Efficient selection of inputs and outputs for robust control

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Efficient selection of inputs and outputs for robust control

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Abstract

A method is proposed to efficiently select controller inputs and outputs that assure a desired level of robust performance. It is based on an efficiently computable and sufficient existence condition test and it uses an effective search strategy. The selection condition is based on D -scales used in μ -analysis, while the search strategy uses a method to generate all so-called minimal dependent sets. The D -scale test is followed up with a more accurate test employing expensive DK -iterations for smaller-sized subproblems. The tests together yield a good compromise between efficiency and effectiveness. This is demonstrated with an application for a large scale active suspension design problem with a ten DOF model of a tractor-semitrailer. In the application, seven actuators and twenty-one sensors are available to choose from.

Keywords: mechanical systems, active suspension, robust control, $\mathcal{H}_\infty$ -control, μ -synthesis, input output selection, combinatorial optimization, maximal independent set, minimal dependent set

1. Introduction

The ultimately achievable performance of a controlled plant depends on plant characteristics and on controller architecture [1]. One of the relevant issues is the type and number of devices for actuation and sensing or, more generally, of input and output signals used for the closed loop. The goal is to select those that will open the opportunity to deliver a desired level of performance. This requires the solution of a feasibility problem.

Selection of the devices or signals based on a complete candidate-by-candidate feasibility test is a combinatorial problem. The selection can be simplified by not using a candidate-by-candidate approach,

but then it is likely to be less effective and favorable combinations of actuators and sensors can be missed. For an overview of approaches for input output selection, see [2] or [3, Chapter 2].

In previous work [4], we introduced an approach that is more refined than brute force methods. The selection is based on a candidate-by-candidate test, but it uses a streamlined rigorous feasibility test combined with an efficient search strategy. Large scale problems may then be tackled in acceptable time, because only a limited number of combinations need to be tested. The search strategy is based on a novel algorithm to generate all maximal independent sets (or minimal dependent sets) [5], which is a standard problem in combinatorial optimization. Although with this approach the problem is still combinatorial, in practice the complexity is polynomial in the number of inputs and outputs and in the size of the solution.

The goal of our control design (and of input output selection) is to achieve a specified level of robust performance. The standard plant setup with a robust performance criterion is selected, because it embraces a lot of control problems, like setpoint regulation, tracking, and disturbance rejection, all in the face of model errors. The feasibility test is based on conditions for the existence of an $\mathcal{H}_\infty$ controller achieving a required level of performance in the presence of model errors. This controller could be generated with μ -synthesis. The conditions are placed on a generalized plant that depends on the controller inputs and outputs.

The μ -synthesis approach to design a robust controller is transformed into an $\mathcal{H}_\infty$ feasibility problem using D -scales. Computing μ , the D -scales, and an $\mathcal{H}_\infty$ controller (or checking the $\mathcal{H}_\infty$ controller existence conditions) in an iterative way, like in the standard DK -iteration, would be a very time consuming

approach. In [4], a more efficient and still exact approach is used to circumvent most *DK*-iterations by reusing (or streaming) *D*-scales. Even this short-cut in combination with the efficient search strategy may prohibit the computation when the size of the sets that characterize the feasible solutions is not small. This is mainly due to the expense of a feasibility test for combinations of inputs and outputs that do not qualify. Here, a repeated *DK*-iteration-like procedure is used to irrefutably assess the unsuitability of these combinations, *i.e.*, to assess that it will not be possible to design a controller achieving the desired performance.

To improve this situation, several simplifications were tested that improve efficiency, by avoiding most of the expensive tests, potentially at the cost of effectiveness. The new approach in this paper is based on a preliminary selection using sufficient conditions for feasibility, avoiding the repeated computation of *D*-scales, see [6]. This is refined with a reduced-size selection using *DK*-iteration along the lines of [4] for those candidates that were seen to be most promising from the preliminary selection.

The contribution of the paper is the following. It addresses the balance between efficient and effective selection criteria for input output selection. It shows that using a more efficient but approximate approach followed by an expensive but exact one yields results that differ only slightly from the results for an exact analysis. The techniques developed are applied on a large scale model of a tractor-semitrailer. The mass of the trailer is uncertain and the required performance of an active suspension control system is allowed to deviate only slightly from a system using all input/output devices.

The paper is structured as follows. First, we discuss the search strategy and feasibility tests. Then we explain how these methods can be applied on a large scale selection problem. Conclusions finish the paper.

2. The IO selection method

To select combinations of inputs and outputs (also called IO sets), we need two things: an algorithm to efficiently search for promising combinations and a feasibility test that assesses a single candidate IO set. The feasibility test should be efficient because it is called often. The test we employ should tell something about robust performance, *e.g.*, it should use conditions derived from μ -synthesis-based robust control design. The remainder of this section addresses the following points

- Strategy for taming the combinatorially explosive search.

- Approaches to circumvent time-consuming steps in the feasibility test.

2.1. Search strategy

The search strategy is based on an algorithm to generate all maximal independent or all minimal dependent sets. The algorithm was proposed in [5]. We briefly explain the problem setup and the usefulness of the algorithm.

Let E be the finite set of all sensors and actuators that are considered, with cardinality $|E| = n$, and let $\mathcal{I}$ be a nonempty family of subsets of E that satisfies the following rule: if $I \in \mathcal{I}$ and $I' \subseteq I$ then $I' \in \mathcal{I}$. Now, $(E, \mathcal{I})$ is called an independence system and $\mathcal{I}$ is its family of independent sets. An independent set I is called maximal if there is no $I' \in \mathcal{I}$ such that $I' \supset I$. Subsets of E that are not in $\mathcal{I}$ are dependent sets. All dependent sets form the family $\mathcal{J}$. A dependent set J is minimal if $J' \in \mathcal{I}$ for all $J' \subset J$.

The IO selection problem with a monotonous selection criterion exactly fits an independence system problem. A monotonous selection criterion is one where the performance always improves, or stays the same, when an IO set is expanded with additional devices. The family of subsets $\mathcal{I}$ gathers all actuator/sensor combinations that are not acceptable and $\mathcal{J}$ characterizes all acceptable ones. The power set $P(E)$ contains all possible combinations of actuators and sensors and $P = \mathcal{I} \cup \mathcal{J}$. The sets can be graphically represented in a so-called Hasse diagram.

Now the problem is to establish the structure of the independence system, *i.e.*, to find $\mathcal{I}$ and/or $\mathcal{J}$. To do this, an oracle is available that decides whether a subset of E belongs to $\mathcal{I}$ or $\mathcal{J}$. The oracle is expensive and its visits should be minimized. In general, it suffices to find the K maximal independent sets of $\mathcal{I}$ or the M minimal dependent sets of $\mathcal{J}$, because with these sets one can generate the families $\mathcal{I}$ and/or $\mathcal{J}$ without visiting the oracle. Because both K and M are bounded by $\binom{n}{n/2}$, one cannot guarantee to obtain a solution in time polynomial in n .

One may wonder if a solution in time polynomial in n and K or M is possible. Lawler *et al.* [7] state that the problem of finding the K maximal independent or M minimal dependent sets is $\mathcal{NP}$ hard and there is no solution possible in time polynomial in n , K , and M . However, in [5] it is shown to be possible to establish all K maximal independent sets and all M minimal dependent sets visiting the oracle only $O(nK + M)$ or $O(K + nM)$ times. This means that a complete solution with visits polynomial in n , K , and M is possible. An algorithm that achieves this has been used.

2.2. Feasibility test

The selection of IO sets with guaranteed robust performance is based on μ -synthesis with DK -iterations. We assume the reader to be familiar with DK -iteration-based robust control synthesis and omit further details. Consult [8] if necessary. There are other methods for the design of robust controllers, but μ -synthesis has the advantages of a sound theoretical foundation and of readily available analysis and synthesis software.

However, there are several reasons to avoid DK -iterations:

1. They are costly, especially for larger problems.
2. They are numerically not very reliable.
3. The iteration may not converge or may not converge to the global optimum, because the optimization problem is nonconvex.
4. They use an upper bound for complex μ which can be arbitrarily conservative [9].
5. Frequency gridding is used for μ -computations, so the worst case μ may go undetected.

Numerical issues were resolved by careful selection of convergence test levels and by conditioning and balancing of the models. Convergence behavior has been improved by starting from a different initial condition in case of unsuccessful tests, but this doubles computation time. The accuracy of the upper bound has been verified for selected IO sets by computing a lower bound and comparing the bounds. Frequency gridding effects can be avoided by using $\mathcal{H}_\infty$ -performance levels of plants with D -scales as criterion and not the computed μ values. No further problems were encountered during the actual computations. So, the last four reasons to avoid DK -iterations are taken care of.

Several approaches can be used to circumvent DK -iterations completely, while still a reasonable approximation to the robust performance problem is obtained. One approach is based on conditions for the existence of a noncausal controller, as proposed in [10] and analyzed in [11]. Other approaches are based on fixed D -scales or fixed worst-case uncertainties and are analyzed in [6]. These approaches are two to three orders of magnitude faster than a DK -iteration-based approach.

In this study, we employ the fixed D -scale approach. The reason is that this approach is based on sufficient conditions, so it is conservative and robust performance can always be guaranteed, which is in the spirit of robust control. The other two approaches are based on necessary or a mix of partly necessary and partly sufficient conditions and no guarantees can be given.

The fixed D -scale approach works as follows. For a selected IO set (most often the full IO set with all

devices or the empty IO set, i.e., the open loop), a μ -synthesis with DK -iteration or a μ -analysis is carried out. The D -scales found are used to augment the plants for all other IO sets. The feasibility test then only consists of checking conditions for the existence of an $\mathcal{H}_\infty$ controller achieving a specified performance level. Efficient tools for this task are available and may be based on Riccati equations [12] or on conditions expressed in terms of linear matrix inequalities [13]. We employ Riccati equations, being more efficient.

The results of IO selection with fixed D -scales can be refined in a second stage with the more effective DK -iteration-based approach. A possibility is to extract from the fixed D -scales results those devices that are most promising, e.g., by selecting those that

- occur often in the minimal dependent sets,
- occur in minimal dependent sets that have a low number of devices.

By eliminating devices that are not expected to add much, the size of the problem is reduced and a selection using DK -iterations may be tractable. In the second stage, the minimal dependent sets found from the analysis with fixed D -scales can be used to eliminate some feasible IO sets without performing a feasibility test, and so speedup the computations. This is possible, because using fixed D -scales is conservative. Also, the results from the first and second stage can be merged to give an improved solution of the full size problem.

3. Example

The methods are illustrated for a tractor-semitrailer active suspension problem with seven actuators and twenty-one sensors, so with $n = 28$ input/output devices, see Fig. 1, making $\approx 2^{28}$ or $\approx 266 \cdot 10^6$ unique combinations possible. This example has been introduced and studied in [4]. We now summarize some results and will present new results for problems that could not be solved in reasonable time with the streaming DK -iteration-based approach followed in [4].

The number of feasibility tests depends on the approach taken to circumvent DK -iterations and on the required performance level. In [4] only 274 candidate combinations were tested for feasibility to completely determine all combinations of sensors and actuators that were guaranteed to reach the specified level of performance ($\mu \leq 1.6$). The number of tests was almost 10^6 times less than a complete candidate-by-candidate search would require. The low number of tests is due to small values for K and M .

For other performance levels the number of feasibility tests may be much larger, due to larger values of

Figure 1: Tractor-semitrailer with potential actuators and sensors indicated

K and M , necessitating the use of approximations to the original robust performance control problem to solve the problem in acceptable time. The number of states of the generalized plant including D -scales or including an $\mathcal{H}_\infty$ controller is rather large (≈ 100), so the computations take their time. Results are presented for the following two stages:

1. The use of fixed D -scales only; this results in an approximate characterization of the independence system but with much faster computations.
2. Searching in potentially better subsets of all devices, coming from, e.g., an initial screening with fixed D -scales from stage 1, but now using DK -iteration; this reduces the number of DK -iterations, because we test for sets of smaller n , while results for different initial subsets can be merged with the results of stage 1 because all results are based on sufficient conditions.

For the full IO set, the achievable value of complex unstructured μ is slightly larger than 1. For required performance levels of 1.6, 1.4, 1.2, and 1.05 (the lower the better), IO selection has been carried out with fixed D -scales associated with the full IO set. For levels 1.6 and 1.4 also D -scales based on the empty IO set were employed. It has been verified that for these IO sets the upper bound is tight. The number of required devices increases with decreasing values of μ . For the performance level of 1.05, using the fixed D -scales results and the two criteria mentioned at the end of Section 2, promising subsets were selected of sizes 10, 13, 15, 18, and 21, where size is the initial number of devices in the subsets. With rigorous DK -iteration tests these sets were analyzed, leading to a reduction of the num-

ber of devices in the minimal dependent sets compared to fixed D -scales. We present an extract of the results. Giving results in Hasse diagrams does not make sense, due to the large number of IO sets. So, a more condensed representation is chosen.

First, Figs. 2-3 present how often certain devices show up in the minimal dependent sets obtained with fixed D -scales and μ requirements of 1.05 and 1.4.

Figure 2: Devices in minimal IO sets for $\mu \leq 1.05$

Remark that some devices, e.g., the ones numbered 2 and 3, occur in all sets and that the number of minimal dependent sets is much larger for the 1.4 level than for 1.05. While the selections for $\mu \leq 1.05$ are based on D -scales for the full IO set only, for $\mu \leq 1.4$ results for two different D -scales, based on the full and on the empty IO set, were computed and merged to get more accurate results.

Figure 3: Devices in minimal IO sets for $\mu \leq 1.40$

Figure 4 shows the distribution of the size of maximal independent and minimal dependent sets for the 1.05 level.

Figure 4: Number of maximal independent IO sets (x) and minimal dependent IO sets (o) of a certain size for $\mu \leq 1.05$

It can be seen that there are five maximal independent sets with 27 devices, one less than the maximal number. This corresponds with the fact that devices 2, 3, 9, 10, and 20 are always used, so sets without one of these devices are not feasible. It also implies that the subsets selected for *DK*-iteration refinement should better include these five devices.

Figure 5 presents a comparison of the distribution of the sizes for several selection levels.

Remark that for lower levels (more stringent requirements) the left tip of the lines shifts to the right, indicating that a larger number of devices is needed to achieve the desired performance level.

Finally, Fig. 6 compares the results of the selection

Figure 5: Comparison of the number of minimal IO sets of a certain size; —: $\mu \leq 1.05$, - -: $\mu \leq 1.20$, - · -: $\mu \leq 1.40$, · · · : $\mu \leq 1.60$

with fixed *D*-scales for the 1.05 level with a merger of these results with the selection results for the smaller subsets with *DK*-iteration, also for the 1.05 level. The smaller subsets initially include the five devices mentioned before that always appear in the minimal IO sets for the fixed *D*-scale approach, extended with devices that occur regularly in the smallest of these IO sets (of size 9 and 10) and with the ones that score high in Fig. 2.

Figure 6: Comparison of sizes of minimal IO sets for $\mu \leq 1.05$; (o): *DK*-iteration based, (x): fixed *D*-scales based

The results show that a smaller number of devices is allowed, which is to be expected because *DK*-iteration is less conservative. The smallest IO sets contain seven devices, namely three actuators and four sensors, and are subsets of several of the smallest IO sets generated with the fixed *D*-scale test. They

are also subsets of the initial subsets with 15 or more devices. The number of devices can be compared with the single sensor/actuator pair that is sufficient for a 1.6 level and with the nine devices, four actuators and five sensors, that the fixed D -scales test needs for level 1.05. For $\mu \leq 1.05$ it has been verified that the minimum number of actuators is three (by input selection with all sensors present) and the minimum number of sensors is four (by output selection with all actuators present). Therefore, there are no smaller IO sets that meet the performance specification and the final result for the smallest IO sets is rigorous. The difference in the minimal number of devices for the results after stage one and two, nine and seven, is not that large, so one can conclude that the approximate but more efficient fixed D -scales method is useful. A reduction with a factor four, from 28 to 7, of the number of devices needed to achieve a performance level on par with the full IO set will give a large saving in investments, maintenance, and reduces instrumentation complexity. A physical interpretation of the results is facilitated by the fact that the design is much oriented towards robustness. Without the model uncertainty, the achievable value of μ is much lower. The uncertainty in the model is related to uncertain semitrailer mass and inertia. Most actuating and sensing devices that show up in the minimal IO sets are physically "close" to the semitrailer:

- actuators are chosen so the influence of model uncertainty on the plant is like a "matched" disturbance,
- sensors are chosen so the relative degree between uncertainty input to the plant and sensed plant output is small.

4. Conclusions

Efficient methods for input output selection were assessed. It was found that a fast but non-effective fixed D -scales feasibility test combined with a effective DK -iteration based test, both using a novel algorithm to search in a Hasse diagram, is sufficient to find promising combinations of devices. Applying this technique on a large scale active suspension control problem shows that the number of actuators and sensors can be reduced by a factor of four, compared with the potential number of devices, without compromising the achievable robust performance. A future goal is to come up with tight necessary existence conditions that are efficiently computable. This could be used to eliminate most of the DK -iterations needed to classify combinations of inputs and outputs as "not good enough."

References

- [1] S. P. Boyd and C. H. Barratt, *Linear Controller Design: Limits of Performance*. Englewood Cliffs, NJ: Prentice Hall, 1991.
- [2] M. van de Wal and B. de Jager, "Control structure design: A survey," in *Proc. of the 1995 American Control Conf.*, vol. 1, (Seattle, WA), pp. 225-229, IEEE, Piscataway, NJ, June 1995.
- [3] M. van de Wal, *Selection of Inputs and Outputs for Control*. PhD dissertation, Eindhoven University of Technology, Fac. of Mechanical Engineering, Apr. 1998.
- [4] B. de Jager, M. van de Wal, and R. Kamidi, "Large-scale rigorous actuator and sensor selection," in *Proc. of the 37th IEEE Conf. on Decision and Control*, (Tampa, FL), pp. 4314-4319, IEEE, Piscataway, NJ, Dec. 1998.
- [5] B. de Jager and O. Toker, "Complexity of input output selection," in *Proc. of the Internat. Symp. on the Mathematical Theory of Networks and Systems (MTNS 98)*, (Padova, Italy), 1998. To appear.
- [6] M. van de Wal and B. de Jager, "Input/output selection for robust control," in *Proc. of the UKACC Internat. Conf. CONTROL'98*, vol. 2, (Swansea, UK), pp. 1023-1028, IEE, Sept. 1998.
- [7] E. L. Lawler, J. K. Lenstra, and A. H. G. Rinnooy Kan, "Generating all maximal independent sets: NP-hardness and polynomial-time algorithms," *SIAM J. Comput.*, vol. 9, pp. 558-565, Aug. 1980.
- [8] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Upper Saddle River, NJ: Prentice-Hall, 1996.
- [9] S. Treil, "The gap between complex structured singular value μ and its upper bound is infinite," tech. rep., MIT, Nov. 1998. Preprint available from <http://www.math.msu.edu/~treil/papers/mu/mu-abs.html>.
- [10] J. H. Lee, R. D. Braatz, M. Morari, and A. Packard, "Screening tools for robust control structure selection," *Automatica—J. IFAC*, vol. 31, pp. 229-235, Feb. 1995.
- [11] M. van de Wal, P. Philips, and B. de Jager, "Actuator and sensor selection for an active vehicle suspension aimed at robust performance," *Internat. J. Control*, vol. 70, pp. 703-720, June 1998.
- [12] K. Glover and J. C. Doyle, "State-space formulae for all stabilizing controllers that satisfy an H_∞ norm bound and relations to risk sensitivity," *Systems Control Lett.*, vol. 11, pp. 167-172, 1988.
- [13] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H_∞ control," *Internat. J. Robust Nonl. Control*, vol. 4, pp. 421-448, Jan.-Feb. 1994.