1/f Noise sources

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1/f Noise Sources

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Invited Paper

Abstract—This survey deals with 1/f noise in homogeneous semiconductor samples. A distinction is made between mobility noise and number noise. It is shown that there always is mobility noise with an $\alpha$ value with a magnitude in the order of $10^{-4}$. Damaging the crystal has a strong influence on $\alpha$. $\alpha$ may increase by orders of magnitude.

Some theoretical models are briefly discussed; none of them can explain all experimental results. The $\alpha$ values of several semiconductors are given. These values can be used in calculations of 1/f noise in devices.

I. NOISE SOURCES

This paper serves as an introduction to several papers in this special issue, which is devoted to noise in devices.

One way of analyzing the noise of a device is to assume noise sources in several parts on the device. The noise measured at the output is the summation of many contributions from the different sources modified by the device characteristics. The device may introduce coupling between different contributions. If one has a reliable model of the device and if one knows the noise sources, it is possible to calculate the output noise. One then concludes that the noise of the device is understood if agreement is observed between calculated and measured noise, as functions of voltages and currents.

It is also possible to go the other way round. Starting from the observed output noise one tries to determine the noise sources. One then estimates properties like concentrations, cross-sections and $\alpha$ values of 1/f noise. This procedure, from observed output to sources, is risky. Apart from the trivial requirement that the model must be treated correctly—not too many simplifying approximations—there is the difficult problem whether the model used, is a correct description of the device under investigation. Noise is much more sensitive to details than the average voltages and currents. This procedure always leads to results, but are they reliable? If one incorrectly assumes a bulk source where in fact a surface source is present, one finds a $\alpha$ value without any meaning. Such $\alpha$ values are nevertheless used as an argument for or against certain theories. The properties for the noise sources derived the second way need independent checking. One must have some idea at least of what realistic values are.

The properties of noise sources can, therefore, best be studied on homogeneous samples. The numerical results can then be used in the much more complicated problem of the inhomogeneous and often nonlinear devices. For the same reason, $\alpha$ values obtained from devices are excluded from Fig. 5 at the end of this paper. There we present reliable data, all obtained from homogenous samples.

Four types of noise are of importance in semiconductors.

1) **Thermal noise.** Any resistance $R$ shows spontaneous current fluctuations or voltage fluctuations according to:

$$S_V = 4kTR$$  \hspace{1cm} (1)

$$S_I = 4kT/R.$$  \hspace{1cm} (2)

This white spectrum is always found, whatever the nature of the conduction process, or the nature of the mobile charge carriers. Thermal noise is generated in any physical resistor that shows dissipation if a current is passed through it. A mathematical property, which is measured in $\Omega$, such as a dynamical resistance $dV/dI$ of a nonlinear device must not be used in (1) or (2).

2) **Shot noise.** The current carried by electrons emitted from a hot cathode in a vacuum diode, or by electrons that cross a potential barrier in a semiconductor, are randomly generated. Random generation leads to fluctuations around the average current $I$:

$$S_I = 2qI.$$  \hspace{1cm} (3)

The details of the emission process have no influence on the noise, provided that there is no interaction between the electrons, and that the statistics is close to Boltzmann.

3) **Generation-recombination noise.** The number of free electrons in the conduction band may fluctuate because of generation and recombination processes between the band and traps. The number fluctuations cause fluctuations in the conductance $G$, and, therefore, in the resistance $R$.

$$\frac{S_R}{R^2} = \frac{S_G}{G^2} = \frac{S_N}{N^2} = \left(\frac{\Delta N}{N}\right)^2 \frac{4\tau}{1 + \omega^2\tau^2}$$  \hspace{1cm} (4)

where $\tau$ is a relaxation time, characteristic of the trap, usually in the range of $10^{-6}$ s to $10^{-3}$ s. If there is one type of trap only, then the variance $(\Delta N)^2$ is given by

$$\frac{1}{(\Delta N)^2} = \frac{1}{N} + \frac{1}{X_n} + \frac{1}{X_p}$$  \hspace{1cm} (5)
where $X_n$ is the average number of occupied traps and $X_p$ the average number of empty traps. The variance thus approximates the smallest of the quantities $N$, $X_n$, and $X_p$.

The complicated problem of a semiconductor with two kinds of traps, $X$ and $Y$, has been solved by van Vliet and Fasset [1]. Later publications [2]-[4] deal with practical questions like, under which conditions will the observed spectrum be the superposition of two Lorentzians, one which would have been found if only $X$ were present, the other one for the case that only $Y$ were present. The condition for this, often naively assumed, situation is

$$\frac{1}{N} \leq \frac{1}{X_n} + \frac{1}{X_p} < \frac{1}{Y_n} + \frac{1}{Y_p}$$

[3] gives simple procedures to determine cross sections and trap concentrations from observed GR spectra too.

4) $1/f$ Noise. This is a fluctuation in the conductance with a power spectral density proportional to $f^{-\gamma}$, where $\gamma = 1.0 \pm 0.1$ in a wide frequency range, usually measured from 1 Hz to 10 kHz. The spectrum cannot be exactly $f^{-\gamma}$ from $f = 0$ to $f = \infty$, since neither the integral of the power density nor the Fourier transform would be able to have finite values. At some higher frequency $f_0$, the slope must be steeper than $-1$. This $f_0$ has never been observed for the simple reason that at higher frequencies the $1/f$ noise disappears in white thermal noise that is always present. Attempts to observe the lower limit $f_0$, below which the spectrum flattens, have always been in vain. Measurements down to $10^{-6}$ Hz showed that even there the spectrum still is $f^{-1}$ [5].

Because of the restrictions on $\gamma \approx 1$ we do not consider $f^{-1/2}$ and $f^{-3/2}$ as $1/f$ noise, like some people do. In semiconductor devices, such spectra usually follow from diffusion processes.

Unlike the first three sources mentioned above, which are well known, the origin of the $1/f$ noise is still open to debate, a debate full of vehement controversies [6]-[11]. Therefore, from here on we will discuss $1/f$ noise only. Due to our present interest in devices, which operate at room temperature, we concentrate on $1/f$ noise at $T = 300$ K. Temperature influence is only discussed if it helps to elucidate the physical nature of $1/f$ noise. For the same reason we need not deal with the noise of hot electrons. In general we shall pay less attention to the theoretical problems of the physical model of the noise, than to reliable numerical values to be used in device models.

II. THE FACTOR $\alpha/N$

The relation

$$\frac{S_R}{f^2} = \frac{\alpha}{fN}$$

was proposed 25 years ago [12] in an effort to systematically collect data on $1/f$ noise from the literature. In that collection the influence of the size of the ohmic homogeneous samples, the carrier concentration, the frequency range of the measurements, etc., had to be eliminated. The only theoretical idea behind the relation was, that whatever the electrons do when producing $1/f$ noise, they do it independently. Thus, $\alpha$ is a normalized measure for the relative noise in different materials, at different temperatures, etc. There was no reason to assume that $\alpha$ was a constant. On the contrary, we were looking for factors influencing $\alpha$. Given the inaccuracies of the individual experimental results—and with hindsight—given the rather poor qualities of the samples no systematic trend in $\alpha$ was found at the time. The $\alpha$ values were not too far apart, and it seemed reasonable to take $\alpha = 2 \times 10^{-3}$ as an average value. Later on it turned out that $\alpha$ depends on the quality of the crystal, and on the scattering mechanisms that determine the mobility $\mu$. In perfect material $\alpha$ can be 2 or 3 orders of magnitude lower than the $2 \times 10^{-3}$ originally proposed.

Before we can further discuss dependences of $\alpha$ on certain parameters, like temperature, dope, etc., or even before we can decide that it is meaningful to take an average of measured values, we should have some idea of what we consider to be essentially the same values. We need an idea of what random errors are, and what may point to a systematic dependence on some parameter. Noise very much depends on the physical conditions during growing, doping, intentional and unintentional surface treatment, and contacting. Fig. 1 gives an example of the best results we can get. The points on a vertical line (at the same value of $\mu$) were measured on different samples made from the same wafer. A spread in $\alpha$ of a factor $1 \cdot 5$ is found. Each point is the average of several measurements, with different currents, on the same sample. The spread is wider if samples are made from different wafers with the same properties nominally. It is wider still if similar samples from different laboratories are compared.

For a meaningful result we need the average over many samples. The best we can hope for is that samples from different sources, measured under different conditions, give $\alpha$ values with the same order of magnitude. As a result, if one wants to find certain numerical values to support or refute a theoretical model one can always find them in literature. Discussing systematic dependences of $\alpha$ on some parameter requires a set of similar samples, expressly made for the purpose, such as the samples used for Fig. 1.

In spite of all uncertainties, Fig. 1 shows that the dependence of $\alpha$ on $\mu$ can be found from a set of carefully prepared samples, where only one parameter is varied. It is even possible to extrapolate the results to samples without impurity scattering, although such samples cannot be made. The inaccuracy in the extrapolated value of $\alpha_{\text{latt}}$ is less than 10%, despite the factor $1 \cdot 5$ between individual $\alpha$ values. The dimensions length, width, and thickness do not appear in (7), proving that $1/f$ noise is a bulk effect. Previous theories that considered $1/f$ noise as a surface effect [13] were thus refuted, at least that is for the homogeneous samples considered. The bulk $1/f$ noise, which has been proven to exist beyond any doubt in homogeneous samples, also occurs in devices where its magnitude can be estimated using the $\alpha$ value determined in homogeneous material. However, there is positive evidence that surface $1/f$ noise exists too. Certainly in MOST's, many
Experimental data are better explained by surface effects than by bulk effects. [14] The noise of MOST's is discussed in the paper by Vandamme, Li, and Rigaud, in this Special Issue.

If the same relation (7) holds good for metals, it is immediately clear that for noise studies in metals very small samples are required. Early experiments on point contacts [15] and thin films [16] showed that the relation does indeed hold good, and that \( \alpha \) has the same order of magnitude as in semiconductors. The old idea that \( 1/f \) noise is exclusively a semiconductor effect was thereby proved to be incorrect.

Arguments against (7) have also been raised, not so much because of the difficulties with \( \alpha \) (a constant or a parameter), but because of the factor \( N \) in the denominator. One argument is rather trivial. If one were to start the discussion of a theoretical model by immediately considering an “average” electron, then the number of electrons \( N \) would not appear in the final result. It has been concluded from such models that relation (7) cannot be correct because of the factor \( 1/N \). This discussion is summarized in [19].

In experiments one always observes a group of \( N \) electrons. In the oversimplified models with an “average” electron, a single electron is considered [17]. A proper theory, however, yields the result averaged over an ensemble of identical samples, electron systems, and so on in full agreement with experiment. Theory does not provide arguments against the use of a normalizing factor \( 1/N \).

Weissman [18] put forward a more serious argument that had already been brought up at some conferences. If one considers the \( 1/f \) noise as a summation of Lorentzians (see Section IV) then the very low frequencies, where \( 1/f \) noise is observed, require Lorentzians with a long characteristic time \( \tau \gg 1 \) s. Therefore, the electrons must stay in the sample much longer than a few seconds, in order to produce these low-frequency Lorentzian distributions. However, even in a sample with a length of 1 cm an electron only stays about 0.1 s as its diffusion coefficient \( D \) is of the order of \( 10^{-3} \) m²/s.

How then can electrons, that stay in the sample for only such a short while, produce noise at frequencies below 1 Hz? The answer is as follows: We have evidence that the \( 1/f \) noise is in the lattice scattering (see Section III). The lattice modes can scatter electrons. The scattering cross-sections fluctuate slowly with a \( 1/f \) spectrum. There are permanently \( N \) electrons—on average—that probe the slowly varying cross sections. We do not follow individual electrons. Individual electrons move in and out the sample; the lattice is permanent.

This idea has been worked out in more detail in [19]. A model was considered where \( N \) electrons were scattered by \( A \) lattice modes. This led to a relation for the noise in the current density where \( S_f/j^2 \) is proportional to \( 1/N \) and independent of \( A \).

Having answered theoretical objections against the factor \( 1/N \), we would nevertheless like to investigate the experimental evidence at this point. We introduced \( N \) as a measure of the size of the sample. In bigger samples the relative noise must average out. Another measure for the size may do as well, like the volume, the number of atoms \( A \), or the number of lattice modes, proportional to \( A \). We therefore made a comparison between \( N \) and \( A \) [19]. If the relative noise is written as

\[
\frac{S_R}{R^2} = \frac{\alpha}{N^2} = \frac{\gamma}{A f}
\]

it turns out that \( \alpha \) is a better “constant” than \( \gamma \) is. Values of \( \alpha \) are found between \( 10^{-4} \) and \( 2 \times 10^{-3} \), whereas \( \gamma \) varies between 10 and \( 10^2 \) in a group of some twenty semiconductors. These experimental data give no argument for preferring \( \gamma/A \) to \( \alpha/N \).

Since it is doubtful whether \( \alpha \) has the same value in different materials, a more direct proof for \( \alpha/N \) being the correct factor is to study the injection of a varying number of electrons in a given volume of a semiconductor, either by electrical injection in a forward diode [20], or by persistent photoconductivity [21]. Such experiments show that the noise is proportional to \( 1/N \). In each experiment \( \alpha \) turns out to be a constant. The observed constants from different experiments are in the normal range \( 10^{-4} \) to \( 10^{-3} \).

III. EXPERIMENTAL DISTINCTION BETWEEN \( \Delta \mu \) AND \( \Delta \mu \)

The \( 1/f \) noise is a fluctuation in the conductivity. There is conclusive experimental evidence for this point of view. Conductivity fluctuations lead to fluctuations in the resistance \( R \) and thus to slow fluctuations in the power density of the thermal noise \( 4kTR \).

\[
S_{S_R} \propto S_R.
\]

Because of (1) one can measure the noise in \( R \) by measuring the noise in \( S_R \). In such measurements no current is passed through the sample [22]. These special measurements prove that \( 1/f \) noise is not generated by the current. In conventional measurements the current is only necessary to transform the already existing conductivity fluctuations into voltage fluctuations that can be measured.
So 1/f noise is a fluctuation in the conductivity. This is the last point on which general agreement can be reached. Any next step causes a great deal of controversy. As the conductivity

\[ \sigma = n q \mu \]  

contains the product of \( n \) and \( \mu \) the next question is: “What is fluctuating with a 1/f spectrum, \( n \) or \( \mu \)?”

In a series of experiments it was shown that there is a type of 1/f noise that is a fluctuation in the mobility. These experiments were done on homogeneous samples, mainly silicon. From these experiments it follows that this mobility 1/f noise is always present. It is described by (7), where the \( \alpha \) value is in the order of magnitude of \( 10^{-4} \) in perfect material. In damaged material the mobility noise may be considerably increased. On top of the mobility noise there may be other types of 1/f noise, e.g., number fluctuations generated at surface states. Number fluctuations caused by trapping processes at the surface play an important part in MOST’s. It seems that we see An noise in an N-channel MOST, whereas in a P-channel Ap noise is observed. The very complicated situation of MOST noise is discussed in this Special Issue in the papers by Vandamme, Li, and Rigaud and by Chang, Abedi, and Viswanathan.

In this section we will concentrate on the fundamental mobility noise in good material. We will consider the noise in thermo EMF, Hall effect, etc. The principle of such an analysis is that in the same sample electrons move because of an applied electric field, and because of some other generalized force, e.g., a temperature gradient.

In the experiment the ratio of the two generalized forces are varied and the change in magnitude of the 1/f noise is observed. In the analysis one first introduces a \( \Delta \mu \) term in the transport equation, and calculates the expected fluctuation in the observed voltage or current. Then such a calculation is done with a \( \Delta n \) source. The observed noise in voltage or current always agrees with \( \Delta \mu \) fluctuations. In most cases the observed noise is far off from the line for \( \Delta n \) fluctuations. Sometimes the results of the calculations for \( \Delta n \) and \( \Delta \mu \) are not that far apart, so that no distinction can be made. There was no case in which the differences between observed and calculated \( \Delta \mu \) values were so large that mobility fluctuations had to be excluded in favor of \( \Delta n \) fluctuations. These experiments have been discussed in a review paper in 1981 [22]. At that time the result of noise measurements in the Hall effect was indecisive. The theoretical lines of \( \Delta n \) and \( \Delta \mu \) as functions of the magnetic induction \( B \) were not that far apart. Recently, results were published on the noise in the Hall effect in \( n \)GaAs, where it is easy to reach high \( \mu B \) values \( (\mu B \gg 1) \). Under these conditions the theoretical lines of \( \Delta n \) and \( \Delta \mu \) versus \( B \) deviate widely; where the experimental points follow the \( \Delta \mu \) line nicely [23]. Such calculations are rather complicated, since they have to start from individual levels in the conduction band. Integration over the whole band then gives the noise magnitude that can be compared with the experimentally observed value.

A much more transparent proof for mobility fluctuations is provided by the analysis of the noise in the conductance of semiconductor samples, where two scattering mechanisms determine the mobility. We consider a semiconductor in which two scattering mechanisms are active: lattice scattering and impurity scattering. The mobility \( \mu_{\text{meas}} \) measured is given by Matthiessen’s rule

\[ \frac{1}{\mu_{\text{meas}}} = \frac{1}{\mu_{\text{latt}}} + \frac{1}{\mu_{\text{imp}}}. \]  

It is certain now that only the lattice scattering generates 1/f noise, whereas the impurity scattering has no appreciable contribution to the noise. In order to obtain simple relations we start by assuming \( \Delta \mu_{\text{imp}} = 0 \). Later we shall introduce a small \( \Delta \mu_{\text{imp}} > 0 \) and discuss the consequences thereof. From (11) it follows that

\[ \Delta \left( \frac{1}{\mu_{\text{meas}}} \right) = \Delta \left( \frac{1}{\mu_{\text{latt}}} \right) + \Delta \left( \frac{1}{\mu_{\text{imp}}} \right). \]

From which follows

\[ \alpha_{\text{meas}} = \left( \frac{\mu_{\text{meas}}}{\mu_{\text{latt}}} \right)^2 \alpha_{\text{latt}} + \left( \frac{\mu_{\text{meas}}}{\mu_{\text{imp}}} \right)^2 \alpha_{\text{imp}} \]

assuming that

\[ (\Delta \mu_{\text{latt}} \cdot \Delta \mu_{\text{imp}}) = 0. \]

The plot of \( \log \alpha_{\text{meas}} \) versus \( \log \mu_{\text{meas}} \) now is somewhat more complicated than Fig. 1. If the term \( (\mu_{\text{meas}}/\mu_{\text{latt}})^2 \alpha_{\text{latt}} \) dominates in (15) we have the situation described by (12) and (13). This corresponds to the line with slope +2 in Fig. 2: the points a to h. If the term \( (\mu_{\text{meas}}/\mu_{\text{imp}})^2 \alpha_{\text{imp}} \) dominates in (15) we have a situation where \( \mu_{\text{meas}} \approx \mu_{\text{imp}} \), so that \( \alpha_{\text{meas}} \approx \alpha_{\text{imp}} \). The noise, characterized by \( \alpha_{\text{imp}} \) is proportional to the number of impurity centers, which in this situation is inversely proportional to \( \mu_{\text{imp}} \approx \mu_{\text{meas}} \).

\[ \alpha_{\text{meas}} \approx \alpha_{\text{imp}} \times \mu_{\text{imp}}^{-1} \approx \mu_{\text{meas}}. \]

We find a line with slope \(-1\) in the plot of \( \log \alpha_{\text{meas}} \) versus \( \log \mu_{\text{meas}} \). These are the points h to j. Fig. 2(b) gives the general situation at a constant temperature \( T_0 \). In all situations investigated only the right hand side of the figure, with the
Fig. 2. (a) log \( \mu_{\text{meas}} \) versus log \( T \). The impurity scattering increases by equal factors in the series of samples \( a \) to \( j \). (b) log \( \alpha_{\text{meas}} \) versus log \( \mu_{\text{meas}} \) at \( T = T_0 \). The noise of the samples \( d \) to \( h \) is determined by \( \alpha_{\text{latt}} \), although \( \mu_{\text{meas}} \approx \mu_{\text{imp}} \).

slope +2, was observed. It is the familiar plot of \( \log \alpha \) versus \( \log \mu \).

Matthiessen's rule is only an approximation. Therefore, the results for \( \alpha_{\text{meas}} \) are also approximations. This does not raise a serious problem. We must distinguish between slopes that are close to \(-1\) and +2. These values are so far apart that the distinction between lattice scattering and impurity scattering is always possible. Whether Tacano [24] is right in interpreting his value \( \alpha_{\text{meas}} = 3 \times 10^{-6} \) as the value of \( \alpha_{\text{imp}} \) of GaAs, depends on the branch his point is on. He assumes it is like point \( j \) in Fig. 2. However, if this value corresponds to point \( c \), the same value \( \alpha_{\text{meas}} = 3 \times 10^{-6} \) then points to \( \alpha_{\text{latt}} \approx 5 \times 10^{-6} \), which is quite a normal value in GaAs at 50 K. From values of \( \alpha_{\text{meas}} \) of samples with less or with more impurity scattering, it can be decided whether the samples are on the branch with slope \(-1\), in which case Tacano is right, or on the branch with slope +2.

In the case of number fluctuations, the following situations occur:

a. \( \alpha \) is proportional to 'GR centers' which create a \( 1/f \) spectrum, and which do not scatter electrons: \( \alpha_{\text{meas}} \propto \mu_{\text{meas}}^{-1} \).

b. \( \alpha \) is proportional to centers that also are scatterers of electrons: \( \alpha \propto \mu_{\text{imp}}^{-1} \). If \( \mu_{\text{meas}} \approx \mu_{\text{imp}} \), \( \alpha_{\text{meas}} \propto \mu_{\text{meas}}^{-1} \).

c. like \( b \), but now with \( \mu_{\text{meas}} \approx \mu_{\text{latt}} \): \( \alpha_{\text{meas}} \propto \mu_{\text{imp}}^{-k} \times \mu_{\text{latt}}^{-1} \), where \( k \gg 1 \).

Number fluctuations will always lead to negative slopes in plots of \( \log \alpha \) versus \( \log \mu \); the slope might be close to zero.

Above, we have presented experimental evidence of the fact that lattice scattering is the source of \( 1/f \) noise. This evidence came exclusively from measurements of electrical noise. Evidence from other experiments than those on electrical noise would, therefore, be most welcome. In a series of optical experiments, Musha et al. [25]--[27] showed that when photons are scattered at acoustic lattice waves the intensity of the scattered light also exhibits \( 1/f \) noise. With quartz it was found that

\[
S_\delta = \langle \phi \rangle^2 8 \times 10^{-5}/f
\]

with water:

\[
S_\delta = \langle \phi \rangle^2 3/f
\]

where \( \langle \phi \rangle \) is the mean phonon number. The essential factor is the term \( 1/f \); the value of the numerical constant is of less importance. It should equal \( (\ln f_1/\ln f_2)^{-1} \), where \( f_1 \) and \( f_2 \) are the lower and higher limits of the \( 1/f \) spectrum. The numerical values were calculated by assuming independent fluctuations in the modes, and using a rough estimate of the number of modes in the illuminated volume.

The consequence for electrical conductivity noise is that what is observed as mobility fluctuations of electrons are essentially number fluctuations of phonons.

IV. \( \Delta n \)-MODELS

All \( \Delta n \) models are based on the same principle: the addition of Lorentzian GR spectra with a special distribution of the relaxation times \( \tau_i \). A very wide range of \( \tau \) values is required. A \( 1/f \) spectrum is obtained between the frequencies \( 1/\tau_1 \) and \( 1/\tau_2 \) if \( \tau_1 < \tau_2 \). Below \( 1/\tau_2 \) the spectrum is white, above \( 1/\tau_1 \) the spectral density is proportional to \( f^{-2} \). We use a normalized distribution function \( g(\tau) \):

\[
\begin{align*}
g(\tau_1) &= 0 & \text{for} & \quad \tau_1 < \tau_2 && (20) \\
g(\tau_1) &= \frac{1}{\ln \tau_2/\tau_1} & \text{for} & \quad \tau_1 < \tau_1 < \tau_2 && (21) \\
g(\tau_1) &= 0 & \text{for} & \quad \tau_2 < \tau_1 && (22)
\end{align*}
\]

\[
\begin{align*}
S_N &= \frac{1}{\Delta N} \int_0^\infty g(\tau) \frac{4\tau}{1 + \omega^2 \tau_1^2} d\tau_i \\
&= \frac{1}{\Delta N} \int_{\ln \tau_2/\tau_1}^{\ln \tau_2} \frac{1}{\tau_1} \frac{\tau}{\tau_1^2 + \omega^2 \tau_1^2} d\tau_i \\
&= \frac{1}{\Delta N} \left[ \frac{4}{\ln \tau_2/\tau_1} \text{arctan} \frac{\omega \tau_1}{\tau \tau_1} \right] \\
&= \omega \ll 1/\tau_2 \ll 1/\tau_1: S_N = \frac{1}{\ln \tau_2/\tau_1} \cdot \frac{4\tau_2}{f} && (24) \\
1/\tau_2 \ll \omega \ll 1/\tau_1: S_N = \frac{1}{\ln \tau_2/\tau_1} \cdot \frac{1}{\pi^2 f^2} && (25) \\
1/\tau_2 \ll 1/\tau_1 \ll \omega: S_N = \frac{1}{\ln \tau_2/\tau_1} \cdot \frac{1}{\pi^2 f^2} && (26)
\end{align*}
\]

where the following approximations were used:

\[
\text{arctan} \delta \approx \delta \quad \text{and} \quad \text{arctan} \frac{1}{\delta} = \frac{\pi}{2} - \delta. \quad (27)
\]

The trap distribution (21) only leads to a \( 1/f \) spectrum if a necessary, but often overlooked, condition is fulfilled: there should be no transitions between traps with different \( \tau \)'s, neither directly nor via the conduction band. See Fig. 3.
**A. Isolated Traps**

The variance \( (\Delta X)^2 \) of the free electrons or of all trapped electrons together is the sum of the variances of each individual kind of trap, characterized by its \( \tau_i \).

\[
(\Delta X)^2 = \int_{\tau_i}^{\infty} g(\tau_i)(\Delta X)^2 \, d\tau_i.
\]

(28)

Each individual spectrum is given by

\[
S_f = \frac{4\tau_i}{1 + (2\pi f\tau_i)^2} = (\Delta X)^2 \frac{4\tau_i}{1 + (2\pi f\tau_i)^2} \, d\tau_i.
\]

(29)

The summation of these spectra leads to

\[
S = \int_0^\infty S_f \, d\tau_i = \frac{4}{(\ln 2\tau_1/\tau_1)^2} \int_0^\infty \frac{1}{1 + x^2} \, dx = \frac{4}{\ln 2\tau_1/\tau_1} \frac{\Delta X^2}{f}
\]

in agreement with (25) and with the variance \( \int_0^\infty S \, df \).

**B. Transitions Between the Traps**

A fluctuation in the number of free electrons now decays by interaction with all traps

\[
- \frac{d\Delta X}{dt} = \Delta X \int_{\tau_i}^{\infty} g(\tau_i) \frac{1}{\tau_i} \, d\tau_i = \Delta X \int_{\ln 2\tau_1/\tau_1}^{\tau_1} \frac{1}{\tau} \, d\tau
\]

\[
\approx \frac{\Delta X}{\ln 2\tau_1/\tau_1} \frac{1}{\tau_1} = \frac{\Delta X}{\tau_0}
\]

(31)

where we define

\[
\tau_0 \equiv \tau_1 \ln 2\tau_1/\tau_1.
\]

(32)

Thus one single Lorentzian is obtained with \( \tau_1 \) close to \( \tau_1 \). The Lorentzian intersects the 1/f spectrum at the frequencies \( f_1 = \tau_1/4\tau_1^2 \) and \( f_2 = 1/\pi^2 \tau_1 \), which are a factor \( 2\pi \) to the left and right from the characteristic frequency \( f_0 = 1/2\pi \tau_0 \).

Fig. 4 shows the two spectra, based on the same distribution function \( g(\tau_i) \): a 1/f spectrum for isolated traps, and a Lorentzian for interacting traps. In this figure we used \( \tau_2/\tau_1 = 5 \times 10^8 = \varepsilon^{20} \), \( \tau_0/\tau_1 = 20 \).

Whatever \( g(\tau) \) one may choose, the result of interacting traps will always be one average \( \tau \), hence a single Lorentzian.

A distribution \( g(\tau) \propto 1/\tau \) is easily realized over a wide range of \( \tau \) values, if \( \tau \) exponentially depends on a quantity that is homogeneously distributed over a limited range.

For instance, in McWhorter’s surface model, traps are assumed to be homogeneously distributed in the oxide layer on a semiconductor. The probability \( 1/\tau \) of an electron in the semiconductor reaching a trap in the oxide layer by tunneling is given by

\[
\tau = \tau_0 e^{x/d}
\]

(33)

where \( x \) is the distance from the trap to the silicon-oxide interface, and \( d \) is a constant characteristic for the tunnel process.

\[
g(\tau) = \frac{dN}{d\tau} = \frac{dN}{d\tau} \cdot \frac{dx}{d\tau} \propto \frac{1}{\tau}
\]

(34)

\( N \) is the number of traps, \( dN/d\tau \) the concentration of traps which is constant.

The St. Petersburg group [28] has proposed a special model for GaAs, with a tail of states below the conduction band. The density of states in the tail is assumed to be exponential

\[
N(E) = N(0) e^{-E/E_t}
\]

(35)

where \( E \) is the distance of the state to the bottom of the band. For the relaxation time of the state they assume

\[
\tau(E) = \tau(0) e^{-E/kT}.
\]

(36)

The result is again \( g(\tau) \propto 1/\tau \).

The required distribution \( g(\tau) \propto 1/\tau \) can be obtained with processes that are thermally activated. If \( E_i \) is homogeneously distributed between \( E_1 \) and \( E_k \), and if \( g(E) \) is zero outside this interval, then an exact 1/f spectrum is obtained at frequencies \( f \) between

\[
\frac{1}{\tau_0} < 2\pi f < \frac{1}{\tau_1}.
\]

(37)

In the Dutta Horn model [29], [30], \( g(E) \) need not be a constant. A peak of \( E \) values a few \( kT \) wide is good enough.
to produce a $1/f$-like spectrum: $f^{-\gamma}$ with $\gamma = 1 \pm 0.3$. The slope will not be constant over the whole frequency range, and there will be a relation between slope and temperature dependence of the spectral power:

$$\gamma(\omega, T) = 1 - \frac{1}{\ln(\omega/\omega_0)} \left[ \frac{d \ln S(\omega, T)}{d \ln T} + 1 \right]. \tag{38}$$

where $\tau_0 \approx 10^{-14}$ s, an attempt time in the order of an inverse phonon frequency.

The essence of the Dutta Horn model is that a wide range of $\tau$'s results from a rather narrow peak of activation energies. The width of the peak is determined by the disorder in the crystal. Our question now is: “Is this of interest for semiconductors at room temperature?”

In applying the model directly to generation recombination noise the activation energies of the traps will be 0.3 eV at most. The width of the peak could then be in the order of 0.03 eV. Hence, $\Delta E/kT$ is of the order 1. So there is no appreciable frequency range in which a $1/f$ spectrum can be observed. The Dutta Horn model is of little interest to number fluctuations at room temperature, but it can be applied to mobility fluctuations (to be discussed in the next section). There we need mobile lattice defects. The movement of lattice atoms requires activation energies in the order of magnitude of binding energies being 1 or 2 eV. A high degree of disorder may give a spread in the activation energy of some 10%–20%. Thus at room temperature $\Delta E/kT$ is of the order of 10, which yields $\tau_0/\tau_0 \approx 10^5$. The resulting $1/f$ spectrum of the electron mobility could then be observed in a frequency range of 5 dec.

The Dutta Horn model can explain mobility noise, certainly at low temperatures.

V. $\Delta\mu$-MODELS

We shall present two theoretical models for mobility $1/f$ noise: 1) local interference noise, 2) quantum $1/f$ noise. No critical discussion of the theories will be attempted; the emphasis is on results which can be used in a discussion of the noise of devices.

A. Local Interference Noise

This is one of the three cases where the Dutta Horn model is applied to mobile defects that act as scattering centers for the electrons [31]. The local interference model has been very successful in the study of noise in metals, especially in disordered metal films [32], [33]. Here our problem is whether it can be of interest for semiconductors at room temperature. We shall mainly be guided by Giordano's review [32].

The principle can be sketched as follows. An electron returns to its original position after a random walk during which it has been scattered by several scattering centers, in this case lattice defects. Each scattering event gives a phase shift. The electron arrives at the original position again with a certain phase shift in its wave function. If it had travelled the same path, but in the opposite direction, its final phase shift would have been the same as for the original direction. So there is constructive interference: the electron density at the original position is higher than for two uncorrelated functions.

Now the conductance between $A$ and $B$ is seen as a summation of all possible paths from $A$ to $B$, via all scattering centers. The phase is preserved over a limited distance $L_q$. In the universal conductance fluctuation model (UCF), the multiple scattering events of all defects in $L_q^2$ contribute to the interference. After a defect has moved to a different position, the conductance is different, which is interpreted as mobility noise. The defects have somewhat different activation energies for jumping, and therefore, different $\tau$ values. The summation of the individual Lorentzians yields a $1/f$ spectrum, according to the Dutta Horn model. The UCF noise is found in a crystal with a high degree of disorder at very low temperatures.

The local interference model (LI) applies to electron waves singly scattered by a few neighboring defects. A special case of LI is the two-level system (TLS), where the scatterer moves from one position to an energetically equivalent second position, by tunneling through an energy barrier. Also in the LI model, the defects move with nearly the same activation energy, giving a $1/f$ spectrum by summation of Lorentzians. The LI model predicts $1/f$ noise at room temperature in weakly disordered metals.

One might think that the degree of disorder required is not to be found in nearly perfect epitaxial semiconductor films. Even though this may be true, it cannot be used as an objection against the LI model. The noise intensity $\alpha$, is proportional to $n^2 n_{md}$, where $n$ is the electron density and $n_{md}$ the density of the mobile defects ((21) in [32]). In metals we find, therefore, $\alpha \propto n^2 n_{md} \propto A n_{md}$, where $A$ is the number of atoms per cm$^3$. In semiconductors we find $\alpha$ proportional to $n$, when we investigate differently doped samples made from the same host material, with the same $n_{md}$ in all samples. So there is no problem with the low value of $n_{md}/A$ in high-quality semiconductors. The problem is rather that $\alpha$ would depend on $n$, contrary to all experimental evidence. LI seems not to apply to semiconductors.

A second argument against LI is that in semiconductors the exponent of the spectrum is 1.0 not $1 \pm \Delta$, as would result from the use of the Dutta Horn procedure in the local interference model. We know of one interesting example where the local interference model might apply to semiconductors: proton irradiated GaAs at low temperature ($T < 150$ K). Ren found a practically temperature independent noise, proportional to the radiation dose. There is a small peak in $\ln \alpha$ versus $1/T$, agreeing with the slope $\gamma$, which is not exactly $-1$. Here (38) holds [34]. The temperatures of the peaks correspond to 0.35 eV.

Irradiation of samples that were originally doped with different donor concentrations show that this is mobility noise, since $\alpha \propto n^2_{md}$ [35]. However, the quadratic dependence is characteristic of lattice scattering, whereas the local interference model is based on impurity scattering.

B. Quantum $1/f$ Noise

All three models UCF, LI, and TLS deal with interference of waves scattered at many centers. When the spatial arrangement of the centers changes, we observe a change in the conductance, noise. Handel's model is more general: there
are 1/f fluctuations in each scattering event at each individual scattering center [36]. In an excellent review paper [37] van Vliet does away with many later additions of the original model, but the essential idea still stands: interference between the Bremsstrahlung and non-Bremsstrahlung part of the electron wave function.

In the scattering process a low-frequency photon is absorbed or emitted. The wave length of such a photon is much longer than the dimensions of the samples and of Faraday cages. The question is: can such photons be present or develop in this limited space? This is the so-called cage effect, which is not accepted as a serious problem by van Vliet [38]. This may be correct, but then there is no way of coming to grips experimentally with these photons. "They are lost to the universe." The model is so general that there are no specific features that lead themselves to experimental confirmation. Due to this general nature and because of the most characteristic participants—the low-frequency photons—cannot be studied, nothing else is left to us than to compare numerical results of model and experiments. Handel’s model predicts the following \( \alpha \) values (see Table 1).

Handel cites some experimental \( \alpha \) values to support the theoretical results. All experimental values, but one, are derived from noise studies of devices. The exception is Bisschop’s work on polysilicon. His values 10\(^{-9} \) – 10\(^{-8}\) are, however, not \( \alpha \) values at all [42].

Comparison of the theoretical values with the experimental values, as shown in Fig. 5, leads to the conclusion that the experimental results do not support Handel’s theory. It could very well be that the theoretical model correctly predicts some kind of 1/f noise, but then this type of 1/f noise is different from the observed noise with a much higher \( \alpha \).

### VI. Empirical Values

In proposing a model for the 1/f noise in devices, two kinds of assumptions for the sources must be made. These are assumptions about 1) their physical nature: \( \Delta n \) or \( \Delta \mu \), isolated or interacting traps, bulk or surface states, etc., 2) the numerical value of \( \alpha \) (when mobility fluctuations are considered).

Because of the latter reason we present experimental values of \( \alpha \) here in Fig. 5 taken from literature. Only results for semiconductors, homogeneous samples and room temperature are included. We did not reject any data that do not fit in the general picture or that run against our own ideas about what \( \alpha \) should be.

Open symbols are used for \( \alpha_{\text{meas}} \), whereas black symbols are used for \( \alpha_{\text{latt}} \). A number next to a symbol refers to the original literature, unless the value is included in an earlier survey [9], [10] and [43]. If no number is given at a point for \( \alpha_{\text{latt}} \), it means that \( \alpha_{\text{latt}} \) was calculated by us from \( \alpha_{\text{meas}} \) given by the original author.

Some of the Si data are much older than the \( n \)-GaAs data. Many GaAs data stem from modern epitaxial material. Today we are much better informed on \( n \)-GaAs than on Si. This is largely due to the work of Ren [44], [45] on epitaxial \( n \)-GaAs, where a large series of samples with the same geometry but different doping levels and different scattering mechanisms were investigated at different temperatures. All data could be summarized in one single expression

\[
\alpha_{\text{latt}} = 0.1 \exp[-0.13 \text{ eV/kT}] + 7 \times 10^{-5}. \tag{39}
\]

A similar study on Si is badly needed. (This study cannot be expected from the Eindhoven group, since we concentrate on III-V compounds in accordance with national science policy.) Fig. 5 shows that \( \alpha_{\text{latt}} \) values are in the range of 10\(^{-5}\) to 10\(^{-3}\). \( \alpha_{\text{meas}} \) can be 10 times lower: \( \alpha_{\text{meas}} \) is in the range of 10\(^{-6}\) to 10\(^{-5}\). If we now have to name one single value, that can be said to be reasonably representative of \( \alpha_{\text{latt}} \) in semiconductors at room temperature, then it is \( \alpha = 10^{-4} \) instead of the historical value 2 \times 10^{-3} of 25 years ago.

In many publications it is stated—often triumphantly—that the low values of \( \alpha \), which are reported there, prove the high quality of the material used by the authors. This is not necessarily correct when \( \alpha_{\text{meas}} \) is considered. The value of \( \alpha_{\text{meas}} \) can be low for two reasons [43]: 1) \( \alpha_{\text{latt}} \) is low, 2) \( \mu_{\text{meas}} < \mu_{\text{latt}} \).

The value of \( \alpha_{\text{meas}} \) is low if scattering mechanisms other than lattice scattering prevail. The factor \( (\mu_{\text{meas}}/\mu_{\text{latt}})^2 \) in (13) then is smaller than 1, and will give a low value of \( \alpha_{\text{meas}} \), even when \( \alpha_{\text{latt}} \) is not low. Only if \( \alpha_{\text{latt}} \) itself is low, it can safely be concluded that the crystal lattice is perfect.

Damaging the crystal by mechanical stress or by irradiation strongly increases the 1/f noise. The electron concentration and the mobility hardly change; nevertheless the noise increases by orders of magnitude. The first explanation

### Table 1

<table>
<thead>
<tr>
<th>( \alpha ) in 10(^{-9} )</th>
<th>Si</th>
<th>GaAs</th>
</tr>
</thead>
<tbody>
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<td>3</td>
<td>6</td>
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<td>phonon</td>
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</tr>
<tr>
<td>phonon</td>
<td>1200</td>
<td>300-3000</td>
</tr>
<tr>
<td>impurity</td>
<td>2-5</td>
<td>70</td>
</tr>
</tbody>
</table>
that comes to mind is that the defects act as generation-recombination centers which in some, as yet unexplained, way generate 1/f noise. If this is correct then the induced noise is a fluctuation in the number of free carriers. However, it might also be possible that the defects act as scattering centers. If they are mobile, they will generate 1/f noise according to the local interference model. Therefore, one would like to see further investigations of the induced noise in damaged material, e.g., a plot of log $\mu_{\text{induced}}$ versus log $\mu_{\text{meas}}$ (like Fig. 1), because that would decide whether the induced noise is mobility noise or number noise. In case of mobility noise it is important to distinguish between impurity scattering—agreeing with the LI model—and lattice scattering, as has possibly been found with proton-irradiated GaAs [35].

The relation between 1/f noise and damage has been critically reviewed in an extensive review by D’yakonova, Levinstein, and Rumyantsev [28]. The model favored by the authors is that the defects create the states in the tail below the conduction band. There is experimental support for this model and this survey, follow the lines of thinking of the survey [46]-[52]. It is immediately assumed by all authors—except one—that the induced noise is number noise. Based on this assumption some model for the generation-recombination centers is then presented without experimental evidence confirming the number fluctuations and excluding mobility fluctuations. The exception is Ren [35] who plotted $\alpha_{\text{meas}}$ versus $\mu_{\text{meas}}$ of the 1/f noise induced by proton irradiation of GaAs. He found that the noise was mobility noise.

VII. CONCLUSION

1) In all semiconductors there always is mobility 1/f noise with an $\alpha$ value of about $10^{-4}$. Other types of 1/f noise may be present and may dominate the mobility noise.

2) $\alpha_{\text{meas}} = \left(\frac{\mu_{\text{meas}}}{\mu_{\text{latt}}}\right)^2 \alpha_{\text{latt}}$

where $\alpha_{\text{meas}}$ directly follows from the empirical relation (7) and $\alpha_{\text{latt}}$ is the value that would have been found if lattice scattering only were present. This means that the lattice scattering is the origin of the mobility 1/f noise.

3) Damaging the crystal increases the 1/f noise considerably, whereas the mobility hardly decreases.

4) Each of the two values, $\alpha_{\text{latt}}$ and $\alpha_{\text{meas}}$, has its own field of application. We need $\alpha_{\text{meas}}$ if we want to propose a noise model of a device. We need $\alpha_{\text{latt}}$ in comparisons with theoretical predictions and in assessments of the quality of semiconductor material.

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F. N. Hooge was born in Amsterdam in 1930. He studied chemistry at the University of Amsterdam, where he received the Doctor’s degree in 1951. He then joined Philips Research Laboratory in Eindhoven to work on semiconductors, and since 1968 he has been studying 1/f noise. In 1971 he was appointed Professor of electronic materials at the Department of Electrical Engineering at Eindhoven University of Technology (EUT). From 1983 to 1985 he was Dean of the department and from 1985 to 1988 was Rector Magnificus of EUT.

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