removed part would have been mapped onto one single token in place pending_complaint.
Given a workflow process definition and a subclass of this process definition constructed by means of transformation rule $PPS$, the transfer of a case from the superclass to the subclass and back yields the original state.

**Property 5.18.** Let $N_0$, $N_1$, $r_{PPS}$, and $r_{PPS}^{-1}$ be as defined in Theorems 5.7 and 5.17. For any $s \in [N_0, [i])$, $r_{PPS}^{-1}(r_{PPS}(s)) = s$.

**Proof.** The property follows immediately from the definitions of the transfer rules in Theorems 5.7 and 5.17. □

Generally, the converse does not hold. Transfer rule $r_{PPS}$ corresponds to specialization and $r_{PPS}^{-1}$ corresponds to generalization. If specialization follows generalization, it may not be possible to reconstruct the original state because information is lost during the generalization step.

Second, we consider the transfer rule for the inheritance-preserving transformation rule $PTS$ applied in the reverse direction. Consider the two workflow process definitions shown in Figure 5.10. The old process definition on the right-hand-side is reduced by removing the alternative branch containing tasks $ignore\_complaint$ and $update\_statistics$. Note that we now have to transfer cases from “right to left” rather than from “left to right.” For a case not in the alternative branch to be removed, the transfer is simple; the case can be transferred without changing its state. For a case in this alternative branch (i.e., places $pending\_complaint$ and $ignored$ are marked), the transfer is more complicated since place $ignored$ is not present in the new process definition. There are two ways to deal with this problem: Either move the corresponding token to the point where the alternative branch started (i.e., place $classified$; conservative approach) or move the corresponding token to the point where the alternative branch ended (i.e., place $ready$; progressive approach). Therefore, we give two valid transfer rules: $r_{PTS,C}^{-1}$ and $r_{PTS,P}^{-1}$.

![Figure 5.19: Transition $u$ introduces a marking not reachable in the workflow process definition without $u$.](image)

Before we formulate $r_{PTS,C}^{-1}$ and $r_{PTS,P}^{-1}$, we need to consider another problem which is illustrated by Figure 5.19. (The net shown in Figure 5.19 without transition $u$ is taken from [21].) Suppose we remove task $u$ by applying rule $PTS$ in the reverse direction, i.e., the old workflow process definition is the process definition with $u$ and the new workflow process definition is the process definition without
u. In the old process definition, the marking with a token in both $p_2$ and $p_0$ is reachable by firing $t_1$ and $u$. Although $p_2$ and $p_0$ are still present in the new workflow process definition, this marking is no longer reachable after removing $u$. This situation is rather exceptional; normally, the removal of an alternative branch of behavior such as the one modeled by transition $u$ does not change the set of reachable markings with respect to the set of places that remain in the new workflow. If such a change does occur, it is not possible to come up with an elegant transfer rule which is valid. Recall that a transfer rule is only valid if every transfer results in a state of the new WF-net that is also reachable from the initial marking in the new workflow process definition. Therefore, to define the transfer rules $r^{-1}_{pts,c}$ and $r^{-1}_{pts,p}$, we add the requirement that the removed part does not change the behavior in the remaining part. To formalize this requirement, we use the virtual transition $u$ defined in Theorem 3.13. Transition $u$ emulates the behavior of the removed part. Therefore, it is required that $u$ does not influence the set of reachable markings.

**Theorem 5.20.** (Transfer rules $r^{-1}_{pts,c}$ and $r^{-1}_{pts,p}$) Let $N_0 = (P_0, T_0, F_0, \ell_0)$ and $N_1 = (P_1, T_1, F_1, \ell_1)$ be two workflow process definitions satisfying the requirements stated in Theorem 3.13. Moreover, let $P^o$, $P^s$, $u$, and $N^o_0$ be as defined in Theorem 3.13. Finally, assume that $[N^o_0, [i]] = [N_0, [i]]$.

Let $r^{-1}_{pts,c}$ be a transfer rule $N_1 \xrightarrow{r^{-1}_{pts,c}} N_0$ such that, for all $s \in B(P_0)$, $r^{-1}_{pts,c}(s) = s$, and, for all $s \in B(P_1) \setminus B(P_0)$, $r^{-1}_{pts,c}(s) = s \upharpoonright P_0 + P^o$. Transfer rule $r^{-1}_{pts,c}$ is valid.

Let $r^{-1}_{pts,p}$ be a transfer rule $N_1 \xrightarrow{r^{-1}_{pts,p}} N_0$ such that, for all $s \in B(P_0)$, $r^{-1}_{pts,p}(s) = s$, and, for all $s \in B(P_1) \setminus B(P_0)$, $r^{-1}_{pts,p}(s) = s \upharpoonright P_0 + P^o$. Transfer rule $r^{-1}_{pts,p}$ is valid.

**Proof.** Let $s \in \{N_1, [i]\}$ and $\sigma \in T^*$ be such that $(N_1, [i]) \xrightarrow{\sigma} (N_1, s)$. We have to prove that $r^{-1}_{pts,c}(s) \in [N_0, [i]]$ and $r^{-1}_{pts,p}(s) \in [N_0, [i]]$. Let $N$ be the labeled P/T net as defined in Theorem 3.13. We show that there are firing sequences $\sigma', \sigma'' \in (T_0 \cup \{u\})^*$ such that $(N^o_0, [i]) \xrightarrow{\sigma'} (N^o_0, r_{pts,c}(s))$ and $(N^o_0, [i]) \xrightarrow{\sigma''} (N^o_0, r_{pts,p}(s))$. Since $[N^o_0, [i]] = [N_0, [i]]$, this result suffices to prove that $r^{-1}_{pts,c}$ and $r^{-1}_{pts,p}$ are valid.

Let $T^I = \{t \in T \mid t \uparrow = P^I\}$ and $T^O = \{t \in T \mid t \downarrow = P^O\}$. Moreover, let $\nu$ and $N^o$ be as defined in Theorem 3.13. In the firing sequence and transitions in $T^I \cap T^O$ have the same effect in $(N_1, [i])$ as transition $u$ in $(N^o_0, [i])$. Let $\sigma_u \in (T_1 \cup \{u\})^*$ be the sequence $\sigma$ with each transition in $T^I \cap T^O$ replaced by transition $u$. As a result, in sequence $\sigma_u$, transitions in $T^I$ and $T^O$ occur alternatingly, i.e., at any point in the sequence the number of times that a transition in $T^I$ has occurred is equal to the number of times that a transition in $T^O$ has occurred or a transition in $T^I$ has occurred one time extra. This property follows from the fact that $(N_1, [i])$ is safe and that $(N^o_0, P^I)$ is live and safe (see Theorem 3.13). Because the transitions in $T^I$ and $T^O$ occur alternatingly in $\sigma_u$, we distinguish two possibilities:

1. Assume that the number of times that a transition in $T^I$ occurs in $\sigma_u$ is equal to the number of times that a transition in $T^O$ occurs in $\sigma_u$. Since $(N^o, P^I)$ is live and safe, firing a transition in $T^O$ removes all tokens in $P^I \setminus (P^I \cup P^O)$. Since, in $(N_1, [i])$, the only way to mark places in $P^I \setminus (P^I \cup P^O)$ is to fire transitions in $T^I$, it follows that $s \in B(P_0)$. Consider all subsequences of $\sigma_u$ that start with a transition in $T^I$, end with an occurrence of a transition in $T^O$, and contain no other occurrences of transitions in $T^I$ or $T^O$. For each such subsequence, replace all occurrences of transitions in $T$ by a single occurrence of transition $u$ at some arbitrary position among the transitions remaining in the subsequence. Let $\sigma'_u \in (T_0 \cup \{u\})^*$ be the resulting firing sequence. Clearly, $\sigma'_u$ is a sequence enabled in $(N^o_0, [i])$ and $(N^o_0, [i]) \xrightarrow{\sigma'_u} (N^o_0, s)$ (i.e., $s \in [N^o_0, [i]]$). Since $s \in B(P_0)$ and, thus, $r^{-1}_{pts,c}(s) = r^{-1}_{pts,p}(s) = s$, this completes the proof in this case.
2. Assume that the number of times that a transition in $T'$ occurs in $\sigma_u$ exceeds the number of times that a transition in $T^O$ occurs in $\sigma_u$ by one. Note that $P \setminus (P' \cup P^O)$ cannot be empty in this case, because then $T'$ would equal $T^O$, which contradicts the assumption. It follows that $s \in B(P_1) \setminus B(P_0)$. Again, consider all subsequences of $\sigma_u$ that start with an occurrence of a transition in $T'$, end with an occurrence of a transition in $T^O$, and contain no other occurrences of transitions in $T'$ or $T^O$. For each such a subsequence, replace all occurrences of transitions in $T$ by a single occurrence of transition $u$. The remaining occurrences of transitions of $T$ in $\sigma_u$ are simply removed. Let $\sigma_u' \in (T_0 \cup \{u\})^*$ be the resulting firing sequence. Sequence $\sigma_u'$ is enabled in $(N_1^u, [i])$ and $(N_0^u, [i])$, by removing the alternative branch containing tasks $\text{ignore\_complaint}$ and $\text{update\_statistics}$. If rule $r_{PTS,C}^{-1}$ is applied to a case which marks places $\text{pending\_complaint}$ and $\text{ignored}$, then the transfer results in the state which marks places $\text{pending\_complaint}$ and $\text{classified}$. Rule $r_{PTS,P}^{-1}$ maps the same case onto the state which marks $\text{pending\_complaint}$ and $\text{ready}$. For all other states, both $r_{PTS,C}^{-1}$ and $r_{PTS,P}^{-1}$ correspond to the identity function.

The following property states that transferring a case from a superclass to a subclass and back yields the original state.

Property 5.21. Let $N_0, N_1, r_{PTS}, r_{PTS,C}^{-1},$ and $r_{PTS,P}^{-1}$ be as defined in Theorems 5.9 and 5.20. For any $s \in [N_1, [i])$, $r_{PTS,C}^{-1}(r_{PTS}(s)) = r_{PTS,P}^{-1}(r_{PTS}(s)) = s$.

Proof. The desired result follows immediately from the definitions of the transfer rules in Theorems 5.9 and 5.20. □

Note that the property does not hold in the opposite direction, i.e., there may be states $s \in [N_1, [i])$ such that $r_{PTS}(r_{PTS,C}^{-1}(s)) \neq s$ and $r_{PTS}(r_{PTS,P}^{-1}(s)) \neq s$. Consider for example Figure 5.10, where the left-hand process definition corresponds to $N_0$ in Property 5.21 and the right-hand process definition corresponds to $N_1$. If $s \in [N_1, [i])$ equals $[\text{pending\_complaint}, \text{ignored}]$, then $r_{PTS}(r_{PTS,C}^{-1}(s)) = [\text{pending\_complaint}, \text{classified}]$ and $r_{PTS}(r_{PTS,P}^{-1}(s)) = [\text{pending\_complaint}, \text{ready}]$. This example shows that transferring a case from a subclass to a superclass and back does not necessarily yield the original state.

Rule $r_{FS}^{-1}$ is a valid transfer rule when inheritance-preserving transformation rule $PJS$ of Theorem 3.15 is used in the reverse direction. Recall that $PJS$ inserts new tasks between a transition $t_p$ and a place $p$. Rule $r_{FS}^{-1}$ removes tokens present in the inserted part and marks place $p$ instead.
Theorem 5.22. (Transfer rule $r_{PJS}^{-1}$) Let $N_0 = (P_0, T_0, F_0, \epsilon_0)$ and $N_1 = (P_1, T_1, F_1, \epsilon_1)$ be two workflow process definitions satisfying the requirements stated in Theorem 3.15. Moreover, let $p$ be the special place defined in Theorem 3.15 and let $r_{PJS}^{-1}$ be a transfer rule $N_1 \xrightarrow{\epsilon_1} N_0$ such that, for all $s \in B(P_0)$, $r_{PJS}^{-1}(s) = s$, and, for all $s \in B(P_1) \setminus B(P_0)$, $r_{PJS}^{-1}(s) = s \upharpoonright P_0 + [p]$. Transfer rule $r_{PJS}^{-1}$ is valid.

Proof. Let $T^0 = \epsilon_1 \cdot p \cap T$. Moreover, let $t_p$ be the special transition defined in Theorem 3.15. If transition $t_p$ is an element of $T^0$, then the theorem reduces to Theorem 5.17, because in this case transformation rule $PJS$ reduces to transformation rule $PPS$ of Theorem 3.11. If $t_p$ is not an element of $T^0$, then the proof is similar to the proof of Theorem 5.20 and is mainly based on the following observation. In the firing sequence $\sigma$, transition $t_p$ and the transitions in $T^0$ occur alternatingly. This property follows from the fact that $(N_1, [i])$ is safe and that $(N, [p])$ is live and safe. \hfill \Box

Figure 5.12 is used to illustrate transfer rule $r_{PJS}^{-1}$. If place $inform\_man$ in the old process definition on the right-hand side is not marked, a case can be transferred without changing its state. If place $inform\_man$ is marked, then the token in $inform\_man$ is moved to ready.

Property 5.23. Let $N_0, N_1, r_{PJS}$, and $r_{PJS}^{-1}$ be as defined in Theorems 5.11 and 5.22. For any $s \in [N_0, [i]]$, $r_{PJS}^{-1}(r_{PJS}(s)) = s$.

Proof. The property follows directly from Theorems 5.11 and 5.22. \hfill \Box

Rule $r_{PJS}^{-1}$ is the remaining transfer rule which can be used to map states to a new workflow process definition where a parallel branch is removed (i.e., rule $PJS$ applied in the reverse direction). This transfer rule simply removes all tokens in the deleted parallel part.

Theorem 5.24. (Transfer rule $r_{PJS}^{-1}$) Let $N_0 = (P_0, T_0, F_0, \epsilon_0)$ and $N_1 = (P_1, T_1, F_1, \epsilon_1)$ be two workflow process definitions satisfying the requirements stated in Theorem 3.18. Moreover, let $r_{PJS}^{-1}$ be a transfer rule $N_1 \xrightarrow{\epsilon_1} N_0$ such that, for all $s \in B(P_1)$, $r_{PJS}^{-1}(s) = s \upharpoonright P_0$. Transfer rule $r_{PJS}^{-1}$ is valid.

Proof. Let $s \in [N_1, [i]]$ and $\sigma \in T_1^+$ be such that $(N_1, [i]) [\sigma] (N_1, s)$. We have to prove that $r_{PJS}^{-1}(s) \in [N_0, [i]]$.

Let $N$ be the labeled P/T net defined in Theorem 3.18. Let $\sigma' \in T_0^*$ be the firing sequence obtained by removing all occurrences of transitions in $T \setminus \{t_i, t_o\}$ from $\sigma$. Sequence $\sigma'$ is enabled in $(N_0, [i])$, because in $N_1$ transitions in $T_0$ can only be blocked by empty places of $N$. It not difficult to verify that the state resulting from firing $\sigma'$ in $(N_0, [i])$ is equal to $s$ with respect to the places in $P_0$, i.e., $r_{PJS}^{-1}(s) = s \upharpoonright P_0 \in [N_0, [i]]$. \hfill \Box

In Figure 5.13, the right-hand-side process definition has been transformed into the left-hand-side process definition by removing task contact\_management. Transfer rule $r_{PJS}^{-1}$ removes any token in contact\_man or man\_contacted (if present) such that the case can be transferred from the subclass to the superclass.

Property 5.25. Let $N_0, N_1, r_{PJS,C}, r_{PJS,P}$, and $r_{PJS}^{-1}$ be as defined in Theorems 5.16 and 5.24. For any $s \in [N_0, [i]]$, $r_{PJS,C}(r_{PJS,C}(s)) = r_{PJS,P}(r_{PJS,P}(s)) = s$.

Proof. It follows directly from Theorems 5.16 and 5.24. \hfill \Box
For each inheritance-preserving transformation rule of Section 3.2, we have defined valid transfer rules for moving a case from a superclass to a subclass and vice versa. These rules are such that if a case is moved from the superclass to the subclass and back, the original state is obtained. (Generally, the converse does not hold.)

If a series of inheritance-preserving transformation rules is applied, the composition of the appropriate corresponding transfer rules yields a composite transfer rule which is valid. Consider for example the two workflow process definitions shown in Figure 5.26. The right-hand-side process definition is a subclass of the left-hand-side process definition under life-cycle inheritance and can be obtained by applying rules PTS, PPS, PJS, and PJJS. Let $r_C(r_P)$ be the transfer rule obtained by composing the transfer rules $r_{PTS}$, $r_{PPS}$, $r_{PJS}$, and $r_{PJJS}$. Consider a case in the superclass which marks pending_complaint and cust_contacted. If $r_C$ is used to transfer this case to the subclass, the places pending_complaint, cust_contacted, and contact_man are marked after the transfer. If $r_P$ is used, man_contacted is marked instead of contact_man. Let $r_C^{-1}(r_P^{-1})$ be the transfer rule obtained by composing the transfer rules $r_{PTS}^{-1}$, $r_{PPS}^{-1}$, $r_{PJS}^{-1}$, and $r_{PJJS}^{-1}$. (Note that for this particular pair of workflow process definitions $r_C^{-1}$ is equivalent to $r_P^{-1}$.)

To conclude, let us return to the problem illustrated by Figure 1.1. Has this problem (i.e., the dynamic-change bug) been solved? One might argue that the problem has not been solved, because the inheritance concepts do not provide a solution for this particular example. However, as explained in the beginning of this section, there is no satisfactory solution but to postpone the transfer (e.g., transfer rule $r_0$). The state with a token in both $s_1$ and $s_4$ (right-hand side of Figure 1.1) cannot be mapped onto a reasonable state in the sequential process definition (left-hand side). Putting a token in $i$, $p_1$, or $p_2$ will result in the double execution of task send_bill. Putting a token in $p_2$, $p_3$, or $o$ will result in the skipping of task send_goods. The transfer rules presented in this section show that, if one restricts change to the inheritance-preserving transformation rules presented in Section 3.2, it is always possible to find a satisfactory transfer rule which is valid and, thus, the dynamic-change problem can be avoided. We have not formalized the term "satisfactory" but it is easy to see that the transfer rules do not lead to the unnecessary skipping or multiple execution of tasks. The transfer rules offer reasonable solutions for the types of changes typically used to adapt a workflow process definition: adding/removing alternative branches (PTS), adding/removing loops (PPS), adding/removing subflows between sequential tasks (PJS), and adding/removing parallel branches (PJJS).
5.4 Related work on dynamic change

There are many similarities between dynamic change in the workflow domain and schema evolution in the database domain. As the requirements of database applications change over time, the definition of the schema, i.e., the structure of the data elements stored in the database, is changed. Schema evolution has been an active field of research in the last decade (mainly in the field of object-oriented databases, cf. [16]) and has resulted in techniques and tools that partially support the transformation of data from one database schema to another. Although dynamic change and schema evolution are similar, there are some additional complications in case of dynamic change. First, as was shown in the example of Figure 1.1, it is not always possible to transfer a case. Second, it is not acceptable to shut down the system, transfer all cases, and restart using the new procedure. Cases should be migrated while the system is running. Finally, dynamic change may introduce deadlocks and livelocks. The solutions provided by today's object-oriented databases do not deal with these complications. Therefore, we need new concepts and techniques.

Several researchers have worked on problems related to dynamic change. Ellis, Keddara, and Rozenberg [22] propose a technique based on so-called "change regions." A change region contains all parts of a workflow process definition that potentially cause problems with respect to the transfer of cases. A change region has two versions; the old situation and the new situation. In this solution, there is one version of the complete process which covers the old and the new situation and changes affect cases as soon as possible. Parts of the workflow (i.e., change regions) become inactive after a while, because all old cases have been handled. This approach has the drawback that the process definition can become very complex. Another, more serious, drawback is the fact that the change regions need to be identified manually thus causing additional efforts and human errors. In [23], the authors improve their approach by introducing jumpers. A jumper moves a case from the old workflow to the new workflow. The jump is postponed if for a state no jumper is available. Again, the authors do not give a concrete technique for the transfer of cases, i.e., jumpers are added manually. Agostini and De Michelis [9] propose a technique for the automatic transfer of cases from an old process definition to a new process definition and also give criteria for determining whether a jump is possible. Unfortunately, the approach only works for a restricted class of workflows (e.g., the modeling language does not allow for iteration, although at runtime iteration can be achieved by backward jumps). Casati, Ceri, and Pernici [18] tackle the problem of dynamic change via a set of transformation rules and partition the state space into a part that is aborted, a part that is transferred, a part that is handled the old way, and parts which are handled by hybrid process definitions (similar to the approach using change regions). Reichert and Dadam [47] use a similar approach. However, semantical issues such as errors introduced by swapping tasks, skipping tasks, or multiple executions of a task are not considered. Voorhoeve and Van der Aalst [55, 56] also propose a fixed set of transformation rules to support dynamic change. However, the rules are not given explicitly at the net level and semantical issues are not considered.

None of the papers mentioned uses an approach based on inheritance. The transfer rules based on the four inheritance-preserving transformation rules guarantee the preservation of the soundness property after a transfer. Moreover, semantical errors such as the swapping of tasks, the skipping of tasks, and the multiple execution of tasks can be avoided by choosing the appropriate inheritance notion, e.g., projection inheritance guarantees that tasks cannot be skipped by transferring a case from the superclass to the subclass. In this section, we have shown that the inheritance-preserving transformation rules of Section 3.2 can be used to avoid the dynamic-change problem; if designers stick to the transformation rules, then situations such as the one shown in Figure 1.1 cannot occur: It is always possible to transfer a case.
6 Management information

The transfer rules defined in the previous section are used to migrate cases from one workflow process definition to another workflow process definition. If all cases are migrated to the most recent workflow process definition, it is easy to provide aggregate management information. For each place \( p \), it is possible to count the number of cases marking \( p \). By indicating these numbers in the WF-net representing the most recent workflow process definition, the manager obtains a condensed view of the work-in-progress. However, in many situations, there are multiple versions and/or variants of the same workflow process. For evolutionary change, the number of versions is typically limited. In fact, if all cases are transferred directly after a change, there is just one active version. However, if the proceed policy (transfers are postponed until the case is handled completely, i.e., transfer rule \( r_0 \) of Definition 5.4) is used or transfers are delayed, there are multiple active versions. As indicated in the introduction, there may be various reasons for using such policies (e.g., legal constraints, technical problems, or managerial considerations). If the average flow time of cases is long and evolutionary changes occur frequently, there can be dozens of versions. Ad-hoc change may result in an even larger number of variants of the same workflow process. In fact, it is possible to end up in the situation where the number of variants is of the same order of magnitude as the number of cases. In Section 4, it has been pointed out that the shift from a “Sellers’ Market” to a “Buyers’ Market” leads to an increase in the number of products and services offered. Consider for example the number of variants of a specific car model (combinations of colors, engines, options, accessories). The number of variants may in fact exceed the number of cars actually sold. Clearly, the result of this shift is an increasing number of variants of a given workflow process. Moreover, today’s customers expect flexibility which may lead to even more new ad-hoc variants. To manage a workflow process with different versions/variants, it is desirable to have an aggregated view of the work-in-progress. Therefore, as indicated in Section 1, it is of the utmost importance to supply a manager with tools to obtain a condensed but accurate view of the status of the cases in the workflow process at hand.

In this section, we show that the inheritance notions introduced in this paper facilitate the construction of aggregate management information. First, we introduce the notion of a management-information net. Second, we define the notions of a Greatest Common Divisor (GCD) and a Least Common Multiple (LCM) of a number of workflow process definitions. Third, we discuss the utilization of the four inheritance-preserving transformation rules of Section 3.2 for constructing aggregate management information. Finally, we discuss several approaches for obtaining aggregate management information in the various application areas identified in Section 4.

6.1 Management-information nets

To provide a manager with succinct information about work-in-progress, we need one diagram summarizing the states of all running cases in all versions and/or variants of a workflow process. That is, the diagram should not show the states of individual cases nor should it show all the versions/variants of a workflow process; the information should be at an aggregate level. Every version/variant of a workflow process is represented as a workflow process definition (i.e., a sound WF-net). Therefore, it is reasonable to use a workflow process definition to present aggregate management information. We use the term Management-Information net (MI-net) to refer to a workflow process definition which is used to visualize work-in-progress.

Definition 6.1. (MI-net) Let \( N_0, N_1, \ldots, N_{n-1} \), where \( n \) is some natural number, be \( n \) workflow process definitions in \( W \), which are versions/variants of the same workflow process. In addition, let
$N$ be a workflow process definition in $\mathcal{W}$. Net $N$ is a Management-Information net (MI-net) for $N_0, N_1, \ldots, N_{n-1}$ if and only if there is a total, valid transfer rule for each variant, i.e., for each $k$, with $0 \leq k \leq n$, there is a transfer rule $r_k$ such that $N_k \xrightarrow{r_k} N_i$, $[N_k, [i]] \subseteq \text{dom}(r_k)$, and $r_k$ is valid.

Given a number of versions and/or variants of a workflow process and an MI-net with accompanying transfer rules, the states of every running case can be mapped onto the MI-net using the appropriate transfer rule. Note that there is no actual transfer of cases; only information about the state of a case is mapped onto the MI-net. Since we want to collect information about all cases, the transfer rules in Definition 6.1 are required to be total. Furthermore, because management information must be as accurate as possible, it is required that the transfer rules are valid.

Note that Definition 6.1 is not very restrictive. In fact, the definition does not impose any restrictions on an MI-net other than the requirement that it is a workflow process definition; any sound WF-net can serve as an MI-net. In principle, we can map cases from any set of workflow process definitions onto an arbitrary MI-net by using one of the trivial transfer rules given in Definition 5.4 (i.e., $r_i$ or $r_o$). Therefore, the quality of management information can be low. In general, it is not easy to find a meaningful MI-net and non-trivial valid transfer rules.

Consider for example the three workflow process definitions shown in Figure 6.2. These process definitions are variants of the same workflow process. These variants can be the result of ad-hoc changes. Process definition $N_0$ represents, for example, the normal process. Process definition $N_1$ yields a slightly changed workflow where the parallel branch containing task $\text{check1}$ is added. Process definition $N_2$ is another ad-hoc variant where task $\text{check2}$ is added in-between $\text{handle}$ and $\text{archive}$. Note that the three workflow process definitions can also be the result of evolutionary changes (in which case they are called versions). Net $N_0$ is for example the oldest process definition, $N_1$ is the successor of $N_0$, and $N_2$ is the successor of $N_1$. Note that $N_2$ is not a subclass of $N_1$ under any of the four inheritance relations of Definition 3.4. This is not a problem since, for now, arbitrary changes are considered, i.e., also changes not respecting the inheritance relations. In the remainder of this section, we only use the term "variant" and not the term "version" when referring to an element of a set of workflow process definitions. However, the concepts are valid for both ad-hoc and evolutionary changes.

The numbers shown in Figure 6.2 are used to indicate the number of cases within a certain state for each variant. For example, in variant $N_1$, there are four cases. For each of these cases, $\text{register}$ has
been executed and for none of these cases *archive* has been executed. Task *handle* has been executed for two of these cases and *check* has been executed for three of these cases. Note that, in total, there are 23 cases in the three variants shown in Figure 6.2: eleven in *N₀*, four in *N₁*, and eight in *N₂*. For these three simple variants, there is no real need for aggregate management information with respect to work-in-progress. However, one can imagine that, if the number of variants increases or the variants themselves become more complex, there is a need for aggregate information rather than separate status reports for each variant (as is shown in Figure 6.2). To accommodate this need, the cases have to be mapped onto an MI-net. Even for the three simple variants shown in Figure 6.2, it is not clear how to construct a meaningful MI-net. Should an MI-net be one of the variants? Should the MI-net emphasize the common parts as much as possible? Should it capture all possible routings? Another non-trivial question is how to obtain meaningful transfer rules.

For these three simple variants, there is no real need for aggregate management information with respect to work-in-progress. However, one can imagine that, if the number of variants increases or the variants themselves become more complex, there is a need for aggregate information rather than separate status reports for each variant (as is shown in Figure 6.2). To accommodate this need, the cases have to be mapped onto an MI-net. Even for the three simple variants shown in Figure 6.2, it is not clear how to construct a meaningful MI-net. Should an MI-net be one of the variants? Should the MI-net emphasize the common parts as much as possible? Should it capture all possible routings? Another non-trivial question is how to obtain meaningful transfer rules.

![Figure 6.3: The Greatest Common Divisor (GCD) and Least Common Multiple (LCM) of the three workflow process definitions of Figure 6.2.](image-url)

Figure 6.3 shows two MI-nets for the three process variants shown in Figure 6.2. The left-hand-side MI-net *N₆₉₈D* emphasizes the common part of the three variants. Each of the three variants contains the steps *register*, *handle*, and *archive* and these steps are always executed in this order. The right-hand-side MI-net *N₇₉₄₉₄M* contains all tasks present in any of the variants in an effort to capture all possible routings. Note that the two MI-nets are augmented with numbers indicating the number of cases in each state. For example, in both MI-nets, place *p₁* is labeled with *7 = 1 + 2 + 4* indicating that seven cases in Figure 6.2 are in a state between *register* and *handle*, one in variant *N₀*, two in variant *N₁*, and four in variant *N₂*. It is interesting to note that each of the three workflow process definitions shown in Figure 6.2 is a subclass under life-cycle inheritance of the left-hand MI-net *N₆₉₈D*. *N₀* is identical to *N₆₉₈D*, *N₁* can be constructed from *N₆₉₈D* by applying transformation rule *P₇₃S* of Theorem 3.18 and *N₂* can be constructed from *N₆₉₈D* by applying rule *P₇₅* of Theorem 3.15. As a result, we can use the transfer rules *r₁₃*, *r⁻¹₃₇*, and *r⁻¹₇₅* of the previous section to map states of the three workflow process definitions of Figure 6.2 onto *N₆₉₈D* of Figure 6.3. It is not difficult to verify that one obtains the numbers given in Figure 6.3 when doing so.

As mentioned, the other MI-net shown in Figure 6.3, *N₇₉₄₉₄M*, attempts to capture all possible behaviors of the three workflow process definitions of Figure 6.2 rather than focussing on the common parts; by hiding *check* and/or *check2*, one can find each of the three variants. This means that each of the three workflow process definitions shown in Figure 6.2 is a superclass of *N₇₉₄₉₄M*: *N₇₉₄₉₄M* can
be constructed from \( N_0 \) by applying the inheritance-preserving transformation rules \( PJS \) and \( PJS_5 \); \( NLCD \) can be constructed from \( N_1 \) by applying \( PJS \) and it can be obtained from \( N_2 \) by applying \( PJS_5 \). Consequently, we can use \( r_{PJS} \) of Theorem 5.11 to map states of \( N_1 \) onto \( NLCD \). Furthermore, states of \( N_2 \) can be mapped onto \( NLCD \) using either \( r_{PJS,C} \) or \( r_{PJS,P} \) of Theorem 5.16. States of \( N_0 \) can be mapped onto \( NLCD \) using \( r_{PJS} \) and either \( r_{PJS,C} \) or \( r_{PJS,P} \). Note that, in Figures 6.2 and 6.3, the four cases in \( N_0 \) in-between \( register \) and \( archive \) as well as the seven cases in \( N_1 \) in-between \( register \) and \( archive \) have been mapped onto place \( p_4 \) of \( NLCD \) using transformation rule \( r_{PJS,P} \). As a result, place \( p_4 \) of \( NLCD \) is labeled with \( 14 = 4 + 3 + 7 \). (Place \( p_4 \) contains also information on the three cases residing in states marking \( p_4 \) in \( N_1 \).)

In the context of management information, there is a good reason to use transfer rule \( r_{PJS,P} \) instead of rule \( r_{PJS,C} \) if both rules are applicable. Technically, it is possible to use a conservative mapping based on \( r_{PJS,C} \) (i.e., in the above example, twelve cases are mapped onto place \( p_3 \) in \( NLCD \)) or a progressive mapping based on \( r_{PJS,P} \) (i.e., just one case is mapped onto place \( p_3 \) in \( NLCD \)). However, for the purpose of management information, it is not meaningful to use \( r_{PJS,C} \). The conservative mapping results in a view which is too pessimistic. The cases in \( N_0 \) and \( N_1 \) in-between \( register \) and \( archive \) do not require the execution of task \( check \), whereas the conservative mapping of these cases onto \( NLCD \) suggests that they do. Transformation rule \( r_{PJS,C} \) does not provide an accurate estimate of work-in-progress. Therefore, we exclude this transformation rule for determining management information. In the remainder, we only use the other transfer rules of the previous section, namely \( r_{PJS}, r_{PJS,P}, r_{PJS,C}, r_{PJS,P}^{-1}, r_{PJS,C}^{-1}, r_{PJS,P}^{-1} \), and \( r_{PJS,C}^{-1} \). Note that it is meaningful to consider both \( r_{PJS,C}^{-1} \) and \( r_{PJS,P}^{-1} \). If an alternative branch present in one variant of a workflow process is not present in the MI-net that is being used, cases for which part of the alternative branch has been executed can be moved to the point where the alternative branch starts (conservative view) or to the point where the alternative branch ends (progressive view). The question which mapping is most accurate depends on the context.

The example of Figures 6.2 and 6.3 illustrates the basic idea of constructing aggregate management information. It shows that the inheritance concepts introduced in this paper can be useful in obtaining meaningful MI-nets and transfer rules. Clearly, the names \( NGCD \) and \( NLCD \) of the two MI-nets in Figure 6.3 are suggestive. In the next subsection, the notion of a Greatest Common Divisor (GCD) of a number of variants of a workflow process and the notion of a Least Common Multiple (LCM) of a number of variants are formalized. In Section 6.3, the role of the inheritance-preserving transformation rules of Section 3.2 and the transfer rules of Section 5 is studied in more detail.

### 6.2 Greatest common divisor/least common multiple of workflow process definitions

The left-hand-side WF-net \( NGCD \) shown in Figure 6.3 is an MI-net which is a superclass of each of the three variants shown in Figure 6.2. As explained, the cases present in the three variants can be mapped onto \( NGCD \) using respectively transfer rule \( r_d \) (see Definition 5.6), transfer rule \( r_{PJS}^{-1} \) (see Theorem 5.24), and transfer rule \( r_{PJS}^{-1} \) (see Theorem 5.22). One can think of \( NGCD \) as the intersection or greatest common divisor (GCD) of the three process definitions shown in Figure 6.2; \( NGCD \) contains the elements which are present in all variants. By coincidence, \( NGCD \) equals one of the variants \( (N_0) \). The right-hand-side MI-net \( NLCD \) shown in Figure 6.3 does not correspond to one of the variants. One can think of this MI-net as the union or least common multiple (LCM) of the three variants shown in Figure 6.2. However, the terms "intersection" and "union" may be misleading, because the straightforward intersection and union of the network structure of a set of workflow process definitions generally does not yield an MI-net. Therefore, we prefer the terms "GCD" and "LCM." The notions of a GCD and an LCM are defined using the life-cycle-inheritance relation of Definition 3.4-4. Recall
that $\cong$, as defined in Definition 2.25, denotes behavioral equivalence of workflow process definitions.

**Definition 6.4.** (GCD, LCM) Let $N_0, N_1, \ldots, N_{n-1}$, where $n$ is some natural number, and $N$ be workflow process definitions in $W$. Net $N$ is a **Greatest Common Divisor** (GCD) of $N_0, N_1, \ldots, N_{n-1}$ if and only if

1. $(\forall k : 0 \leq k < n : N_k \leq_k N)$ and,
2. for any workflow process definition $N'$ such that $(\forall k : 0 \leq k < n : N_k \leq_k N')$ and $N' \leq_k N$, $N' \cong N$.

Net $N$ is a **Least Common Multiple** (LCM) of $N_0, N_1, \ldots, N_{n-1}$ if and only if

1. $(\forall k : 0 \leq k < n : N \leq_k N_k)$ and,
2. for any workflow process definition $N'$ such that $(\forall k : 0 \leq k < n : N' \leq_k N_k)$ and $N \leq_k N'$, $N' \cong N$.

Note that the notions of a GCD and an LCM are defined with respect to life-cycle inheritance and not with respect to the size of workflow process definitions (where the size of a workflow process definition is determined by its number of tasks). If $N_{GCD}$ is a GCD of two workflow process definitions $N_0$ and $N_1$, then $N_{GCD}$ typically contains fewer tasks than $N_0$ and $N_1$, which conforms to the intuitive notion of a GCD. On a first reading, the definition of a GCD of a number of process variants might be counterintuitive because a GCD is required to be a superclass of the process variants. Similarly, if $N_{LCM}$ is an LCM of $N_0$ and $N_1$, then $N_{LCM}$ typically contains more tasks than $N_0$ and $N_1$. Moreover, although it is straightforward to show that $N_{LCM} \leq_k N_{GCD}$ ($\leq_k$ is transitive; see Property 3.6), $N_{LCM}$ is typically larger than $N_{GCD}$ in terms of their numbers of tasks. Consider for example the process definitions shown in Figure 6.3. According to Definition 6.4, $N_{GCD}$ is a GCD of the three variants shown in Figure 6.2 and $N_{LCM}$ is an LCM of these three variants. Although $N_{LCM} \leq_k N_{GCD}$, $N_{LCM}$ has two tasks more than $N_{GCD}$.

So far, we have used the terms “a GCD” and “an LCM” rather than “the GCD” and “the LCM.” This raises the following two questions:

1. Is there always a GCD and an LCM for any set of workflow process definitions?
2. In case there exist a GCD and an LCM, are they unique?

In the remainder, we answer these two questions. The following property is needed to answer the first question. A set of totally ordered (according to the life-cycle-inheritance relation $\leq_k$) workflow process definitions is called a **chain**.

**Property 6.5.** Let $N_0$ and $N_1$ be two workflow process definitions in $W$ such that $N_0 \leq_k N_1$. There is no infinite chain $N^0 \leq_k N^1 \leq_k \ldots$ of different (with respect to $\cong$) workflow process definitions $N^0, N^1, \ldots \in W$ such that $N_0 \leq_k N^0 \leq_k N^1 \leq_k \ldots \leq_k N_1$.

**Proof.** Let $N$ and $N'$ be two workflow process definitions in $W$ such that $N \leq_k N'$. The following three observations are important. First, $\alpha(N') \subseteq \alpha(N)$. Second, if $N \not\cong N'$, then $\alpha(N') \subset \alpha(N)$. Third, $\alpha(N) \setminus \alpha(N')$ is finite.

Let $N^0 \leq_k N^1 \leq_k \ldots$ be an infinite chain of different workflow process definitions $N^0, N^1, \ldots \in W$ such that $N_0 \leq_k N^0 \leq_k N^1 \leq_k \ldots \leq_k N_1$. It follows from the first two of the above observations that $\alpha(N_1) \subseteq \ldots \subseteq \alpha(N^1) \subset \alpha(N^0) \subseteq \alpha(N_0)$. The third observation above states that $\alpha(N_0) \setminus \alpha(N_1)$ is finite. However, this yields a contradiction, which proves the property.

51
The following theorem answers the first question raised above affirmatively.

**Theorem 6.6. (Existence of a GCD and an LCM)** Let $N_0, N_1, \ldots, N_{n-1}$, where $n$ is some natural number, be $n$ workflow process definitions in $W$. There exists a GCD of $N_0, N_1, \ldots, N_{n-1}$ and there exists an LCM of $N_0, N_1, \ldots, N_{n-1}$.

**Proof.** Recall Property 6.5. It states that there are no infinite chains in-between any two workflow process definitions related by $\leq_B$. Consequently, to prove the existence of a GCD, it suffices to show that there exists a workflow process definition that is a superclass of all variants $N_0, N_1, \ldots, N_{n-1}$. Similarly, to prove the existence of an LCM, it suffices to show that there is a workflow process definition that is a subclass of all variants.

Let $N_\tau$ be the workflow process definition containing one task labeled $\tau$, i.e., $N_\tau = \langle \{i, o\}, \{\tau\}, \{(i, \tau), (\tau, o)\}, \{(\tau, \tau)\}\rangle$. Clearly, $N_\tau$ is a superclass of any workflow process definition. Thus, $N_\tau$ is a superclass of each of the variants, which proves the existence of a GCD.

Let $N_0$ be the net which is constructed from all the variants $N_0, N_1, \ldots, N_{n-1}$ as follows. The source place $i$ of $N_0$ has $n$ output transitions, one for each variant. Each of these new transitions has a unique task label that does not occur in the alphabets of any of the variants. The source place of each variant is given a new identifier and connected as an output place to one of the $n$ new transitions. In this way, the new transitions act as guards for the $n$ original variants. The sink places of the $n$ variants are simply fused together, yielding the sink place $o$ of $N_0$. Clearly, $N_0$ is a subclass of each variant; by blocking all new transitions except one which is hidden, one obtains a workflow process definition branching bisimilar to one of the variants. Therefore, we conclude that also an LCM of the $n$ variants exists. \(\Box\)

The answer to the first question raised above is positive: For any set of workflow process definitions, there is a GCD and an LCM. Unfortunately, the answer to the second question is negative. Given a set of workflow process definitions, there can be more than one GCD and more than one LCM. Consider for example the two process definitions shown in Figure 1.1. There are at least two GCDs. Both the sequential process definition consisting of task prepare_shipment, task send_goods, and task record_shipment and the sequential process definition consisting of task prepare_shipment, task send_bill, and task record_shipment are GCDs of $N_0$ and $N_1$. It is easy to verify that both workflow process definitions are GCDs. Each of them is a superclass of both $N_0$ and $N_1$ and, in both cases, there is not a smaller (with respect to $\leq_B$) candidate. Similarly, the two workflow process definitions $N_0$ and $N_1$ shown in Figure 1.1 have more than one LCM. Consider the process definition $N_0$ mentioned in the proof of Theorem 6.6. That is, consider a workflow process definition consisting of $N_0$ and $N_1$ and starting with two additional guard transitions. Each of the guard transitions has a unique label, say $l_0$ and $l_1$, respectively. If $l_0$ is blocked and $l_1$ hidden, then $N_0$ is branching bisimilar to $N_1$; if $l_1$ is blocked and $l_0$ hidden, then $N_0$ is branching bisimilar to $N_1$. Therefore, $N_0$ is a subclass of both variants. There is no workflow process definition which is a subclass of both variants, a superclass of $N_0$, and not branching bisimilar to $N_0$. Therefore, $N_0$ is an LCM of $N_0$ and $N_1$. However, the labels $l_0$ and $l_1$ were chosen arbitrarily, i.e., any pair of labels not used in $N_0$ and $N_1$ will do. Therefore, $N_0$ and $N_1$ have as many LCMs as there are combinations of labels not used in $N_0$ and $N_1$.

Based on the two variants shown in Figure 1.1, we conclude that a given set of workflow process definitions can have several GCDs and LCMs. In the example of Figure 1.1, the reason is that $N_0$ and $N_1$ do agree on the presence of the tasks send_goods and send_bill, whereas they do not agree on their ordering. However, in many situations, there is one unique GCD and one unique LCM (modulo branching bisimilarity). For example, the three variants shown in Figure 6.2 have a unique GCD and
a unique LCM, namely the nets $N_{\text{GCD}}$ and $N_{\text{LCM}}$ of Figure 6.3, respectively. The following theorem states requirements for the existence of a unique GCD/LCM modulo branching bisimilarity.

**Theorem 6.7.** Let $N_0, N_1, \ldots, N_{n-1}$, where $n$ is some natural number, be $n$ workflow process definitions in $\mathcal{W}$. If there is a workflow process definition $N$ in $\mathcal{W}$ such that

1. $(\forall k : 0 \leq k < n : N_k \leq_{le} N)$ and,

2. for any workflow process definition $N'$ in $\mathcal{W}$, $(\forall k : 0 \leq k < n : N_k \leq_{le} N')$ implies $N \leq_{le} N'$,

then $N$ is a GCD of $N_0, N_1, \ldots, N_{n-1}$ and every workflow process definition $N_{\text{GCD}}$ in $\mathcal{W}$ that is also a GCD of $N_0, N_1, \ldots, N_{n-1}$ is branching bisimilar to $N$ (i.e., $N_{\text{GCD}} \cong N$).

If there is a workflow process definition $N$ in $\mathcal{W}$ such that

1. $(\forall k : 0 \leq k < n : N \leq_{le} N_k)$ and,

2. for any workflow process definition $N'$ in $\mathcal{W}$, $(\forall k : 0 \leq k < n : N' \leq_{le} N_k)$ implies $N' \leq_{le} N$,

then $N$ is an LCM of $N_0, N_1, \ldots, N_{n-1}$ and every workflow process definition $N_{\text{LCM}}$ in $\mathcal{W}$ that is also an LCM of $N_0, N_1, \ldots, N_{n-1}$ is branching bisimilar to $N$ (i.e., $N_{\text{LCM}} \cong N$).

**Proof.** The proofs of the two parts of the theorem are very similar. Therefore, we only prove that there exists a unique GCD. Let $N$ be a workflow process definition satisfying the first pair of requirements stated in the theorem.

Consider Definition 6.4 (GCD). Assume that $N'$ is a workflow process definition such that $(\forall k : 0 \leq k < n : N_k \leq_{le} N')$ and $N' \leq_{le} N$. It follows that $N \leq_{le} N'$, which in combination with $N' \leq_{le} N$ implies that $N \cong N'$ ($\leq_{le}$ is a partial order; see Property 3.6). Thus, net $N$ satisfies the requirements in Definition 6.4, which means that it is a GCD.

Assume that $N_{\text{GCD}}$ is an arbitrary GCD of the $n$ variants. It follows from the assumptions on $N$ that $N \leq_{le} N_{\text{GCD}}$. Consequently, Definition 6.4 yields that $N \cong N_{\text{GCD}}$, which completes the proof. $\square$

So far, we have formalized the notions of a GCD and an LCM of a set of workflow process definitions. It has been shown that such a GCD and LCM always exist, but that they are not necessarily unique. Theorem 6.7 shows under what assumptions a GCD and an LCM are unique modulo branching bisimilarity.

The reason for studying the notions of a GCD and an LCM of a set of workflow process definitions is that they are suitable to aggregate management information. That is, any GCD or LCM of a set of workflow process definitions is a suitable MI-net, as defined in Definition 6.1, for these process definitions. However, in general, it is not straightforward to determine a GCD or an LCM of a set of workflow process definitions. In addition, even given a GCD or an LCM, it is not always possible to find meaningful transfer rules for mapping cases in the various workflow process definitions onto this GCD or LCM. However, there are situations where it is quite easy to pinpoint a GCD or LCM. The remainder of this subsection is devoted to explaining a number of situations, where the GCD or LCM of a set of workflow process definitions is unique (modulo branching bisimilarity) and can be determined straightforwardly. In the next subsection, we return to the topic of finding appropriate transfer rules.

Consider some number of workflow process definitions that are variants of a single workflow process. First, if all the variants are equivalent (according to the behavioral equivalence relation $\cong$),
an arbitrary variant is the (unique modulo branching bisimilarity) GCD as well as the LCM. Second, if the variants form a chain, i.e., the variants are totally ordered according to the $\leq$ relation, then the smallest element is the LCM and the largest element is the GCD. Third, if one variant is a superclass of all the other variants, then this variant is the GCD. Note that the three workflow process definitions of Figure 6.2 satisfy this requirement. Process definition $N_0$ is a superclass of both $N_1$ and $N_2$, which means that it is the GCD of the three variants. This result conforms to our earlier claims. Fourth, if one variant is a subclass of all the other variants, then this variant is the LCM. Fifth, if two variants have no tasks in common, then the GCD equals the empty workflow process definition $\emptyset$ as introduced in the proof of Theorem 6.6. Finally, if the variants have nothing in common (i.e., with respect to internal places, transitions, and labels) and always start with a real task (i.e., a non-$\tau$-labeled transition), then the LCM is simply the union of all workflow process definitions. Note that, in all six cases, the GCD and LCM are unique modulo branching bisimilarity. The following property formalizes the above claims.

**Property 6.8.** Let $N_0, N_1, \ldots, N_{n-1}$, where $n$ is some natural number, be $n$ workflow process definitions in $\mathcal{W}$; let $N_{GCD} \in \mathcal{W}$ be a GCD of $N_0, N_1, \ldots, N_{n-1}$ and let $N_{LCM} \in \mathcal{W}$ be an LCM of $N_0, N_1, \ldots, N_{n-1}$.

1. If $N_0 \cong N_1 \cong \ldots \cong N_{n-1}$, then, for all $k$ with $0 \leq k < n$, $N_k \cong N_{GCD} \cong N_{LCM}$.
2. If $N_0 \leq_k N_1 \leq_k \ldots \leq_k N_{n-1}$, then $N_0 \cong N_{LCM}$ and $N_{n-1} \cong N_{GCD}$.
3. If, for all $k$ with $0 \leq k < n$, $N_k \leq_k N_0$, then $N_0 \cong N_{GCD}$.
4. If, for all $k$ with $0 \leq k < n$, $N_0 \leq_k N_k$, then $N_0 \cong N_{LCM}$.
5. If, for some $j$ and $k$ with $0 \leq j < k < n$, $\alpha(N_j) \cap \alpha(N_k) = \emptyset$, then $N_t = \{(i, o), (\tau, \tau)\}$.
6. If, for all $j$ and $k$ with $0 \leq j < k < n$, $\alpha(N_j) \cap \alpha(N_k) = \emptyset$ and $(P_j \cup T_j) \cap (P_k \cup T_k) = \{i, o\}$ and, for all $k$ with $0 \leq k < n$ and all transitions $t \in i^{N_k}$, $t$ has a label different from $\tau$, then $N_0 = \bigcup_{0 \leq k < n} N_k \cong N_{LCM}$.

**Proof.** The first four properties follow immediately from Theorem 6.7.

To prove the fifth property, consider two workflow process definitions $N_j$ and $N_k$, with $0 \leq j < k < n$, such that $\alpha(N_j) \cap \alpha(N_k) = \emptyset$. Since $N_j \leq_k N_{GCD}$, it follows that $\alpha(N_{GCD}) \subseteq \alpha(N_j)$; similarly, $\alpha(N_{GCD}) \subseteq \alpha(N_k)$. Hence, it follows that $\alpha(N_{GCD}) \subseteq \alpha(N_j) \cap \alpha(N_k) = \emptyset$, which means that $\alpha(N_{GCD}) = \emptyset$. Consequently, $N_{GCD} \cong N_t$.

To prove the last property, we first show that $N_0$ is a subclass of each of the variants. Consider a variant $N_k$, for some $k$ with $0 \leq k < n$. Since for all $j$ with $0 \leq j < n$ and $j \neq k$, $\alpha(N_j) \cap \alpha(N_k) = \emptyset$, $(P_j \cup T_j) \cap (P_k \cup T_k) = \{i, o\}$, and all transitions $t \in i^{N_j}$ have a label different from $\tau$, blocking all transitions in $i^{N_j} \cup \bullet$ in $N_0$, yields a process branching bisimilar to $N_k$. Hence, $N_0 \leq_k N_k$, which means that it is a subclass of all $n$ variants. Second, we prove that any workflow process definition $N'$ in $\mathcal{W}$ that is a subclass of all the variants is also a subclass of $N_0$. Assume that $N' \subseteq \mathcal{W}$ is a subclass of all variants. Let, for all $k$ with $0 \leq k < n$, $I_k$ and $H_k$ be sets of task labels such that $(\varphi_k \circ \vartheta_k(N'), [i]) \sim_h (N_k, [i])$. Let $I = \bigcup_{0 \leq k < n} I_k$ and $H = \bigcup_{0 \leq k < n} H_k$. Clearly, $(\varphi \circ \vartheta(N'), [i]) \sim_h (N_0, [i])$, because each label in $H$ or $I$ appears in the alphabet of precisely one of the $n$ variants. Hence, $N' \leq_k N_0$. Combining the two results derived so far, it follows from Theorem 6.7 that $N_0$ is a unique LCM of the set of variants $N_0$ through $N_{n-1}$. Hence, $N_0 \cong N_{LCM}$. □
6.3 Inheritance-preserving transformation rules and management information

In Section 6.1, the notion of an MI-net has been introduced as a means to collect aggregate management information on the status of cases in a number of variants of a workflow process. It is essential that a set of total and valid transfer rules is available to map information of running cases onto an MI-net. Section 6.2 has introduced the notions of a GCD and an LCM of a number of workflow process definitions. It has been argued that a GCD or an LCM are MI-nets that are good candidates for collecting aggregate management information. However, it is not always easy to determine a GCD or an LCM nor is it always straightforward to obtain a useful set of transfer rules. However, in Section 5, it has been shown that it is always possible to transfer a case from one workflow process definition to another one if the latter is constructed from the former by means of one of the inheritance-preserving transformation rules of Section 3.2. Thus, the inheritance-preserving transformation rules and accompanying transfer rules can also be used to extract aggregate management information.

Observation 6.9. Consider a set of workflow process definitions that are created from each other by means of the inheritance-preserving transformation rules presented in Section 3.2 (in both directions). For each pair of elements of this set, the transfer rules of Section 5 can be used to construct a total valid transfer rule which maps cases from one element of this pair to the other element.

Observation 6.9 implies that any workflow process definition of a set of process definitions that are created from each other by means of the inheritance-preserving transformation rules of Section 3.2 forms a meaningful MI-net. Consider again the three workflow process definitions in Figure 6.2. Workflow process definition \( N_1 \) can be obtained from \( N_0 \) by means of inheritance-preserving transformation rule \( PJJS \) of Theorem 3.18, whereas \( N_2 \) can be obtained from \( N_0 \) with transformation rule \( PJJS \) of Theorem 3.15. This means that all cases in \( N_1 \) and \( N_2 \) can be mapped onto \( N_0 \) by means of transfer rules \( r_{P_JS}^{-1} \) of Theorem 5.24 and \( r_{P_JS}^{-1} \) of Theorem 5.22, respectively. However, it is more interesting to see how cases can be mapped onto \( N_1 \) and \( N_2 \). Assume that \( N_1 \) is used as an MI-net. All cases in \( N_0 \) can be mapped onto \( N_1 \) by means of the rule \( r_{P_JS, p} \) of Theorem 5.16. (Recall that we have explicitly excluded rule \( r_{P_JS, C} \) in the context of aggregating management information.) Cases in \( N_2 \) can be mapped onto \( N_1 \) by means of the composite transfer rule \( r_{P_JS, p} \circ r_{P_JS}^{-1} \). That is, net \( N_0 \) is used as an intermediate to map cases from \( N_2 \) onto \( N_1 \). Similarly, all cases can be mapped onto \( N_2 \) by means of transfer rules \( r_{P_JS} \) of Theorem 5.11 and \( r_{P_P} \circ r_{P_JS}^{-1} \).

Observation 6.9 has several important consequences. Consider a set of workflow process definitions satisfying the requirement in Observation 6.9. As already mentioned, any process definition in this set can be chosen as an MI-net. The transfer rules of Section 5 based on the four inheritance-preserving transformation rules (i.e., \( r_{P_T}, r_{P_P}, r_{P_J}, r_{P_J S} \)) provide mappings for running cases in any of the workflow process definitions onto states in the MI-net. These states are as close to the actual states of the cases as possible, which is very important for the quality of the management information. Note that the chosen MI-net is not necessarily a GCD or an LCM of the set of process definitions (see also the example discussed above and illustrated in Figures 6.2 and 6.3). However, if the set of workflow process definitions satisfies any of the conditions of Property 6.8, then a suitable choice for the MI-net yields a GCD or an LCM (see, again, the example illustrated in Figures 6.2 and 6.3).

Figure 6.10 illustrates a slightly larger example. It shows four workflow process definitions. The two process definitions \( N_0 \) and \( N_1 \) are the two variants of the complaints-handling process. These two variants hold 38 cases, 14 in \( N_0 \) and 24 in \( N_1 \). The other two process definitions are MI-nets. Net \( N_{GCD} \) (top) is the (unique) GCD of \( N_0 \) and \( N_1 \); net \( N_{LCM} \) (bottom) is the LCM of \( N_0 \) and \( N_1 \). Net \( N_{GCD} \) can be obtained from \( N_0 \) by means of transformation rules \( PPS \) and \( PTS \), both applied
Figure 6.10: Two workflow process definitions $N_0$ and $N_1$ and two aggregate views $N_{GCD}$ and $N_{LCM}$. 
in reverse direction; \( N_{GCD} \) is obtained from \( N_1 \) by means of rules \( PJS \) and \( PJ3S \) in reverse direction. Furthermore, net \( N_0 \) yields \( N_{LCM} \) via rules \( PJS \) and \( PJ3S \), whereas \( N_1 \) yields \( N_{LCM} \) via rules \( PPS \) and \( PTS \).

The transfer rules of Section 5 are used to map cases of the variants \( N_0 \) and \( N_1 \) onto the two MI-nets; the composition of \( r_{PPS}^{-1} \) and \( r_{PTSC}^{-1} \) (or \( r_{PTSP}^{-1} \) which equals \( r_{PTSC}^{-1} \) for this example) is used to map cases of \( N_0 \) onto \( N_{GCD} \), the composition of \( r_{PS}^{-1} \) and \( r_{PJ3S}^{-1} \) is used to map cases of \( N_1 \) onto \( N_{GCD} \), the composition of \( r_{PJ5} \) and \( r_{PJ3S.P} \) is used to map cases of \( N_0 \) onto \( N_{LCM} \), and the composition of \( r_{PPS} \) and \( r_{PTS} \) is used to map cases of \( N_1 \) onto \( N_{LCM} \).

Consider, for example, place \textit{ready} in \( N_{GCD} \). The label \( 10 = 3 + 7 \) indicates that ten of the 38 cases are in the state corresponding to place \textit{ready}. In \( N_0 \), three cases are ready and, in \( N_1 \), two cases are ready, i.e., just five cases are actually ready. However, there are five cases in the state corresponding to \textit{inform\_man} in \( N_1 \). If we abstract from task \textit{inform\_management}, these cases are also ready. This brings the total to ten cases in state ready in \( N_{GCD} \). Note that, in \( N_{LCM} \), there are just five cases in a state corresponding to place \textit{ready}, because the LCM distinguishes between \textit{inform\_man} and \textit{ready}. Figure 6.10 is a good example illustrating that the LCM is more complex and contains more detailed information, whereas the GCD is more succinct and only contains information which is relevant for all variants. Which one is most suitable as an MI-net depends on the context.

6.4 Management information in the workflow-management domain

To end this section on management information, let us return to Section 4. In that section, we have discussed the relevance of inheritance in four (application) domains: \textit{ad-hoc change}, \textit{evolutionary change}, \textit{workflow templates}, and \textit{E-commerce}. In the remainder, we discuss for each of these domains possible approaches for obtaining aggregate management information using the inheritance-preserving transformation rules and the transfer rules presented in this paper.

Ad-hoc change typically results in many slightly different variants of a predefined workflow process. These variants are usually the result of an error, a rare event, or special demands of a customer. The predefined workflow process can be seen as a template. If all variants are constructed by extending the template workflow using the inheritance-preserving transformation rules and the template itself is also a variant, then the template workflow is the GCD of all variants (see Property 6.8-3). If the variants are constructed by applying the inheritance-preserving transformation rules in the reverse direction only and the template itself is also a variant, then the template workflow is the LCM of all variants (see Property 6.8-4). If the variants are constructed by applying the inheritance-preserving transformation rules in both directions, the template workflow is not the GCD nor the LCM but it is still a suitable MI-net for presenting aggregate management information. If change is restricted according to the inheritance-preserving transformation rules, then the transfer rules of Section 5 can be used to obtain transfer rules from the ad-hoc variants to the template workflow process (see Observation 6.9).

Evolutionary change typically results in a limited set of versions of a workflow process. If every time a change occurs, all cases are transferred immediately, there is just one active version. Only if transfers are postponed (e.g., transfer rule \( r_9 \)), there is a need to aggregate management information. In case of evolutionary change, the most recent version of a workflow process is the most likely candidate for presenting aggregate management information. If all changes are restricted to the inheritance-preserving transformation rules, it is no problem to map the cases onto the most recent version of the workflow process (Observation 6.9). Note that, if all changes in the past were \textit{extensions} (i.e., the transformation rules were only applied in the forward direction), then the most recent version of the workflow process definition is the LCM of all variants (see Property 6.8-2).
When using a workflow template as the starting point for designing workflows, the template is the most likely candidate for projecting aggregate management information. Again, by restricting modifications of the template to the four inheritance-preserving transformation rules presented in this paper, all cases can be mapped onto the workflow template without any problems.

For E-commerce, it is important that business partners agree on some common workflow process (see Section 4.4). For each of the business partners, it is useful to have aggregated management information at the level of the common workflow process. In Section 4.4, it has been suggested that each of the local workflow processes should be a subclass of (part of) the common workflow process under projection inheritance. If local extensions are restricted to transformation rules $PPS$, $PJS$, and $PJ3S$, then the transfer rules $r_{PPS}^{-1}$, $r_{PJS}^{-1}$, and $r_{PJ3S}^{-1}$ can be used to map cases onto the common process definition. Moreover, the common process definition is, under certain restrictions, the GCD of the workflow processes perceived by the business partners (i.e., local and global view).

7 Tool support

In the preceding sections, we have shown that inheritance concepts can be used to tackle many of the problems related to ad-hoc change and evolutionary change of workflow processes. Moreover, the concepts can be used to enhance the application of workflow templates and may be beneficial in the design and enactment of interorganizational workflows (see Section 4.4). Unfortunately, today's workflow management systems do not support workflow inheritance as discussed in this paper. Some workflow management systems have adopted object-oriented concepts. For example, InConcert [35] allows for building workflow class hierarchies. However, in these class hierarchies, inheritance is restricted to the static interface (i.e., attributes and/or is-part-of relationships). To our knowledge, there is not a single workflow management system taking the dynamics of the workflow process into account when defining inheritance. In this section, we briefly discuss how the results presented in this paper can be used to aid existing workflow management systems in supporting dynamic change and supplying aggregate management information.

In the remainder of this section, first, we describe Woflan which allows for the verification of soundness. (Recall that soundness is pivotal to the notions of inheritance, the transformation rules, and the transfer rules.) Then, we discuss tool support to verify the inheritance notions introduced in Section 3. Finally, we consider ways to integrate change facilities, i.e., services to support dynamic change and to construct aggregate management information, in existing workflow management systems.

7.1 Verification of soundness

Throughout this paper, we considered workflow process definitions. Recall that a workflow process definition is a sound WF-net. That is, a workflow process definition determines not an arbitrary process but a process with desirable properties such as proper completion, completion option, etc. (see Definitions 2.19 (WF-net) and 2.22 (Soundness)). Most of today's workflow management systems can only enact workflow processes having these properties. However, they do not support advanced techniques to verify the correctness of workflow process definitions [3]. At design-time, there are hardly any checks to verify whether the properties stated in Definitions 2.19 and 2.22 are fulfilled. Violations of these properties typically result in serious run-time errors such as deadlocks or livelocks. Contemporary workflow management systems typically restrict themselves to a number of (trivial) syntactical checks. Therefore, serious errors such as deadlocks and livelocks may remain undetected.
This means that an erroneous workflow may go into production, thus causing dramatic problems for the organization. An erroneous workflow may lead to extra work, legal problems, angry customers, managerial problems, and depressed employees. Therefore, it is important to verify the correctness of a workflow process definition before it becomes operational. If there are frequent ad-hoc or evolutionary changes, then the role of verification becomes even more important. This is the reason that we developed Woflan (WOrkFLow ANalyzer) [6, 53, 54]. Woflan is a stand-alone verification tool specifically designed for workflow analysis. Woflan is product independent, i.e., it is possible to analyze processes designed with various workflow products of different vendors. Woflan is able to handle complex workflows with up to hundreds of tasks. Woflan provides to-the-point diagnostic information for repairing the errors detected. Pivotal to Woflan is the notion of soundness as defined in Definition 2.22. In fact, Woflan uses a slightly weaker version of soundness where the safeness requirement is omitted (cf. [3, 54]). However, Woflan also analyzes the stronger notion used in this paper. The soundness notion expresses the minimal requirements any workflow should satisfy and includes properties such as proper termination and the absence of deadlock and livelocks. The current version of Woflan can analyze workflows designed with the following four workflow products: COSA, Staffware, METEOR, and Protos. COSA [51] is one of the leading workflow management systems on the Dutch workflow market. COSA uses Petri nets as a modeling language and thus allows for the modeling and enactment of complex workflow processes which use advanced routing constructs. However, COSA does not support verification. Fortunately, Woflan can analyze any workflow process definition constructed by using CONE (COSA Network Editor), the design tool of the COSA system. Woflan can also import process definitions made with Staffware [52], METEOR [50], or Protos [43]. Staffware is one of the most used workflow management systems in the world. METEOR is a workflow management system based on CORBA and supports transactional workflows ([26]). Protos is a Business-Process-Reengineering tool which can be used to (re)design and document workflow processes.

To illustrate the use of Woflan, consider the WF-net shown in Figure 2.21. Figure 7.1 shows this workflow modeled with COSA and Figure 7.2 shows some of the diagnostics provided by Woflan when analyzing this workflow. Woflan reports that the workflow modeled with COSA is not sound and that the connection between tasks ignore_complaint and inform_management is the source of the error. (Note that the boundedness and safeness property in the diagnosis of Woflan refer to the short-circuited workflow net; see Definition 2.19 (WF-net) and Theorem 2.23 (Characterization of soundness).)

For more information on Woflan, we refer to [54]; the interested reader can also download a version of Woflan via the World-Wide-Web [53].

7.2 Verification of inheritance relations

In Section 4, we have shown several application areas where it is desirable to limit possible changes by imposing inheritance relationships, e.g., the designed workflow process definition should be extended in such a way that the result is a subclass of a predefined workflow process definition (e.g., a workflow template or existing workflow) under life-cycle inheritance. Recall that we have defined four inheritance relations: protocol/projection inheritance, protocol inheritance, projection inheritance, and life-cycle inheritance. Basically, there are two ways to support these inheritance relations.

1. Enumerative verification method

For any two workflow process definitions, it is decidable whether one workflow process definition is a subclass of the other workflow process definition under one of the four inheritance
notions. By comparing the state spaces of two process definitions, it is possible to decide whether the process definitions are branching bisimilar. Therefore, a brute-force approach can be used by systematically blocking and hiding tasks, enumerating all reachable states of the resulting nets, and comparing the state spaces. There are several tools that can check branching bisimilarity using enumerative methods. It is well-known that deciding whether two finite processes are branching bisimilar can be done in polynomial time, where the size of the problem is defined as the number of states and transitions of the two processes [31]. However, even a workflow process with a limited number of tasks can have many states. Therefore, there are two practical problems when using a separate verification tool based on enumerative methods. First, it is difficult to provide an interface between the workflow editor (i.e., the workflow design tool) and the verification tool. The workflow editor has to construct the state space typically containing thousands of states and send it to the verification tool. Second, it is very difficult to translate the results generated by the verification tool to diagnostics understandable by the workflow designer.

2. Workflow editor supporting inheritance-preserving transformation rules

In Section 3.2, we have identified four inheritance-preserving transformation rules. Instead of using an enumerative method to verify inheritance relations afterwards, it is possible to limit the changes in the workflow editor to the four inheritance-preserving transformation rules identified in this paper. Note that the transformation rules correspond to the design constructs typically used when constructing/adapting a workflow process definition. Using an editor augmented with these rules has two benefits. First, using the rules instead of an enumerative method is
more efficient from a computational point-of-view. Second, the user is forced to make correct
designs with respect to the selected inheritance relation (correctness by design). Therefore,
there is no need to provide diagnostics to locate the source of an error. Unfortunately, the
editors of current workflow management systems do not provide facilities to enforce design
rules and the conditions for the transformation rules are quite complex to check by tools not
dedicated to Petri-net analysis. Therefore, it will not be easy to extend the existing workflow
tools with inheritance-preserving transformation rules.

The ideal situation would be an editor which automatically checks inheritance relationships or limits
change to the inheritance-preserving transformation rules. At the moment, such tools are missing.
The current version of Woflan could be used as a basis for such a tool. Woflan already constructs
the state space for a single workflow process definition. Moreover, most of the requirements stated
when formulating the inheritance-preserving transformation rules can be verified (indirectly) with the
current version of Woflan. Unfortunately, it is not likely that workflow management systems will
provide sophisticated editors supporting the inheritance notions in the very near future. However,
the transformation rules can also be used as a method to be employed in combination with existing
workflow management systems. This means that the workflow designer has to check the appropriate
conditions stated in Section 3.2 before making a change. For this purpose, it is useful to translate the
rules to the specific terminology of the workflow management system. Moreover, the rules can be
simplified by further restrictions. Such a method would enable workflow designers to benefit from the
results presented in this paper using current technology.

Figure 7.2: Some of the diagnostics provided by Woflan when verifying the correctness of the COSA
design shown in Figure 7.1.
7.3 Supporting dynamic change

Most of today's workflow management systems provide a versioning mechanism, i.e., it is possible to enact multiple versions of the same workflow process at the same time. However, each case (i.e., workflow instance) refers to one version and it is not possible to migrate a case from one version to another. In addition, such a mechanism is not suitable for ad-hoc change. Some workflow management systems such as InConcert [35] and Ensemble [25] provide support for ad-hoc changes, i.e., while executing a case it is possible to adapt the corresponding process definition; each case has a private copy of the workflow process definition which can be modified without any problems.

None of today's commercial workflow management systems support dynamic change, i.e., it is not possible to transfer a case from one process definition to another. Yet, for many applications such dynamic changes are a necessity. In Section 5, we presented several transfer rules under the assumption that changes are limited to the application of the inheritance-preserving transformation rules of Section 3.2 (both directions). To support the transfer of cases from one version of a process to another, the workflow enactment service ([39]) needs to be extended. If change is limited to the inheritance-preserving transformation rules, the implementation of a transfer facility is rather straightforward since there is no need to postpone transfers (i.e., there is always just one active version of the workflow process). The transfer of cases can be handled by the workflow engine(s) or by a separate service. If the workflow engine is extended with a transfer facility, then the engine is notified every time there is a new version of a workflow process. For each case which is not active (i.e., no tasks are being executed), the transfer is a simple database update: Change the reference of the case and create a new workflow state (i.e., marking). If a task is being executed (for a case which needs to be transferred), the transfer is delayed until completion of the task or the running task is aborted and rolled back before the case is transferred. If a separate service is used to transfer the cases (i.e., a service not integrated in the engine), all relevant cases need to be blocked (i.e., all instances which need to be transferred are frozen) to avoid concurrency problems.

7.4 Providing aggregate management information

If there are multiple versions or variants of the same workflow process, it is desirable to have an aggregated view of the work-in-progress, i.e., condensed management information showing the statuses of all cases in one diagram (i.e., an MI-net). In Section 6, we have shown that if change is limited to the inheritance-preserving transformation rules (applied in both directions), then it is possible to construct such a view. For this purpose, the following information is needed: The states of all cases involved (including version/variant information), the transformation rules used to move from one version or variant to another, and the MI-net. For a suitably chosen MI-net, the transfer rules can be calculated automatically and all cases can be mapped onto a single diagram. Clearly, today's workflow management systems do not provide such a facility and show aggregate management information at the level of versions/variants rather than processes. (In fact, many workflow management systems provide hardly any management information.) Although the implementation of such a facility is far from trivial, there are two circumstances which simplify the realization of the ideas presented in Section 6. First, the information needed to distill the management information can be extracted without interfering with the enactment service, because cases are not actually transferred. Second, much of the functionality needed to implement dynamic change (e.g., the transfer rules) can be used for this facility.
8 Conclusion

This paper tackles two notorious problems related to adaptive workflow: (1) supporting dynamic change and (2) providing management information at the right aggregation level. The solution is based on an approach using inheritance. Since the inheritance notions used in this paper focus on the dynamic behavior of processes rather than their static structure, they are of particular relevance for workflow management. We have provided four inheritance relations (protocol/projection inheritance, protocol inheritance, projection inheritance, and life-cycle inheritance), four inheritance-preserving transformation rules which can be applied in two directions (PTS, PPS, PJS, and PJ3S), and ten transfer rules (\( r_{PTS} \), \( r_{PPS} \), \( r_{PJS} \), \( r_{PJ3S} \), \( r_{PJS,C} \), \( r_{PTS,C} \), \( r_{PPS} \), \( r_{PJS,C} \), \( r_{PJ3S} \)). The transformation rules can be used to restrict changes in workflow process definitions in such a way that the new workflow process definition inherits certain properties of the old workflow process definition. Such restrictions are useful when dealing with ad-hoc workflow, evolutionary change, workflow templates, and interorganizational workflows. Moreover, the transformation rules combined with the transfer rules enable dynamic change and aggregation of management information. If process changes are restricted to the transformation rules, then the typical problems related to adaptive workflow can be avoided. The transfer rules which are used to transfer cases from one workflow process definition to another can also be used to generate condensed management information showing an aggregate view of the work-in-progress. The inheritance notions are interesting both from a theoretical and practical perspective. On the one hand, the inheritance relations lead to interesting concepts such as the GCD and LCM of a set of process definitions. On the other hand, they provide concrete solutions for problems today's workflow management systems are faced with.

An interesting topic for future research is the application of the inheritance rules in various domains. We already mentioned the application of projection inheritance to E-commerce. Another application would be the application of our inheritance concepts to component-based software architectures. Another future challenge is to deal with the dynamic-change problem in case there is no inheritance relationship between the old and the new process definition. One solution would be to calculate a change region. If the change region is empty for a given case (i.e., a designated part of the WF-net contains no tokens), the case is transferred. Otherwise, the transfer is postponed until the region is empty. However, given arbitrary changes, it is difficult to calculate a minimal change region which does not lead to the anomalies discussed in this paper. Finally, we continue to enhance Woflan. At the moment, we are improving the diagnostics provided by Woflan. In the future, we hope to extend Woflan with the inheritance concepts presented in this paper.

Acknowledgement We want to thank Robert van der Toorn and Eric Verbeek for their careful reading of early versions of this paper.

References


64


### Computing Science Reports

**Department of Mathematics and Computing Science**  
**Eindhoven University of Technology**

**In this series appeared:**

<table>
<thead>
<tr>
<th>Number</th>
<th>Title and Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>96/01</td>
<td>M. Voorhoeve and T. Basten</td>
</tr>
<tr>
<td>96/02</td>
<td>P. de Bra and A. Aerts</td>
</tr>
<tr>
<td>96/03</td>
<td>W.M.P. van der Aalst</td>
</tr>
<tr>
<td>96/04</td>
<td>S. Mauw</td>
</tr>
<tr>
<td>96/05</td>
<td>T. Basten and W.M.P. v.d. Aalst</td>
</tr>
<tr>
<td>96/06</td>
<td>W.M.P. van der Aalst and T. Basten</td>
</tr>
<tr>
<td>96/07</td>
<td>M. Voorhoeve</td>
</tr>
<tr>
<td>96/08</td>
<td>A.T.M. Aerts, P.M.E. De Bra, J.T. de Munk</td>
</tr>
<tr>
<td>96/09</td>
<td>F. Dignum, H. Weigand, E. Verharen</td>
</tr>
<tr>
<td>96/10</td>
<td>R. Bloo, H. Geuvers</td>
</tr>
<tr>
<td>96/11</td>
<td>T. Laan</td>
</tr>
<tr>
<td>96/12</td>
<td>F. Kamareddine and T. Laan</td>
</tr>
<tr>
<td>96/13</td>
<td>T. Borghuis</td>
</tr>
<tr>
<td>96/14</td>
<td>S.H.J. Box and M.A. Reniers</td>
</tr>
<tr>
<td>96/15</td>
<td>M.A. Reniers and J.J. Vereijken</td>
</tr>
<tr>
<td>96/17</td>
<td>E. Boiten and P. Hoogendijk</td>
</tr>
<tr>
<td>96/18</td>
<td>P.D.V. van der Stok</td>
</tr>
<tr>
<td>96/19</td>
<td>M.A. Reniers</td>
</tr>
<tr>
<td>96/20</td>
<td>L. Feijs</td>
</tr>
<tr>
<td>96/21</td>
<td>L. Bijluma and R. Nederpelt</td>
</tr>
<tr>
<td>96/22</td>
<td>M.C.A. van de Graaf and G.J. Houben</td>
</tr>
<tr>
<td>96/23</td>
<td>W.M.P. van der Aalst</td>
</tr>
<tr>
<td>96/24</td>
<td>M. Voorhoeve and W. van der Aalst</td>
</tr>
<tr>
<td>96/25</td>
<td>M. Vaccari and R.C. Backhouse</td>
</tr>
<tr>
<td>97/01</td>
<td>B. Knaack and R. Gelh</td>
</tr>
<tr>
<td>97/02</td>
<td>J. Hoornen and O. v. Roosmalen</td>
</tr>
<tr>
<td>97/03</td>
<td>J. Blanco and A. v. Deursen</td>
</tr>
<tr>
<td>97/04</td>
<td>J.C.M. Baeten and J.A. Bergstra</td>
</tr>
<tr>
<td>97/05</td>
<td>J.C.M. Baeten and J.J. Vereijken</td>
</tr>
<tr>
<td>97/06</td>
<td>M. Franken</td>
</tr>
</tbody>
</table>

- **Example specifications in phi-SDL**.
- **A Process-Algebraic Approach to Life-Cycle Inheritance**  
  \[\text{Inheritance} = \text{Encapsulation} + \text{Abstraction}\], p. 15.
- **Life-Cycle Inheritance: A Petri-Net-Based Approach**, p. 18.
- **Structural Petri Net Equivalence**, p. 16.
- **AUTOMATH and Pure Type Systems**, p. 30.
- **A Correspondence between Nuprl and the Ramified Theory of Types**, p. 12.
- **Priori Tenre Logics in Modal Pure Type Systems**, p. 61.
- **The $i^2$ C-bus in Discrete-Time Process Algebra**, p. 25.
- **Completeness in Discrete-Time Process Algebra**, p. 139.
- **Nested collections and polymorphism**, p. 11.
- **Static Semantics of Message Sequence Charts**, p. 71
- **Algebraic Specification and Simulation of Lazy Functional Programs in a concurrent Environment**, p. 27.
- **Structural Characterizations of sound workflow nets**, p. 22.
- **Conservative Adaption of Workflow**, p. 22
- **Deriving a syntactic regular language recognizer**, p. 28
- **A Discretisation Method for Asynchronous Timed Systems**.
97/07 J.C.M. Baeten and J.A. Bergstra Bounded Stacks, Bags and Queues, p. 15.
97/08 P. Hoogendijk and R.C. Backhouse When do datatypes commute? p. 35.
97/13 W.M.P. van der Aalst Exploring the Process Dimension of Workflow Management, p. 56.
97/14 J.F. Groote, F. Monin and J. Springintveld A computer checked algebraic verification of a distributed summation algorithm, p. 28
97/15 M. Franssen λP: A Pure Type System for First Order Logic with Automated Theorem Proving, p. 35.
97/16 W.M.P. van der Aalst On the verification of inter-organizational workflows, p. 23.
97/17 M. Vaccari and R.C. Backhouse Calculating a Round-Robin Scheduler, p. 23.
98/02 M. Voorhoeve State/Event Net Equivalence, p. 25.
98/03 J.C.M. Baeten and J.A. Bergstra Deadlock Behaviour in Split and ST Bisimulation Semantics, p. 15.
98/05 D. Dams Flat Fragments of CTL and CTL*: Separating the Expressive and Distinguishing Powers, p. 22.
98/10 T. Basten and J. Hooman Process Algebra in PVS
98/11 J. Zwanenburg The Proof-assistant Yarrow, p. 15.
98/14 T. Verheoef (artikel volgt)
99/01 V. Bos and J.I.T. Kleijn Structured Operational Semantics of χ, p. 27.
99/02 H.M.W. Verbeek, T. Basten and W.M.P. van der Aalst Diagnosing Workflow Processes using Woflan, p. 44.
<table>
<thead>
<tr>
<th>Date</th>
<th>Authors</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>99/03</td>
<td>R.C. Backhouse and P. Hoogendijk</td>
<td>Final Dialgebras: From Categories to Allegories</td>
<td>26</td>
</tr>
<tr>
<td>99/04</td>
<td>S. Andova</td>
<td>Process Algebra with Interleaving Probabilistic Parallel Composition</td>
<td>81</td>
</tr>
<tr>
<td>99/05</td>
<td>M. Franssen, R.C. Veltkamp and W. Wesselink</td>
<td>Efficient Evaluation of Triangular B-splines</td>
<td>13</td>
</tr>
<tr>
<td>99/06</td>
<td>T. Basten and W. v.d. Aalst</td>
<td>Inheritance of Workflows: An Approach to tackling problems related to change</td>
<td>66</td>
</tr>
</tbody>
</table>