Measurement of low-frequency base and collector current noise and coherence in SiGe heterojunction bipolar transistors using transimpedance amplifiers

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Measurement of Low-Frequency Base and Collector Current Noise and Coherence in SiGe Heterojunction Bipolar Transistors Using Transimpedance Amplifiers

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Abstract—Transimpedance amplifiers have been used for direct study of current noise in silicon germanium (SiGe) heterojunction bipolar transistors (HBT's) at different biasing conditions. This has facilitated a wider range of resistances in the measurement circuit around the transistor than is possible when using a voltage amplifier for the same kind of measurements. The ac current amplification factor $10^4$ and the sum of the base and emitter series resistances $40$ and $114$ have been extracted from the noise. It has been established that the dominant noise source is situated in the base emitter junction at the emitter side and is not related to contact resistance noise. The simultaneous measurement of both the base-lead noise and the collector-lead noise and the calculation of the coherence between the signals has facilitated the pinpointing of the dominant noise source in the device and the extraction of $40$ and $114$.

Index Terms—HBT, low-frequency noise, silicon germanium (SiGe), transimpedance amplifiers.

I. INTRODUCTION

The major source of phase noise in microwave oscillators is the transistor low-frequency noise which modulates the carrier [1]. Therefore low-frequency investigations on prototype microwave transistors are a first step in trying to optimize the phase noise of an oscillator. The general approach up to now for studying the $1/f$ noise in bipolar transistors is to use voltage amplifiers to measure the voltage fluctuations over a resistor which forms part of the measurement setup. By dividing the fluctuations in the voltage drop by the resistance value, a corresponding current noise is calculated. A first successful attempt has been carried out [2], [3] to use low-noise voltage amplifiers in Si bipolar transistors. Their configuration limits however the range of source and load resistances that can be used within the measurement. Their configuration limits however the range of source and load resistances that can be used within the measurement. The present work aims at studying the current noise in base and collector current and their coherence in silicon germanium (SiGe) heterojunction bipolar transistors (HBT's) in a direct way, making use of ultra low-noise transimpedance amplifiers with or without internal biasing possibilities for measuring the current noise. These particular devices have been chosen for their pronounced noise in order to facilitate distinct measurements. Analysing the results shows where the dominant noise source is located. We will point out benefits and shortcomings with this technique.

II. THEORY

The noise sources of a bipolar transistor can schematically be presented as in Fig. 1 where the representation is made using resistance and current noise sources. $S_{r_B}, S_{r_E}, S_{I_B},$ and $S_{I_C}$ are the resistance fluctuations of the base and emitter resistances and the current fluctuation in the base-emitter junction and in the collector, respectively. Their configuration limits however the range of source and load resistances that can be used within the measurement. The present work aims at studying the current noise in base and collector current and their coherence in silicon germanium (SiGe) heterojunction bipolar transistors (HBT's) in a direct way, making use of ultra low-noise transimpedance amplifiers with or without internal biasing possibilities for measuring the current noise. These particular devices have been chosen for their pronounced noise in order to facilitate distinct measurements. Analysing the results shows where the dominant noise source is located. We will point out benefits and shortcomings with this technique.

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$$S_{I_B} = 2\mu I_B + S_{I_B}^{1/f} + S_{I_B}^{GR}$$  
(1)

$$S_{I_C} = 2\mu I_C + S_{I_C}^{1/f} + S_{I_C}^{GR}.$$  
(2)
Similarly, in general the sources due to conductance fluctuations \( (S_{r_b} \text{ and } S_{r_e}) \) can be composed of GR noise, random telegraph signal (RTS) noise and \( 1/f \) noise. The resistances \( r_b \text{ and } r_e \) also contribute with Nyquist noise. The Nyquist noise can be seen as voltage sources in series with \( I_B^2 S_{r_b} \) and \( I_E^2 S_{r_e} \), but due to the heavy doping of both the base and the emitter, facilitated by the heterostructure, this noise is very low and has not been detected in measurements. They are thus not included in Fig. 1. Also the GR and RTS noise have not been observed and are therefore ignored in the discussion. From the equivalent circuit in Fig. 1 the theoretical expressions for the measured noise current spectra in the base \( (S_{I_{BE}}) \) and the collector-lead \( (S_{I_{CE}}) \) have been derived in the Appendix and are expressed as

\[
S_{I_{BE}} = \frac{1}{2}\left[ I_B^2 S_{r_b} + r^2 S_{I_B} + I_E^2 S_{r_e} + r_S^2 S_{I_C} \right] \\
S_{I_{CE}} = \frac{1}{2}\left[ I_B^2 S_{r_b} + Z_S^2 S_{I_B} + I_E^2 S_{r_e} \right. \\
+ \frac{Z}{Z_{S}} \left. S_{I_{CE}} \right] \\
\]

(3) (4)

The correlated spectrum between the two signals is expressed as

\[
S_{I_{BE}I_{CE}} = \frac{h_{fe}}{2}\left[ I_B^2 S_{r_b} - r \pi r_e S_{I_B} + I_E^2 S_{r_e} \right. \\
- \frac{(Z + r_b + r_e) r_e}{h_{fe}} S_{I_{CE}} \right] \\
\]

(5)

where

\[
Z = (R_S + r_b + r_e + (1 + h_{fe}) r_e) \\
Z_S = (R_S + r_b + r_e) = Z - r \pi - h_{fe} r_e \\
I_B = (1 + \beta) I_B.
\]

(6) (7)

Normally, in the active region, \( (r_b/\text{mean}) \approx 1 \); and \( \beta \) is the dc part of the current amplification factor. The minus sign in (5) is related to the different directions of the currents that this source generates in the two measurement leads. For the coherence defined as (the full expression can be found in the Appendix)

\[
\gamma^2 \text{ def } \frac{S_{I_{BE}I_{CE}}^2}{S_{I_{BE}} \cdot S_{I_{CE}}}
\]

(8)

holds \( \gamma^2 = 1 \) when a single noise source is dominant in both measurement leads, independent of \( R_S \) and \( R_I \).

In practice, the correlated spectrum \( (S_{I_{BE}I_{CE}}) \) is calculated through a multiplication in the frequency domain between the complex Fourier transforms of the the two measured signals whose spectral power densities are \( S_{I_{BE}} \) and \( S_{I_{CE}} \). The coherence function is then calculated using its definition in (8).

### III. Measurement Setup

Using a measurement setup like the one in Fig. 2(a), current noise can be directly measured for a variety of lead resistances of the base \( [R_S \text{ in Fig. 1 and Fig. 2(a)]. Also the lead resistance of the collector can be varied, in order to find information regarding the dynamic output resistance. In this investigation the external resistance in the collector lead has been held as low as possible which means that the nonnegligible internal resistance of the transimpedance amplifier in this lead has been dominating the resistance. This ensures that the whole noise current is measured. Due to the very low feedback from collector to base, the variation in impedance when changing range setting of the amplifier in the collector-lead does not affect the electrical behavior of the circuit and can thus be neglected.

Assuming the noise sources as shown in Fig. 1, the measured noise in the base-lead \( (S_{I_{BE}}) \) and in the collector-lead \( (S_{I_{CE}}) \) can be used for extracting information regarding the different noise sources within the device. Some advantages and drawbacks of transimpedance amplifiers will be discussed.

In order to keep control of the measurements, two different types of transimpedance amplifiers have been used for converting the measured current to a corresponding voltage: Brookdeal 5002 and Stanford Research Systems SR570. The latter facilitates internal voltage bias with a high resolution which can be controlled by a computer. This showed very useful when performing sweeps of the base current. However, caution must be taken not to unintentionally induce noise in the system by using this internal voltage source. The Brookdeal 5002 does not have an internal biasing and induces very little noise into the measurement setup. An external voltage source with variable ac impedance [see Fig. 2(b)] has been realized, mainly based on a battery, a high-quality potentiometer and
a variable resistance in series with a large capacitance. If
the measuring of dc part of the current can be omitted,
the transimpedance amplifier can be inserted in direct series
with the capacitance and the variable resistance which can
enable a higher amplification factor as the amplifier does not
need to sink the dc current. A higher amplification factor
results however also in a higher internal impedance within
the amplifier which must be accounted for.

Measuring the noise currents directly with transimpedance
amplifiers enables us to use a large range in source resistances.
The only limitation is that the impedance of the measured
circuit should be larger than the input impedance of the current
amplifier. This is easily obtained with transistors where the
dynamic resistance always is much higher than the input
impedance of the amplifier for all current levels. This enables
simultaneous measurements of both the base and collector
current noise and calculation of their coherence directly.

The measurements would be very difficult to perform using
voltage amplifiers at low values of the source resistances since
the signal would be drowned in the Nyquist noise of the
resistor itself and the background noise of the amplifier. As
an example, a measurement of the shot noise level, calculated
as \(2qI\), using a 1 k\(\Omega\) resistor and a voltage amplifier means
the current \(I\) must be higher than 50 \(\mu\)A in order not to
be drowned by the internal Nyquist noise of the resistor.

This is a high current for small devices. Lower current
noise levels would thus not be possible to detect without
severely degrading the coherence calculation between the two
measured signals due to uncorrelated noise contributions of
the resistances \(R_L\) and \(R_S\).

By changing the sensitivity of the transimpedance amplifier,
its internal impedance changes which appears as a variation
in source impedance for the device under test. Attention to
this behavior is given during measurements of current sweeps
since both \(S_{IBE}\) and \(S_{IBC}\) can be affected by variation in
impedance.

High current sensitivity goes hand in hand with a higher
input impedance and a lower bandwidth. In addition to a
limited bandwidth, the transimpedance amplifier exhibits an
inductive behavior at the input which can introduce a re-
sonance with a (parasitic) capacitance at the input. For this
work, either a frequency of 20, 25, or 170 Hz has been chosen
when comparing results of different bias conditions; mainly
in order to avoid disturbance from harmonics of the 50 Hz
power line signal, background noise and bandwidth problems.
The reason for choosing the higher frequency of 170 Hz is
due to the series capacitance for the ac impedance in the base
lead [see Fig. 2(b)].

The internal bias supply of the SR570 amplifier enables a
very compact solution for the measurement setup which makes
it less prone to picking up undesired signals. However, owing
to the very high open loop gain of the amplifier in the internal
transimpedance stage, any fluctuation in the reference voltage
for the bias will appear as an almost ideal voltage noise source
at, e.g., the base of the transistor. This will affect both \(S_{IBE}\)
and \(S_{ICE}\) and in worst case induce an undesired full coherence
between the two measured noise signals. The same undesired
noise source can also be unevenly partitioned between the
two channels, especially for low values of \(R_S\), where \(S_{ICE}\) is
affected, which can result in a low coherence. Here the results
of coherence between \(S_{IBE}\) and \(S_{ICE}\) without measurement
artifacts will be presented by the usage of the Brookdeal 5002
where the SR570 is found insufficient after comparing the
results with a Brookdeal 5002.

IV. RESULTS

Measured current noise spectra of a 20-\(\mu m^2\) device for the
base-lead and collector-lead is shown in Fig. 3. The curves
indicate a spectra with \(1/f^{0.8}\) dependence below 10 kHz
which is in accordance with results for similar devices [4].
The level of the noise is too high for any white noise to
be seen within the measured frequency range. Furthermore,
the noise measurements at low currents are limited by the
bandwidth of the amplifier which can be seen as a distinct
roll-off above 10 kHz. A plot of coherence between the two
noise spectra measured at any given base current shows a value
very close to unity, see Fig. 3. This indicates that the measured
noise originates from a common source, thus excluding any
dominant source in the collector-lead and hence \(S_{IB}, S_{IR},\) or
\(S_{IC}\) are the possible candidates.

In order to discriminate, we measure the current spectra at
approximately constant values of base and collector current
for a range of source resistances (\(R_S\) in Fig. 2). We deduce
from (4) and (7) and our experimental results that \(S_{IB}\) is
the single dominant source for almost all values of \(R_S\). A more
thorough explanation will follow.

As can be seen in Fig. 4, \(S_{ICE}\) shows a square depen-
dence on \(R_S\) for values of 300 \(\Omega\) \(\leq R_S \leq \pi\) which indicates
strongly that \(S_{IB}\) can be the only source, as can be seen
from the second term in (4). All other components in (4)
are for these values of \(R_S\) either independent of, or only
very weakly dependent on \(R_S\). In this region of \(R_S\), the
coherence (\(\gamma\)) is very close to unity, thus indicating that
the same source is totally dominant also in \(S_{IBE}\). This means
however that \(S_{IB}\) is dominant in \(S_{IBE}\) for all values of \(R_S\)
since the interdependence between the different terms in \(S_{IBE}\)
is independent of \(R_S\), [see (3)].

For low values of \(R_S\), the coherence goes down (see Fig. 4)
which indicates that at least one additional source (here called
\(S_X\)) becomes significant in \(S_{ICE}\). The origin of this source
is difficult to distinguish as the measured noise levels at these
values of \(R_S\) are very low and thus external noise may enter,
but from (5) or (36), shown at the bottom of the page, it can
be indicated that \(S_{IX}\) is a candidate since \(\gamma^2\) never drops to
zero. To pinpoint the source, a more thorough investigation
would however be necessary which is beyond the scope of
this paper. Assuming that this source is independent of \(R_S\)
and bearing in mind that \(S_{IBE}\) is completely dominated by
\(S_{IB}\), \(S_X\) is calculated as

\[
S_X = (1 - \gamma^2)S_{ICE}.
\]

The coherence does however not reach zero for the given
range of \(R_S\) thus indicating that \(S_{IB}\) still contributes to \(S_{ICE}\).
This level is calculated (here called \(S_{ICE}\)) as

\[
S_{ICE} = \gamma^2 S_{ICE}.
\]
Fig. 3. Measured current noise spectrum through the base and collector-lead for a 20 μm² device. Coherence between the two signals is included. The source resistance is set to $R_S = 10$ kΩ and $I_B$ was set to 1 μA. The roll-off at higher frequencies for the curves is due to the limited bandwidth of the transimpedance amplifiers.

Fig. 4. Measured and calculated data of $S_{ICE}$ versus $R_S$ for a 24 μm² device at $V_{CE} = 1.5$ V, $I_B = 2$ μA and $f = 170$ Hz. Calculated data ($S_{ICE\text{calculated}}$) using (13). Coherence is also included.

and the value of $(R_S + \eta_b + \eta_e)$ can be extracted as a factor in front of $S_{IB}$, see (3). Note that in this region, $R_S \ll r_\pi$.

For the extraction, a measurement (here called $S_{ICE_{P_2}}$) of $S_{ICE}$ is taken where $S_{IB}$ is dominant and $R_S \gg (\eta_b + \eta_e)$. The quotient between $S_{ICE_{P_1}}$ and $S_{ICE_{P_2}}$ will then be (neglecting insignificant terms)

$$\frac{S_{ICE_{P_1}}}{S_{ICE_{P_2}}} = \left(\frac{(R_S + \eta_b + \eta_e)(R_S + r_\pi)}{R_S r_\pi}\right)^2 \tag{11}$$

and $(\eta_b + \eta_e)$ is found as

$$(\eta_b + \eta_e) = \sqrt{\frac{S_{ICE_{P_1}}}{S_{ICE_{P_2}}} \left(\frac{R_S r_\pi}{R_S + r_\pi}\right)} - R_S \tag{12}$$

where $R_S$ and $R_S$ are the values of $R_S$ for the measurements of $S_{ICE_{P_1}}$ and $S_{ICE_{P_2}}$, respectively. An average of the extracted values for $R_S = 1$, 11, 19, and 34 Ω (indicating values of $S_{ICE}$ where the coherence clearly differs from unity) and $R_S = 3.4$ kΩ gives $S_X = 4.4 \cdot 10^{-24}$ A²/Hz at $f = 170$ Hz and $(\eta_b + \eta_e) = 28$ Ω.

Under the assumption that only the sources $S_{IB}$ and $S_X$ contribute to $S_{ICE}$, a simplified expression is found as

$$S_{ICE_{calculated}} = S_{ICE_0} \left(\frac{Z^*}{Z}\right)^2 + S_X. \tag{13}$$

Thus, only one additional parameter, here called $S_{ICE_0}$, needs to be extracted to be able to calculate $S_{ICE}$. From (4) it
is found that this corresponds to $h_{be}^2 S_{IB}$. However, in order to gain as much information as possible from the measured $S_{ICE}$ and not include uncertainty from additional circuit parameters, a calculation of this term is omitted in favor of a direct extraction. In a region where $(\tau_e + r_e) \ll R_S < r_\pi/2$, $S_{IB}$ is dominant, $S_{ICE0}$ is found as

$$S_{ICE0} = S_{ICE} \left( \frac{R_S + r_\pi}{R_S + (\tau_e + r_e)} \right)^2. \quad (14)$$

For $300 \Omega \leq R_S \leq 6 \kOmega$ and $f = 170$ Hz an average of $S_{ICE0}$ was found to be $1.8 \cdot 10^{-16} \text{A}^2/\text{Hz}$. This compares very well with the calculated value found using extracted $h_{be}$ and $S_{IB}$.

The curve of $S_{ICE_{calcualted}}$ has been included in Fig. 4 as a full line and compares very well with measured data (dots), thus showing that the assumptions are correct. From the shape of $S_{ICE}$, it can be seen that there is a lower limit to which there is any point in setting the biasing impedance, $R_S$, and this is above the value of $(\tau_e + r_e)$. Furthermore, as biasing impedance increases, $S_{ICE}$ rolls off, indicating that bias impedances chosen above the value of $r_\pi$ does not further increase $S_{ICE}$. An implication of this is that devices with a high $\beta$ will have a high range of $S_{ICE}$ depending on which biasing impedance is chosen since $I_B$ here is small and thus $r_\pi$ becomes large, giving a large span of values between $(\tau_e + r_e)$ and $r_\pi$.

Using (4) and (5), a value for $h_{be}$ is found as

$$h_{be} = \frac{S_{ICE}}{S_{IB_{EL}}} \frac{\tau_e}{R_S} \quad (15)$$

assuming $R_S \gg (1 + h_{be})r_e$, $S_{IB}$ is the dominant noise source and $(\tau_o/(R_L + \tau_o)) \rightarrow 1$. Extracted values using (15) and data from the Gummel plot are compared in Table I.

A value of $S_{IB}$ is calculated as

$$S_{IB} = S_{IBE} \frac{(R_S + r_\pi)^2}{r_\pi^2} \quad (16)$$

for values of $R_S \gg (\tau_e + (1 + h_{be})r_e)$. In particular, for $(\tau_e + (1 + h_{be})r_e) \ll R_S \ll r_\pi$ we have $S_{IB} \approx S_{IBE}$.

A dependence on base current of approximately $I_B^{-1}$ has been measured throughout the available bias range, see Fig. 5. For currents below 250 nA, the frequency dependence if the noise is weaker and bandwidth limitations play a role. The reduced frequency dependence of $S_{IB}$ in conjunction with the current dependence indicate that the noise source is not of pure $1/f$-type and any real predictions on corner frequencies where the low-frequency noise disappears in the shot noise are difficult to make [5].

The current dependence of $S_{IB}$ gives space for interpretations of the origin of the noise. As shown earlier, the noise originates within the base-emitter junction which here means not in the base resistance and not in the emitter resistance. Assuming a true area dependent effect, the mobility fluctuation model or the number fluctuation model seem to be reasonable candidates. The Kleinpenning model considers mobility fluctuations and hence fluctuations in the diffusion coefficient for the minority carriers. A simplified relation, which indicates a linear dependence of $S_{IB}$ on current, is given as follows [6]:

$$S_{IB} = \frac{2e\mu I_B D_P}{fW_E^2} \quad (17)$$

where $D_P$ is the hole diffusion coefficient in the emitter of an NPN transistor and $W_E$ is the length of the undepleted part of the emitter. With $D_P = 5 \text{cm}^2/\text{s}$, $W_E = 1.2 \times 10^{-5}$ cm and

---

Table I

<table>
<thead>
<tr>
<th>$I_B$</th>
<th>$h_{be}$</th>
<th>$\frac{S_{IB}}{h_{be}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 nA</td>
<td>35</td>
<td>38</td>
</tr>
<tr>
<td>200 nA</td>
<td>48</td>
<td>58</td>
</tr>
<tr>
<td>1 μA</td>
<td>64</td>
<td>79</td>
</tr>
<tr>
<td>2.5 μA</td>
<td>110</td>
<td>108</td>
</tr>
<tr>
<td>10 μA</td>
<td>120</td>
<td>177</td>
</tr>
</tbody>
</table>
Hz, calculated for $I_B = 250$ nA and $I_B = 25$ µA give values of $5 \times 10^{-5}$ and $2 \times 10^{-3}$, respectively, which is within reasonable limits [7]. Due to the variation in base current, the depletion region in the emitter vary and thus also the region where the fluctuation in mobility will be of greatest importance. The fact that $\alpha$ is nonuniform can speak for a position dependence on $\alpha$ in the $W_E$ region just at the rim of the depletion region [8].

The number fluctuation model considers a fluctuation in the surface recombination velocity, $s$. With the relation for short diodes [9]

$$\frac{S_I}{f^2} = C \cdot S_s$$

(18)

where $C$ is independent of current and $S_s$ is the spectral density of the fluctuations in $s$ with a $1/f$-type of spectrum. It can be seen that this model predicts a square-law dependence on current just like series resistance fluctuations. This model thus fits to the “near-square-law” dependence of $S_{IB}$ which has been observed.

V. CONCLUSION

We have used transimpedance amplifiers for direct study of current noise in a silicon germanium HBT at different biasing conditions. The possibility of measuring the device using a wider range of external resistances than is possible when using a voltage amplifier has facilitated extraction of the ac current amplification factor $h_{fe}$ and the sum of the base and emitter series resistances $(r_b + r_e)$. The simultaneous measurement of both the base-lead noise and the collector-lead noise has enabled a calculation of the coherence between the signals. From the high coherence and the dependence of collector-lead noise on source resistance, it has been established that the dominant noise source is situated in the base emitter junction, in the $W_E$ region at the emitter side and not in the contact resistances $r_o$ and $r_e$. The use of the coherence function has been essential for the discrimination of the dominant noise source and for the extraction of $(r_b + r_e)$.

APPENDIX

DERIVATION OF $S_{IC'E}, S_{IB'E},$ AND $S_{IB'EICE'}$ AND $\gamma^2$ AND ESTIMATION OF THE IMPACT OF $S_{IC}$ ON THE MEASUREMENTS

This Appendix gives a more thorough description of how the important expressions for $S_{IC'E}, S_{IB'E},$ and $S_{IB'EICE'}$ and $\gamma^2$ [(3)–(5) and (8)] are derived. The expressions will be derived in steps, concentrating on one source at a time and finally collecting the different parts to the full expressions.

A. Derivation of $S_{IC'E}, S_{IB'E},$ and $S_{IB'EICE'}$ with Respect to $S_{IC}$

The emphasis is first put on finding the impact of a noise source in the collector lead on the noise in the base lead. From Fig. 1, we define the following currents when $S_{IC}$ is the only contributor:

$$\begin{align*}
\dot{i}_b &= -i_{SC} + \frac{r_o}{R_L + (R_S + r_b + r_e) + h_{fe}} \\
\dot{j} &= i_{SC} \left( \frac{r_e}{R_S + r_b + r_e} \right)
\end{align*}$$

(19)

Elimination then gives (20) and (21), shown at the bottom of the page, where

$$
\frac{(R_S + r_b + r_e) + h_{fe}}{(R_S + r_b + r_e) + h_{fe}} = \frac{(R_S + r_b + r_e) + h_{fe}}{(R_S + r_b + r_e) + h_{fe}} .
$$

(22)

For a common emitter configuration, $r_o, r_e \gg r_e$ and $R_L$ can be chosen very small and it is thus possible to assume that $r_o, r_e \gg (r_e + R_L)$, thus

$$\begin{align*}
\dot{i}_b &= -i_{SC} \frac{r_e}{Z} \\
\dot{j} &= i_{SC} \frac{R_S + r_b + r_e}{Z}
\end{align*}$$

(23)

(24)

where

$$Z = (R_S + r_b + r_e + (1 + h_{fe}) r_e) .$$

(25)

It can be seen in Fig. 1 that $\dot{j}$ is the same as the current flowing through $R_L$. Straightforward circuit analysis thus gives

$$\begin{align*}
S_{IB'E} &= \frac{r_e^2 S_{IC}}{Z^2} \\
S_{IC'E} &= \frac{h_{fe}^2 (R_S + r_b + r_e + r_e)^2}{h_{fe}^2} \cdot
\end{align*}$$

(26)

Again it has been assumed that $r_o \gg (r_e + R_L)$.

The correlated spectrum between the two signals is expressed as [in the text (5)]

$$S_{IB'EICE'} = \frac{h_{fe}^2 (R_S + r_b + r_e + r_e)^2}{h_{fe}^2} S_{IC'} .$$

(27)

Can the source in the collector lead (i.e., $S_{IC}$) be dominant?

If this is the case, we should find for the quotient between the power spectral densities of the measured signals

$$\frac{S_{IC'E}}{S_{IB'E}} = \left( \frac{R_S + r_b + r_e + r_e}{r_e} \right)^2 .$$

(28)
\[
\gamma^2 = \frac{[P_B S_{rb} - r_{\pi} Z'_S S_{IB} + P_E S_{re} - (R_S + r_b + r_{\pi} + r_e) r_e S_{IC}]^2 \left[ P_B S_{rb} + (r_{\pi} + h_{be} r_e)^2 S_{IB} + P_E S_{re} + r_e^2 S_{IC} \right]}{[P_B S_{rb} + (r_{\pi} + h_{be} r_e)^2 S_{IB} + P_E S_{re} + r_e^2 S_{IC}]}.
\]

which yields
\[
S_{IBE} = S_{IB} \frac{(r_{\pi} + h_{be} r_e)^2}{Z^2}.
\]

and in total for all sources
\[
S_{IBE} = \frac{1}{Z^2} \left[ P_B^2 S_{rb} + (r_{\pi} + h_{be} r_e)^2 S_{IB} + P_E^2 S_{re} + r_e^2 S_{IC} \right].
\]

Continuing with \( S_{IC} \), using the already found expressions ([26] and [28]), the impact on \( S_{IC} \) from \( S_{rb} \) and \( S_{re} \) can be found by noticing that in these cases \(-i_{be} = \dot{i}_b\) and the current through \( R_S \) is \( i_b h_{be} \). The impact of \( S_{IB} \) on \( S_{IC} \) can be found by using (30) and (32)
\[
\dot{i}_b = -i_{Steb} \left[ \frac{R_S + r_b + r_e}{Z} \right].
\]

Thus, \( S_{IC} \) will be
\[
S_{IC} = \frac{h_{be}}{Z^2} \left[ \frac{P_B^2 S_{rb} + Z'_S^2 S_{IB} + P_E^2 S_{re}}{h_{be}^2} \right],
\]

where
\[
Z'_S = (R_S + r_b + r_e)
\]

and \( S_{IBE^L} \) can be found as (noticing the direction of the different current components)
\[
S_{IBE^L} = \frac{h_{be}}{Z^2} \left[ \frac{P_B^2 S_{rb} - r_{\pi} Z'_S S_{IB} + P_E^2 S_{re}}{h_{be}^2} \right].
\]

\[\text{C. The Full Expression of } \gamma^2\]

Given the expressions for \( S_{IBE} \), \( S_{IC} \), and \( S_{IBE^L} \), the expression for \( \gamma^2 \) can be calculated from its definition (8) and is found to be (36), shown at the top of the page.

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