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Energy Spectra for Decaying 2D Turbulence in a Bounded Domain

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New results are presented for the energy spectra of decaying 2D turbulence in a square container with no-slip walls for integral-scale Reynolds numbers up to 20000. The one-dimensional energy spectra measured close to the walls reveal a \( k^{-5/3} \) inertial range, instead of a \( k^{-3} \) direct enstrophy cascade, due to the production of small-scale vorticity near no-slip boundaries. During the intermediate decay stage a \( k^{-3} \) spectrum starts to emerge and the change in location of the injection scale of small-scale vorticity is explained in terms of an average boundary-layer thickness.

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Thirty years ago the first phenomenological theory of two-dimensional (2D) turbulence was presented by Kraichnan [1] and by Batchelor [2]. Following this theory, the energy spectrum shows an inverse energy cascade (the \( k^{-5/3} \) spectrum) for \( k < k_i \), with \( k_i \) the wave number associated with the injection scale of the forcing, and a direct enstrophy cascade (the \( k^{-3} \) spectrum) for \( k > k_i \). Since these pioneering theoretical studies many numerical investigations have been carried out in order to find supporting evidence for the presence of an inverse energy cascade and a direct enstrophy cascade, and the associated inertial range spectra, in 2D turbulence [3–8]. For the same reasons several experimental investigations have been carried out [9–12].

Numerical studies of forced 2D turbulence with periodic boundary conditions more or less support the Kraichnan-Batchelor picture [3,7,8], although Legras et al. [7] observed steeper spectra for large wave numbers (small scales). Numerical simulations of decaying 2D turbulence with periodic boundary conditions show a more intricate behavior of the energy spectra: the inverse energy cascade is usually not very clearly observed and the direct enstrophy cascade is often established only as a transient state before the viscous range starts to dominate [4,5]. Additionally, the appearance of coherent vortices complicates a comparison of the spectra of decaying 2D turbulence with the Kraichnan-Batchelor theory. It is assumed that due to the presence of a hierarchy of coherent vortices the energy spectrum becomes more steep [4]. Experiments [9–12] to confirm the presence of the inverse energy cascade, the direct enstrophy cascade, or both cascades simultaneously are even more complicated. This is due to the restriction to flows with intermediate or low integral-scale Reynolds numbers (Re \( \leq 2000 \)) in 2D turbulence experiments in thin, magnetically forced, fluid layers [9,10], or due to the lack of 2D incompressibility in soap film experiments which is a consequence of thickness fluctuations [11,12] (note that in the latter experiments higher integral-scale Reynolds numbers can be achieved). Despite the complications associated with soap film experiments, Kellay et al. [11] were able to show evidence for the presence of a direct enstrophy cascade in decaying 2D turbulence by means of homodyne photon correlation spectroscopy and by optical fiber velocimetry. Recently, Rutgers [12] measured the simultaneous presence of the \( k^{-5/3} \) and the \( k^{-3} \) spectrum in forced 2D turbulence using laser Doppler velocimetry. All experimental setups mentioned above disregard the role of (no-slip) boundaries. Moreover, the arrangement of the experiments to measure the spectrum is often not suitable to obtain 1D spectra close to boundaries.

In this Letter we report new results of energy spectra for decaying 2D turbulent flows in a square container with no-slip walls. The numerical simulations of the 2D Navier-Stokes equations on a bounded domain were performed with a 2D dealiased Chebyshev pseudospectral method, with a maximum of 513 Chebyshev modes in each direction for Re = 20000 (361 modes for Re = 10000 and 257 modes for Re = 5000). The numerical computations of decaying 2D Navier-Stokes turbulence with periodic boundary conditions were carried out with a standard 2D Fourier pseudospectral method with a maximum of 341 active Fourier modes in each direction. In both cases neither hyperviscosity nor any other artificial dissipation has been used. The integral-scale Reynolds number of the flow is defined as Re = U/W/\( \nu \) with U the rms velocity of the initial flow field, W the half-width of the container, and \( \nu \) the kinematic viscosity of the fluid. Time has been made dimensionless by W/U and vorticity by U/W. The initial microscale Reynolds number is defined as Re\(_{\text{micro}}\) = 2 Re/\( \omega_0 \), with \( \omega_0 \) the (dimensionless) initial rms vorticity. In our numerical experiments \( \omega_0 = 38.0 \pm 0.5 \) thus corresponding with Re\(_{\text{micro}}\) ≈ 263, 526, and 1052, respectively. The time \( t \) is defined as \( \tau = \frac{t \nu}{W} \), with \( t \) the dimensionless time and \( N^2 \) the number of vortices present in the initial flow field, and \( \tau \approx 1 \) corresponds approximately with the (initial) eddy turnover time.

The initial condition for the velocity field consists of 100 nearly equal-sized Gaussian vortices (thus \( N = 10 \)). The vortices have a dimensionless radius of 0.05 and a dimensionless absolute vortex amplitude \( |\omega_{\text{max}}| \approx 100 \). Half of the vortices have positive circulation, and the other vortices have negative circulation. The vortices are placed on a regular lattice, initially well away from the boundaries,
with a random displacement of the vortex centers equal to
approximately 6% of the dimensionless lattice parameter
$\lambda$, with $\lambda \approx 0.17$. A smoothing function, similar to the
one employed in Ref. [13], has been used in order to en-
sure the no-slip condition exactly. The initial conditions
for the simulations with periodic boundary conditions are
the same.

The spectra discussed in this Letter are computed from
an ensemble average of 12 runs for $\text{Re} = 5000$, an
ensemble average of 8 runs for $\text{Re} = 10000$, and from an
average of 2 runs for the simulations with $\text{Re} = 20000$.

Figure 1 shows some snapshots of the vorticity distribu-
tion of decaying turbulence for $\text{Re} = 20000$ in a container
with no-slip boundaries (Figs. 1a and 1b) and for the case
with periodic boundary conditions (Figs. 1c and 1d). It can
be concluded that already at early times in the flow ev-
olution strong vortex-wall interactions can be observed and
that huge amounts of small-scale vorticity are produced
near the no-slip walls.

A geometry with walls is no longer well approximated
as isotropic and homogeneous in nature and so the dimen-
sional arguments leading to the $k^{-5/3}$ and $k^{-3}$ slope lose
their validity and such spectra are not expected a priori.
One of the tools to understand the role of the boundaries
on the evolution of turbulence is by comparing the energy

\textbf{no-slip, Re=20,000}

![Image](a) \(\tau = 8\) \hspace{1cm} (b) \(\tau = 40\)

\textbf{periodic, Re=20,000}

![Image](c) \(\tau = 8\) \hspace{1cm} (d) \(\tau = 40\)

FIG. 1. Vorticity contour plots of the simulations with no-slip
(a), (b) and with periodic (c), (d) boundary conditions. The con-
tour level increment in units of $\omega_0$ is (a) and (c) 0.5, (b) and
(d) 0.125.
In Figure 3 we have plotted the ensemble averaged 1D spectra, computed near the no-slip boundary ($a = 0.95$), for $Re = 5000$, 10000, and 20000. The inertial range spectra all behave like $k^{-n}$ with $n = 2.4 \pm 0.1$, $1.9 \pm 0.1$, and $1.7 \pm 0.1$, respectively. It shows the gradual disappearance of the direct enstrophy cascade when the Reynolds number is increased. From this observation one can conclude that the absence of the direct enstrophy cascade is not a low Reynolds number artifact. On the contrary, the higher the Reynolds number, the more the energy spectrum shows a $k^{-5/3}$ spectrum, underlining the role of the boundaries as a source of small-scale vorticity. The corresponding inertial range spectra for the runs with periodic boundary conditions behave like $k^{-n}$ with $n = 3.1 \pm 0.2$, $3.0 \pm 0.2$, and $2.8 \pm 0.2$, respectively, which differs considerably from the confined container case. It should be mentioned that the spectra for the runs with periodic boundary conditions do not behave like pure power laws. The computed power law exponent $n$ is a best estimate; as a consequence slight variations around the value $n = 3$ might be expected. The disappearance of the direct enstrophy cascade near no-slip boundaries has also consequences for the spectra measured at $a = 0$, although not so striking. Estimates of the power law exponents of the ensemble averaged 1D spectra are summarized in Table I. It is clear that the spectra measured in the center of the container with no-slip walls show inertial range spectra with $2.1 \leq n \leq 2.5$, and that a clear direct enstrophy cascade is absent.

The inverse cascade, for $\tau < 10$ observed up to the smallest resolved scales, is actually a consequence of the production of small-scale vorticity at the no-slip boundaries. The injection scale $k_i$ of small-scale vorticity appears to be associated with an average boundary-layer thickness $\delta(\tau)$ which is defined as the ratio between the vorticity and the normal gradient of the vorticity at the boundary. It is expected that $\delta(\tau)$ grows in the course of time. This is supported by the time evolution of the spectra shown for $\tau = 8$, 20, 32, and 120 (from upper to lower curve) in Fig. 4a which shows that for $\tau \geq 20$ the

FIG. 2. The 1D energy spectra for runs with no-slip walls and with periodic boundary conditions (the steeper spectrum in the inertial range). The best power law fits for the Chebyshev and the Fourier spectra are represented by the drawn and dashed lines, respectively.

FIG. 3. The 1D energy spectra as function of the wave number for runs with no-slip boundaries. The steepest inertial range spectrum corresponds to the run with $Re = 5000$ ($n = 2.4$). The inertial range spectrum with $n = 5/3$ corresponds with the $Re = 20000$ run. The spectra are measured for $\tau = 4$ and at position $a = 0.95$. 
high wave number part of the $k^{-5/3}$ spectrum slowly transforms into a $k^{-3}$ spectrum, i.e., the injection scale $k_i$ moves to smaller wave numbers (or, equivalently, $\lambda_1 \sim k_i^{-1}$ becomes larger) when the average boundary-layer thickness increases. Note that all spectra shown in Fig. 4a reveal a clear $k^{-5/3}$ slope for $k < k_i$. The strong correlation between the wavelength $\lambda_i$ corresponding to the location of the kink in the spectrum and the average boundary-layer thickness is obvious from the data shown in Fig. 4b. Actually, the following relation is found: $\lambda_i(\tau) = 2\delta(\tau)$ (for clearness we did not plot the data on top of each other). The data displayed in Fig. 4b for Re = 20,000 is much more spiky than for Re = 10,000 and 5000 because only two runs were available for averaging. Note that for $\tau < 20$ it is very difficult to observe a kink in the spectrum: for $\tau < 10$ no kink is observed at all and for $\tau \approx 10$ the $k^{-5/3}$ slope tends to become slightly steeper for large wave numbers. From $\tau = 20$ the position of the kink can be determined with reasonable accuracy.

In conclusion, we have shown the absence of the direct enstrophy cascade in 1D spectra computed near no-slip walls during the initial decay stage ($\tau < 20$) of 2D turbulence in a bounded domain for Re $\leq 20,000$. It is conjectured that the direct enstrophy cascade also disappears during the initial decay stage for simulations with higher initial integral-scale Reynolds numbers. However, numerical confirmation is not possible yet due to the large amount of necessary CPU time on a Cray Y-MP C916 for these high Reynolds number runs in domains with no-slip walls. After approximately 20 initial eddy turnover times the spectrum slowly evolves towards the form known from forced 2D turbulence: a $k^{-5/3}$ slope for $k < k_i$ and a $k^{-3}$ slope for $k > k_i$ with $k_i$ the wave number associated with the average boundary-layer thickness. At this wave number small-scale vorticity is injected into the flow. This observation for the late-time spectrum evolution is nevertheless rather surprising because homogeneity and isotropy, necessary assumptions for the Kraichnan-Batchelor theory of 2D turbulence, are absent for turbulent flows in bounded domains.

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TABLE I. Power law exponents $n$ for runs with no-slip and with periodic boundary conditions for different integral-scale Reynolds numbers. The power law exponents are computed at positions $a = 0.00$ and $a = 0.95$ for $\tau = 4$ and $\tau = 8$.

<table>
<thead>
<tr>
<th>Re</th>
<th>$\tau$</th>
<th>$a$</th>
<th>$n_{\text{no-slip}}$</th>
<th>$n_{\text{periodic}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000</td>
<td>4</td>
<td>0.95</td>
<td>1.7</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.00</td>
<td>2.1</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.95</td>
<td>1.8</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>2.2</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>10,000</td>
<td>4</td>
<td>0.95</td>
<td>1.9</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>2.4</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.95</td>
<td>1.9</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>2.5</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>4</td>
<td>0.95</td>
<td>2.4</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>2.5</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.95</td>
<td>2.5</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>2.5</td>
<td>3.0</td>
<td></td>
</tr>
</tbody>
</table>

FIG. 4. Time evolution of 1D spectra for runs with no-slip walls (Re = 20,000) (a) and the growth of $2\delta(\tau)$ (drawn lines) compared to the growth of $\lambda_i(\tau)$ ($\protect\text{---}$ for Re = 5000, $\protect\text{---}$ for Re = 10,000, and $\protect\text{---}$ for Re = 20,000) (b).