Finite element model simplification methods for stacks of superconducting tapes

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Finite Element Model Simplification Methods for Stacks of Superconducting Tapes

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This paper presents a simplification method for finite element models for the calculation of AC losses in stacks of superconducting tapes. A fast and accurate model of these losses can be exploited in the design of large-scale superconducting applications, where the coils are represented as stacks of tapes in 2D models. The simplification method aggregates the superconducting, substrate, and copper layers of several windings of the superconducting tape to form bulk elements. The resulting model is able to evaluate the AC losses in the superconducting layer faster than the full model. The accuracy of the proposed method and decrease in computation time are shown.

Index Terms—Superconducting coils, High Temperature Superconductors, Finite element modeling

I. INTRODUCTION

Superconducting tapes are often wound into pancake or racetrack coils for large-scale devices. These devices include magnetic energy storage, motors and generators, and high-field magnets. Ramping up of a DC current or the application of AC currents to the superconducting coils results in AC losses [1]. In a 2D representation, the pancake or racetrack coils are represented as a stack of superconducting tapes. Evaluation of the AC losses for geometries including nonlinear magnetization is currently only possible using finite element models [2]. A drawback of these models is the long computation time if each winding of the coil is individually modeled.

Superconducting tapes consist of a magnetic or non-magnetic substrate layer on which a superconducting layer is deposited. These layers are enclosed in a copper sheath. A magnetic substrate is found in Bi-2223 and Bi-2212 tapes [3], while the substrate in YBCO tapes is only very weakly magnetic. Finite element analyses of coils which include the superconductor layer, the magnetic substrate layer, and the copper sheath have been shown previously [4]. The computation time of these models is in the order of hours, for a 2D representation of a coil with 40 windings. In [5] a method with a reduced computation time of the AC losses has been developed, which averages the properties of the superconducting layer over the volume of the coil. However, this approach neglects the effect of the magnetic substrate layer and the copper sheath on the AC losses, as well as the AC losses in the copper sheath itself.

In this paper, a model simplification method is presented for stacks of superconducting tapes which can include the magnetic substrate and the copper sheath. The method is based on aggregating the layers of multiple windings into a single new winding with layers of increased thickness. The AC losses in the superconducting layer and copper layer, calculated with the simplified and full models are compared for a range of frequencies and applied current amplitudes. Finally, the discrepancy of the simplified and the full models as well as the computation times are compared.

II. MODEL SIMPLIFICATION METHOD

The computation time of the finite element models increases approximately quadratically with the number of elements [6]. To reduce the number of elements without reducing the accuracy of the calculated AC losses, degrees of freedom in the model should be provided only where variations of quantities occur. A method to reduce the number of degrees of freedom and the number of required mesh elements is to represent groups of \( N \) windings by a single layer of which the thickness of all layers is multiplied by a factor \( N \).

![Fig. 1. Model simplification method for stack of superconducting tapes. (a) full geometry, (b) simplified geometry where \( N \) layers are represented by a single layer of which the thickness of all layers is multiplied by a factor \( N \).](image)

In this paper, a model simplification method is presented for stacks of superconducting tapes which can include the magnetic substrate and the copper sheath. The method is based on aggregating the layers of multiple windings into a single new winding with layers of increased thickness. The AC losses in the superconducting layer and copper layer, calculated with the simplified and full models are compared for a range of frequencies and applied current amplitudes. Finally, the discrepancy of the simplified and the full models as well as

The geometrical parameters of the considered tapes are given in Table I.
The relative permeability presented in [9] as
\[ \mu_r = 1 + 30600(1 - e^{-(H/295)^{2.5}})H^{-0.81} + 450e^{-(H/120)^{2.5}}, \]
is applied for the magnetic substrate layer, where \( H \) is the magnetic field intensity. The superconducting material is described by a current-density dependent resistivity, \( \rho \), as
\[ \rho = \frac{E_c}{J_c} \left( \frac{J}{J_c} \right)^n, \]
where \( E_c \) is the threshold voltage, \( J_c \) is the critical current density, and \( n \) is the n-value of the material. The magnetic-field dependency of the critical current is approximated by an extension of the Kim-model
\[ J_c = \frac{J_{c0}}{1 + \left( \frac{2\mu_r|B|}{\mu_0} - 1 \right)^\alpha}, \]
where \( J_{c0} \) is the field-free critical current density, \( B_0 \) the critical field parameter. The parameters \( \alpha \), \( B_0 \), and \( k \) are obtained by least square fitting of the model to measurement data [10]. The values of these parameters are given in Table II.

### III. Finite Element Modelling

Finite element models are implemented for the full model of stacks of superconducting tapes with magnetic and non-magnetic substrate, as well as simplified models with magnetic and non-magnetic substrates, with and without including the copper sheath. In the models with non-magnetic substrates, the relative permeability of the substrate is set to unity. In the models where the copper sheath is neglected, the copper regions are omitted. Therefore, the model without the magnetic sheath has slightly less mesh elements.

The modeled stacks of tapes consist of 40 windings. The finite element models of the full and simplified geometries are implemented in the H-formulation. The time dependent problem is solved using the backwards differential formula method, and the solver tolerance is equal for all models.

Constraints are added in the model forcing the total current in the superconducting layers to be equal to the transport current, and the total current in the copper layer to be zero. A mapped mesh is applied in regions where this is possible. The number of elements in the height of the superconducting layer is equal to 40 for the full models and equal to 80 for the bulk models. Three elements are used in the width of the copper, substrate, and air layers, and one in the width of the superconducting layer. In all models, the superconducting tape stack is surrounded by an air region, with a radius of 0.1 meter.

Losses are calculated for sinusoidal transport currents with various amplitudes and frequencies. In the superconducting and copper layers, the losses are calculated from the electric field and current density. The ferromagnetic hysteresis losses in the substrate are not included, since they are only a small part of the total losses, except for very low currents [11]. The AC losses are reported as function of the current ratio \( I_0 \), which the ratio of the peak current \( I_{pk} \) to the critical current
\[ I_0 = I_{pk}/I_c. \]
The frequency of the applied currents is set at 50, 500 and 1000 Hz. The calculated losses are periodic after the first full period of the applied current. Therefore, the losses in the second full period of the applied current are compared among all models. The losses are reported in Joule per meter length of the coil stack per cycle of the applied current.

The implemented models are the full models with magnetic and non-magnetic substrate, and the simplified ones with and without magnetic substrate, and with and without copper sheath.

### IV. Results and Analysis

#### A. AC Losses

First, the dependency of the losses in the full model on peak current and frequency is analyzed. Losses calculated for the full model with magnetic substrate are shown in Fig. 2. The copper losses are much more dependent on frequency than the hysteresis losses, and increase with applied frequency. The hysteresis losses in the superconducting layer decrease with

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**TABLE I**

**Geometrical parameters of the tape.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of copper layer</td>
<td>( d_{cu} )</td>
<td>37 ( \mu \text{m} ) [7]</td>
</tr>
<tr>
<td>Thickness of SC layer</td>
<td>( d )</td>
<td>1.4 ( \mu \text{m} ) [7]</td>
</tr>
<tr>
<td>Thickness of substrate layer</td>
<td>( d_{sub} )</td>
<td>100 ( \mu \text{m} ) [8]</td>
</tr>
<tr>
<td>Height of copper layer</td>
<td>( h_{cu} )</td>
<td>37 ( \mu \text{m} ) [8]</td>
</tr>
<tr>
<td>Tape pitch</td>
<td>( L )</td>
<td>300 ( \mu \text{m} )</td>
</tr>
</tbody>
</table>

**TABLE II**

**Parameters of the superconducting tape.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical current</td>
<td>( I_c )</td>
<td>120 A</td>
</tr>
<tr>
<td>Critical field parameter</td>
<td>( B_0 )</td>
<td>0.33 T</td>
</tr>
<tr>
<td>Angular dependency parameter</td>
<td>( k )</td>
<td>0.078</td>
</tr>
<tr>
<td>n-value</td>
<td>( n )</td>
<td>25</td>
</tr>
<tr>
<td>Scaling parameter</td>
<td>( \alpha )</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Fig. 2. AC losses in the superconductor layer (sc) and copper layer (cu), calculated for the full model with magnetic substrate.
The magnetic properties of the substrate are crucial. Indications show that for the calculation of the losses in coils with a magnetic substrate, tapes with a non-magnetic substrate are more than 100% higher than the losses for tapes with a non-magnetic substrate. This suggests that the losses in both the copper and superconducting layers, for tapes with a magnetic substrate, are relevant.

Next, the effect of the magnetic substrate on the losses is investigated. The losses calculated with the full model with magnetic substrate and the full model with non-magnetic substrate, are shown in Fig. 3. It can be seen that the magnetic substrate increases both the hysteresis losses in the superconducting layer and the losses in the copper layer. The losses in both the copper and superconducting layers, for tapes with a magnetic substrate are more than 100% higher than the losses for tapes with a non-magnetic substrate. This indicates that for the calculation of the losses in coils with superconducting tapes with magnetic substrate, including the magnetic properties of the substrate is crucial.

The optimum value of the aggregation factor $N$, is determined by comparing the AC losses in the superconducting layer for 50 Hz sinusoidal currents of the full model to aggregated models with various values of $N$, as shown in Fig. 4. The figure shows that for an aggregation factor of 4 or lower, and a current ratio higher than $I_0 = 0.2$, the difference between the full model and the aggregated models is less than 30%. For the remainder of the paper, an aggregation factor of 4 is used, such that the aggregated models consist of 10 bulk elements.

Subsequently, the validity of the simplified model is determined. The losses calculated by the simplified model of the coil with magnetic substrate are compared to the full model with magnetic substrate in Fig. 5. In the figure, the losses calculated in the copper by the aggregated model are divided by a factor $N = 4$. For a current ratio of $I_0 = 0.3$, the aggregated model predicts a 46% higher loss in the superconducting layer than the full model. For a current ratio of $I_0 = 0.6$, the difference between the full and aggregated models is lower than 3% for the losses calculated in the superconducting layer.

For models with non-magnetic substrate, the simplified model can also be applied. Losses calculated for the full model with a non-magnetic substrate, the aggregated model with copper layer, and the aggregated model without copper layer, are shown in Fig. 6. The losses calculated in the copper by the aggregated model are divided by a factor $N = 4$. The

<table>
<thead>
<tr>
<th>$I_0$</th>
<th>$10^{-6}$</th>
<th>$10^{-5}$</th>
<th>$10^{-4}$</th>
<th>$10^{-3}$</th>
<th>$10^{-2}$</th>
<th>$10^{-1}$</th>
<th>$10^{0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>cu 50 Hz</td>
<td>sc 50 Hz</td>
<td>cu 50 Hz</td>
<td>sc 50 Hz</td>
<td>cu 50 Hz</td>
<td>sc 50 Hz</td>
<td>cu 50 Hz</td>
<td>sc 50 Hz</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Fig. 3. Comparison of losses calculated with the full model with magnetic substrate and the full model with non-magnetic substrate.

Fig. 4. AC losses in the superconductor layer, calculated for the full model with magnetic substrate, and the aggregated model with different values of the aggregation factor $N$.

Fig. 5. Losses calculated for the full model with magnetic substrate (dots and lines), compared to the aggregated model with magnetic substrate with corrected copper losses (circles and diamonds).

Fig. 6. Losses calculated for the full model with non-magnetic substrate (crosses and lines), the aggregated model with copper sheath (circles and diamonds), and the aggregated model without copper sheath (squares).
TABLE III
COMPUTATION TIME OF DIFFERENT MODELS

<table>
<thead>
<tr>
<th>Model</th>
<th>Substrate</th>
<th>Copper sheath</th>
<th># Elements</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>magnetic</td>
<td>yes</td>
<td>39702</td>
<td>35 m 14 s</td>
</tr>
<tr>
<td>Simplified</td>
<td>magnetic</td>
<td>yes</td>
<td>20914</td>
<td>10 m 24 s</td>
</tr>
<tr>
<td>Full</td>
<td>non-magnetic</td>
<td>yes</td>
<td>39702</td>
<td>22 m 52 s</td>
</tr>
<tr>
<td>Simplified</td>
<td>non-magnetic</td>
<td>yes</td>
<td>20914</td>
<td>6 m 04 s</td>
</tr>
<tr>
<td>Full</td>
<td>magnetic</td>
<td>no</td>
<td>23476</td>
<td>32 m 57 s</td>
</tr>
<tr>
<td>Simplified</td>
<td>magnetic</td>
<td>no</td>
<td>11538</td>
<td>17 m 40 s</td>
</tr>
<tr>
<td>Full</td>
<td>non-magnetic</td>
<td>no</td>
<td>23476</td>
<td>32 m 17 s</td>
</tr>
<tr>
<td>Simplified</td>
<td>non-magnetic</td>
<td>no</td>
<td>8276</td>
<td>3 m 07 s</td>
</tr>
</tbody>
</table>

losses calculated by all models for the superconducting layer are within 30% for current ratios higher than $I_0 = 0.2$. For this model, the agreement between the full model and the aggregated models also increases with increasing peak current. For low current ratios, the losses in the copper sheath are higher than those in the superconductor layer.

In the implemented finite element model in the H-formulation, the current density in each mesh element is homogeneous. In the finite element models, the highest losses occur in regions in which the current density exceeds the critical current density. These regions are at the edges of the tapes for the cases shown in this paper. If the mesh elements are substantially larger than these regions, the average current density in the mesh elements is lower than the critical current density, resulting in erroneously low calculated AC losses. Therefore, the number of mesh elements in the height of the superconducting layer should be larger than $2/I_0$. Decreasing the size of the mesh elements, especially near the top and bottom edges of the tapes, will result in more accurate results.

B. Computation time

The computation times of the full and aggregated models are compared for a current ratio of $I_0 = 0.3$, and a frequency of 500 Hz. Calculations are performed on a PC with an Intel i7-3770 processor and 16 GB RAM. The models are implemented in COMSOL 5.1, where the H-formulation is implemented in the general form PDE solver. The computation time of the various models is shown in Table III. The decrease in computation time between the full model and the simplified model, of the tape stack with magnetic substrate and copper sheath, is equal to 70%. The decrease for a magnetic substrate without copper sheath is equal to 46%. If the magnetic substrate and copper sheath are not taken into account, the simplified model is almost 12 times faster than the full model. For the last case, the approach is very similar to the previously published methods, where a similar decrease in computation time has been shown.

V. CONCLUSIONS AND ANALYSIS

This paper has presented a model simplification method for calculating the hysteresis losses in the superconducting layer of tapes with a magnetic substrate. The losses in the superconducting layer are dominant to the losses in the copper layer for frequencies up to 1 kHz. The decrease in computation time for coils with a magnetic substrate, achieved by the simplified model, is 70% for the test case. Losses estimated by the simplified models for the copper layer agree if they are divided by the factor $N$. To further validate the simplification method, losses in case of externally applied field should be investigated.

REFERENCES