Statistical analysis of cross-talk accumulation in WDM networks

Citation for published version (APA):

DOI:
10.1109/50.809661

Document status and date:
Published: 01/01/1999

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the author's version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

Take down policy
If you believe that this document breaches copyright please contact us:
openaccess@tue.nl
providing details. We will immediately remove access to the work pending the investigation of your claim.
Statistical Analysis of Crosstalk Accumulation in WDM Networks
H. J. S. Dorren, H. de Waardt, and I. Tafur Monroy

Abstract—Accumulation of inband crosstalk in all-optical networks is studied. By applying statistical methods, we have investigated how inband crosstalk accumulation influences the performance of optical networks of different configurations. Our study shows that there exists a delicate dependence between network topology and robustness with respect to accumulation of inband crosstalk. A method is proposed to design optical networks with optical paths satisfying a certain level of inband crosstalk performance.

I. INTRODUCTION

OPTICAL wavelength division multiplexing (WDM)-networks offer a large transport capacity and are regarded as a promising solution to the increasing demand of bandwidth in future telecommunication systems. In all optical WDM-networks, routing, switching and amplification is performed in the optical domain. An important component in the WDM-network is the multiwavelength optical cross-connect which is schematically presented in Fig. 1. Suppose we consider a signal at wavelength \( \lambda_3 \) which propagates from input 1 to output 2. It is well known that due to an imperfect switching array, the output signal is corrupted with leakage of other input signals. This phenomena is called crosstalk. If the contamination has the same nominal wavelength as the signal, we speak about in-band crosstalk. This type of crosstalk can not be removed by optical filters and it is therefore necessary to design optical networks with an optimum crosstalk performance. Transparent optical networks impose strict requirements on the crosstalk performance of the network elements involved [1], [2].

Up until now considerable effort is invested in developing an adequate understanding of crosstalk in optical switches (see [2] and the references therein) as well as improving the crosstalk performance of optical switches. Nevertheless, the applicability of cross-connects in optical networks depends on the crosstalk properties of the device. Modern integrated optical switches introduce crosstalk which is in the order of \(-35\) dB. A signal propagating through an optical network in general passes through more than one switch. As a result of this the overall crosstalk on the signal is larger than \(-35\) dB. It is therefore important to design the optical network in such a way that the effects of crosstalk are minimized. In this paper we deal with the question whether we can reduce the crosstalk accumulation by using a good network design. This will be done by selecting four network designs and investigate, by using statistical methods, what effects are important on the crosstalk accumulation. The results are derived taking into account that realistic networks are always subject to upgrades. With this we mean that we have assumed that during the course of time more nodes are added in the network. We ask ourselves the question whether particular network designs have a better crosstalk performance with respect to upgrades, and which parameters play an important role in this.

We investigate the problem presented above by using statistical methods. This will be done on two levels. First, we describe the crosstalk in every node by a statistical model in which is accounted for the data-statistics, linear random polarization and a nonperfect extinction ratio. On the other hand we also introduce statistics to cover the properties of the complete network layout. The latter has been done to make the results obtained in this paper as independent as possible for a particular network design. We aim to formulate general criteria that cover large generic classes of optical networks. We will show that there is a delicate relationship between the number of optical links in a network and the signal quality with respect to crosstalk. Introducing more links leads to a shorter connection, but on the other hand the number of crosstalk sources increases leading to a worse crosstalk performance.
This paper is structured as follows. In Section II, the investigated networks are discussed as well as the numerical scheme to obtain a probability density function (pdf) which represents the crosstalk performance of the network design. In Section III, we discuss how the crosstalk accumulates through optical networks, and what parameters are governing this process. The paper is concluded with a discussion. Technical matters with respect to the crosstalk model are added as a comprehensive appendix.

II. CONCEPTS

Since realistic optical networks are continuously subject to upgrades, it is inadequate to investigate crosstalk accumulation in one particular network and extrapolate the results to arbitrary networks. We apply statistical techniques to obtain generic results about large classes of networks. To do this we investigate four different network designs which are presented in Fig. 2. We do not have the intention to present an optimum network design. The networks which are presented in the following have to be regarded as examples to illustrate and understand the effects which play a role in the crosstalk performance of optical networks. The particular networks have been chosen as presented below, because in agreement with realistic networks, they include examples with rings, grids, and combinations of both. The first network design (Network 1) is presented in Fig. 2(a). The network consists of interconnected ring networks. The nodes in Fig. 2 represent cross-connects of the type as presented in Fig. 1. Network 1 represents a central core network which is connected to several regional ring-networks. The second network design (Network 2) which is investigated in this paper is presented in Fig. 2(b). Network 2 only differs from Network 1 by the fact that all the nodes in the central core-network are interconnected. In the third network (Network 3), which is presented in Fig. 2(c), a large outer-ring is implemented to interconnect the subnetworks of Network 1. Finally, in the last network design (Network 4), which is presented in Fig. 2(d), all the nodes in the core-network and subnetworks are interconnected.

If we compare for instance Network 1 and Network 3, we can conclude that the distance between two nodes in adjacent subnetworks is shorter in Network 3. Throughout this paper, we define the “distance” as the number of nodes which are passed by an optical signal. The path-length than equals the number of nodes crossed by an optical signal while traveling though the network. In Network 3, the signal can use the outer-ring, while in Network 1, the signal has to take a path which includes more nodes through the inner core. As a result of this one can expect that the signal in Network 1 is subject to much more crosstalk contamination than a signal in Network 3. Making the path-length between two nodes shorter requires that the remaining optical cross-connects can handle more connections which also introduces new sources of crosstalk. This makes clear that there is a delicate balance between the path-length and the number of interfering crosstalk sources. Before we proceed describing the method which is used to compute crosstalk accumulation in these networks, we want to remark that in Fig. 2 only only the lay-out of the network-design is presented. For investigating the crosstalk performance, we have developed a recipe consisting of four steps which are described below.

Step 1: We generate, for each of the network design presented in Fig. 2, 25 different samples with a size of the inner-core between five and 30 nodes. Every node is connected to a ring-shaped subnetwork. The number of nodes in the subnetwork is chosen randomly with a uniform distribution, but it has a minimum of three nodes and the maximum number of nodes equals half the number of nodes in the core. Network 2, can be constructed form Network 1, by connecting all the nodes in the core. For Network 3, the nodes which determine the outer ring are also chosen randomly. These networks can be formulated as a directed graph which is represented as a matrix. Network 4, follows from network 2, by simply connecting all the nodes in the subnetworks to each other.

Step 2: As a first step Floyd’s path-search algorithm is used to compute the shortest paths between two nodes in the network [3]. With the shortest path we mean the shortest connection (measured in nodes) between two points in the network. If the optical network is presented as a graph, Floyd’s algorithm provides an efficient tool to compute these paths. When the shortest paths are identified, we can also compute the number of crosstalk interferers. It follows from Fig. 2 that the number of crosstalk sources in every node equals the number of neighbors of every node; i.e., it is assumed that the signal is contaminated in every node with corruption from only the neighboring nodes. This assumption
means that we only consider crosstalk originating from switches. The present model can be expanded to account for (de)multiplexing crosstalk. However, it should be mentioned that integrated optical cross-connects can be designed in such a way that crosstalk caused by the switches becomes the dominant crosstalk contribution [4]. Because of reasons of limited computation time, higher order crosstalk accumulation is not taken into account.

We compute the number of crosstalk sources for all shortest connections between all pairs of nodes in the graph.

Step 3: The procedure described in Step 1 and Step 2 is repeated 80 times and an ensemble average is taken. An example is given in Fig. 3, where the normalized ensemble averages for the number of crosstalk interferers is plotted for all the network designs at a core-size of 30 nodes. Similar histograms are generated for every core-size between 5 and 30 nodes. The normalized histogram has the interpretation of a probability density function (pdf) for the number of crosstalk interferers of a particular network design. Taking ensemble averages is important to make the results independent for particular network configurations, and to obtain generic results.

Step 4: By using the mathematical model presented in Appendix A, we relate the number of interferers to the bit error rate (BER). The method is described in more detail in the next section, but the results are already presented in Fig. 4. This also implies that we can compute the pdf for the BER for a certain network design. An example is shown in Fig. 4 for a core-size of 30 nodes.

The recipe described above helps us to compute generic results. The fact that we take averages over large ensembles of networks guarantees that the results are independent for specific realizations. Once we have determined the pdf's for the BER, we can formulate a criterion on what conditions a particular ensemble of networks can guarantee a satisfactory QoS (quality of service). A criterion could be for instance that the BER should be below a $10^{-9}$ level. By repeating this procedure for an increasing number of nodes in the core, one can compute how sensitive a particular network design is for crosstalk accumulation after upgrades. In the next section, it will be shown that the network designs which are presented in Fig. 2, all have characteristic properties with respect to crosstalk accumulation, and an optimum design can be chosen.

III. RESULTS

In this section, we present the results with respect to crosstalk accumulation in the network designs as presented in Fig. 2, following the recipe described in the previous section. In Fig. 5, the BER is plotted as a function of the number of interfering crosstalk sources for $-33$ dB, $-37$ dB, $-50$ dB and $-80$ dB of crosstalk isolation. The BER is computed by using (A6). We have chosen the parameters so that if no crosstalk is present a BER of $10^{-12}$ is obtained. It is witnessed from Fig. 5 that in the case of $-80$ dB crosstalk isolation, nearly no degradation for the BER takes place. However, in the case of $-33$ dB crosstalk isolation (this corresponds to presently available optical switches) less than ten interfering crosstalk sources can be handled. This result implies that the crosstalk per optical switch has to be improved to $-50$ dB or preferable $-80$ dB, before this kind of switches can be used in realistic optically transparent networks.

In Fig. 3, as an example, the pdf of the four network designs is computed for a core-size of 30 nodes. The results are obtained after an ensemble averaging of 80 realizations, following the recipe as presented in the previous section. A similar behavior also takes place for different number of nodes in the core. It can be concluded that Network 3 has the best crosstalk performance since the probability to find paths having more that 12 crosstalk sources is negligible. Network 4 has the poorest crosstalk performance since it has a large number of paths with more than 40 crosstalk sources (this result is not visible in Fig. 3, but is follows from the raw data and manifests itself as a large BER in Fig. 4). The performance with respect to the number of crosstalk sources is shown.
sources of Network 1 and Network 2 is slightly worse than the performance of Network 3. One may conclude from the results presented in Fig. 3 that by counting the number of interfering crosstalk sources (i.e., the number of neighboring nodes) insight can be obtained about the relationship between the network lay-out and the crosstalk performance. It is clearly visible in Fig. 3 that the network designs of Fig. 2 have a characteristic crosstalk performance.

By using the results presented in Fig. 5, we can also compute the performance with respect to the BER. In Fig. 4 the probability of the BER is presented for the same network as presented in Fig. 3. We have assumed that the crosstalk is \(-37\) dB. Due to the nonlinear relation between the number of interferers and the BER, our conclusions derived from Fig. 3 have to be changed. If we relate the crosstalk performance to the decay of the BER-tail in Fig. 4, we would draw a similar conclusions as drawn in the previous paragraph. On the other hand if we implement a criterion that only paths with a BER performance below \(10^{-9}\) are satisfactory, it appears that Network 1 has the best performance. To distinguish this, we compute the surface underneath the PDF presented in Fig. 4. We first define

\[ S = \log \int_{x_{\min}}^{x_{\max}} f(x) \, dx \]  

(1)

where \(f(x)\) is the pdf presented in Fig. 4. If we put \(x_{\min} = 0\) and \(x_{\max} = 10^{-9}\), the quantity \(S\) represents a measure for the number of nodes in the network which can be crossed while still having a BER smaller than \(10^{-9}\). A larger value of \(S\) means a larger probability for a path having an adequate BER \((\leq 10^{-9})\).

If we plot the surface \(S\) as a function of the number of nodes in the core, we obtain Fig. 6. It follows clearly from Fig. 6 that every network design has its characteristic properties with respect to an increasing number of nodes. This implies that we can use Fig. 6 to compare the performance of different network designs. Clearly, Network 1 has the best properties. This is related to the fact that the number of interfering crosstalk sources is low as well as the number of paths which crosses a large number of nodes. The other extreme is Network 4 in which all the rings are completely connected. As a result of this in every node a large amount of crosstalk contamination takes place. It is also interesting to remark that up to 15 nodes, Network 2 and Network 3 have a comparable crosstalk performance.

### IV. Conclusion

We have introduced a method to determine the robustness of optical networks with respect to crosstalk accumulation. An underlying consideration for the work conducted in this paper is that integrated optical switches always introduce a certain level of crosstalk. It is therefore necessary to identify network configurations which minimize crosstalk accumulation. We have approached the problem sketched above by implementing statistical methods. By taking averages over large ensembles of networks with a similar topology, we obtain results which are generic and representative for large classes of networks.

We can conclude that for crosstalk accumulation in optical networks two “parameters” are important. The first parameter is the number of nodes in a network. This can be understood easily since more nodes means more sources of crosstalk. One wants to design a network that minimizes the number of nodes to be crossed from an arbitrary signal traveling through the network. Minimizing the number of nodes can be done by introducing more connections. The second parameter is the number of crosstalk interferers per connection. This implies that after enlarging the network the number of crosstalk interferers increases, and hence a poorer crosstalk performance takes place. It is therefore crucial to find the optimum number of connections per node.

The work conducted is far from complete and the network topologies only form an illustration for the effect we describe. An important issue which is not considered in this paper, is the impact of fiber-cuts. As broken fiber implies that the traffic has to be rerouted, in general more nodes have to be crossed. In [5], it is shown that the impact of broken fibers does not affect the results drawn in this paper.
APPENDIX
CROSSTALK MODEL

This Appendix presents the crosstalk model used in the performance analysis. The receiver considered is an ASK/DD receiver whose schematic diagram is presented in Fig. 7.

The proposed crosstalk model is a balance between accuracy and complexity. We want the model to be as accurate as possible, but it must be not prohibitive in terms of computing time needed to run the simulation for the set of networks under investigation. We choose the commonly used Gaussian approximation for the bit-error probability evaluation. Although crosstalk has been shown to exhibit non-Gaussian statistics, i.e., [6], the Gaussian approximation, which is numerically simple, gives us in particular a good indication of how crosstalk affects the system performance if the number of crosstalk sources is large [1]. We proceed by making the following assumptions and definitions:

- perfect signal and crosstalk bit-alignment (worse case scenario);
- extinction ratio is denoted by \( \varepsilon \);
- it is assumed that \( m \) photons per bit are received for a transmitted “one” while \( qm \) are for a transmitted “zero”;
- the ratio of leakage crosstalk to signal power is denoted by \( \epsilon \);
- there are \( N \) crosstalk sources operating at the same nominal wavelength as the informative signal;
- we assume that each interferer has the same relative crosstalk power \( \epsilon \) (worse case scenario);
- the crosstalk statistics is assumed to be Gaussian having zero-mean with a normalized variance which is equal to one-half;
- the signal and crosstalk polarization are linearly random and its induced intensity noise is assumed to be Gaussian with zero-mean and with a normalized variance which is equal to one-half;
- a postdetection filter which is of the “integrate-and-dump” type;
- the detection threshold is fixed to be a midway point between the signal level for a transmitted “zero” and “one”;
- the receiver thermal-noise is assumed to be Gaussian distributed, zero-mean and with variance \( \sigma_{th}^2 \).

A. Performance Analysis

The question is to evaluate the average error rate \( P_e \) of the system under discussion. We are going to treat the case of amplitude shift keying (ASK) modulation format. To account for all possible combinations of signal and crosstalk interfering bits we proceed by assuming that \( \mu \) interferers are simultaneously a logical “one”, thus \( N - \mu \) interferers are “zero”. The error probability analysis is conducted by a weighted statistical average of the error probability for each value \( \mu \). This probability is given by the binomial distribution

\[
P(\mu) = \frac{N!}{(N-\mu)!\mu!} 2^{-N}.
\]

Hence, the average error probability \( P_e \), for a given threshold \( \alpha \), is given by

\[
P_e = \langle P_e | \mu \rangle P(\mu).
\]

Using the Gaussian approximation \( P_e | \mu \) can be written as

\[
P_e | \mu = \frac{1}{2} Q\left(\frac{E_1 - \alpha}{\sigma_1}\right) + \frac{1}{2} Q\left(\frac{\alpha - E_0}{\sigma_0}\right)
\]

where \( E_{0,1} \) is the mean of the received signal when a “single zero,” and a “single one” is sent, respectively. The variance is denoted by \( \sigma_{0,1}^2 \), and the decision threshold by \( \alpha \). The function \( Q(\cdot) \) is the standard Gaussian probability tail function.
With (A3), and assuming that the symbols are \textit{a priori} equally probable, the average BER (A4) can be written as
\[
P_e = \frac{1}{2N} \sum_{\mu=0}^{N} \left( \binom{N}{\mu} \right) \times \left\{ \frac{1}{2} Q\left( \frac{E_0(\mu) - \alpha}{\sigma_1(\mu)} \right) + \frac{1}{2} Q\left( \frac{\alpha - E_0(\mu)}{\sigma_0(\mu)} \right) \right\}.
\]
(A6)

If we consider the case for a transmitted symbol “one” ($l_0^* = 1$), then the mean ($E_1$) and variance ($\sigma_1^2$) for the decision variable $Z$ are presented in (A7) and (A8).
\[
E_1 = m + cm(\mu + (N - \mu)\varphi)
\]
(A7)
\[
\sigma_1^2 = \mu(2m\sqrt{\varphi})^2 \frac{1}{4} + (N - \mu)(2m\sqrt{\varphi})^2 \frac{1}{4}
+ (2mc)^2 \frac{1}{2} \mu(\mu - 1) + (2mc)^2 \frac{1}{2} \mu(N - \mu)
+ (2mc\varphi)^2 \frac{1}{4} \frac{(N - \mu)(N - \mu - 1)}{2} + E_0 + \sigma_0^2
\]
(A8)

where the first term in (A8) represents the beat terms between the signal and the $\mu$ crosstalk sources which are “one”. The factors one-fourth arise from the variance of the polarization, and crosstalk, respectively. The second term accounts for the beating terms which are “zero”. The next three terms represent the crosstalk-crosstalk beating terms. First, the $\mu(\mu - 1)/2$ possible combinations of “one-one” beating terms are considered. Second, there are $(N - \mu)(N - \mu - 1)/2$ combinations of “zero-zero” beating terms. Last, beating terms for “one” and “zero” are accounted for in the fourth term. This implies that all possible crosstalk-crosstalk combination are covered in this simple formula. The variance of the shot-noise is according to the Poisson statistics for photo-detection equal to the mean of the photo-current. In a similar manner one could derive that under the assumption that the signal contains a “zero” ($l_0^* = 0$), the mean ($E_0$) and variance ($\sigma_0^2$) are
\[
E_0 = m\varphi + cm(\mu + (N - \mu)\varphi)
\]
(A9)
\[
\sigma_0^2 = \mu(2m\sqrt{\varphi})^2 \frac{1}{4} + (N - \mu)(2m\sqrt{\varphi})^2 \frac{1}{4}
+ (2mc)^2 \frac{1}{2} \mu(\mu - 1) + (2mc)^2 \frac{1}{2} \mu(N - \mu)
+ (2mc\varphi)^2 \frac{1}{4} \frac{(N - \mu)(N - \mu - 1)}{2} + E_0 + \sigma_0^2
\]
(A10)

Given a number $N$ of crosstalk sources, the error probability is expeditiously evaluated by (A6). In the computations used in this paper we used an extinction ratio of $\varphi = 8$ dB while $m_1$ is $1.3 \times 10^4$ photons/s for a BER of $10^{-12}$. This corresponds to a signal power of $-24$ dBm for a receiver with responsivity equal to 1 A/W, operating at a wavelength of 1.55 $\mu$m. The receiver resistance load is equal to 50 $\Omega$.

**ACKNOWLEDGMENT**

The authors would like to thank J. Siffels for making figures available.

**REFERENCES**


H. J. S. Dorren, photograph and biography not available at the time of publication.

H. de Waardt, photograph and biography not available at the time of publication.

I. Tafur Monroy, photograph and biography not available at the time of publication.