

## Toward Unfolding Doubly Covered $n$ -Stars

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# Toward Unfolding Doubly Covered $n$ -Stars

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Open Problem 22.3 from [1] asks: does every closed polyhedron  $P$  have a general unfolding to a non-overlapping polygon? A *general unfolding* is produced by cutting the surface along the edges of a cut tree spanning the vertices of  $P$  and flattening it to a connected, planar piece without overlap (here the cuts are not restricted to the edges of the polyhedron). It is known that convex polyhedra always admit general unfoldings [2]. This is also true for non-convex but nearly flat polyhedra [4] and for various classes of orthogonal polyhedra where the cuts are restricted to be parallel to the polyhedron edges [3]. At CCCG in August 2017, Stefan Langerman posed the question of finding general unfoldings for doubly covered polygons, and more specifically for doubly covered  $n$ -stars: regular  $n$ -gons with identical isosceles triangular “spikes” attached to their edges (all spikes have the same base angle  $\alpha \in (0, \pi/2)$ ). Does every doubly covered  $n$ -star admit a general unfolding?

In this paper, we explore the space of doubly covered  $n$ -stars in search of families of general unfoldings. We show that general unfoldings of doubly covered  $n$ -stars exist for:

- any base angle  $\alpha \in (0, \pi/2)$ , for  $n \in \{3 \dots 10, 12\}$
- any  $n$ , for base angle  $\alpha < \frac{\pi}{3} \left( \frac{n+3}{n} \right)$ .

We prove existence by construction, providing families of general unfoldings within specific subdomains of  $n$  and  $\alpha$ . Figure 1 shows representative constructions from some relevant families.

## References

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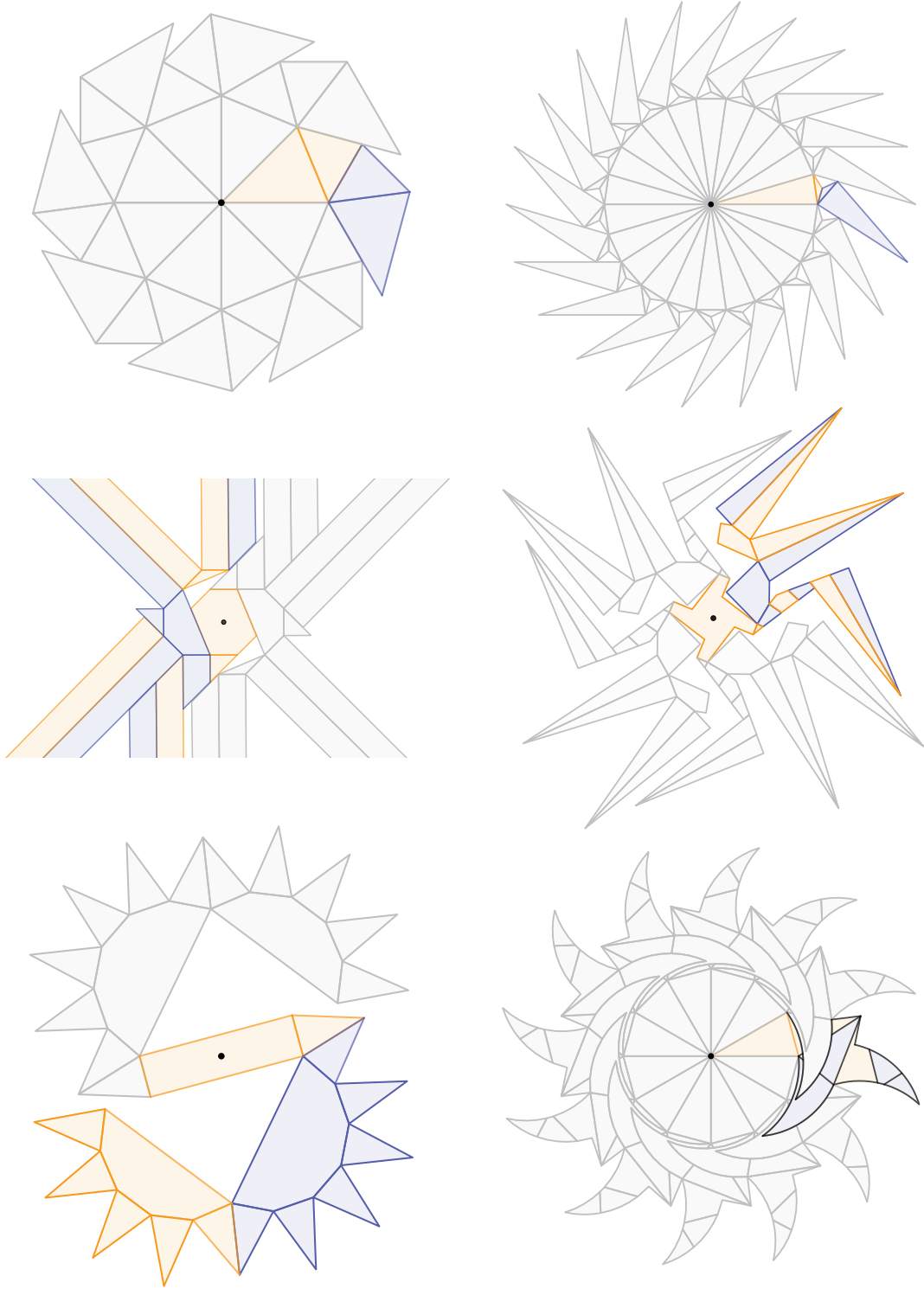


Figure 1: Top row: naively unfolding each spike provides valid unfoldings up to  $\alpha \leq \frac{\pi}{6} \left( \frac{n+6}{n} \right)$  for  $n \leq 12$  (left shows  $n = 8$ ), and up to  $\alpha \leq \pi \left( \frac{3}{n} \right)$  for  $12 < n$  (right shows  $n = 22$ ); cuts are made along edges only; representative top and bottom surface pieces are color-coded. Middle row: unfolding families for  $n = 8$  (left) and  $n = 12$  (right) for any  $\alpha \in (0, \pi/2)$ ; the latter case uses a more complex cut tree to avoid overlap. Bottom row: improvements over the naive unfoldings for  $\alpha \leq \frac{\pi}{3} \left( \frac{n+2}{n} \right)$  for even  $n$  (left), and for  $\alpha < \frac{\pi}{3} \left( \frac{n+3}{n} \right)$  for all  $n$  (right); while the right unfolding supersedes the left, both families are constrained by the same asymptotic upper bound  $\alpha < \pi/3$ , in the limit of large  $n$ .