A validation of the quadriphasic mixture theory for intervertebral disc tissue

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by

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The swelling and shrinking behaviour of soft biological tissues is described by a quadriphasic mixture model. In this model four phases are distinguished: a charged solid, a fluid, cations and anions. A description of the set of coupled differential equations of this quadriphasic mixture model is given. These equations are solved by the finite element method using a weighted residual approach. The resulting non-linear integral equations are linearized and solved by the Newton-Raphson iteration procedure. We performed some confined swelling and compression experiments on intervertebral disc tissue. These experiments are simulated by a one-dimensional finite element implementation of this quadriphasic mixture model. In contrast to a triphasic mixture model, physically realistic diffusion coefficients can be used to fit the experiments when the fixed charge density is relatively large, because in the quadriphasic mixture model electrical phenomena are not neglected.

Key words: porous media, swelling, coupled flow, electro-osmosis, streaming potential

1 Introduction

Biological tissues, like intervertebral disc tissue, exhibit swelling or shrinking behaviour. This behaviour depends on the external salt concentration. For example, in pure water the volume of a sample made out of the nucleus pulposus,
can increase until it reaches 200% of the original volume [11]. Even in a physiological salt solution, the tissue swells substantially. Not only the geometry but also the composition of the tissue change by swelling. This swelling behaviour is caused by osmotic pressure. Thus, the tissue deformation depends not only on mechanical pressures, but also on chemical pressures. A mixture theory is needed to describe this tissue behaviour.

Lai et al. [7] developed a triphasic theory for soft hydrated tissues. They applied the theory to cartilage while neglecting electrical fluxes and geometric non-linearities. Snijders [10] developed a finite element formulation of a simplified version of the triphasic theory, neglecting except electrical fluxes also electrical potential gradients. From confined compression experiments, performed by de Heus [2], Snijders [11] and Houben [5], and their triphasic finite element simulations, it appeared that fitting the experimental data is only possible with a diffusion coefficient many times larger than the diffusion coefficient of the ions in water [9]. As the fixed charge density is large, the electrical effects inside the tissue may not be neglected. A quadriphasic mixture model, derived by Huyghe and Janssen [6], includes electrical fluxes and potential gradients. In this mixture model, four phases are distinguished: a charged solid, a fluid, cations and anions.

In this paper we present a finite element formulation of the quadriphasic mixture theory and a validation of the theory by means of a one-dimensional swelling and compression experiment. But first, short descriptions of intervertebral disc tissue and the confined compression experiment are given.

2 Intervertebral disc tissue

An intervertebral disc consists of a gelatinous centre, the nucleus pulposus, surrounded by several concentrically arranged lamellae, the anulus fibrosus (figure 1). In these lamellae, fiber networks can be seen. The orientation of these fibers differs per lamella. Due to this orientation, the material is anisotropic.

The material mainly consists of a collagen network embedded in a hydrated proteoglycan gel. Collagen is a rod-like protein molecule built of long polypeptide chains. Proteoglycans are large molecules consisting of many glycosaminoglycans linked to core proteins. These glycosaminoglycans are made up of long chains of polysaccharides. Due to the physiological pH and the ionic strength of the interstitial fluid, the carboxyl and sulfate groups of the polysaccharides are ionized. The density of these charges is called fixed charge density. Due to this ionization the proteoglycans are capable of retaining water up to a 50-fold of their own weight [2].
3 Experimental set-up

Out of the anulus fibrosus of the intervertebral disc of large dogs, some cylindrical samples are made in the same way as in Drost et al. [3]. To make these samples, the spine of a dog is removed, post mortem, and immediately frozen. Next, the intervertebral discs are cut out of the spine and cylindrical samples are taken out in radial direction. The diameter of these samples is 4 mm and the height is about 1 mm. The samples are kept in the freezer until they are used to perform the experiments. We performed the experiments in a testing chamber (figure 2) at room temperature. In the experiment, the sample is kept in an impermeable confining ring. A loading piston, by which a mechanical load can be applied, is put on top of the sample. This piston can move in the vertical direction. A linear variable displacement transducer measures the piston displacements during the experiments. These measurements are recorded by a computer every 8 seconds. At the bottom of the sample is a porous glass filter through which a salt solution flows. By altering the concentration of this salt solution, the chemical load changes. The permeability of the glass filter is much larger than the sample permeability, to ensure well defined boundary conditions along the filter-sample interface. Because of an insulating silicon oil film, we assume that there is no electrical current through the sample.

During the measurements, four different periods of 5 or 6 hours can be distinguished: a conditioning period, a swelling period, a consolidation period and a control period. In each period different boundary conditions are prescribed (table 1).
Table 1

<table>
<thead>
<tr>
<th></th>
<th>conditioning</th>
<th>swelling</th>
<th>consolidation</th>
<th>control</th>
</tr>
</thead>
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<tr>
<td>mechanical load</td>
<td>0.078</td>
<td>0.078</td>
<td>0.195</td>
<td>0.078</td>
</tr>
<tr>
<td>[MPa]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>concentration</td>
<td>0.469±0.026</td>
<td>0.159±0.014</td>
<td>0.159±0.014</td>
<td>0.469±0.026</td>
</tr>
<tr>
<td>[kmol m(^{-3})]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that in the conditioning period and in the control period, the boundary conditions are the same.

4 Quadriphasic mixture model

In a quadriphasic mixture model, as derived by Huyghe and Janssen [6], four phases are distinguished: a charged solid (s), a fluid (f), cations (+) and anions (−). These phases interact with each other. We summarize the main equations for the quadriphasic mixture theory. For a more extensive treatment, we refer to [6].

As we assume that all phases are intrinsically incompressible and that there are no (chemical) reactions between the phases, the mass balance of each phase...
is written as

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0, \quad \alpha = s, f, +, -.$$  \hfill (1)

In this equation $n_\alpha$ is the volume fraction of phase $\alpha$ and $\mathbf{v}_\alpha$ is the velocity of phase $\alpha$. As we assume saturation ($n_s + n_f + n^+ + n^- = 1$) and as we assume that the volume fractions of the ions are relatively small compared to the other volume fractions, we can derive one equation for the mass balances of the solid and the fluid, because $n^s = 1 - n^f$:

$$\nabla \cdot \mathbf{v}^s + \nabla \cdot (n^f(\mathbf{v}^f - \mathbf{v}^s)) = 0. \quad \hfill (2)$$

The mass balances of the cations and anions are written as differential equations by using equation (1) and an equation describing the ion volume flux ($\mathbf{j}_\alpha = n^\alpha(\mathbf{v}^f - \mathbf{v}^s)$, $\alpha = +, -$). The mass balances of the cations and anions are given by

$$n^f \frac{d}{dt} \left( \frac{n^\alpha}{n^f} \right) + n^f (\mathbf{v}^f - \mathbf{v}^s) \cdot \nabla \left( \frac{n^\alpha}{n^f} \right) = \nabla \cdot \mathbf{j}_\alpha, \quad \alpha = +, -, \quad \hfill (3)$$

where $\frac{d}{dt}$ denotes the solid material time derivate.

By neglecting the inertial terms and the body forces, the momentum equation is given by

$$\nabla \cdot \mathbf{\sigma}_{eff} = \nabla p = 0. \quad \hfill (4)$$

In this equation $\mathbf{\sigma}_{eff}$ is the effective stress tensor and $p$ is the hydrodynamic fluid pressure.

Electro-neutrality of the mixture is expressed by:

$$z^+ c^+ + c^{fe} + z^- c^- = 0. \quad \hfill (5)$$

In this equation $z^\alpha$ is the valence of ion $\alpha$ and $c^\alpha$ is the concentration of ion $\alpha$ per unit fluid volume. By $c^{fe}$ the fixed charge density per unit fluid volume is meant. For simplicity, we assume that the ions are monovalent ($z^+ = 1$, $z^- = -1$).

Thus, the material behaviour is described by the coupled equations (2) – (4) and some constitutive equations.

The first constitutive equation is an extended Darcy equation:

$$n^f(\mathbf{v}^f - \mathbf{v}^s) = -\mathbf{F} \cdot \mathbf{K} \cdot \mathbf{F}^c \cdot \left( \nabla (p - \pi) + \left( \frac{n^+}{n^f} \right) \nabla \mu^+ + \left( \frac{n^-}{n^f} \right) \nabla \mu^- \right), \quad \hfill (6)$$
where $\mathbf{F}$ is the deformation tensor, $\mathbf{K}$ is the permeability tensor, $\pi$ is the osmotic pressure:

$$\pi = RT \phi (c^+ + c^-) + \pi_0,$$

and $\mu^\alpha$ are the electro-chemical potentials:

$$\mu^+ = \mu_0^+ + \frac{1}{V^\alpha} (RT \ln (f^+ c^+) + F \xi),$$

$$\mu^- = \mu_0^- + \frac{1}{V^\alpha} (RT \ln (f^- c^-) - F \xi).$$

In these equations $\pi_0$ and $\mu_0^\alpha$ are reference values, $\phi$ is an osmotic coefficient, $V^\alpha$ are partial molar volumes, $R$ is the universal gas constant, $T$ is the absolute temperature, $f^\alpha$ are activity coefficients, $F$ is Faraday's constant and $\xi$ is the electrical potential. The deformation tensor $\mathbf{F}$ appears in equation (6) to ensure objectivity of the extended Darcy equation under rigid body rotation.

The second constitutive equation is a linear relation between the effective stress $\sigma^{eff}$ and the Green-Lagrange strain $\mathbf{E}$:

$$\sigma^{eff} = H \mathbf{E}$$

with stiffness $H$.

The relation between the ion volume flux and the electro-chemical potentials is given by Fick's law

$$\mathbf{j}^\alpha = \frac{V^\alpha}{RT} \mathbf{F} \cdot \mathbf{D}^\alpha \cdot \mathbf{F}^c \cdot \nabla \mu^\alpha, \quad \alpha = +, -, \tag{11}$$

where $\mathbf{D}^\alpha$ is the diffusion tensor of ion $\alpha$. Here also, the objectivity is ensured by the deformation tensor $\mathbf{F}$.

Because the carboxyl and sulfate groups of the proteoglycans are ionized, the solid is charged. As the charges are fixed to the solid, the concentration varies with the tissue deformation. Owing to the intrinsic incompressibility, the tissue only deforms when fluid is expelled. Thus, the fixed charge density in deformed state is expressed by

$$c^{Ic} = \frac{n_0^I c_0^{Ic}}{n_0^I - 1 + J}, \tag{12}$$

where $J$ is the relative volume change. By the subscript 0 a reference situation is meant. In the same way an expression for the porosity $n^I$ in the deformed state is derived:

$$n^I = 1 - \left(1 - \frac{n_0^I}{J}\right). \tag{13}$$
The boundary conditions are given by

\[
[(\sigma^{eff} - p\mathbf{I}) \cdot \mathbf{n}] = 0, \quad (14)
\]
\[
[p - \pi] = 0, \quad (15)
\]
\[
[\mu^+] = 0, \quad (16)
\]
\[
[\mu^-] = 0, \quad (17)
\]

where \(\mathbf{n}\) is the unit normal vector on the boundary.

As an equilibrium is reached, the ion volume flux and the difference between the fluid velocity and the velocity of the solid are equal to zero. From equations (6) and (11) it follows that \(\nabla \mu^+, \nabla \mu^-\) and \(\nabla(p - \pi)\) are zero.

5 Finite element model

To compute a solution of this quadriphasic problem, we write the differential equations (2) - (4) in a form that is suitable for a finite element solution procedure. The differential equations are written in an integral form by using the weighted residual method. Therefore we multiply these equations with arbitrarily chosen weighting functions. We multiply the momentum balance equation (4) with the weighting vector \(\mathbf{w}\), the mass balance equation for the solid and fluid (2) with the weighting function \(\mathbf{g}\) and the mass balance equation for the cations and anions (3) respectively with weighting functions \(h^+\) and \(h^-\). Then we integrate over the volume \(\Omega\). Next, we look at the equations with respect to a reference situation, denoted by the subscript 0. By using Gauss' theorem, the extended Darcy equation (6) and Fick's law (11), and

\[
\nabla = \mathbf{F}^{-c} \cdot \nabla_0,
\]
\[
dV = JdV_0,
\]
\[
ndA = JF^{-c} \cdot n_0 dA_0,
\]

the coupled differential equations are written as

\[
\int_{\Omega_0} (\nabla_0 \mathbf{w})^c : (\mathbf{S} \cdot \mathbf{F}^{-c} - (p - \pi)\mathbf{F}^{-1}J)dV_0 = \int_{\Gamma_0} \mathbf{t}_0 \cdot \mathbf{w} dA_0, \quad (19)
\]

\[
\int_{\Omega_0} g(\mathbf{F}^{-c} \cdot \nabla_0) \cdot \mathbf{v}^s JdV_0 + \int_{\Omega_0} \nabla_0 \mathbf{g} \cdot \mathbf{K} \cdot (\nabla_0(p - \pi) + \frac{n^+}{n_f} \nabla_0 \mu^+ + \frac{n^-}{n_f} \nabla_0 \mu^-)JdV_0 = \int_{\Gamma_0} g\mathbf{n}_0 \cdot \mathbf{K} \cdot (\nabla_0(p - \pi) + \frac{n^+}{n_f} \nabla_0 \mu^+ + \frac{n^-}{n_f} \nabla_0 \mu^-)JdA_0; \quad (20)
\]
\[ \int_{\Omega_0} h^+ n^f \frac{d}{dt} \left( \frac{n^+}{n^f} \right) J dV_0 + \int_{\Omega_0} h^+ n^f (F^{-1} \cdot (\nu^f - \nu^e)) \cdot \nabla \left( \frac{n^+}{n^f} \right) J dV_0 + \\
+ \int_{\Omega_0} \nabla \nabla^+ \frac{D^+}{RT} \cdot \nabla \mu^+ J dV_0 = \int_{\Gamma_0} h^+ n_0 \cdot \frac{V^+ D^+}{RT} \cdot \nabla \mu^+ J dA_0, \quad (21) \]

\[ \int_{\Omega_0} h^- n^f \frac{d}{dt} \left( \frac{n^-}{n^f} \right) J dV_0 + \int_{\Omega_0} h^- n^f (F^{-1} \cdot (\nu^f - \nu^e)) \cdot \nabla \left( \frac{n^-}{n^f} \right) J dV_0 + \\
+ \int_{\Omega_0} \nabla \nabla^- \frac{D^-}{RT} \cdot \nabla \mu^- J dV_0 = \int_{\Gamma_0} h^- n_0 \cdot \frac{V^- D^-}{RT} \cdot \nabla \mu^- J dA_0, \quad (22) \]

where

\[ S = J F^{-1} \cdot (\sigma^{\sigma ff} - \pi I) \cdot F^{-c}, \quad (23) \]
\[ t_0 = J (F^{-c} \cdot n_0) \cdot (\sigma^{\sigma ff} - p I). \quad (24) \]

Here \( \Omega_0 \) is the undeformed volume and \( \Gamma_0 \) the undeformed boundary of the sample. The relative volume fractions can also be written as

\[ \frac{n^\alpha}{n^f} = V^\alpha c^\alpha, \quad \alpha = +, -. \quad (25) \]

The material behaviour is described by equations (19) – (22). The independent unknowns are: the displacements \( u \), the fluid pressure \( p \), and the electrochemical potentials \( \mu^+ \) and \( \mu^- \).

Next, the mixture volume is divided into a number of subvolumes or elements. The volume and surface integrals over the total volume are replaced by the sum of volume and surface integrals over these elements. At all time, an element consists of the same material points. Therefore, the element only changes in shape during deformation.
The following approximations are used:

\[
\begin{align*}
\mathbf{u}(\zeta, t) &= \sum_{i=1}^{n} \sum_{j=1}^{3} \phi^i(\zeta) u^i_j(t) \mathbf{e}_i, \\
\mathbf{p}(\zeta, t) &= \sum_{i=1}^{n} \psi^i(\zeta) p^i(t), \\
\mathbf{\pi}(\zeta, t) &= \sum_{i=1}^{n} \psi^i(\zeta) \pi^i(t), \\
\mathbf{\mu}^+(\zeta, t) &= \sum_{i=1}^{n} \gamma^i(\zeta) \mu^{+i}(t), \\
\mathbf{\mu}^-(\zeta, t) &= \sum_{i=1}^{n} \gamma^i(\zeta) \mu^{-i}(t),
\end{align*}
\]

where the shape functions have the properties

\[
\begin{align*}
\phi^i(\zeta^K) &= \delta^i_K, \\
\psi^i(\zeta^K) &= \delta^i_K, \\
\gamma^i(\zeta^K) &= \delta^i_K.
\end{align*}
\]  

(26)

(27)

In these functions \(n\) is the number of nodes, \(\mathbf{e}_i\) is a direction vector and \(\zeta\) a local coordinate. For simplicity, we have chosen the shape functions for the electro-chemical potentials of the cations and the anions to be equal. The weighting functions \(w, g, h^+\) and \(h^-\) are chosen \(\sum_{i=1}^{3} \phi^i(\zeta) \mathbf{e}_i, \psi^i(\zeta), \gamma^i(\zeta)\) and \(\gamma^i(\zeta)\) respectively. Thus the linearized weighted differential equations (19) - (22), can be written as

\[
\mathbf{M} \frac{d\mathbf{x}}{dt} + \mathbf{N}\mathbf{x} = \mathbf{r},
\]

(28)

where \(\mathbf{M}\) and \(\mathbf{N}\) are the mass matrix and the stiffness matrix; \(\mathbf{x}\) and \(\mathbf{r}\) are column vectors. The vector \(\mathbf{x}\) is defined by

\[
\mathbf{x} = (\mathbf{u}^T (\mathbf{p} - \mathbf{\pi})^T \mathbf{\mu}^+ \mathbf{\mu}^-)^T, \quad I = 1, ..., n.
\]

(29)

The matrix \(\mathbf{M}\) is singular, because the momentum equation (19) does not have any time derivate.

Next, the resulting equations are discretized in time. We use an implicit iteration scheme to solve system (28). So, the problem to solve can be written as

\[
\mathbf{A}\mathbf{x} = \tilde{\mathbf{b}}.
\]

(30)

The resulting matrix \(\mathbf{A}\) is non-singular. For simplicity, we choose the shape function to be piecewise linear. The Newton-Raphson iteration procedure is used to solve these equations.
6 Results

In this section numerical results are compared to experimental data. The protocol of the confined compression experiment, we performed, is described earlier in this article. An example of experimental data of such an experiment, is shown in figure 3. The sample used in this experiment, consists of anulus fibrosus of intervertebral disc L6-L7. The axial direction of this sample, inside the intervertebral disc, is ventrolateral. In the conditioning period, we start with unknown begin conditions and wait until an equilibrium is reached. In the second period the sample swells due to the osmotic pressure. In the consolidation period water is expelled by applying a larger mechanical load. In the last period the same boundary conditions as in the first period are prescribed. Figure 3 shows that the sample height at the end of the control period is 2 % lower than the sample height at the end of the conditioning period, although the same boundary conditions are applied. This is probably caused by fixed charges that have leached out during the experiment due to damage inside the sample.

In the numerical simulations, only the swelling and the consolidating periods are computed. The conditioning period is not considered, because the starting conditions are unknown. For the numerical simulation, a one-dimensional finite element formulation is used to solve the non-linear equations described in the previous section. Therefore the 'sample' is divided into 10 uniform elements. The weighting functions $\phi$, $\psi$ and $\gamma$ are chosen to be piecewise linear within each element. Non-linearity is accounted for by using the Newton-Raphson

![Fig. 3. Results of a confined swelling and compression experiment.](image-url)
iterative process. For simplicity, we assume the diffusion coefficients $D^+$ and $D^-$, the permeability $K$ and the stiffness $H$ deformation-independent. In figure 4 the computed data (dashed line) and the experimental data are plotted. At time $t = 0$ h, the external concentration $\tilde{c}$ in the glass filter increases from 0.458 mol l$^{-1}$ to 0.146 mol l$^{-1}$. At time $t = 6$ h, the mechanical load increases from 0.078 MPa to 0.195 MPa. The fixed charge density is measured afterwards by using a radioactively labeled NaCl-solution. Also the porosity $n_f$ is determined afterwards by measuring the sample weight before and after water is removed by freeze-drying.

The initial conditions are given by the Donnan equilibrium:

$$c^+ = \frac{1}{2} \left( -c^f + \sqrt{(c^f)^2 + 4 \left( \frac{f^+ f^-}{f^+ f^-} \right)^2 \tilde{c}^2} \right), \quad (31)$$

$$c^- = \frac{1}{2} \left( +c^f + \sqrt{(c^f)^2 + 4 \left( \frac{f^+ f^-}{f^+ f^-} \right)^2 \tilde{c}^2} \right), \quad (32)$$

$$p = RT(\phi(c^+ + c^-) - 2\tilde{c}). \quad (33)$$

The overlined symbols are the values in the outer solution in the conditioning period.
The electrical potential is described by

\[ \xi = \frac{RT}{F} \ln \left( \frac{j^+ c^+}{j^+ c^+} \right) + \xi_0, \]  

(34)

where \( \xi_0 \) is the electrical potential in the glass filter. This value is chosen to be zero. We assume that we have an ideal solution. So, the activity coefficients are equal to one.

In the one-dimensional experiment, the boundary conditions on top of the sample are

\[
\begin{align*}
\sigma^f - pI \cdot n &= p_1 n, \\
(V_f - V_B) \cdot n &= 0, \\
\bar{J}_\alpha \cdot n &= 0, \quad \alpha = +, -, \\
\end{align*}
\]

(35)

where \( p_1 \) is the external mechanical load.

The boundary conditions at the side of the glass filter are

\[
\begin{align*}
u &= 0, \\
p - \pi &= -2\bar{\phi}RTc, \\
\mu^\alpha &= \frac{RT}{V^\alpha} \ln(\bar{J}^\alpha c), \quad \alpha = +, -. \\
\end{align*}
\]

(36)

Although \( D^+, D^-, K \) and \( H \) are constants, the fitting of the experimental data is reasonable. We also fitted four other experiments. In table 2 the results of these fitting procedures are shown.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fitting Values</th>
<th>Literature Values</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H ) [N m(^{-2})]</td>
<td>0.75-1.08 ( \cdot 10^6 )</td>
<td>0.36-0.96 ( \cdot 10^6 )</td>
<td>[3]</td>
</tr>
<tr>
<td>( K ) [m(^4) N(^{-1}) s(^{-1})]</td>
<td>0.15-0.40 ( \cdot 10^{-15} )</td>
<td>0.22-0.40 ( \cdot 10^{-15} )</td>
<td>[1, 4]</td>
</tr>
<tr>
<td>( D^+ ) [m(^2) s(^{-1})]</td>
<td>0.45-0.54 ( \cdot 10^{-9} )</td>
<td>0.48 ( \cdot 10^{-9} )</td>
<td>[8]</td>
</tr>
<tr>
<td>( D^- ) [m(^2) s(^{-1})]</td>
<td>0.70-0.85 ( \cdot 10^{-9} )</td>
<td>0.78 ( \cdot 10^{-9} )</td>
<td>[8]</td>
</tr>
</tbody>
</table>
7 Discussion

From the simple one-dimensional finite element model, it appears that physically realistic diffusion coefficients can be used to fit the data of the confined compression experiments, when using a quadriphasic mixture model. The electrical potential, which is included in the quadriphasic mixture model, causes a slower fluid outflow in the beginning of the swelling period. The micromechanical phenomenon which causes this slowing down, is the friction between water and the counter-ions which are electrostatically linked to the fixed charges (figure 5). In the extended Darcy equation (6), rewritten by substitution of

\[ n^f(v^f - \nu^s) = -F \cdot K \cdot Fc \cdot (\nabla(p - \pi) + RT(f^+\nabla c^+ + f^-\nabla c^-) - Fc^+c^-\nabla \xi), \]

(37)

this effect is accounted for by the electrical potential gradient, which contributes negatively to the fluid flux. At the beginning of the swelling period, the electrical potential gradient is high (figure 6). In this figure a negative sample height means that these points are in the glass filter and not in the sample. From its simulation and equation (37) it is clear that the potential gradient contributes negatively to the fluid flow. So, this phenomenon must not be neglected.

The finite element model used in this study, is a one-dimensional implementation of the quadriphasic mixture model. Further research will be done on two- and three-dimensional implementations of this mixture model and deformation dependence of stiffness \( H \), permeability \( K \) and diffusion coefficients \( D^+ \).
Fig. 6. Electrical potential at the beginning of the swelling period ($t = 5 \text{ [s]}$) relatively to the potential of the electrolyte solution in the glass filter.

and $D^-$. 

Acknowledgement

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A Nomenclature

c$^\alpha$ = concentration of ion $\alpha$ per unit fluid volume [mol m$^{-3}$];
D$^\alpha$ = diffusion tensor of ion $\alpha$ [m$^2$ s$^{-1}$];
E = Green-Lagrange strain [-];
f$^\alpha$ = activity coefficient of phase $\alpha$ [m$^3$ mol$^{-1}$];
F = Faraday’s constant [C mol$^{-1}$];
\( \mathbf{F} \) = deformation tensor \([-]\);
\( j^\alpha \) = ion volume flux of ion \( \alpha \) \( [\text{m s}^{-1}] \);
\( H \) = elastic stiffness \( [\text{N m}^{-2}] \);
\( J = \det(\mathbf{F}) \) = relative volume change \([-]\);
\( \mathbf{K} \) = permeability tensor \( [\text{m}^4 \text{N}^{-1} \text{s}^{-1}] \);
\( n^\alpha \) = volume fraction of phase \( \alpha \) \([-]\);
\( p \) = fluid pressure \( [\text{N m}^{-2}] \);
\( R \) = universal gas constant \( [\text{J mol}^{-1} \text{K}^{-1}] \);
\( t \) = time \([\text{s}]\);
\( T \) = absolute temperature \([\text{K}]\);
\( u \) = displacement \([\text{m}]\);
\( v^\alpha \) = velocity of phase \( \alpha \) \( [\text{m s}^{-1}] \);
\( \nabla^\alpha \) = molar volume of component \( \alpha \) \( [\text{m}^3 \text{mol}^{-1}] \);
\( z^\alpha \) = valence of component \( \alpha \) \([-]\);
\( \zeta \) = local coordinate \([\text{m}]\);
\( \mu^\alpha \) = electro-chemical pressures of phase \( \alpha \) \( [\text{N m}^{-2}] \);
\( \xi \) = electrical potential \([\text{V}]\);
\( \pi \) = osmotic pressure \( [\text{N m}^{-2}] \);
\( \sigma^{\text{eff}} \) = effective stress \( [\text{N m}^{-2}] \);
\( \phi \) = osmotic coefficient \([-]\).

Superscripts:

\( f \) = fluid;
\( fc \) = fixed charge;
\( s \) = solid;
\( + \) = cation;
\( - \) = anion.

References


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<td>High order divergence-form elliptic operators on Lie groups</td>
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