Reference Spreading

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Reference Spreading: Tracking Performance for Impact Trajectories of a 1DoF Setup

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Abstract—In this brief, trajectory tracking control for nonperiodic motions with impacts is analyzed by means of physical experiments. This brief aims at demonstrating the effectiveness and robustness of the control method called reference spreading (RS) on a physical setup. An actuated rebounding pendulum, a one-degree-of-freedom system specifically designed for performing trajectories with partially elastic impacts, acts as a test bench. RS control is compared to both classic proportional derivative (PD) feedback control and distance function-based control, showing superior performance. Evidence of RS's robustness to delayed impact detection, deteriorated velocity estimation accuracy, and model inaccuracies is moreover reported.

Index Terms—Impact, reference spreading (RS), restitution, trajectory tracking.

I. INTRODUCTION

THE analysis and control of mechanical systems performing motions with intended impacts is an active field of research in the robotics and control communities [1]–[6]. Motions with planned collisions naturally occur, for example, in robotic locomotion [1] and dynamic pick-and-place tasks [7], [8]. The control of this type of motions is complicated by the rapid velocity changes that are seen during an impact. The contact forces that cause the sudden changes are usually either modeled using a (nonlinear) spring-damper combination [9] or are considered impulsive. The latter method (effectively) abstracts the velocity changes as state jumps as we will do in this brief [10], [11]. Combining the smooth (flow) dynamics with these discrete (jump) effects gives a hybrid system [12].

Tracking a reference trajectory with jumps is challenging as the closed-loop system will likely experience the impacts at different times than expected. In such a setting, employing the classical error notion leads to an undesired phenomenon called *peaking* in [13] and [14] (not to be confused with the peaking in high-gain nonlinear observers). Even when convergence is occurring, the error is artificially large in the interval between an actual impact and the time where the impact was supposed to happen. In this brief, we illustrate peaking experimentally together with its detrimental effect in control.

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Fig. 1. Photograph of the ARP, the experimental setup considered in this brief.

A few approaches have been proposed in the literature to avoid peaking for the sake of stability analysis, state estimation, and for control [1], [14]–[18]. In [15], in tracking a periodic reference of a linear system with jumps, the authors disregard a small time interval about the reference impact times and switch off the velocity-dependent feedback in those intervals. Periodic trajectories/orbits with jumps are also considered in the hybrid zero dynamics approach [1], [2]. The tracking problem for nonperiodic motions with elastic impacts is addressed in [17] for polyhedral billiards by making use of both the reference trajectory and its mirrored version in the proximity of the billiard's boundary. In [13] and [14], and prior work referenced therein, the distance between two trajectories with jumps is quantified in a way that it is invariant to jumps of the reference and closed-loop tracking system. The method can be applied just for motions with partially elastic impacts. Morarescu and Brogliato [16] proposed a switching controller that can be employed to drive the system toward a unilateral constraint undergoing an infinite number of restitutions in the transition. In [18], another approach is discussed where the hybrid system dynamics are reformulated on a manifold embedded in a higher dimensional state space such that they can be represented by transformed continuous dynamics only. The approach bears similarities with the older works [10], [19] and with the more recent and complete projection-based formulation in [20]: in both the latter approaches, alternative states are introduced to transform the hybrid/nonsmooth dynamics into a continuous formulation.

In this brief, tracking control for systems with state-triggered jumps is tackled employing *reference spreading* (RS) [21], [22]. RS is a control strategy that is based on continuously extending the reference trajectory about its

impact times resulting in multiple reference segments defined for the same time interval. Switching from one such segment to another is triggered by the detection of an impact of the system, thus enforcing coincidence of state and (extended) reference jump times. The method does not require periodicity of the reference trajectory and envelopes the mirroring approach in [17]. In addition, it allows impacts to be (partially) elastic or inelastic [6] and allows for simultaneous impacts [23]. The basic assumption is that impacts are isolated, excluding Zeno behavior which is instead addressed in [12], [16], and [24], the latter proposing an online reference adaptation strategy.

Whereas different theoretical frameworks exist for the trajectory tracking of mechanical systems performing impacting motions, the experimental validation of these control methods is somewhat lacking (see [2] and [5] for experimental examples of robots performing impacting motions). This brief aims to fill that gap for RS and builds upon a previous analysis [25] in which the preliminary experimental results using RS were presented. In [25], limitations of the setup such as dominant (unmodeled) flexural dynamics and limited stroke, appeared to undermine the controller's performance preventing the authors to execute a thorough performance analysis. In this brief, instead, we consider an experimental one-degree-of-freedom (1DOF) test-bench that has been specifically designed for performing trajectories with partially elastic impacts (see Fig. 1). Only quantized position data from an optical encoder will be used. This choice is made to demonstrate that RS can successfully be implemented in situations with limited sensory information. The technique known as jump aware (JA) filtering [26] is employed for estimating position and velocity as well as for detecting impacts. On the other hand, the goal of [25] was to provide an experimental proof of principle, the objective of this brief is to analyze the control strategy from a performance and robustness perspective. We introduce three quantitative measures for assessing performance.

The key contributions of this brief are: 1) an experimental study on the performance of RS control in tracking trajectories with partially elastic impacts and 2) a study on how this performance is affected by state estimation accuracy, delay in impact detection, and model inaccuracies. A comparative analysis is presented, investigating how RS control performs against classic feedback control and, for the sake of completeness, against the method presented in [13]. The method in [13] is considered here as: 1) it is a representative of the mirroring approach [17]; 2) similar to RS, it can be applied to nonperiodic trajectories (as opposed to, e.g. [2]); and 3) it has been developed by colleagues of the authors, making its adoption natural and quick.

This brief is structured as follows. In Section II, the notation and mathematical preliminaries are introduced. The trajectory tracking problem is discussed in Section III and three different feedback control methods are briefly described. The experimental setup and tracking results are presented in Section IV. The conclusions of this brief are given in Section V.

II. PRELIMINARY

Mechanical systems performing motions with hard impacts are necessarily modeled using the framework of nonsmooth

mechanics [10], [27] in which complementarity conditions restrict the configuration space and, at the same time, enforce the feasibility of contact forces. In that framework, a velocity reset map is used for modeling impacts in zero time [10]. In many cases of practical interest, e.g., for vibroimpact dynamics with no persistent contact as in this brief, the non-smooth dynamics can be fitted in the hybrid systems framework, an approach that can also be taken in trajectory tracking of mechanical systems with state-triggered jumps [2]. Our framework takes inspiration from [12] combining continuous dynamics with discrete state jumps. The continuous part satisfies

$$\dot{x} = f(x, u, t), \quad x \in C \quad (1a)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, and $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^n$ is the vector field. As indicated in (1a), the flow dynamics applies only when the state is within the flow set $C \subset \mathbb{R}^n$. The rapid velocity changes at the time of an impact are modeled as instantaneous and occur when the state reaches the set $D \subseteq \partial C$ called the jump set. The impact effects are captured by a state-reset map $g : D \rightarrow C$ according to

$$x^+ = g(x^-), \quad x^- \in D \quad (1b)$$

where x^+ and x^- denote the right, respectively, left limit of x at the impact time. In parameterizing solutions to (1), we employ the notion of hybrid time (t, j) , where continuous time t and discrete time j are joined together [12].

Consider a trajectory $x(t, j)$ that is the solution to (1) for a given input $u(t, j)$ and initial condition $x(t_0, 0) = x_0 \in C$. Let t_j indicate the time corresponding to the j th discrete event. Then, from (1b), we have $x(t_{j+1}, j+1) = g(x(t_{j+1}, j))$. The hybrid domain of x [12] is defined as $\text{dom } x = \bigcup_{j=0}^{N_x} [t_j, t_{j+1}] \times \{j\}$ with N_x the number of impacts. All jump event times t_j are assumed to be strictly separated in time.

We denote the reference trajectory as $\alpha(t, j)$ and assume that α is known in advance and is the unique solution to (1) for input $u = \mu(t, j)$ and initial condition $\alpha(t_0, 0) = \alpha_0 \in C$. The j th event time of α is denoted τ_j .

In the following, $\alpha(t)$ should be read as $\alpha(t, j_\alpha(t))$ where $j_\alpha(t) := \max \{j \mid (t, j) \in \text{dom } \alpha\}$ is a jump counter function for the trajectory α . Similarly, for brevity, we occasionally write $\mu(t, j_\alpha(t))$ simply as $\mu(t)$ and $x(t, j_x(t))$ as $x(t)$.

III. CONTROL OF SYSTEMS WITH IMPACTS

The trajectory tracking problem is that of finding an input $u = \kappa(x, t, j)$ such that, when applied to the system (1), the trajectory of interest is stabilized (locally and asymptotically [28]). When the input law is incorporated in the dynamics (1a), we say the system is in closed loop. The common approach to drive a system to a desired reference trajectory is to apply a feedforward input μ to the system and penalize the error by incorporating feedback control. In this brief, we compare different control methods that all follow this approach, but consider different quantifications of error. These different control approaches will be briefly described next.

Fig. 2. Illustration of the reference trajectory extension approach. For $\alpha(\cdot, j)$, the left dashed line illustrates the postevent extension about event time τ_j and the right dashed line is the ante-event extension about τ_{j+1} [22].

A. Classic Linear Feedback Control

The classic linear feedback control is

$$u = \kappa_{cl}(x, t) := \mu(t) - K(x - \alpha(t)) \quad (2)$$

with $K \in \mathbb{R}^{m \times n}$ a (possibly time-varying) feedback gain. In (2), the error is obtained by taking the difference between x and α , for the same continuous time t only, disregarding their respective jump counters. As anticipated in the introduction, because discrete jumps are state-triggered, a nonzero error is likely to result in a time mismatch between the closed-loop impact times t_j and the reference impact times τ_j . Consequently, a mismatch in the discrete time j of the closed-loop and reference trajectories at time t is witnessed in the neighborhoods of the nominal event times τ_j , $j \in \{1, 2, \dots\}$. Since either x or α may have experienced one jump more than the other for those times, large errors are typically encountered close to the reference event times, i.e., *peaking* occurs [14], [27], [29]. As illustrated by means of simulation examples in, e.g., [17] and [22] and as quantified experimentally in Section IV-C of this brief, employing an error signal exhibiting peaking is likely to result in reduced tracking performance and large peaks in the feedback signal.

B. Reference Spreading Control

In RS, peaking is eliminated by employing a different notion of error [21], [22]. Fig. 2 illustrates the key concept behind RS. In this, the reference trajectory is spreaded in the sense that every branch $\alpha(\cdot, j)$ is extended such that it is defined beyond each single interval $[\tau_j, \tau_{j+1}]$. Ideally, the spreading could be done such that the branches are defined for all time $t \in \mathbb{R}$. In practice, it suffices to extend the domain to $[\tau_j - \delta, \tau_{j+1} + \delta]$ for sufficiently large expected maximum time mismatch $\delta > 0$. Let $\bar{\mu}(t, j)$ be a chosen/designed continuous extension of $\mu(t, j)$ on that domain. For each j , the extension is then achieved by forward integration of (1a) using $u = \bar{\mu}(t, j)$, starting from initial condition $\alpha(\tau_{j+1}, j)$ and, similarly, by backward integration using the same vector field and input curve, starting from $\alpha(\tau_j, j)$. During the forward and backward integrations, no state jumps are applied and

flow *outside* the flow set C is allowed. The resulting *extended reference trajectory* is denoted $\bar{\alpha}$ and its (ideal) domain is $\bar{I}_\alpha := \mathbb{R} \times \{0, 1, \dots, N_\alpha\}$. As long as $N_\alpha \geq N_x$, it follows that $\text{dom } x \subset \text{dom } \bar{\alpha} = \bar{I}_\alpha$. Consequently, for every time t , we are able to compare the trajectory x , that has experienced j jumps, to the reference branch $\bar{\alpha}(\cdot, j)$ with the same jump counter. This allows to define the following linear feedback law plus feedforward:

$$u = \kappa_{rs}(x, t, j) := \mu(t, j) - K(x - \bar{\alpha}(t, j)). \quad (3)$$

The tracking law (3) is tested experimentally in Section IV and compared against the classical feedback (2). The reader is referred to [21], [22], and [28] for more details on RS.

C. Distance Function-Based Control

Aside from (2), in this brief, we also compare RS to what we will refer to as the *distance function (DF)* control. By this, we mean the control law for 1 DoF systems with partially elastic impacts in [13]. The controller in [13] is similar to the mirroring approach in [17] but uses scaling to allow for impacts that are not perfectly elastic. Note that, for the system of interest, DF control is analogous to RS control for a particular extension method as we will show in Section IV-C (Remark 3).

The DF approach eliminates peaking by employing a DF $d : (C \cup D) \times (C \cup D) \rightarrow \mathbb{R}_{\geq 0}$ that is insensitive to state jumps as it is tailored to the hybrid system of interest (particularly, to the jump map g). The DF controller [13, eqs. (11) and (12) therein] builds upon state-jump-insensitivity. It is designed for systems of the form (1) possessing the control-affine mechanical structure $f(x, u, t) = [x_2 \ l(t, x) + u]^T$, $g(x) = [x_1 \ -\epsilon x_2]^T$, $C = [0, \infty) \times \mathbb{R}$, and $D = \{0\} \times (-\infty, 0)$. In this, $l(t, x) : \mathbb{R} \times \mathbb{R}^2 \rightarrow \mathbb{R}$ represents all external forces except for the input u and $\epsilon \in (0, 1]$ is a constant coefficient of restitution (CoR). The DF controller uses a different input depending on whether the reference and closed-loop system have had the same number of impacts or not. Three different modes can be discerned, discriminated on the basis of the Lyapunov functions

$$V_a(\alpha, x) = |x - \alpha|_P^2, \quad V_b(\alpha, x) = \left| x + \frac{1}{\epsilon} \alpha \right|_P^2 \quad (4)$$

$$V_c(\alpha, x) = \left| \frac{1}{\epsilon} x + \alpha \right|_P^2 \quad (5)$$

where the matrix $P > 0$ is chosen to satisfy

$$P \begin{bmatrix} 0 & 1 \\ -k_p & -k_d \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -k_p & -k_d \end{bmatrix}^T P = -Q \quad (6)$$

for some $Q > 0$, position feedback gain $k_p > 0$, and velocity feedback gain $k_d > 0$ [13]. From simulations, we found the mode switching strategy to be insensitive to the choice of Q and in Section IV, we, therefore, chose Q as identity.

Fig. 3. Exploded view of the experimental setup.

TABLE I
ACTUATION AND SENSING HARDWARE FOR THE ARP

Motor	Maxon RE 35, 90W, 42V, bushed DC
Encoder	HP HEDL-5540-A11 (500 CPR)
Current amplifier	ELMO VIO-5/60
Power supply	Siemens Simatic S7 6ES7 307-1EA00-0AA0

The DF control law [13, eqs. (11) and (12) therein] is given by $u = \kappa_{df}(x, t)$ with

$$\kappa_{df}(x, t) = \begin{cases} -l(t, x) + \tilde{\mu}(t) - K(x - \alpha(t)) & \text{if } V_a(\alpha(t), x) \leq V_b(\alpha(t), x) \wedge \alpha_2(t) \geq 0 \\ & \text{or } V_a(\alpha(t), x) \leq V_c(\alpha(t), x) \wedge \alpha_2(t) < 0 \\ -l(t, x) - 1/\epsilon \tilde{\mu}(t) - K(x + 1/\epsilon \alpha(t)) & \text{if } V_b(\alpha(t), x) \leq V_a(\alpha(t), x) \wedge \alpha_2(t) \geq 0 \\ -l(t, x) - \epsilon \tilde{\mu}(t) - K(x + \epsilon \alpha(t)) & \text{if } V_c(\alpha(t), x) \leq V_a(\alpha(t), x) \wedge \alpha_2(t) < 0 \end{cases} \quad (7)$$

where $\tilde{\mu}(t) := \mu(t) + l(t, \alpha(t))$ and $K = [k_p \ k_d]$.

Remark 1: In (7), a typographical error has been corrected. That is, contrary to [13, eq. (12) therein], in the second case also the feedforward term $\tilde{\mu}$ is scaled with a factor $-1/\epsilon$. Δ

IV. EXPERIMENTAL ANALYSIS

Fig. 3 depicts the exploded view of the *actuated rebounding pendulum* (ARP), a picture which was shown in Fig. 1. The ARP consists of a pendulum that is actuated at the axle using an electric motor and that bounces back when its tip collides with a metal block. The electronic components used for actuation and sensing of the ARP are listed in Table I. In the experiments, the sampling time is taken 1/500 s.

A. Dynamic Model

The modeling and identification of the setup's hybrid dynamics will be discussed next. The continuous part of the dynamics is modeled as

$$I\ddot{q} + d\dot{q} + c \operatorname{sign}(\dot{q}) + M_g \sin(q + \phi) = M \quad (8)$$

Fig. 4. Schematic of the experimental setup.

TABLE II
CONTINUOUS DYNAMICS SYSTEM PARAMETERS

Parameter	Value	Unit
I	$4.7 \cdot 10^{-4}$	kg m^2
d	$2.6 \cdot 10^{-4}$	N m s
c	$2.5 \cdot 10^{-3}$	N m
M_g	0.0472	N m
ϕ	0.116	rad

where q indicates the position of the pendulum as defined in Fig. 4, I is the inertia about the rotation axis, d is the viscous friction coefficient, c is the Coulomb friction coefficient, M_g is the maximal moment due to gravity, ϕ is the angle between the vertical and a line through the axis of rotation and center of mass (see Fig. 4), and M is the motor torque. Note that, as a reference trajectory without stick phases will be considered in this brief, it suffices to use the single-valued sign function in (8) as opposed to a set-valued Coulomb friction law (see [27, Sec. 5.3]) for modeling friction in the joint. The two friction models will only differ for a finite number of time instances as the considered motion has no static phases. The continuous dynamics parameters have been identified experimentally using linear regression on measurement data and the result is given in Table II.

As can be seen from (7), the DF control law contains a feedback linearization part, i.e., the component $-l(t, x)$, with in this case $x = [x_1 \ x_2]^T := [q \ \dot{q}]^T$. For fair comparison of the controllers (2), (3), and (7), instead, we feedback linearize the dynamics before applying the controllers. That is, using the model (8), we apply the torque $M = M_{fl}(u, \hat{x})$ with

$$M_{fl}(u, \hat{x}) := Iu + d\hat{x}_2 + c \operatorname{sign}(\hat{x}_2) + M_g \sin(\hat{x}_1 + \phi) \quad (9)$$

such that the continuous dynamics can be approximated by the double integrator $\ddot{q} = u$, artificially setting $l(t, x) = 0$. In (9), $u \in \mathbb{R}$ is a new control variable, and $\hat{x} = [\hat{x}_1 \ \hat{x}_2]^T = [\hat{q} \ \hat{v}]^T$, where $\hat{q}$ and $\hat{v}$ are the estimations for position, respectively, for velocity. Using the state x , the (approximate) continuous dynamics can be expressed as in (1a) with $f(x, u) = [x_2 \ u]^T$ and $C = \{x \in \mathbb{R}^2 \mid x_1 \geq 0\}$.

For performing the trajectory tracking experiments in Section IV-C, we use the JA filtering method introduced in [26] to estimate position $\hat{q}$ and velocity $\hat{v}$ from the encoder data. The method is employed because it not only accurately estimates state in the presence of velocity jumps but also includes an impact detection feature. The used filter settings

Fig. 5. Measured CoR as a function of angular velocity at impact.

can be found in this brief's supplementary material (available at <http://ieeexplore.ieee.org>).

A Newton restitution model is considered for describing the system's response to an impact. When the pendulum collides with the impact block, the velocity is reset according to $\dot{q}^+ = -e(\dot{q}^-)\dot{q}^-$ where $e : \mathbb{R} \rightarrow [0, 1]$ is the true velocity-dependent CoR. In terms of the state x , the discrete dynamics satisfy

$$x^+ = \begin{bmatrix} 1 & 0 \\ 0 & -e(x_2^-) \end{bmatrix} x^- = g(x^-)$$

and are triggered when the state reaches the jump set $D = \{x \in \mathbb{R}^2 \mid x_1 = 0, x_2 \leq 0\}$. The CoR e is identified experimentally by releasing the pendulum from different initial positions and measuring the response. By fitting the third-order polynomials to the data of each bounce right before and right after impact, finding the intersection of the polynomials, and subsequently computing the derivative of the polynomials at this time of intersection, the relation between preimpact and postimpact velocities can be approximated. The result of this procedure is depicted in Fig. 5 for the first three impacts after each release of the pendulum. Fig. 5 shows that the CoR decreases with increasing impact velocity and extrapolation of the data suggests that the restitutions become fully elastic (i.e., $e = 1$) for negligible preimpact velocity (see [10, p. 145]).

Remark 2: As can be seen from (4) and (7), a *constant* CoR parameter ϵ is required in the DF control law, whereas Fig. 5 shows that the system's CoR is velocity dependent. However, since in Section IV-C, we will consider a reference trajectory with impacts occurring within a small preimpact velocity range, for simplicity, we approximate the velocity-dependent restitution characteristics with a constant $\epsilon = 0.56$. $\triangle$

B. Performance Measures

A quantitative measure is required to objectively compare the performance of RS control to that of classic control and of DF control. Commonly, this is done using a norm on the (state) error. However, since the three controllers (2), (3), and (7) all employ a different error notion, the choice of a suitable performance measure is far from trivial. The peaking phenomenon results from the jumps in velocity

Fig. 6. Reference trajectory with the corresponding required actuation torque (solid line) and the extended reference (dashed line). The numbered, segmented bar on the top indicates the discrete time j .

for mechanical systems with impacts. The position on the other hand is a continuous function of time. Keeping the cause of peaking and the continuity of position in mind and considering that the goal in trajectory tracking is to make the system follow the reference trajectory $\alpha(t)$ (and not the extensions) as closely as possible, we opt to use the performance measure

$$p_1 := \frac{1}{T} \int_0^T |x_1(t) - \alpha_1(t)| dt \quad (10)$$

indicating the mean absolute position error, where T is the measurement duration. We employ the L_1 -norm in p_1 instead of the L_2 -norm, for example, since it gives both low and high errors equal weight in the performance index.

Ideally, the position error is kept small and is quickly suppressed in the case of a state perturbation, but “how hard the system has to work” to do so is another important aspect to consider when assessing a controller's performance. The feedback effort is related to the deviation of the torque supplied to the system from the ideal torque that is expected based on the system model and reference motion. Consequently, we introduce a second performance measure

$$p_2 := \frac{1}{T} \int_0^T |M_{\text{fl}}(u(t), \hat{x}(t)) - M_{\text{fl}}(\mu(t), \alpha(t))| dt \quad (11)$$

with $M_{\text{fl}} : \mathbb{R} \times \mathbb{R}^2 \rightarrow \mathbb{R}$ given by (9).

Finally, to quantify the effect that a peaking error has on the motor torques, we introduce a third measure of performance

$$p_3 := \max_{t \in [0, T]} |M_{\text{fl}}(u(t), \hat{x}(t)) - M_{\text{fl}}(\mu(t), \alpha(t))| \quad (12)$$

indicating the peak feedback effort over the period $[0, T]$.

C. Trajectory Tracking Controller Comparison

For the trajectory tracking experiments in this section, we consider the reference trajectory shown in Fig. 6.

Fig. 7. Trajectory tracking results using classic feedback (2) for natural frequency $\omega_n = 2$ rad/s (dashed, orange), $\omega_n = 8$ rad/s (dashed-dotted, green), and $\omega_n = 14$ rad/s (solid, dark blue). Light blue: reference trajectory. The bars on the top indicate discrete time j .

Also, other reference trajectories have been considered in tracking experiments, showing similar results. Those are, therefore, not detailed here. Fig. 6 depicts the motor torque M , position α_1 , and velocity α_2 as a function of time. By construction, the motor torque is continuous at the impact times τ_j . The steps in the torque M , away from the impact times, are the result of the friction term $c \text{sign}(\alpha_2)$ [see (8)]. The reference trajectory has duration $T = 2.5$ s and the initial condition is $\alpha_0 = [\pi/4 \ 0]^T$. The dashed lines in the second and third subplots shown in Fig. 6 illustrate the extensions $\bar{\alpha}$ obtained via integrating (8) forward and backward from the preimpact state and postimpact state, applying for each counter j the same motor torque curve M . Mathematically, the latter entails that $\bar{\mu}$ is chosen such that, for all $i, j \in \{0, 1, \dots, N_a\}$, $M_{\bar{n}}(\bar{\mu}(\cdot, i), \bar{\alpha}(\cdot, i)) = M_{\bar{n}}(\bar{\mu}(\cdot, j), \bar{\alpha}(\cdot, j))$.

We will use RS control to track this hybrid reference trajectory [28] and, as anticipated earlier, compare its performance to that of standard PD feedback control and of DF control. To verify convergence to the reference and to analyze settling time, we perturb the system's initial condition and consider $x_0 = [0.3 \ 0]^T$. The feedback gain K is chosen of the form

$$K = \begin{bmatrix} \omega_n^2 & 2\omega_n \end{bmatrix} \quad (13)$$

such that the continuous part of the error dynamics represents a *critically damped* mass-spring-damper system with natural frequency $\omega_n \in \mathbb{R}$. Overshoot and oscillations due to insufficient damping are therewith avoided, which otherwise could result in undesired impacts for a system as the ARP. Experiments are conducted for different ω_n and the results using controllers (2), (3), and (7) are depicted in Figs. 7–9, respectively. The figures illustrate the responses for a low ($\omega_n = 2$ rad/s), a medium ($\omega_n = 8$ rad/s), and a high ($\omega_n = 14$ rad/s) closed-loop frequency. The performance measures p_1 , p_2 , and p_3 are

Fig. 8. Trajectory tracking results using RS feedback (3) for natural frequency $\omega_n = 2$ rad/s (dashed, orange), $\omega_n = 8$ rad/s (dashed-dotted, green), and $\omega_n = 14$ rad/s (solid, dark blue). Light blue: (extended) reference trajectory. The bars on the top indicate discrete time j .

Fig. 9. Trajectory tracking results using DF feedback (7) for natural frequency $\omega_n = 2$ rad/s (dashed, orange), $\omega_n = 8$ rad/s (dashed-dotted, green), and $\omega_n = 14$ rad/s (solid, dark blue). Light blue: (virtually extended) reference trajectory. The bars on the top indicate discrete time j .

computed from the system responses for an even larger set of controller bandwidths and the results are shown in Fig. 10.

Remark 3: In a neighborhood of the reference α , the DF controller (7) is analogous to RS control, for a specific reference extension method (see Fig. 9). To clarify this statement, consider (7). If the reference and closed-loop systems have experienced the same number of impacts, RS and DF controls are, indeed, the same as can be seen from (3) and the first line of (7), considering $l(t, x) = 0$.

Fig. 10. Performance versus closed-loop natural frequency. The subplots show the average position error performance index p_1 , the average control effort performance index p_2 , and the peak feedback effort performance index p_3 (from top to bottom).

In the case that the closed-loop system experiences an impact prior to the corresponding reference impact, the error used for feedback is reshaped as seen from the third line in (7) and can be interpreted as comparing the state to a mirrored and scaled version of α . A similar operation is performed on the feedforward. It follows that for each $j \geq 1$, the DF control law can be considered equivalent to RS control with reference extension $\bar{\alpha}(t, j) = -\epsilon\alpha(t, j-1)$ and feedforward $\bar{\mu}(t, j) = -\epsilon\mu(t, j-1)$, for $t \leq \tau_j$.

Specularly, if the closed-loop system experiences an impact too late, the reference trajectory is mapped back [see the second line in (7)], i.e., for each $j \geq 0$ and $t \geq \tau_{j+1}$, $\bar{\alpha}(t, j) = -\frac{1}{\epsilon}\alpha(t, j+1)$ and $\bar{\mu}(t, j) = -\frac{1}{\epsilon}\mu(t, j+1)$. $\triangle$

Fig. 7 shows the disastrous effect that a mismatch in impact time can have on tracking performance, if not accounted for. For the lower feedback gain setting, the classic control approach is not able to make the system converge to the desired motion and this results in extra, unplanned impacts (ten events are encountered whereas seven were desired). The reference is stabilized when sufficiently high gains K are chosen, but these higher feedback control settings result in large peaks in the motor torque. The almost unavoidable mismatch between impact time t_j of the closed-loop system and τ_j of the reference trajectory is translated to “kicks” to the system.

These needless torque peaks are not seen when RS control is applied as Fig. 8 clearly shows. RS control is capable of

steering the system to the considered reference motion, even for the low feedback gain parameter $\omega_n = 2$ rad/s. Also, DF control avoids propagation of event time mismatches to motor torque peaks, as can be deduced from Fig. 9 and, similar to RS, makes the system converge to the reference for low ω_n .

Similar conclusions can be drawn in Fig. 10. The figure shows the performance measures p_1 , p_2 , and p_3 (see Section IV-B) for the three control methods as a function of closed-loop natural frequency ω_n . From the results, it can be concluded that the DF and RS controls perform similarly, which was expected based on their analogous methodology. The position error norm p_1 is significantly larger for classic feedback control when low gains are considered and the position performance for the three controllers becomes indistinguishable for increasing ω_n . To attain this same position error, however, the control based on a classic error notion requires larger feedback effort as illustrated by p_2 shown in Fig. 10. The classic control effort is higher for all considered feedback gains and is even more than twice as large as that of RS and DF controls when $\omega_n \leq 4$ rad/s. An interesting result is illustrated by the trend of p_3 versus ω_n for classic control (2). The peak feedback effort keeps increasing with closed-loop bandwidth and is much larger than that found when RS or DF control is applied. The peak feedback torque is already larger than the maximum absolute feedforward torque for $\omega_n \geq 8$ rad/s (see Fig. 6). These excessive torques are due to the peaking phenomenon that the error displays.

Note that, at some point, the peak control effort also increases with ω_n for RS and DF controls. This is due to the fact that the tracking experiments are intentionally started from a fixed perturbed initial condition. The initial error translates to an increasingly large feedback effort when the gains increase.

Next, we investigate how robust RS control is to inaccuracies in state information, impact detection, and model parameters. This is done by performing tracking experiments, considering the feedback gain (13) with $\omega_n = 8$ rad/s, where we introduce inaccuracies in the quantities of interest. The experiments and results are discussed in detail in this brief’s supplementary material (available at <http://ieeexplore.ieee.org>). The main findings are briefly discussed here.

When a second-order low-pass filter is used for estimating velocity, as opposed to a method tailored to discontinuous velocity signals [26], the inevitable lag in the response to an impact causes large (velocity) errors right after impact. These fictitious errors are fed back to the system via feedback which, in turn, results in larger tracking errors.

A similar effect is witnessed when there is a delay in the impact detection. Such delay causes dissynchrony between the jump times of the closed-loop system and switching times of the reference. It can, thus, be concluded that, in order to get the best tracking performance using RS control, both accurate state estimates and event detection are required.

Robustness of RS control to model inaccuracies is investigated by modifying the parameters M_g and I by 30% and correspondingly adjusting the feedforward $\bar{\mu}$. Naturally, such severe model imperfections cause the closed-loop responses to deviate from the reference trajectory. However, by the use of extended references, no peaking effect is encountered in the

feedback effort. This aids the control method's robustness to model inaccuracies and results in acceptable tracking errors.

V. CONCLUSION

In this brief, trajectory tracking for motions with partially elastic impacts has been considered from an experimental and performance perspective. Particularly, RS has been analyzed using an *ad hoc* built experimental setup. For different feedback gains, the controller's performance in tracking a nonperiodic impacting reference motion has been compared to that when using classic PD control, as well as to employing a DF control approach. Moreover, the controller's robustness to inaccuracies in velocity estimates, impact detection, and model parameters, has been investigated.

The tracking experiments have confirmed that the (almost inevitable) mismatch between the impact times and the reference event times can deteriorate performance and destabilize the system when a classic control strategy is employed. Large peaks in the feedback torque are typically encountered near impact times for such method. RS effectively solves these issues. As expected, we showed that RS performs similarly as a DF/mirroring-based method. RS is, however, a more versatile strategy as it can also be applied in inelastic impact scenarios, can be easily applied to more complex systems, and can deal with simultaneous impacts. It can be concluded that considering an extended reference is a sensible approach when tracking trajectories with expected impacts.

The effectiveness of RS for multiple DoF systems has been already demonstrated via numerical simulations. An experimental validation in this context is under investigation and will be presented in future publications. In this brief, the reference is fully known and extended offline. We expect a receding horizon implementation of RS to be straightforward. This should require to generate online a look-ahead portion of the reference including, at least, the next expected impact.

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