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Extending the Magic Formula and SWIFT tyre models for inflation pressure changes

Dr. Ir. A.J.C. Schmeitz\textsuperscript{1}, Dr. Ir. I.J.M. Besselink\textsuperscript{1,2}, Ir. J. de Hoogh\textsuperscript{1}, Prof. Dr. H. Nijmeijer\textsuperscript{1},
\textsuperscript{1}TU Eindhoven, Eindhoven, \textsuperscript{2}TNO Automotive, Helmond, The Netherlands

Summary

The Magic Formula and SWIFT tyre models are well-known semi-empirical tyre models for vehicle dynamic simulations. Up to now, the only way to account for inflation pressure changes is to identify all model parameters for each inflation pressure that has to be considered. Since this is a time consuming and consequently expensive activity, research at TNO and Eindhoven University of Technology was started to extend the Magic Formula and SWIFT tyre model so that tyre inflation pressure changes can be accounted for. This paper discusses the influence of inflation pressure changes on the quasi-static force and moment characteristics and on the enveloping properties of tyres.

1 Introduction

The Magic Formula and SWIFT tyre models are well-known tyre models that are widely used in vehicle dynamic simulations. These models have been developed at Delft University of Technology and TNO Automotive supplies them under the product names MF-Tyre and MF-Swift for all main vehicle simulation codes (amongst others ADAMS, Virtual.Lab/Motion, DADS, SIMPACK and Matlab/Simulink) as part of the Delft-Tyre product line, see www.delft-tyre.com.

After the first publications (Bakker et al., 1987 and Bakker et al., 1989), the Magic Formula has quickly gained broad acceptance as a very accurate model to describe the measured steady-state forces and moments developed by a rolling tyre under various slip
conditions. The model is typically used in vehicle handling simulations, where an accurate representation of the nonlinear dependency of the tyre forces and moments on slip is a primary requirement. In its latest version (Pacejka, 2002), the Magic Formula is capable of dealing with combined slip, camber and turn slip and transient responses up to about 8 Hz.

![Diagram of the SWIFT model](image)

**Figure 1:** Schematic representation of the SWIFT model.

The SWIFT model (see Figure 1), which is an acronym for Short Wavelength Intermediate Frequency Tyre model, is a dynamic tyre model with an extended validity range for higher frequencies (up to 60-100 Hz), short wavelengths (larger than 0.1 m) excitations and rolling over arbitrary road unevenness, while retaining the Magic Formula as slip model. Besides the Magic Formula, the model consists of a rigid ring, a contact patch model and a contact model to account for the tyre enveloping properties. Three PhD students have worked on the development of this model under the supervision of professor Pacejka and great emphasis was put on experimental validation using laboratory experiments (Zegelaar, 1998, Maurice, 2000, and Schmeitz, 2004). The model is typically used in ride comfort and durability simulations (Schmeitz et al., 2004b) and for studying the behaviour of control systems (Pauwelussen et al., 2003). A recent overview of the model is given in (Besselink et al., 2004).

Both the Magic Formula and the SWIFT tyre model are semi-empirical tyre models, which means that for parameter identification several tyre measurements have to be carried out. Up to now, the only way to account for inflation pressure changes is to identify all model parameters for each inflation pressure that has to be considered. Since this is a time consuming
and consequently expensive activity, research at TNO and Eindhoven University of Technology was started to extend the Magic Formula and SWIFT tyre model so that tyre inflation pressure changes can be accounted for.

This paper describes the research that was conducted to extend the Magic Formula expressions for inflation pressure changes. Furthermore, it focuses on the influence of inflation pressure changes on the enveloping properties of the tyre. To obtain insight in which Magic Formula characteristics should be extended to include the influences of inflation pressure changes, both a physical brush tyre model and measurement results from the TNO Tyre Test Trailer have been used. For investigating the influence of tyre pressure changes on the enveloping properties of a tyre, experiments were carried out at the Flat Plank test facility of Eindhoven University of Technology.

The paper is organised as follows. First, the influence of inflation pressure changes is discussed with regard to the force and moment characteristics; after that, with regard to the enveloping properties of the tyre.

2 Force and moment characteristics

To investigate the influence of inflation pressure, the force and moment characteristics of five passenger car tyres ranging from a small 155/70 R13 to a large 225/55 R16 tyre are investigated. These characteristics have been measured on the road at a velocity of 60 km/h with the Tyre Test Trailer of TNO Automotive (see Figure 2). For each tyre, the pure lateral and longitudinal slip characteristics have been measured for at least four vertical loads and three inflation pressures. Table 1 gives an overview of the measurements.
Table 1: Tyres, inflation pressures and vertical loads used in the investigation.

<table>
<thead>
<tr>
<th>Tyre</th>
<th>Inflation pressures [bar]</th>
<th>Vertical loads [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>155/70 R13</td>
<td>2.0, 2.4, 2.8</td>
<td>1, 2, 4, 6</td>
</tr>
<tr>
<td>185/60 R14</td>
<td>2.0, 2.4, 2.8</td>
<td>2, 4, 6, 8</td>
</tr>
<tr>
<td>195/65 R15</td>
<td>2.0, 2.4, 2.8</td>
<td>2, 4, 6, 8</td>
</tr>
<tr>
<td>205/55 R16</td>
<td>2.1, 2.4, 2.8</td>
<td>1, 2.5, 4, 5.5, 7</td>
</tr>
<tr>
<td>225/55 R16</td>
<td>1.8, 2.1, 2.4</td>
<td>1, 2.5, 4, 5.5, 7</td>
</tr>
</tbody>
</table>

In order to not solely rely on these measurements and to get more insight in the physical tyre behaviour, an advanced brush model (Treadsim) is used in addition. This model is described in the next section. To investigate which effects must be included in the Magic Formula, first a thorough investigation of tyre force and moment dependency on inflation pressure has been carried out by studying both the measured and simulated characteristics. The results of this investigation are presented in section 2.2. The proposed extensions of the Magic Formula model are discussed in section 2.3.

2.1 The tread simulation model

After the development of the original ‘brush’ model of Fromm and of Julien, many authors have applied and extended this model since the early 1940s. For references and a historic overview it is referred to (Pacejka, 2002).

The advanced brush model (see Figure 3) used in this study is based on the tread simulation model (Treadsim) described in (Pacejka, 2002). Besides the brush/tread elements with different longitudinal and lateral stiffnesses representing the tread rubber, the model consists of a flexible carcass and belt.

Figure 3: Schematic representation of the tread simulation model.
In the model, the carcass and belt are only compliant in lateral direction; consequently the longitudinal carcass/belt compliance is neglected. The lateral carcass compliance is modelled with springs and the belt is represented by two-dimensional beam (finite) elements. The bending stiffness of the beam and the lateral carcass compliance linearly depend on the inflation pressure.

The contact patch shape is approximated with a rectangular shape that changes size with tyre vertical deflection. The tyre deflection itself depends on the applied vertical load, the inflation pressure and the forward velocity and is determined with empirical relations. Following (Sjahdanulirwan and Yang, 1995), a normal pressure distribution model with an ‘inverted-boat’ shape in the longitudinal direction and uniform in the lateral direction is used to predict the maximum allowable total shear stresses of a brush element. In the model the following velocity and pressure dependent friction law is used:

\[
\mu = \left(\frac{p}{p_0}\right)^k \frac{\mu_0}{1 + a_\mu V_b}
\]

with \( \mu \) the friction coefficient, \( p \) the contact pressure, \( p_0 \) the reference contact pressure, \( V_b \) the ‘sliding’ velocity of point B (as an approximation of the actual sliding velocity of point P) and \( k, \mu_0 \) and \( a_\mu \) constants.

The model works basically as follows. One brush element is followed while it travels through the length of the contact patch. During such a passage, the carcass deflections are kept constant. The bristle deflections are established and the corresponding bristle forces are calculated. With these forces the carcass deflections for the next iteration step are obtained. The iteration process continues till an error criterion is satisfied. Finally, the forces and moments at the wheel centre are obtained from the bristle forces.

Although Treadsim is a physical tyre model, its parameters have been determined from the tyre measurements presented in Table 1. An optimisation routine is used that minimises the error between the force and moment measurements and Treadsim output by adjusting the physical model parameters. An example of a Treadsim fit result is shown in Figure 4. As can be observed, the Treadsim results agree qualitatively well with measurements. In Figure 4 also a plot of the combined slip characteristics is added to show that the model also generates reasonable combined slip data.

2.2 Tyre force and moment dependency on inflation pressure

As mentioned before, a thorough investigation of tyre force and moment dependency on inflation pressure has been carried out by studying both the measured and simulated basic tyre handling characteristics: pure lateral force (\( F_y \) versus slip angle \( \alpha \)), pure aligning torque (\( M_z \) versus slip angle \( \alpha \)), and pure longitudinal force (\( F_x \) versus longitudinal slip \( \kappa \)). Below, the results are discussed per characteristic.
Although in section 2.1 it is shown that the Treadsim results agree well with measurements for one set of parameters, in this section various optimised parameter sets for one tyre are used to increase the fit accuracy for the studied tyre property. The following approach is used: The model parameters for one tyre are determined only for the reference/nominal inflation pressure (middle value in Table 1). Simulations at other inflation pressures are carried out with these parameters. For increasing the accuracy of the studied property weighting functions are used. For example, if the cornering stiffness is considered, the error of the lateral force for small slip angles is weighted more. Since in this section various tyre properties are studied separately to analyse the influence of inflation pressure, this approach is justified.
**Lateral force**

For the lateral force characteristic, the effect of inflation pressure changes are investigated on the cornering stiffness and the peak lateral friction coefficient.

With regard to the cornering stiffness an increase of inflation pressure has two counteracting effects resulting in a lower cornering stiffness at low vertical loads and a higher cornering stiffness at high vertical loads. These effects can be found in both the measurement and Treadsim results. Figure 5 for example illustrates this.

![Figure 5: Cornering stiffness dependency on inflation pressure for the 185/60 R14 tyre.](image)

The first effect is caused by the decreasing contact length as a result of the increased vertical stiffness. This can be understood if one considers the analytical solution for the cornering stiffness of the basic brush model (without carcass compliance) (Pacejka, 2002):

\[
C_{fa} = \left( \frac{\partial F_y}{\partial \alpha} \right)_{\alpha=0} = 2c_{py}a^2. \tag{2}
\]

In this equation \( c_{py} \) is the lateral tread element stiffness per unit length of circumference, which is in Treadsim independent of inflation pressure, and \( a \) equals half of the contact length. A decrease of contact length results in a decrease of cornering stiffness.

An increase of inflation pressure will also lead to an increase of carcass stiffness, i.e. a decrease of carcass compliance. If the carcass compliance decreases, the slip angle experienced by the contact patch (brush elements) will become larger resulting in a higher lateral force from the deflected brush elements (note that for an infinitely stiff carcass, the slip angle of the contact patch equals that of the tyre). A higher lateral force for the same tyre slip angle will result in a higher cornering stiffness. This effect is dominant at high vertical loads.
The effects discussed above are in accordance with the conclusions drawn on cornering stiffness dependency on inflation pressure in the final report of the TIME project (Oosten, 1999) and the internal TNO TEK-tyre project (Erp and Verhoeff, 2004). Therefore, these effects will be included in the extensions of the Magic Formula.

With regard to the peak lateral friction coefficient, the force and moment measurements do not show a clear tyre independent relation for tyre inflation pressure. For an intermediate inflation pressure some tyres show a minimum peak lateral friction coefficient at low vertical loads that shifts to an optimum at high vertical loads. For other tyres the opposite behaviour is observed.

The peak lateral friction coefficients from the Treadsim results show a decrease in peak lateral friction at all vertical loads for increasing tyre inflation pressure. Not much importance is attached to this, because Treadsim does not generate accurate results for the peak friction coefficients, probably caused by the inaccurate modelling of the contact pressure distribution and contact patch shape at large slip angles. As an example, Figure 6 illustrates this for the 225/55 R16 tyre.

![Figure 6](image)

**Figure 6:** Peak friction coefficient dependency on inflation pressure for the 225/55 R16 tyre.

Since the Treadsim results are not reliable and the measurement results show different behaviour for various tyres, it is concluded that the relation between lateral friction coefficient, inflation pressure and vertical load is tyre dependent. This will be considered in the Magic Formula extensions.

Also results from other studies are not unambiguous. For example, (Augustin and Unrau, 1997) investigated the influence of inflation pressure on the peak lateral friction coefficient for three tyres as part of the TIME project. They concluded from measurements that an increase of inflation pressure causes a higher lateral force value at higher vertical loads. However, this
conclusion is not supported in the main report of Work-package II of the TIME project (Hogt, 1998).

**Aligning torque**

For all tyres, measurement and Treadsim results show a clear influence on the aligning torque characteristics. As can be observed in Figure 7, the aligning torque curves have higher peak values for lower inflation pressure. This can be easily understood if we again consider the basic brush model. In this model the pneumatic trail, i.e. the ratio of aligning torque and lateral force, is linearly dependent on the contact length (Pacejka, 2002). Higher inflation pressures lead to a shorter contact length, thus a smaller pneumatic trail. The changes of contact length are relatively large compared to the changes of lateral force, causing the aligning torque amplitudes to decrease with increasing inflation pressure.

![Figure 7: Aligning torque characteristics for the 155/70 R13 tyre.](image)

**Longitudinal force**

As the longitudinal force characteristic is similar to the lateral force characteristic, the effect of inflation pressure changes are investigated on the slip stiffness and the peak friction coefficient. In addition, the effect of inflation pressure on the wheel lock friction is considered.

In accordance with the cornering stiffness, it is expected that the longitudinal slip stiffness exhibits a similar behaviour. Since there is no carcass/belt compliance in longitudinal direction, the longitudinal slip stiffness of Treadsim depends solely on the contact length according to (Pacejka, 2002):
\[ \frac{\partial F}{\partial \kappa} \right|_{\kappa=0} = 2c_p \kappa^2, \]  

(3)

where \( c_p \) is the longitudinal tread element stiffness per unit length of circumference. It is thus expected that higher inflation pressure gives lower slip stiffness. The slip stiffnesses of three tyres indeed behave like this, see for example Figure 8. However, other tyres exhibit a different behaviour. Figure 9 for example depicts the longitudinal slip stiffnesses for the 155/70 R13 tyre. Nevertheless, for the Magic Formula extensions first a linear relation is tried.

**Figure 8:** Longitudinal slip stiffness for the 225/55 R16 tyre.

**Figure 9:** Longitudinal slip stiffness for the 155/70 R13 tyre.
A peaking longitudinal friction coefficient is found in the measurements of almost all tyres, see for example Figure 10. Only the 225/55 R16 tyre shows a different behaviour. The Treadsim results show again a decreasing peak friction coefficient with increasing inflation pressure. This behaviour is similar to that for the peak lateral friction. The inaccurate modelling of the contact pressure distribution is probably again the cause of the divergent Treadsim results.

![Figure 10: Peak longitudinal friction coefficient for the 155/70 R13 tyre.](image-url)

The findings reported here are supported by the findings of (Marshek and Cuderman, 2002), who determined the effect of inflation pressure on deceleration for a series of emergency braking tests with different vehicles equipped with anti-lock braking systems (ABS). As the deceleration is assumed to be proportional to the peak longitudinal friction coefficient with an active ABS, the tests give an indication of the effect of inflation pressure on peak longitudinal friction. Their conclusion with respect to inflation pressure is that the effect on the braking performance is only slight. Especially for large tyres there is no noticeable effect. For smaller tyres there is an optimal inflation pressure, meaning that with higher and lower pressures the deceleration decreases. This corresponds with our findings.

The Treadsim simulation results show a constant ratio between the lock and peak friction for the various inflation pressures, see Figure 11. In Figure 11, we also see that the ratio lock/peak friction of the measurements is nearly constant for all inflation pressures for the 155/70 R13 tyre. However, measurements of the other four tyres show a randomly varying ratio. As we are not able to find a clear relation between tyre pressure and lock/peak friction, it is assumed that this ratio is constant for various tyre pressures when extending the Magic Formula.
2.3 Extending the Magic Formula for inflation pressure changes

Based on the observations described in the previous section, extensions of the Magic Formula equations to include inflation pressure effects are proposed. With these extensions it is possible to account for inflation pressure changes in a limited range of typically 1 bar. The relations developed here are intended to be valid for ‘interpolation’ only, which means that the range is limited by measurements at the highest and lowest inflation pressure considered.

To evaluate the proposed Magic Formula extensions, the measurements presented in Table 1 are again used. Since it is desired to have as few as possible parameters and since the measurements are conducted at only three inflation pressures, the extensions are preferably limited to linear relations. Second degree polynomials are used only when significantly better results are obtained.

To assess the proposed relations, criteria are used that are based on the fit error. The fit error \( \varepsilon \) for one characteristic is obtained with:

\[
\varepsilon_A = 100 \sqrt{\frac{\sum_{i=1}^{n} (A_{MF,i} - A_{Meas,i})^2}{\sum_{i=1}^{n} A_{Meas,i}^2}}. \tag{4}
\]

In this equation \( A \) is either \( F_y \), \( M_z \) or \( F_x \), \( n \) is the number of measurement points and the subscripts \( MF \) and \( Meas \) are used to denote the fit and measurement results, respectively. The overall error results are obtained by averaging the fit errors per characteristic, i.e. pure \( F_y \), pure

Figure 11:  Longitudinal slip characteristics showing relation between lock friction and peak friction for the 155/70 R13 tyre at a vertical load of 6 kN.
and pure $F_z$, for all five tyres considered. Thus per characteristic only one fit error is evaluated. The following criteria are used:

- fit error with the new Magic formula relations,
- fit error lower bound (i.e. average fit error for separate Magic Formula fits at each inflation pressure),
- fit error upper bound (i.e. fit error of one Magic Formula fit of all measurement data without including pressure effects).

Since it is not expected that the Magic Formula fit including the new relations will be better than separate fits at each inflation pressure, the average fit error for the separate fits is set as lower bound. The upper bound is the fit error obtained by fitting the characteristics neglecting the relations to include inflation pressure effects. The lower and upper bounds are used to assess the quality of the extensions. Table 2 gives an overview of the obtained fit errors. Below the improvements will be quantified with a percentage, where 100 % is the room for improvement, i.e. the difference between the upper and lower bounds.

Table 2: Average fit error results for the five tyres considered.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Lower bound [%]</th>
<th>Upper bound [%]</th>
<th>New [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>pure $F_y$</td>
<td>2.1</td>
<td>3.0</td>
<td>2.4</td>
</tr>
<tr>
<td>pure $M_z$</td>
<td>6.9</td>
<td>11.0</td>
<td>8.5</td>
</tr>
<tr>
<td>pure $F_z$</td>
<td>2.2</td>
<td>5.1</td>
<td>3.6</td>
</tr>
</tbody>
</table>

To include inflation pressure $p_i$ in the Magic Formula, the normalised change in inflation pressure $dp_i$ is introduced:

$$dp_i = \frac{p_i - p_{i0}}{p_{i0}},$$  \hspace{1cm} (5)

where $p_{i0}$ is the nominal or reference inflation pressure. Further, the normalised change in vertical load $df_z$ is introduced:

$$df_z = \frac{F_z - F_{z0}}{F_{z0}}.$$  \hspace{1cm} (6)

$F_{z0}$ and $F_z$ are the nominal and current vertical load, respectively. These normalised quantities are used in the relations described below. These relations are parts of the Magic Formula model described in (Pacejka, 2002). In this paper, only the extensions are given. Furthermore, camber and turn slip are not considered here.

**Lateral force**

In section 2.2 it is shown that the cornering stiffness is dependent on the inflation pressure and vertical load. An increase of inflation pressure results in a lower cornering stiffness at low
vertical loads and a higher cornering stiffness at high vertical loads. To include the inflation pressure effects in the Magic Formula, two linear relations are added. These relations are underlined in the expression for the cornering stiffness $K_{y\alpha}$:

$$K_{y\alpha} = p_{Ky1}(1 + p_{py1}dp_1)F_{z0} \sin \left[ 2 \arctan \left( \frac{F_z}{p_{Ky2}(1 + p_{py2}dp_1)F_{z0}} \right) \right],$$

where the $p$’s are parameters.

In section 2.2 it is discussed that the maximum lateral friction $\mu_y$ is inflation pressure and tyre dependent. For an intermediate inflation pressure some tyres show a minimum peak lateral friction coefficient at low vertical loads that shifts to an optimum at high vertical loads. For other tyres the opposite behaviour is observed. To be able to fit the optimum/minimum at least a second degree polynomial with regard to the inflation pressure is required. Therefore the following relation is proposed:

$$\mu_y = \left( p_{Dy1} + p_{Dy2}df_z \right) \left( 1 + p_{py3}dp_i + p_{py4}dp_i^2 \right) > 0. \quad (8)$$

It appeared that making the introduced parameters ($p_{py3}$ and $p_{py4}$) dependent on the vertical load did not significantly reduce the fit error. Therefore, these extensions are not included in (8).

To assess these relations, (7) and (8) are implemented in the Magic Formula, all pure $F_y$ characteristics are fitted and the average fit error for the five tyres is calculated. As is shown in Table 2, this fit error is 2.4 %, which is an improvement of 67 %.

### Aligning torque

As discussed in section 2.2, a clear relation between aligning torque and inflation pressure exists. Higher inflation pressure gives lower amplitudes. In the Magic Formula, the aligning torque is calculated by multiplying the lateral force with the pneumatic trail. The magnitude of the pneumatic trail is determined by the parameter $D_z$. Therefore, a linear relation is added (underlined) to include inflation pressure changes:

$$D_z = F_z \left( \frac{r_0}{F_{z0}} \right) \left( q_{Dz1} + q_{Dz2}df_z \right) \left( 1 - q_{pz1}dp_i \right) \text{sgn} V_{cx} .$$

In this equation the $q$’s are parameters, $r_0$ the tyre free radius and $V_{cx}$ the forward velocity of the contact centre.

To assess this relation, the average fit error for the aligning torque is determined. It appears that with only one additional parameter, an improvement of 61 % is achieved. The obtained average fit error for the five tyres is 8.5 %, see Table 2.
Longitudinal force

In section 2.2, it is shown that the longitudinal force exhibits a similar behaviour as the lateral force. The standard Magic Formula equations for the cornering stiffness and longitudinal slip stiffness, however, are different. The equations for the maximum longitudinal and lateral friction coefficient are similar. Therefore the following extension (underlined) is proposed for the longitudinal friction coefficient $\mu_l$:

$$
\mu_l = \left( p_{D1} + p_{D2} df_z \right) \left( 1 + p_{px1} dp_i + p_{px2} dp_i^2 \right) ( > 0 ).
$$

(10)

For the longitudinal slip stiffness, first a linear relation with regard to inflation pressure was proposed. However, it appeared that a second degree term significantly improves the fit error. Therefore, the following extension (underlined) is proposed for the longitudinal slip stiffness $K_{xx}$ expression:

$$
K_{xx} = F_z \left( p_{Kx1} + p_{Kx2} df_z \right) e^{p_{px1} dp_i} \left( 1 + p_{px1} dp_i + p_{px2} dp_i^2 \right).
$$

(11)

Finally, since no clear relation for the ratio lock/peak friction is found, the shape factor of the Magic Formula is not modified.

To assess the proposed relations, the equations are again implemented, the results are fitted and the average fit error for the five tyres is calculated. The extensions presented here give an improvement of 52%. The average fit error for the five tyres is 3.6 % for the longitudinal force.

Summary and discussion

By introducing five inflation pressure dependent empirical relations for some basic tyre properties (cornering stiffness, longitudinal slip stiffness, peak friction coefficient and pneumatic trail), the Magic Formula is extended for inflation pressure changes. In total nine additional parameters are introduced. In Figure 12, 13 and 14, the force and moment characteristics of the extended Magic Formula are compared with measurements for three vertical loads (2, 4 and 6 kN) and three inflation pressures (2.0, 2.4 and 2.8 bar). It is observed that the Magic Formula results correspond qualitatively rather well with measurements. It is also observed that the lateral and longitudinal force characteristics agree better than the aligning torque characteristics, which corresponds with the fit errors given before.

In this section, it is shown that with the proposed extensions it is possible to reduce the overall fit error approximately 60 % with regard to the standard Magic Formula that does not account for inflation pressure changes, but this also means that there is still room for improvement. There are however several problems that have to be considered.
Figure 12: Magic Formula pure lateral force results and measurements at three inflation pressures and vertical loads (2, 4 and 6 kN) for the 155/70 R13 tyre.

Figure 13: Magic Formula pure aligning torque results and measurements at three inflation pressures and vertical loads (2, 4 and 6 kN) for the 155/70 R13 tyre.
First of all, to reduce the remaining error, more Magic Formula factors, like the curvature and shape factors must be made inflation pressure dependent. So far, no clear similarity between these factors has been found for several tyres. Another aspect is that the number of parameters will probably increase drastically. Last but not least, the tyre characteristics only change slightly in the considered inflation pressure ranges (see Figure 12, 13 and 14). When no inflation pressure changes are considered, the fit error (upper bound) for the five tyres is for example less than 3%, 11% and 5.1% for the pure lateral force, aligning torque and longitudinal force characteristics, respectively. This means that accuracy and especially repeatability of measurements becomes quite important. Unfortunately, the tyre characteristics were only measured once, so no conclusions with regard to repeatability can be drawn in this research.

For further research it is therefore recommended to carry out an investigation with regard to repeatability and accuracy of measurements. In addition the force and moment characteristics of at least two types of tyres should be measured at more than three inflation pressures to further validate the proposed extensions based on second degree polynomials. Preferably these measurements should be carried out in a controlled laboratory environment in order to minimise repeatability problems.

Figure 14: Magic Formula pure longitudinal force results and measurements at three inflation pressures and vertical loads (2, 4 and 6 kN) for the 155/70 R13 tyre.
3 Enveloping properties

The enveloping properties of a tyre comprehend the capability of the tyre to envelop short sharp road irregularities, like bumps, potholes, cobblestones, etc. Like the suspension system, the tyre enveloping properties are responsible for isolating the driver from vibrations occurring as a result of driving over uneven road surfaces. Since the tyre is the interface between the vehicle and the road, it is the first component that ‘filters’ the road unevenness.

Since the tyre enveloping properties are related to the tyre deformation on road irregularities, they are only important when driving over road surfaces containing short sharp irregularities (short wavelengths). On smooth road surfaces, where the tyre deformation is virtually the same as on a flat road surface, the enveloping properties can be neglected. It is impossible to give an exact criterion, but for road surfaces containing wavelengths shorter than about 1.5 m the tyre enveloping properties should be considered (Schmeitz, 2004).

3.1 Enveloping properties and the SWIFT model

In the SWIFT model, the conceptual basis is that the tyre enveloping properties and the tyre dynamics can be dealt with separately. It is assumed that the tyre contact zone, where the large deformations due to envelopment of the unevenness occur, dynamically deforms mainly in the same way as it does quasi-statically and that local dynamic effects can be neglected. This automatically implies that the enveloping properties can be obtained from the quasi-static response of a tyre on road unevenness and that the enveloping properties can be used to excite the dynamic rigid ring model.

In the SWIFT model, the enveloping properties are ‘translated’ into an effective road surface, see Figure 1. This effective road surface consists of an effective height, forward slope, forward curvature and camber angle. In this paper, only the in-plane tyre enveloping properties are considered, i.e. all obstacles are positioned perpendicular to the driving direction and cover the whole tyre with. Consequently, effective road camber angles are not considered here.

The enveloping model used is the so-called tandem-cam model of Schmeitz (Schmeitz, 2004), see Figure 15. The model consists of two rigid elliptical cams (in tandem configuration) following each other at a distance \( l_s \), which is related to the tyre contact length (typically about 80% of the contact length). Each cam contacts the road surface in at least one point and is allowed to move vertically. The shape of the cams corresponds to the contour of the tyre in the contact zone (in side view). The cam dimensions follow from assessing the best approximation of low speed responses of a tyre rolling over steps of different heights at a number of constant vertical loads. Once determined, the cam dimensions are not affected anymore (also not by the applied vertical load). The effective height \( w' \) and slope \( \beta_s \), required for the effective road surface description, are obtained from the height of the midpoint and the inclination of the connecting line between the bottom points of the cams. In Figure 15 a simple quasi-static tyre model is created by adding a nonlinear spring representing the tyre vertical stiffness.
3.2 Inflation pressure changes

To account for inflation pressure changes, the following approach is considered. Since the cam dimensions are not affected by vertical load, it is also not expected that inflation pressure changes will affect these dimensions. This means that the only property that will change with inflation pressure is the length \( s \) that is related to the contact length of the tyre. In the SWIFT model the contact patch dimensions are fitted with relations that are a function of vertical load. Following (Smiley and Horne, 1960), Besselink (Besselink, 2000) conversely uses a relation that is a function of vertical tyre deflection. This relation for the half contact length \( a \) reads:

\[
a = p_{a1}r_0 + p_{a2}\sqrt{\frac{\rho_z}{r_0}}
\]

(12)

where \( \rho_z \) is the vertical tyre deflection, \( r_0 \) the unloaded tyre radius and \( p_{a1} \) and \( p_{a2} \) fit parameters. Besselink found that the contact length of a tyre for different inflation pressures can be described well using the same coefficients. This implies that the only tyre property that changes with inflation pressure is the tyre vertical stiffness. Thus, the tyre outside contour is primarily determined by the vertical tyre deflection. This approach is used to predict the tyre enveloping properties at different inflation pressures.

The tyre vertical stiffness is obtained from the following empirical relation describing the vertical force \( F_z \) as function of vertical deflection \( \rho_z \) and the normalised change in inflation pressure \( dp_i \):

\[
F_z = (1 + q_{F_z}dp_i)\left(q_{F_z1}\rho_z + q_{F_z2}\rho_z^2\right).
\]

(13)

In this equation the \( q_{F_z} \)'s are parameters. It is assumed that the relation between inflation pressure and vertical stiffness is linear. The parameter \( q_{F_z1} \) indicates the ratio of the vertical
stiffness that is due to the inflation pressure at the nominal inflation pressure. Figure 16 shows that this relation can be used to describe the vertical force characteristics for various inflation pressures.

\[ p_i = 1.4, 1.8, 2.2, 2.6 \text{ and } 3.0 \text{ bar} \]

Figure 16: Vertical force vs. deflection characteristics for five inflation pressures (higher inflation pressure gives higher stiffness).

3.3 Validation of the proposed approach

To validate the model, experiments have been carried out at the Flat Plank test facility of Eindhoven University of Technology for two types of tyres, three inflation pressures and various obstacle shapes. The experiments are carried out at a fixed axle height corresponding to the nominal tyre load on a flat road surface. In addition, for one tyre (205/60 R15) and one obstacle, experiments have been carried out at a constant vertical load as well.
Figure 17: Measured force variations for three inflation pressures for a 205/60 R15 tyre rolling over a cleat (10x50 mm) at fixed axle height conditions corresponding to an initial vertical load of 4 kN.

Figure 18: Simulated force variations for three inflation pressures for a 205/60 R15 tyre rolling over a cleat (10x50 mm) at fixed axle height conditions corresponding to an initial vertical load of 4 kN.
Figure 19: Measured force variations for three inflation pressures for a 205/60 R15 tyre rolling over a cleat (10x50 mm) at a constant vertical load of 4 kN.

Figure 20: Simulated force variations for three inflation pressures for a 205/60 R15 tyre rolling over a cleat (10x50 mm) at a constant vertical load of 4 kN.
Figures 17 and 18 present the measurement and simulation results respectively for a 205/60 R15 tyre rolling over a cleat (height 10 mm and length 50 mm) at a fixed axle height corresponding to an initial vertical load of 4 kN. Results are shown for three inflation pressures: 1.8, 2.2 and 2.6 bar. It is observed that measurement and simulation results agree qualitatively rather well. However, a closer look also reveals that the vertical force build-up of the model is too fast. Regarding the influence of inflation pressure, it is observed that a higher inflation pressure results in higher forces and a shorter response. The fact that the response is shorter can be explained by considering the contact length of the tyre. For higher inflation pressures the contact length is smaller, which results in a shorter response (length $l_s$ in Figure 15 is shorter). Both the increase of vertical stiffness and the fact that the front and rear edge of the tyre are closer for higher inflation pressures (in the model: the distance between the cams is shorter) lead to higher force levels. Finally, when comparing Figures 17 and 18, it is observed that the model can predict the changes of the responses as result of inflation pressure variations rather well.

The measured and simulated responses of the 205/60 R15 tyre rolling over the same cleat, but now at constant vertical load, are depicted in Figure 19 and 20, respectively. The figures show that the results again qualitatively agree rather well and that inflation pressure changes can be represented well by the model. In conclusion it can be said the enveloping model is well able to account for inflation pressure changes. Last but not least, it must be noticed that for including inflation pressure influences no additional enveloping model parameters are required. Only the vertical stiffness characteristic is made inflation pressure dependent and a different approach is used for obtaining the contact length.

4 Summary and concluding remarks

This paper discusses the influence of inflation pressure changes on the force and moment characteristics and enveloping properties of tyres. It focuses on extensions/modifications of the Magic Formula and SWIFT models. It is the first step in developing a semi-empirical tyre model that can account for tyre inflation pressure changes.

In both the Magic Formula and SWIFT model, the empirical Magic Formula equations are used for obtaining the forces and moments as a result of slip. Therefore, considering inflation pressure in these equations is important. In this paper, extensions of the Magic Formula equations are proposed by introducing five inflation pressure dependent empirical relations for a number of basic tyre properties (cornering stiffness, longitudinal slip stiffness, peak friction coefficient and pneumatic trail). These extensions are proposed after studying the force and moment characteristics of several tyres and of a physical tyre model.

With the proposed Magic Formula extensions it is possible to simulate the effect of inflation pressure changes in a limited range of typically 1 bar. Besides the benefit that the tyre force and moment characteristics at different inflation pressures can be obtained with only nine additional parameters, instead of the about 100 Magic Formula parameters that are required for each new Magic Formula fit, it is also possible to obtain the force and moment characteristics...
for inflation pressures that are not measured via interpolation. This last aspect is considered important for vehicle handling optimisation.

In this paper it is shown that with the proposed extensions, the Magic Formula results correspond qualitatively rather well with measurements. General trends are predicted well. Quantitatively the overall fit error is improved approximately 60% with regard to the standard Magic Formula that does not account for inflation pressure changes. However, it is also concluded that there is still room for improvement. Aspects that limit further improvement in this research are the limited similarity between the behaviour of several tyres and the fact that the characteristics only change slightly, which implies that especially repeatability of measurements becomes quite important. This last aspect has not been considered in this research.

To account for the enveloping properties of the tyre, the tandem-cam model is used as part of the SWIFT model. To include inflation pressure changes, a new approach is proposed that is based on the principle that enveloping properties and contact length are geometrically determined and depend solely on the tyre deflection. It is shown that by introducing an inflation pressure dependent relation for the vertical tyre stiffness and by making the contact length a function of vertical deflection, instead of vertical load, the enveloping properties can be represented well by the model. Thus the effect of inflation pressure changes can be accounted for without changing the enveloping model, i.e. without introducing additional enveloping model parameters.

Finally, the Magic Formula equations and the tandem-cam enveloping model are only two components of the SWIFT model. Therefore, the effect of inflation pressure changes on other tyre properties like the natural frequencies and damping ratios of the tyre will be investigated in future research.

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