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Measurement of Low-Frequency Base and Collector Current Noise and Coherence in SiGe Heterojunction Bipolar Transistors Using Transimpedance Amplifiers

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Abstract—Transimpedance amplifiers have been used for direct study of current noise in silicon germanium (SiGe) heterojunction bipolar transistors (HBT's) at different biasing conditions. This has facilitated a wider range of resistances in the measurement circuit around the transistor than is possible when using a voltage amplifier for the same kind of measurements. The ac current amplification factor h_{fe} and the sum of the base and emitter series resistances ($r_b + r_e$) have been extracted from the noise. It has been established that the dominant noise source is situated in the base emitter junction at the emitter side and is not related to contact resistance noise. The simultaneous measurement of both the base-lead noise and the collector-lead noise and the calculation of the coherence between the signals has facilitated the pinpointing of the dominant noise source in the device and the extraction of ($r_b + r_e$).

Index Terms—HBT, low-frequency noise, silicon germanium (SiGe), transimpedance amplifiers.

I. INTRODUCTION

THE major source of phase noise in microwave oscillators is the transistor low-frequency noise which modulates the carrier [1]. Therefore low-frequency investigations on prototype microwave transistors are a first step in trying to optimize the phase noise of an oscillator. The general approach up to now for studying the $1/f$ noise in bipolar transistors is to use voltage amplifiers to measure the voltage fluctuations over a resistor which forms part of the measurement setup. By dividing the fluctuations in the voltage drop by the resistance value, a corresponding current noise is calculated. A first successful attempt has been carried out [2], [3] to use low-noise voltage amplifiers in Si bipolar transistors. Their configuration limits however the range of source and load resistances that can be used within the measurement. The present work aims at studying the current noise in base and collector current and their coherence in silicon germanium (SiGe) heterojunction bipolar transistors (HBT's) in a direct way, making use of ultra

Fig. 1. Equivalent small signal circuit for an HBT in a common emitter measurement setup indicating the fundamental noise sources. The circuit inside the dotted rectangle corresponds to the transistor itself. I_B and I_E indicate steady state currents.

low-noise transimpedance amplifiers with or without internal biasing possibilities for measuring the current noise. These particular devices have been chosen for their pronounced noise in order to facilitate distinct measurements. Analysing the results shows where the dominant noise source is located. We will point out benefits and shortcomings with this technique.

II. THEORY

The noise sources of a bipolar transistor can schematically be presented as in Fig. 1 where the representation is made using resistance and current noise sources. S_{r_b} , S_{r_e} , S_{I_B} , and S_{I_C} are the resistance fluctuations of the base and emitter resistances and the current fluctuation in the base-emitter junction and in the collector, respectively. The steady state currents are indicated by I_B and I_E and cause a voltage fluctuation as a result of S_{r_b} and S_{r_e} ; h_{fe} is the ac current amplification factor; r_π and r_o are the dynamic input and output resistances of the transistor; r_b and r_e are the base and emitter resistances within the transistor and R_S and R_L are the source and load resistances within the measurement setup.

These sources can be further decomposed into different types of noise. S_{I_B} and S_{I_C} consist of shot noise, generation-recombination (GR) noise and $1/f$ noise

$$S_{I_B} = 2qI_B + S_{I_B}^{1/f} + S_{I_B}^{GR} \quad (1)$$

$$S_{I_C} = 2qI_C + S_{I_C}^{1/f} + S_{I_C}^{GR} \quad (2)$$

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Similarly, in general the sources due to conductance fluctuations (S_{r_b} and S_{r_e}) can be composed of GR noise, random telegraph signal (RTS) noise and $1/f$ noise. The resistances r_b and r_e also contribute with Nyquist noise. The Nyquist noise can be seen as voltage sources in series with $I_B^2 S_{r_b}$ and $I_E^2 S_{r_e}$, but due to the heavy doping of both the base and the emitter, facilitated by the heterostructure, this noise is very low and has not been detected in measurements. They are thus not included in Fig. 1. Also the GR and RTS noise have not been observed and are therefore ignored in the discussion. From the equivalent circuit in Fig. 1 the theoretical expressions for the measured noise current spectra in the base ($S_{I_{BE}}$) and the collector-lead ($S_{I_{CE}}$) have been derived in the Appendix and are expressed as

$$S_{I_{BE}} = \frac{1}{Z^2} [I_B^2 S_{r_b} + r_\pi^2 S_{I_B} + I_E^2 S_{r_e} + r_e^2 S_{I_C}] \quad (3)$$

$$S_{I_{CE}} = \frac{h_{fe}^2}{Z^2} \left[I_B^2 S_{r_b} + Z_S'^2 S_{I_B} + I_E^2 S_{r_e} + \frac{(R_S + r_b + r_\pi + r_e)^2}{h_{fe}^2} S_{I_C} \right]. \quad (4)$$

The correlated spectrum between the two signals is expressed as

$$S_{I_{BE}I_{CE}} = \frac{h_{fe}}{Z^2} \left[I_B^2 S_{r_b} - r_\pi Z_S' S_{I_B} + I_E^2 S_{r_e} - \frac{(R_S + r_b + r_\pi + r_e)r_e}{h_{fe}} S_{I_C} \right] \quad (5)$$

where

$$Z = (R_S + r_b + r_\pi + (1 + h_{fe})r_e) \quad (6)$$

$$Z_S' = (R_S + r_b + r_e) = Z - r_\pi - h_{fe}r_e \quad (7)$$

$$I_E = (1 + \beta)I_B.$$

Normally, in the active region, $(r_o/(R_L + r_o)) \approx 1$; and β is the dc part of the current amplification factor. The minus sign in (5) is related to the different directions of the currents that this source generates in the two measurement leads. For the coherence defined as (the full expression can be found in the Appendix)

$$\gamma^2 \stackrel{\text{def}}{=} \frac{S_{I_{BE}I_{CE}}^2}{S_{I_{BE}} \cdot S_{I_{CE}}} \quad (8)$$

holds $\gamma^2 = 1$ when a single noise source is dominant in both measurement leads, independent of R_S and R_L .

In practice, the correlated spectrum ($S_{I_{BE}I_{CE}}$) is calculated through a multiplication in the frequency domain between the complex Fourier transforms of the two measured signals whose spectral power densities are $S_{I_{BE}}$ and $S_{I_{CE}}$. The coherence function is then calculated using its definition in (8).

III. MEASUREMENT SETUP

Using a measurement setup like the one in Fig. 2(a), current noise can be directly measured for a variety of lead resistances of the base [R_S in Fig. 1 and Fig. 2(a)]. Also the lead resistance of the collector can be varied, in order to find information

Fig. 2. Measurement setup. (a) The current passing through the transimpedance amplifiers is converted to an equivalent voltage which is measured using a double-channel low-frequency spectrum analyzer with the possibility to calculate the coherence between the two channels. (b) A close up of the biasing of the base current facilitating a precise control of the dc bias current and of the ac load impedance.

regarding the dynamic output resistance. In this investigation the external resistance in the collector lead has been held as low as possible which means that the nonnegligible internal resistance of the transimpedance amplifier in this lead has been dominating the resistance. This ensures that the whole noise current is measured. Due to the very low feedback from collector to base, the variation in impedance when changing range setting of the amplifier in the collector-lead does not affect the electrical behavior of the circuit and can thus be neglected.

Assuming the noise sources as shown in Fig. 1, the measured noise in the base-lead ($S_{I_{BE}}$) and in the collector-lead ($S_{I_{CE}}$) can be used for extracting information regarding the different noise sources within the device. Some advantages and drawbacks of transimpedance amplifiers will be discussed.

In order to keep control of the measurements, two different types of transimpedance amplifiers have been used for converting the measured current to a corresponding voltage: Brookdeal 5002 and Stanford Research Systems SR570. The latter facilitates internal voltage bias with a high resolution which can be controlled by a computer. This showed very useful when performing sweeps of the base current. However, caution must be taken not to nonintentionally induce noise in the system by using this internal voltage source. The Brookdeal 5002 does not have an internal biasing and induces very little noise into the measurement setup. An external voltage source with variable ac impedance [see Fig. 2(b)] has been realized, mainly based on a battery, a high-quality potentiometer and

a variable resistance in series with a large capacitance. If the measuring of dc part of the current can be omitted, the transimpedance amplifier can be inserted in direct series with the capacitance and the variable resistance which can enable a higher amplification factor as the amplifier does not need to sink the dc current. A higher amplification factor results however also in a higher internal impedance within the amplifier which must be accounted for.

Measuring the noise currents directly with transimpedance amplifiers enables us to use a large range in source resistances. The only limitation is that the impedance of the measured circuit should be larger than the input impedance of the current amplifier. This is easily obtained with transistors where the dynamic resistance always is much higher than the input impedance of the amplifier for all current levels. This enables simultaneous measurements of both the base and collector current noise and calculation of their coherence directly. The measurements would be very difficult to perform using voltage amplifiers at low values of the source resistances since the signal would be drowned in the Nyquist noise of the resistor itself and the background noise of the amplifier. As an example, a measurement of the shot noise level, calculated as $2qI$, using a 1 k Ω resistor and a voltage amplifier means the current I must be higher than 50 μ A in order not to be drowned by the internal Nyquist noise of the resistor. This is a high current for small devices. Lower current noise levels would thus not be possible to detect without severely degrading the coherence calculation between the two measured signals due to uncorrelated noise contributions of the resistances R_L and R_S .

By changing the sensitivity of the transimpedance amplifier, its internal impedance changes which appears as a variation in source impedance for the device under test. Attention to this behavior is given during measurements of current sweeps since both $S_{I_{BE}}$ and $S_{I_{BC}}$ can be affected by variation in impedance.

High current sensitivity goes hand in hand with a higher input impedance and a lower bandwidth. In addition to a limited bandwidth, the transimpedance amplifier exhibits an inductive behavior at the input which can introduce a resonance with a (parasitic) capacitance at the input. For this work, either a frequency of 20, 25, or 170 Hz has been chosen when comparing results of different bias conditions; mainly in order to avoid disturbance from harmonics of the 50 Hz power line signal, background noise and bandwidth problems. The reason for choosing the higher frequency of 170 Hz is due to the series capacitance for the ac impedance in the base lead [see Fig. 2(b)].

The internal bias supply of the SR570 amplifier enables a very compact solution for the measurement setup which makes it less prone to picking up undesired signals. However, owing to the very high open loop gain of the amplifier in the internal transimpedance stage, any fluctuation in the reference voltage for the bias will appear as an almost ideal voltage noise source at, e.g., the base of the transistor. This will affect both $S_{I_{BE}}$ and $S_{I_{CE}}$ and in worst case induce an undesired full coherence between the two measured noise signals. The same undesired noise source can also be unevenly partitioned between the

two channels, especially for low values of R_S , where $S_{I_{CE}}$ is affected, which can result in a low coherence. Here the results of coherence between $S_{I_{BE}}$ and $S_{I_{CE}}$ without measurement artifacts will be presented by the usage of the Brookdeal 5002 where the SR570 is found insufficient after comparing the results with a Brookdeal 5002.

IV. RESULTS

Measured current noise spectra of a 20- μ m² device for the base-lead and collector-lead is shown in Fig. 3. The curves indicate a spectra with $1/f^{0.8}$ dependence below 10 kHz which is in accordance with results for similar devices [4]. The level of the noise is too high for any white noise to be seen within the measured frequency range. Furthermore, the noise measurements at low currents are limited by the bandwidth of the amplifier which can be seen as a distinct roll-off above 10 kHz. A plot of coherence between the two noise spectra measured at any given base current shows a value very close to unity, see Fig. 3. This indicates that the measured noise originates from a common source, thus excluding any dominant source in the collector-lead and hence S_{I_B} , S_{r_b} , or S_{r_e} are the possible candidates.

In order to discriminate, we measure the current spectra at approximately constant values of base and collector current for a range of source resistances (R_S in Fig. 2). We deduce from (4) and (7) and our experimental results that S_{I_B} is the single dominant source for almost all values of R_S . A more thorough explanation will follow.

As can be seen in Fig. 4, $S_{I_{CE}}$ shows a square dependence on R_S for values of $300 \Omega \leq R_S \leq r_\pi$ which indicates strongly that S_{I_B} can be the only source, as can be seen from the second term in (4). All other components in (4) are for these values of R_S either independent of, or only very weakly dependent on R_S . In this region of R_S , the coherence (γ) is very close to unity, thus indicating that the same source is totally dominant also in $S_{I_{BE}}$. This means however that S_{I_B} is dominant in $S_{I_{BE}}$ for all values of R_S since the interdependence between the different terms in $S_{I_{BE}}$ is independent of R_S , [see (3)].

For low values of R_S , the coherence goes down (see Fig. 4) which indicates that at least one additional source (here called S_X) becomes significant in $S_{I_{CE}}$. The origin of this source is difficult to distinguish as the measured noise levels at these values of R_S are very low and thus external noise may enter, but from (5) or (36), shown at the bottom of the page, it can be indicated that S_{I_C} is a candidate since γ^2 never drops to zero. To pinpoint the source, a more thorough investigation would however be necessary which is beyond the scope of this paper. Assuming that this source is independent of R_S and bearing in mind that $S_{I_{BE}}$ is completely dominated by S_{I_B} , S_X is calculated as

$$S_X = (1 - \gamma^2)S_{I_{CE}}. \quad (9)$$

The coherence does however not reach zero for the given range of R_S thus indicating that S_{I_B} still contributes to $S_{I_{CE}}$. This level is calculated (here called $S_{I_{CE1}}$) as

$$S_{I_{CE1}} = \gamma^2 S_{I_{CE}} \quad (10)$$

Fig. 3. Measured current noise spectrum through the base and collector-lead for a $20 \mu\text{m}^2$ device. Coherence between the two signals is included. The source resistance is set to $R_S = 10 \text{ k}\Omega$ and I_B was set to $1 \mu\text{A}$. The roll-off at higher frequencies for the curves is due to the limited bandwidth of the transimpedance amplifiers.

Fig. 4. Measured and calculated data of S_{ICE} versus R_S for a $24 \mu\text{m}^2$ device at $V_{CE} = 1.5 \text{ V}$, $I_B = 2 \mu\text{A}$ and $f = 170 \text{ Hz}$. Calculated data ($S_{ICE\text{calculated}}$) using (13). Coherence is also included.

and the value of $(R_S + r_b + r_e)$ can be extracted as a factor in front of S_{IB} , see (3). Note that in this region, $R_S \ll r_\pi$.

For the extraction, a measurement (here called S_{ICE_2}) of S_{ICE} is taken where S_{IB} is dominant and $R_S \gg (r_b + r_e)$. The quotient between S_{ICE_1} and S_{ICE_2} will then be (neglecting insignificant terms)

$$\frac{S_{ICE_1}}{S_{ICE_2}} = \left(\frac{(R_{S_1} + r_b + r_e)(R_{S_2} + r_\pi)}{R_{S_2} r_\pi} \right)^2 \quad (11)$$

and $(r_b + r_e)$ is found as

$$(r_b + r_e) = \sqrt{\frac{S_{ICE_1}}{S_{ICE_2}}} \left(\frac{R_{S_2} r_\pi}{R_{S_2} + r_\pi} \right) - R_{S_1} \quad (12)$$

where R_{S_1} and R_{S_2} are the values of R_S for the measurements of S_{ICE_1} and S_{ICE_2} , respectively. An average of the extracted values for $R_{S_1} = 1, 11, 19$, and 34Ω (indicating values of S_{ICE} where the coherence clearly differs from unity) and $R_{S_2} = 3.4 \text{ k}\Omega$ gives $S_X = 4.4 \cdot 10^{-21} \text{ A}^2/\text{Hz}$ at $f = 170 \text{ Hz}$ and $(r_b + r_e) = 28 \Omega$.

Under the assumption that only the sources S_{IB} and S_X contribute to S_{ICE} , a simplified expression is found as

$$S_{ICE\text{calculated}} = S_{ICE_0} \left(\frac{Z'}{Z} \right)^2 + S_X. \quad (13)$$

Thus, only one additional parameter, here called S_{ICE_0} , needs to be extracted to be able to calculate S_{ICE} . From (4) it

Fig. 5. Extracted values of S_{I_B} versus I_B on a $20 \mu\text{m}^2$ device. Measurements done at $f = 25 \text{ Hz}$, $V_{CE} = 1.5 \text{ V}$ and $R_S = 100 \text{ k}\Omega$.

is found that this corresponds to $h_{fe}^2 S_{I_B}$. However, in order to gain as much information as possible from the measured $S_{I_{CE}}$ and not include uncertainty from additional circuit parameters, a calculation of this term is omitted in favor of a direct extraction. In a region where $(r_b + r_e) \ll R_S < r_\pi/2$, S_{I_B} is dominant, $S_{I_{CE_0}}$ is found as

$$S_{I_{CE_0}} = S_{I_{CE}} \left(\frac{R_S + r_\pi}{R_S + (r_b + r_e)} \right)^2. \quad (14)$$

For $500 \Omega \leq R_S \leq 6 \text{ k}\Omega$ and $f = 170 \text{ Hz}$ an average of $S_{I_{CE_0}}$ was found to be $1.8 \cdot 10^{-16} \text{ A}^2/\text{Hz}$. This compares very well with the calculated value found using extracted h_{fe} and S_{I_B} .

The curve of $S_{I_{CE_{\text{calculated}}}}$ has been included in Fig. 4 as a full line and compares very well with measured data (dots), thus showing that the assumptions are correct. From the shape of $S_{I_{CE}}$, it can be seen that there is a lower limit to which there is any point in setting the biasing impedance, R_S , and this is above the value of $(r_b + r_e)$. Furthermore, as biasing impedance increases, $S_{I_{CE}}$ rolls off, indicating that bias impedances chosen above the value of r_π does not further increase $S_{I_{CE}}$. An implication of this is that devices with a high β will have a high range of $S_{I_{CE}}$ depending on which biasing impedance is chosen since I_B here is small and thus r_π becomes large, giving a large span of values between $(r_b + r_e)$ and r_π .

Using (4) and (5), a value for h_{fe} is found as

$$h_{fe} = \frac{S_{I_{CE}}}{S_{I_{BCI_{CE}}}} \frac{r_\pi}{R_S} \quad (15)$$

assuming $R_S \gg (1 + h_{fe})r_e$, S_{I_B} is the dominant noise source and $(r_o/(R_L + r_o)) \rightarrow 1$. Extracted values using (15) and data from the Gummel plot are compared in Table I.

A value of S_{I_B} is calculated as

$$S_{I_B} = S_{I_{BE}} * \frac{(R_S + r_\pi)^2}{r_\pi^2} \quad (16)$$

TABLE I
EXTRACTED VALUES OF h_{fe} FOR A $20 \mu\text{m}^2$ DEVICE AT DIFFERENT VALUES OF I_B USING (15). $f = 25 \text{ Hz}$, $R_S = 100 \text{ k}\Omega$ AND $V_{CE} = 1.5 \text{ V}$. THE COLUMN $\delta I_C / \delta I_B$ INDICATES EXTRACTED DATA FROM A GUMMEL PLOT

I_B		h_{fe}	$\frac{\delta I_C}{\delta I_B}$
20	nA	35	38
200	nA	48	58
1	μA	64	79
2.6	μA	110	108
10	μA	120	177

for values of $R_S \gg r_b + (1 + h_{fe})r_e$. In particular, for $(r_b + (1 + h_{fe})r_e) \ll R_S \ll r_\pi$ we have $S_{I_B} \approx S_{I_{BE}}$.

A dependence on base current of approximately $I_B^{1.7}$ has been measured throughout the available bias range, see Fig. 5. For currents below 250 nA, the frequency dependence if the noise is weaker and bandwidth limitations play a role. The reduced frequency dependence of S_{I_B} in conjunction with the current dependence indicate that the noise source is not of pure $1/f$ -type and any real predictions on corner frequencies where the low-frequency noise disappears in the shot noise are difficult to make [5].

The current dependence of S_{I_B} gives space for interpretations of the origin of the noise. As shown earlier, the noise originates within the base-emitter junction which here means not in the base resistance and not in the emitter resistance. Assuming a true area dependent effect, the mobility fluctuation model or the number fluctuation model seem to be reasonable candidates. The Kleinpenning model considers mobility fluctuations and hence fluctuations in the diffusion coefficient for the minority carriers. A simplified relation, which indicates a linear dependence of S_{I_B} on current, is given as follows [6]:

$$S_{I_B} = \frac{2\alpha q I_B D_p}{f W_E^2} \quad (17)$$

where D_p is the hole diffusion coefficient in the emitter of an NPN transistor and W_E is the length of the undepleted part of the emitter. With $D_p = 5 \text{ cm}^2/\text{s}$, $W_E = 1.2 \cdot 10^{-5} \text{ cm}$ and

$f = 25$ Hz, calculated α for $I_B = 250$ nA and $I_B = 25$ μ A give values of $5 \cdot 10^{-5}$ and $2 \cdot 10^{-3}$, respectively, which is within reasonable limits [7]. Due to the variation in base current, the depletion region in the emitter vary and thus also the region where the fluctuation in mobility will be of greatest importance. The fact that α is nonuniform can speak for a position dependence on α in the W_E region just at the rim of the depletion region [8].

The number fluctuation model considers a fluctuation in the surface recombination velocity, s . With the relation for short diodes [9]

$$\frac{S_I}{I^2} = C \cdot S_s \quad (18)$$

where C is independent of current and S_s is the spectral density of the fluctuations in s with a $1/f$ -type of spectrum. It can be seen that this model predicts a square-law dependence on current just like series resistance fluctuations. This model thus fits to the “near-square-law” dependence of S_{I_B} which has been observed.

V. CONCLUSION

We have used transimpedance amplifiers for direct study of current noise in a silicon germanium HBT at different biasing conditions. The possibility of measuring the device using a wider range of external resistances than is possible when using a voltage amplifier has facilitated extraction of the ac current amplification factor h_{fe} and the sum of the base and emitter series resistances $(r_b + r_e)$. The simultaneous measurement of both the base-lead noise and the collector-lead noise has enabled a calculation of the coherence between the signals. From the high coherence and the dependence of collector-lead noise on source resistance, it has been established that the dominant noise source is situated in the base emitter junction, in the W_E region at the emitter side and not in the contact resistances r_b and r_e . The use of the coherence function has been essential for the discrimination of the dominant noise source and for the extraction of $(r_b + r_e)$.

APPENDIX

DERIVATION OF $S_{I_{CE}}$, $S_{I_{BE}}$, $S_{I_{BEICE}}$, AND γ^2 AND ESTIMATION OF THE IMPACT OF S_{I_C} ON THE MEASUREMENTS

This Appendix gives a more thorough description of how the important expressions for $S_{I_{CE}}$, $S_{I_{BE}}$, $S_{I_{BEICE}}$, and γ^2 [(3)–(5) and (8)] are derived. The expressions will be derived in steps, concentrating on one source at a time and finally collecting the the different parts to the full expressions.

A. Derivation of $S_{I_{CE}}$, $S_{I_{BE}}$, and $S_{I_{BEICE}}$ with Respect to S_{I_C}

The emphasis is first put on finding the impact of a noise source in the collector lead on the noise in the base lead. From Fig. 1, we define the following currents when S_{I_C} is the only contributor:

$$\begin{cases} i' = i_{S_{I_C}} - i_{r_o} + h_{fe} i_b \\ i_b = -i' \frac{R_S + r_b + r_\pi + r_e}{R_L + (R_S + r_b + r_\pi) || r_e} \\ i_{r_o} = i' \frac{r_e}{R_L + (R_S + r_b + r_\pi) || r_e} \end{cases} \quad (19)$$

Elimination then gives (20) and (21), shown at the bottom of the page, where

$$(R_S + r_b + r_\pi) || r_e \stackrel{\text{def}}{=} \frac{(R_S + r_b + r_\pi) * r_e}{(R_S + r_b + r_\pi) + r_e}. \quad (22)$$

For a common emitter configuration, $r_o, r_\pi \gg r_e$ and R_L can be chosen very small and it is thus possible to assume that $r_o, r_\pi \gg (r_e + R_L)$, thus

$$i_b = -i_{S_{I_C}} \frac{r_e}{Z} \quad (23)$$

$$i' = i_{S_{I_C}} \frac{R_S + r_b + r_\pi + r_e}{Z} \quad (24)$$

where

$$Z = (R_S + r_b + r_\pi + (1 + h_{fe})r_e).$$

It can be seen in Fig. 1 that i' is the same as the current flowing through R_L . Straightforward circuit analysis thus gives

$$\begin{aligned} S_{I_{BE}} &= \frac{r_e^2 S_{I_C}}{Z^2} \\ S_{I_{CE}} &= \frac{h_{fe}^2 (R_S + r_b + r_\pi + r_e)^2}{Z^2 h_{fe}^2} S_{I_C} \end{aligned}$$

Again it has been assumed that $r_o \gg (r_e + R_L)$.

The correlated spectrum between the two signals is expressed as [in the text (5)]

$$S_{I_{BEICE}} = \frac{h_{fe}}{Z^2} \frac{(R_S + r_b + r_\pi + r_e)r_e}{h_{fe}} S_{I_C}.$$

Can the source in the collector lead (i.e., S_{I_C}) be dominant?

If this is the case, we should find for the quotient between the power spectral densities of the measured signals

$$\frac{S_{I_{CE}}}{S_{I_{BE}}} = \left(\frac{R_S + r_b + r_\pi + r_e}{r_e} \right)^2. \quad (25)$$

$$i_b = -i_{S_{I_C}} \left[\frac{r_e}{R_S + r_b + r_\pi + r_e} \right] \left\{ \frac{1}{1 + \frac{R_L + (R_S + r_b + r_\pi) || r_e}{r_o} + h_{fe} \frac{r_e}{R_S + r_b + r_\pi + r_e}} \right\} \quad (20)$$

$$i' = i_{S_{I_C}} \frac{1}{1 + \frac{R_L + (R_S + r_b + r_\pi) || r_e}{r_o} + h_{fe} \frac{r_e}{R_S + r_b + r_\pi + r_e}} \quad (21)$$

$$\gamma^2 = \frac{\left[I_B^2 S_{r_b} - r_\pi Z'_S S_{I_B} + I_E^2 S_{r_e} - \frac{(R_S + r_b + r_\pi + r_e)r_e}{h_{fe}} S_{I_C} \right]^2}{\left[I_B^2 S_{r_b} + (r_\pi + h_{fe}r_e)^2 S_{I_B} + I_E^2 S_{r_e} + r_e^2 S_{I_C} \right] \left[I_B^2 S_{r_b} + Z_S'^2 S_{I_B} + I_E^2 S_{r_e} + \frac{(R_S + r_b + r_\pi + r_e)^2}{h_{fe}^2} S_{I_C} \right]}. \quad (36)$$

For a base current $I_B = 1 \mu\text{A}$, $r_\pi \approx 25 \text{ k}\Omega$. Assuming $r_e < 10 \Omega$ which is well above the value found from dc analysis and neglecting R_S in order to get a worst case value, this gives from (25)

$$\frac{S_{I_{CE}}}{S_{I_{BE}}} > 6 \cdot 10^6.$$

However, at $I_B = 1 \mu\text{A}$ (see Fig. 3) the ratio between $S_{I_{CE}}$ and $S_{I_{BE}}$ is less than $5 \cdot 10^4$ and S_{I_C} can thus not be the dominant noise source.

B. Derivation of $S_{I_{CE}}$, $S_{I_{BE}}$, and $S_{I_{BE}I_{CE}}$ with respect to S_{r_b} , S_{r_e} and S_{I_B} , Ignoring S_{I_C}

Call current flowing through R_S from basenode to ground as i_{be} . Assume that the value of r_o is high which means it does not shunt any significant current in comparison with $h_{fe}i_b$.

To begin with, we study the different components in $S_{I_{BE}}$. Looking first at the impact of S_{r_b} , i.e., this being the only source. Call voltage caused by $I_B^2 S_{r_b}$ as $u_{S_{r_b}}$. Kirchoff's law gives

$$u_{S_{r_b}} - i_b r_\pi - (i_b + h_{fe}i_b)r_e + i_{be}R_S - i_b r_b = 0. \quad (26)$$

Notice that $i_{be} = -i_b$. This gives immediately

$$S_{I_{BE}} = \frac{I_B^2 S_{r_b}}{Z^2}. \quad (27)$$

Second, looking at the impact of S_{r_e} , i.e., this being the only source. Call voltage caused by $I_E^2 S_{r_e}$ as $u_{S_{r_e}}$. Kirchoff's law gives

$$u_{S_{r_e}} - (i_b + h_{fe}i_b)r_e + i_{be}R_S - i_b r_b - i_b r_\pi = 0. \quad (28)$$

Notice that $i_{be} = -i_b$. This gives immediately

$$S_{I_{BE}} = \frac{I_E^2 S_{r_e}}{Z^2}. \quad (29)$$

Third, looking at the impact of S_{I_B} , i.e., this being the only source. Call current caused by S_{I_B} as $i_{S_{I_B}}$. Notice that

$$-i_{be} - i_{S_{I_B}} - i_b = 0. \quad (30)$$

Kirchoff's law gives

$$i_{be}R_S + i_{be}r_b - i_b r_\pi - (i_{S_{I_B}} + i_b + h_{fe}i_b)r_e = 0. \quad (31)$$

Substitution of i_b using (30) gives

$$\begin{aligned} i_{be}(r_b + R_S + r_\pi + (1 + h_{fe})r_e) \\ + i_{S_{I_B}}(r_\pi + h_{fe}r_e) = 0 \end{aligned} \quad (32)$$

which yields

$$S_{I_{BE}} = S_{I_B} \frac{(r_\pi + h_{fe}r_e)^2}{Z^2} \quad (33)$$

and in total for all sources

$$S_{I_{BE}} = \frac{1}{Z^2} [I_B^2 S_{r_b} + (r_\pi + h_{fe}r_e)^2 S_{I_B} + I_E^2 S_{r_e} + r_e^2 S_{I_C}]. \quad (34)$$

Continuing with $S_{I_{CE}}$, using the already found expressions [(26) and (28)], the impact on $S_{I_{CE}}$ from S_{r_b} and S_{r_e} can be found by noticing that in these cases $-i_{be} = i_b$ and the current through R_L is $i_b h_{fe}$. The impact of S_{I_B} on $S_{I_{CE}}$ can be found by using (30) and (32)

$$i_b = -i_{S_{I_B}} \left[\frac{R_S + r_b + r_e}{Z} \right]. \quad (35)$$

Thus, $S_{I_{CE}}$ will be

$$\begin{aligned} S_{I_{CE}} = \frac{h_{fe}^2}{Z^2} \left[I_B^2 S_{r_b} + Z_S'^2 S_{I_B} + I_E^2 S_{r_e} \right. \\ \left. + \frac{(R_S + r_b + r_\pi + r_e)^2}{h_{fe}^2} S_{I_C} \right] \end{aligned}$$

where

$$Z'_S = (R_S + r_b + r_e)$$

and $S_{I_{BE}I_{CE}}$ can be found as (noticing the direction of the different current components)

$$\begin{aligned} S_{I_{BE}I_{CE}} = \frac{h_{fe}}{Z^2} \left[I_B^2 S_{r_b} - r_\pi Z'_S S_{I_B} + I_E^2 S_{r_e} \right. \\ \left. - \frac{(R_S + r_b + r_\pi + r_e)r_e}{h_{fe}} S_{I_C} \right]. \end{aligned}$$

C. The Full Expression of γ^2

Given the expressions for $S_{I_{BE}}$, $S_{I_{CE}}$, and $S_{I_{BE}I_{CE}}$, the expression for γ^2 can be calculated from its definition (8) and is found to be (36), shown at the top of the page.

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