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Gap Closing for Cooperative Driving in Automated Vehicles using B-splines for Trajectory Planning

Robbin van Hoek¹, Jeroen Ploeg² and Henk Nijmeijer³

Abstract—Recently, increasing interest has been shown in cooperative driving and platooning, as they show great potential for increasing the road throughput, by both increasing road capacity and preventing so called 'ghost' traffic jams. These cooperative vehicles make use of controllers or trajectory planners that achieve a certain spacing policy. However, these systems may result in large accelerations when the host vehicle is trying to close a gap to a preceding vehicle, leading to uncomfortable responses. In this work, a trajectory planning method is presented that is able to smoothly close a large gap, by means of a time-varying spacing policy. The resulting trajectory planner is capable of both maintaining a desired spacing as well as closing a gap comfortably to a preceding vehicle.

I. INTRODUCTION

The increase in the demand for mobility has been steadily growing in recent years. As a result, the current infrastructure encounters more traffic which often results in traffic jams. One major contribution to traffic jams are the so called 'ghost' traffic jams, in which a following vehicle needs to brake harder than the preceding vehicle, due to the reaction time of the driver. This amplification of disturbances is referred to as string instability.

A modern development that tackles this problems is Cooperative Adaptive Cruise Control (CACC), in which the velocity of a vehicle is adjusted based on the distance to the preceding vehicle and V2V communication [1] [2]. This control system yields string stable vehicle platoons, at very short inter vehicle distances, significantly increasing the capacity of the road, while preventing 'ghost' traffic jams. However, the application domain of CACC is rather limited, as the main purpose of CACC is platooning on highways. For vehicle automation that is capable of navigating various traffic scenarios, typically trajectory planners are used [3] [4]. Using these trajectory planners, the vehicle is capable of navigating both highways and urban areas. Additionally, they can be used in various scenarios, such as merging or overtaking, rather than only vehicle following. In contrast to CACC, however, these planners are typically not focussed on achieving string stability. To achieve string stability, cooperative trajectory planning is required, in which planned trajectories are communicated between vehicles. These trajectories are based on the trajectories of preceding vehicles as

presented in [5], which developed a method to use B-splines for cooperative trajectory planning, thus enabling a single trajectory planning framework to be used for both urban driving as well as platooning on highways. Cooperative driving controllers, based on Model Predictive Control (MPC), such as presented in [6], could also be viewed as trajectory planning for cooperative driving. However, trajectories generated by an MPC approach communicate all the samples of the trajectory. This results in a higher required communication bandwidth than communicating only the parameters of the B-spline. Moreover, closed-form computations of the latter are beneficial for real time implementation.

Although this method works well for vehicle following, large accelerations are generated in gap closing scenarios, due to large initial spacing error. These result in discomfort for the passengers. Various solutions have been proposed for the traditional feedback CACC system. In [7], CACC is evaluated in real life traffic situations. Three controllers are developed for the tasks of velocity keeping, gap closing and distance keeping. In [8], a single controller is used for both distance keeping as well as gap closing and gap making, in which the control objective is modified accordingly. In [9] [10], Artificial Potential Fields are used for the synthesis of a non-linear controller, while maintaining a single control objective. However, these methods do not make use of the benefits of cooperative trajectory planning.

This work presents a unified method for cooperative trajectory planning, that integrates both comfortable gap closing as well as string-stable distance regulation. The main contribution is a time-varying spacing policy, that is used for the construction of B-spline trajectories. Both varying time gap and gap distance are considered and compared with the constant spacing policy. B-splines trajectories are used such that trajectories can be communicated using a small number of parameters, facilitating a small communication bandwidth. A closed form computation ensures that a trajectory is available in time for execution. The gap closing method is easily tunable such that gap closing rate and minimum closing velocity can be adapted to user preference.

This paper is organised as follows. Section II presents the adapted framework and the use of B-splines for trajectory planning. Section III outlines the gap closing method. Section IV compares the results of the gap closing approach to the trajectory planner without gap closing strategy by means of simulation. Finally, Section V presents the conclusions.

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Fig. 1: Reference path described by $\mathbf{X}(s)$, deviated path constructed via lateral offset d in the moving Frenet frame $\mathbf{e}^2(s)$, which is parametrized to curvilinear distance, s .

II. PRELIMINARIES

The framework as presented in [3], [5] is adopted, in which a method for real time trajectory planning relative to a reference path is constructed. Planning is performed relative to this reference path by means of a Frenet frame, as illustrated in Figure 1. Employing this coordinate frame the planning problem is reformulated in the curvilinear coordinate s and the lateral offset d . Trajectories are constructed by parametrizing these coordinates in time, thus obtaining $s(t)$ and $d(t)$. In this work, vehicle following scenarios are emphasised. The vehicles are following the same reference path without the need to change lanes. Therefore only $s(t)$ is considered. Notice however, that the same work can still be applied to the full planner in which both $s(t)$ and $d(t)$ are considered.

B-splines of degree p [11] are used to construct parametrized trajectories. The trajectories under consideration express the curvilinear distance $s_i(t)$ of vehicle i :

$$s_i(t) = \mathbf{N}_{n,p}(t) \mathbf{P}_i, \quad t \in [t_{c,i}, t_{c,i} + T_i] \quad (1)$$

$$\mathbf{N}_{n,p}(t) = [N_{0,p}(t) \quad N_{1,p}(t) \quad \dots \quad N_{n,p}(t)] \quad (2)$$

where $\mathbf{N}_{n,p}(t)$ is a row vector of basis functions, $\mathbf{P}_i = [P_{0,i}, P_{1,i}, \dots, P_{n,i}]^T \in \mathbb{R}^{n+1}$ are $n+1$ control points, $t_{c,i}$ is the current planning time, that increments with planning interval t_p , and T_i the planning horizon for vehicle i . The basis functions $N_{j,p}(t)$ are based on the knotvector $\mathbf{U}_i$. A uniform knotvector that spans from $t_{c,i}$ to the end of the horizon $t_{c,i} + T_i$ is used, that is defined as

$$\mathbf{U}_i = [\underbrace{t_{c,i}, \dots, t_{c,i}}_{p+1}, \underbrace{u_{p+1}, \dots, u_n}_{n-p}, \underbrace{T_i + t_{c,i}, \dots, T_i + t_{c,i}}_{p+1}],$$

$$u_j = \frac{j-p}{n-p+1} T_i + t_{c,i}, \quad j = \{p+1, \dots, n\} \quad (3)$$

using the knots u_j , basis functions $N_{j,k}(u)$, $k \in [0, \dots, p]$ are recursively defined as,

$$N_{j,0}(t) = \begin{cases} 1 & \text{if } u_j \leq t < u_{j+1} \\ 0 & \text{otherwise} \end{cases}, \quad (4)$$

$$N_{j,k}(t) = \frac{t - u_j}{u_{j+k} - u_j} N_{j,k-1}(t) + \frac{u_{j+k+1} - t}{u_{j+k+1} - u_{j+1}} N_{j+1,k-1}(t)$$

An advantage of using B-splines as trajectories, is that the derivative of a B-spline is again a B-spline. As a result, the

Fig. 2: B-spline trajectory ($n=7, p=5$) $s_i(t)$ (—), control points ($\mu_j, P_{j,i}$) (■), control polygon (---) and knots (|).

trajectory $s(t)$ as well as the derivatives $\dot{s}_i(t)$ and $\ddot{s}_i(t)$ is expressed as a function of the original control points

$$\dot{s}_i(t) = \mathbf{N}_{n-1,p-1}(t) \mathbf{\Gamma}^1(\mathbf{U}_i) \mathbf{P}_i \quad (5)$$

$$\ddot{s}_i(t) = \mathbf{N}_{n-2,p-2}(t) \mathbf{\Gamma}^2(\mathbf{U}_i) \mathbf{\Gamma}^1(\mathbf{U}_i) \mathbf{P}_i \quad (6)$$

where $\mathbf{\Gamma}^{(k)}(\mathbf{U}_i)$ is a matrix to transform the control points $\mathbf{P}_i^{(k-1)}$ to the control points of the k -th derivative of the B-spline, i.e., $\mathbf{P}_i^{(k)} = \mathbf{\Gamma}^{(k)}(\mathbf{U}_i) \mathbf{P}_i^{(k-1)}$. The expression for $\mathbf{\Gamma}^{(k)}(\mathbf{U}_i)$ is omitted here for brevity but can be derived from the hodograph relation (for $k \geq 1$)

$$P_{j,i}^{(k)} = \frac{p-k+1}{u_{j+p+k+1} - u_{j+1+k}} (P_{j+1,i}^{(k-1)} - P_{j,i}^{(k-1)}) \quad (7)$$

This is shown in Figure 2, which displays the trajectory and its derivatives together with the corresponding control polygons, which connect the control points. The control polygon is used to shape the spline. For a 1D curve, this control polygon can be constructed using control points $P_{j,i}$ and the Greville abscissae [11]

$$\mu_j = \frac{1}{p} \sum_{k=1}^p u_{j+k}, \quad j \in \{0, \dots, n\} \quad (8)$$

describing the location of the control points in the control polygon. Note that by using B-splines, the trajectory $s_i(t)$ can be fully described using the $n+1$ control points $\mathbf{P}_i$, current planning time $t_{c,i}$ and time horizon T_i , such that less communication bandwidth is required to inform the other vehicles about the trajectory, resulting in more reliable communication.

For the vehicle following scenario the same spacing policy that is commonly used in CACC applications [2] is used. The desired inter vehicle distance $d_{i,r}$ between vehicle i and vehicle $i-1$ and the distance error e_i of vehicle i are

$$d_{i,r}(t) = c_{r,i}(t) + h_{r,i}(t) \dot{s}_i(t), \quad (9)$$

$$e_i(t) = s_{i-1}(t) - s_i(t) - c_{r,i}(t) - L_i - h_{r,i}(t) \dot{s}_i(t) \quad (10)$$

in which $h_{r,i}(t)$ is the desired time gap, $c_{r,i}(t)$ the desired gap size, L_i the vehicle length. Note that, $h_{r,i}$ and $c_{r,i}$ are not assumed to be constant.

The desired trajectory for $s_i(t)$ is constructed based on the planned trajectory $s_{i-1}(t)$ of the preceding vehicle. By construction, $\mathbf{N}_{k,p}(t_{c,i}) = [1 \ 0 \ \dots \ 0] \in \mathbb{R}^{k+1}$, $\forall p$, such that the control points $P_{j,i}, j \in \{0, 1, 2\}$ are fixed to start the trajectory in the initial condition via (5)(6). This is due to the vehicle model as described in [2] not being able to change acceleration instantaneously. The remaining control points for the spline trajectory are found by solving a linear system of equations, resulting from substituting (1) in (10) and equating it to zero at the time stamps $\mu_i, i \in \{3, \dots, n\}$.

$$\begin{bmatrix} P_{3,i} \\ P_{4,i} \\ \vdots \\ P_{n,i} \end{bmatrix} = \mathbf{\Omega}_{12}^{-1} \left(\begin{bmatrix} s_{i-1}(\mu_3) - c_{r,i}(\mu_3) - L_i \\ s_{i-1}(\mu_4) - c_{r,i}(\mu_4) - L_i \\ \vdots \\ s_{i-1}(\mu_n) - c_{r,i}(\mu_n) - L_i \end{bmatrix} - \mathbf{\Omega}_{11} \begin{bmatrix} P_{0,i} \\ P_{1,i} \\ P_{2,i} \end{bmatrix} \right) \quad (11)$$

$$\begin{bmatrix} \mathbf{\Omega}_{12} & \mathbf{\Omega}_{11} \end{bmatrix} = \begin{bmatrix} \mathbf{N}_{n,p}(\mu_3) + h_{r,i}(\mu_3) \mathbf{N}_{n-1,p-1}(\mu_3) \mathbf{\Gamma}^1(\mathbf{U}_i) \\ \mathbf{N}_{n,p}(\mu_4) + h_{r,i}(\mu_4) \mathbf{N}_{n-1,p-1}(\mu_4) \mathbf{\Gamma}^1(\mathbf{U}_i) \\ \vdots \\ \mathbf{N}_{n,p}(\mu_n) + h_{r,i}(\mu_n) \mathbf{N}_{n-1,p-1}(\mu_n) \mathbf{\Gamma}^1(\mathbf{U}_i) \end{bmatrix}$$

where $\mathbf{\Omega}_{11}, \mathbf{\Omega}_{12}$ are matrix partitions of corresponding sizes. More timestamps could be included in the optimization, in which case a Moore-Penrose inverse should be used.

III. VARIABLE SPACING POLICY

The objective is to construct B-spline trajectories such that gap closing is comfortable and independent of distance to the preceding vehicle, while distance keeping is unaffected. To achieve this, the spacing policy is adjusted as a function of time, such that the planner will not attempt to close the gap all at once. To this extent, either the speed-independent gap size $c_{r,i}(t)$ or the time gap $h_{r,i}(t)$ can be adjusted.

A. Variable time gap

First, a varying time gap $h_{r,i}(t)$, is considered. The initial time gap $h_i(t_{c,i})$ is determined by rewriting (10) with $e_i = 0$ and $c_{r,i}(t) = \underline{c}_i$ as,

$$h_i(t_{c,i}) = \frac{s_{i-1}(t_{c,i}) - s_i(t_{c,i}) - L_i - \underline{c}_i}{\dot{s}_i(t_{c,i})} \quad (12)$$

The desired time gap is defined over time interval $t \in [t_{c,i}, t_{c,i} + T_i]$ as a linearly decreasing function starting in $h_i(t_{c,i})$ and lower bounded by the final value $\underline{h}_i$ as typically used in vehicle-following situations:

$$h_{r,i}(t) = \min [h_i(t_{c,i}) + \phi_i \cdot (t - t_{c,i}), \underline{h}_i] \quad (13)$$

in which, ϕ_i is a parameter used to control how $h_{r,i}(t)$ changes over time.

1) *Constant ϕ_i :* Initially, a user defined constant is used for $\phi_i = \underline{\phi}_i$, which can be used for comfort tuning. Since only the gap closing aspect is modified and distance keeping is maintained, $h_{r,i}(t)$ should never drop below $\underline{h}_i$, which is illustrated in Figure 3(a). The value for ϕ_i can be used to determine the velocity with which the gap is closed. This can be seen by writing down the dynamics in which a constant rate of change is assumed for the time gap $\dot{h}_{r,i} = \phi_i$,

Fig. 3: Desired time gap $h_{r,i}(t)$ (13). (a) fixed change rate of time gap $\phi_i = \underline{\phi}_i$, (b) variable rate of change of time gap ϕ_i according to (17)

$$\dot{e}_i = v_{i-1} - v_i - \phi_i v_i - h_{r,i} a_i \quad (14)$$

$$\dot{v}_i = a_i \quad (15)$$

where $v_i := \dot{s}_i(t)$. This system has the following equilibrium for a given velocity v_{i-1} of vehicle $i-1$ and $a_i = 0$:

$$v_i = \frac{1}{1 + \phi_i} v_{i-1} \quad (16)$$

Thus $\phi_i \in (-1, 0)$ can be used to control the velocity difference with which the gap to the preceding vehicle is closed. This is also somewhat intuitive, as it would be desirable to close the gap on a highway with a larger velocity difference than in urban areas. However, using a constant $\phi_i = \underline{\phi}_i$ requires $\dot{s}_i(t)$ to be discontinuous, where the saturation in (13) occurs. This discontinuity originates from the vehicle being requested to maintain a higher velocity (to reduce the time gap) and then instantaneously switch to maintaining a constant time gap at a lower velocity. Due to the continuity requirements of the B-spline, this introduces oscillations when fitting the spline which is undesirable.

2) *Variable ϕ_i :* To realise a continuous gap closing velocity, a second method is presented that uses (13), but in which the slope ϕ_i is adjusted, such that $h_i(t_{c,i}) + \phi_i T_i \geq \underline{h}_i$, via

$$\phi_i = \begin{cases} \underline{\phi}_i, & \text{if } h_i(t_{c,i}) + \underline{\phi}_i T_i \geq \underline{h}_i \\ \frac{h_i(t_{c,i}) - \underline{h}_i}{T_i}, & \text{otherwise} \end{cases} \quad (17)$$

and not saturation is required. Note that in doing so, no discontinuity in velocity is required in the planning horizon, which can also be seen when comparing Figure 3(a) and 3(b). This discontinuity is instead shifted to the end of the B-spline, such that no oscillations occur in the fitting process. Note that this naturally introduces a smoothing effect as the vehicle starts to slow down slightly as soon as the minimum time gap comes within the horizon at the current velocity, according to the second equation in (17). Then, every next planning update, the initial time gap has decreased further, such that the slope ϕ_i decreases even more, converging to zero as $t \rightarrow \infty$. Clearly, this spacing strategy changes every time step due to (17). This is illustrated in Figure 4(a), which shows the desired time gap $h_{r,i}(t)$ constructed by in consecutive planning cycles. This shows that the newly created desired time gap does not coincide with the one that was planned in the previous time step. As a result temporal consistency is lost, and the planned trajectories will not coincide with those of the previous planning cycle. This is

Fig. 4: Desired time gap, $h_{r,i}(t)$. (a) Consecutive planning cycles using (13) and (17), (b) $h_{r,i}$, as defined in (18)

undesirable, as the communicated trajectories are used for planning in following vehicles as well.

3) *Dynamics for $h_{r,i}$* : To achieve temporal consistency, observe that an identical smoothing effect can be achieved by creating dynamics for $h_{r,i}$ as following

$$\dot{h}_{r,i}(t) = \begin{cases} \underline{\phi}_i, & \text{if } h_{r,i}(t) + \underline{\phi}_i T_i > \underline{h}_i \\ \frac{h_{r,i}(t) - \underline{h}_i}{T_i}, & \text{otherwise} \end{cases} \quad (18)$$

The time gap h_{tr} where the dynamics (18) transition, is found by equating the condition in (18), and noting that it cannot be bigger than the initial time gap $h_i(t_{c,i})$. The time corresponding time t_{tr} is then found via

$$h_{tr} = \min [\underline{h}_i - \underline{\phi}_i T_i, h_i(t_{c,i})], \quad t_{tr} = \frac{h_i(t_{c,i}) - h_{tr}}{-\underline{\phi}_i} + t_{c,i}$$

This transition is also illustrated in Figure 4 (b). Using this transition, the trajectories, $h_{r,i}(t)$ for $t \in [t_{c,i}, t_{c,i} + T_i]$, with initial condition $h_{r,i}(t_{c,i}) = h_i(t_{c,i})$, can be computed as

$$h_{r,i}(t) = \begin{cases} h_i(t_{c,i}) + \underline{\phi}_i(t - t_{c,i}), & \text{if } t < t_{tr} \\ (h_{tr} - \underline{h}_i) \exp\left(\frac{t_{tr} - t}{T_i}\right) + \underline{h}_i, & \text{otherwise} \end{cases} \quad (19)$$

This is illustrated in Figure 4(b), which plots the resulting $h_{r,i}(t)$. As a result, the strategy of (19) does result in temporal consistency, as the desired time gap $h_{r,i}$ will follow the same curve in the next planning update.

B. Variable gap

An important drawback of using strategies for desired time gap $h_{r,i}(t)$ is that at low velocities or standstill, the current time gap from (12) is very large or undefined, respectively. A desired gap $c_{r,i}(t)$ will not suffer from these issues. In this section, the time gap $h_{r,i}(t) = \underline{h}_i$ will be assumed constant. Instead, the current gap size is determined as

$$c_i(t) = s_{i-1}(t) - s_i(t) - \underline{h}_i \dot{s}_i(t) - L_i \quad (20)$$

Similarly to (18), dynamics for desired gap $c_{r,i}(t)$ can be derived as

$$\dot{c}_{r,i}(t) = \begin{cases} \underline{\psi}_i, & \text{if } c_{r,i}(t) + \underline{\psi}_i T_i > \underline{h}_i \\ \frac{c_{r,i}(t) - \underline{c}_i}{T_i}, & \text{otherwise} \end{cases} \quad (21)$$

in which $\underline{\psi}_i$ is the rate at which the gap size decreases. This rate is determined with three considerations. The resulting velocity should be no smaller than that of (16) for comparison with strategy (19). In case the vehicle already drives faster than (16), no unnecessary hard braking should occur

for user comfort. Finally, in case both vehicles are driving too slow or are at standstill, a minimum gap size rate of $\underline{v}_{cl}$ is utilized instead, to ensure the gap can always be decreased. These lower bounds on $\underline{\psi}$ yield:

$$\underline{\psi}_i = -\max \left[\left(\frac{1}{1+\underline{\phi}_i} - 1 \right) \bar{V}_{i-1}, \underline{v}_{cl}, \dot{s}_i(t_{c,i}) - \bar{V}_{i-1} \right] \quad (22)$$

$$\bar{V}_{i-1} = \frac{1}{T_i} (s_{i-1}(t_{c,i}) + s_{i-1}(t_{c,i} + T_i)) \quad (23)$$

The transition time t_{tr} and gap size c_{tr} in the dynamics (21) can be determined similar as before via

$$c_{tr} = \min [\underline{c}_i - \underline{\psi}_i T_i, c_i(t_{c,i})], \quad t_{tr} = \frac{c_i(t_{c,i}) - c_{tr}}{-\underline{\psi}_i} + t_{c,i}$$

which can be used to compute the trajectories for $c_{r,i}(t)$

$$c_{r,i}(t) = \begin{cases} c_i(t_{c,i}) + \underline{\psi}_i(t - t_{c,i}), & \text{if } t < t_{tr} \\ (c_{tr} - \underline{c}_i) \exp\left(\frac{t_{tr} - t}{T_i}\right) + \underline{c}_i, & \text{otherwise} \end{cases} \quad (24)$$

The gap closing strategy and resulting trajectory planner should be able to operate in all conditions in which the vehicle can reasonably be expected to operate. The handling of low velocities or standstill is thus an important criterion, as well as the lead vehicle not driving at a constant velocity. These gap closing strategies are all defined relative to the trajectory of the preceding vehicle via (11), such that a disturbance from the lead vehicle should not pose a problem.

IV. RESULTS

In this section, simulations are used to compare the four gap closing strategies in terms of user comfort, measured by acceleration and jerk, and temporal consistency. The simulations are set up with $p = 5$, $n = 6$, $T_{1,2} = 5$ s and planning is updated with planning interval $t_p = 0.2$ s. In all cases, the user defined values are chosen as $\underline{\phi}_2 = -0.1$, $\underline{h}_2 = 0.5$ s, $\underline{c}_2 = 5$ m, $L_{1,2} = 0$ m and $\underline{v}_{cl} = 1$ m s⁻¹.

Two vehicles are considered, that are driving at a forward velocity of $\dot{s}_{1,2}(0) = 15$ m s⁻¹. Note that if more vehicles are added, these vehicles would simply perform vehicle following behind the vehicle that is closing the gap. The initial spacing error of the following vehicle is set to $e_2(0) = 25$ m. An overview of the vehicle response in terms of distance $d_2(t)$ speed $\dot{s}_2(t)$ and $\ddot{s}_2(t)$, is given in Figure 5. In this figure the distance $d_2(t) = s_1(t) - s_2(t)$, the velocity $\dot{s}_2(t)$ and acceleration $\ddot{s}_2(t)$ are given for all the introduced spacing policies. The original B-spline trajectory planning strategy with constant spacing policy is also shown. This system successfully regulates the initial error towards zero. However, the corresponding control inputs are excessively large. A typical vehicle is not capable of achieving such large forward accelerations (here $\|\ddot{s}_2(t)\| = 9.19$ m s⁻²) due to limited drive train power. Moreover, this is highly uncomfortable for the passengers in terms of both acceleration and high approaching velocity (here $\|\dot{s}_2(t)\| = 24.67$ m s⁻¹).

The first gap-closing strategy constructs the adjusted time gap, $h_{r,i}$ via (13) and uses a constant rate of change for the time gap $\phi_i = \underline{\phi}_i$. Clearly, the gap is closed more gradually,

Fig. 5: Comparison of gap closing strategies. Original $h_{r,2} = \underline{h}_2$ and $c_{r,2} = \underline{c}_2$ (---), $h_{r,2}$ via (13) $\phi_2 = \underline{\phi}_2$ (—), $h_{r,2}$ via (13) ϕ_2 via (17) (—), $h_{r,2}$ via (19) (---), $c_{r,2}$ via (24) (—).

with far smaller accelerations, than is the case for the system without gap-closing strategy. The closing velocity can indeed be seen to be related to the parameter $\underline{\phi}_i$ according to (16). When the vehicle comes near the preceding vehicle, the time gap is saturated to the minimum value $h_{r,i}(t) = \underline{h}_i$ and the vehicle starts decelerating. However, the discontinuity in the time gap change rate results in oscillatory behaviour in terms of acceleration. The resulting response is fairly jerky and uncomfortable for passengers.

These oscillations are absent in the response given by the gap closing strategy with variable change rate ϕ_i (17) based on the current time gap. As a result, a more gradual deceleration is obtained, which is far more comfortable for the passengers. The planned trajectories of consecutive planning updates that are constructed via this gap closing strategy are shown in Figure 6a. It is clear that the consecutive planned trajectories in blue all converge to slightly lower constant velocities given in (16), corresponding to the lower time gap rates, ϕ_i . This implies temporal consistency is indeed lost.

This temporal consistency is regained when using the gap closing strategy of (19). This is illustrated in Figure 6b, where it can be seen that all the consecutive planned trajectories in the deceleration phase all remain close to the previously planned trajectory, such that temporal consistency is achieved. As a result, the executed trajectory also follows these planning cycles closely. This makes the vehicle behave more predictable, which is beneficial when other vehicles base their planning around the previously communicated trajectories. In Figure 5, the response of this strategy is can be seen to nearly coincide with the strategy using variable ϕ_i (13) (17). The small difference is due to the dynamics of $h_{r,i}$ being specified in continuous time via (18), whereas the strategy using a variable ϕ_i via (17), can be viewed as discrete time with the planning update time, t_p being the

Fig. 6: Results for planned (—) and executed (—) B-spline trajectories. Comparison of temporal consistency.

sample time of this discrete time system.

Finally, the the gap closing strategy of (24), in which a variable gap size $c_{r,i}(t)$ (24) was used, is shown in Figure 5. The response can be seen to be fairly similar to the response of strategy (19). It can be seen that $\underline{\psi}$ via (22) is indeed a proper analogue to ϕ as the same approaching velocity is obtained. However, this approaching velocity is realised much faster due to a higher initial acceleration, which is less comfortable for the passengers.

Since the strategy of (24) is also capable of working well at low velocities, this strategy seems best suitable for implementation. This is shown in Figure 7a, where the initial velocities of both vehicles are set to $\dot{s}_{1,2}(0) = 0 \text{ ms}^{-1}$ and the second vehicle has an initial error $e_2(0) = 15 \text{ m}$ back with respect to the spacing policy. The lead vehicle remains stationary for 4 seconds, after which it accelerates to $\dot{s}_1(t) = 10 \text{ ms}^{-1}$. It can be seen that the strategy using $c_{r,i}(t)$ is capable of handling standstill, closes the distance initially with predefined velocity $\underline{v}_{c1}$. When the lead vehicle accelerates the planning of the host vehicle is updated accordingly, even though the newly communicated trajectory of the preceding is not consistent with the previously communicated trajectory. This can be seen by the sudden change in planned trajectories, where initially the host vehicle plans to maintain its velocity until $t = 8.5 \text{ s}$, but reacts accordingly once the preceding vehicle starts moving. This demonstrates the versatility of the approach.

Another important situation is the approach of a stationary vehicle, which could occur for instance while approaching a traffic light. In this scenario, the host vehicle approaches with a velocity of $\dot{s}_2(0) = 10 \text{ ms}^{-1}$ and error $e_2(0) = 50 \text{ m}$ backwards of the desired position. As can be seen from Figure 7b, the vehicle gradually brakes towards the stationary vehicle. This is due to the added term $\dot{s}_i(t_{c,i}) - \bar{V}_{i-1}$ in (22), that prevents the vehicle from hard initial braking and closing the remaining gap with $\underline{v}_{c1}$. These examples demonstrate the the gap closing strategy using (24) and (22), is not only

(a) Depart from standstill with spacing error $e_2(0) = 25$ m

(b) Approach of a standstill vehicle, spacing error $e_2(0) = 15$ m

Fig. 7: Gap closing using $c_{r,i}(t)$ via (21) and (22)

capable of performing normal gap closing, but also is able to operate in various perturbed situations, hence making it suitable for practical implementation. Note that with these examples all three modes of (22) are demonstrated and simulating at more velocities does not lead to new insights.

V. CONCLUSIONS AND RECOMMENDATIONS

This paper demonstrates the use of B-splines for gap closing in cooperative driving. The proposed method makes use of trajectory planning with B-splines, which was already demonstrated to yield string stable platoons. In this work, a strategy for gap closing was integrated in this B-spline trajectory planning method. Both a variable time gap, as well as a variable gap size over the planning horizon are considered. By limiting the rate of change with which the time gap or gap size changes, control is gained over the velocity difference with which the host vehicle closes the gap. This method improves comfort over both the original B-spline cooperative trajectory planning method as well as the traditional CACC control, as the accelerations in the new method do not increase with linearly with the initial gap. In contrast, in the presented approach the acceleration does not increase with increasing initial gap. Thus this presents a unified method for cooperative driving, that is capable of both gap closing as well as distance keeping, while maintaining driver and passenger comfort.

The strategy utilizing dynamics for the desired time gap demonstrated the most suitable response, with lower accel-

erations for increased passenger comfort, while maintaining temporal consistency. However, this method shows issues when the vehicle is at a near standstill. Therefore, the recommended strategy for implementation is the strategy utilizing dynamics for the desired gap size, which functions well at any velocity. Experimental validation and rigorous string stability analysis is left for future work.

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