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A New Loop-based Hybrid Analytical Modeling Formulation and the Selection of its Nonlinear Solver

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This paper presents a comprehensive comparative study between two nonlinear solvers for the hybrid analytical modeling (HAM) formulation. The Newton-Raphson method (NRM) and the fixed-point method (FPM) are compared in terms of their convergence rate, computational time, and accuracy. A new HAM formulation using the loop-based magnetic equivalent circuit instead of the node-based one is proposed to improve the convergence and condition number. The loop-based formulation is coupled with both NRM and FPM nonlinear solvers to perform the magnetostatic analysis of a 12/10 variable flux reluctance machine (VFRM) under local magnetic saturation. It is shown that both methods can achieve convergence for various saturation levels, mesh, and harmonic refinements. However, FPM exhibits a 0.4 larger convergence rate than NRM. It is also observed that the accuracy of NRM decreases under deep magnetic saturation and the number of required iterations of NRM increases with the model refinement. However, FPM is able to converge for all analyzed refinements with less than six iterations.

Index Terms—Hybrid analytical modeling, loop-based magnetic equivalent circuit, nonlinear analysis, Newton-Raphson method, fixed-point method, variable flux reluctance machine.

I. INTRODUCTION

THE hybrid analytical modeling (HAM) method uses a strong coupling between the Fourier analysis (FA) in the airgap and the magnetic equivalent circuit (MEC) in the stator and rotor [1]. HAM is favorable for the analysis of electromagnetic systems compared to other mesh-based modeling techniques such as the finite element method (FEM) or MEC, since the magnetic field in the airgap is calculated analytically using FA without mesh elements [2]. In particular, this property reduces the required number of degrees-of-freedom (dof) and thereby the size of the system matrix. The mesh-free airgap also enables the relative motion of the rotor without changing the mesh structure. The authors have investigated the advantages of the method compared to FEM recently in [3] and [4]. Although the classical numerical methods, such as FEM and MEC, are well-known even under the local magnetic saturation, the comparison of the nonlinear solvers for HAM is not available in the literature. On one hand, HAM provides high accuracy with less number of dof; but on the other hand, the coupling between numerical and analytical models causes the system of equations to become ill-conditioned. The main reason is that the values of the unknown coefficients of the coupled system might exhibit relatively large differences, especially under the finer mesh and harmonic refinements. Although this does not cause any problem in the linear case with a direct solver, iterative nonlinear procedures suffer from the ill-conditioning [5]. Moreover, [6] shows that the number of nonlinear iterations can be significantly reduced for MEC using the loop-based approach with the voltage Kirchhoff law instead of the node-based one with the current Kirchhoff law. Particularly, the node-based MEC models do not work properly with NRM as demonstrated in [7]. The coupling between the node-based MEC and Fourier analysis causes an even more ill-conditioned system matrix for the node-based HAM, which leads to losing the convergence of the nonlinear solver. In this study, a new HAM formulation with

the loop-based MEC is proposed to improve the convergence.

The nonlinear characteristics of the soft-magnetic material are included in the developed loop-based HAM using two different techniques: the Newton-Raphson method (NRM) and the fixed-point method (FPM). In [8], two methods are compared, and NRM is found to be more advantageous than FPM to solve magnetic hysteresis problems using FEM. This paper presents a comprehensive comparison between these two nonlinear solvers in the context of HAM. A 12/10 variable flux reluctance machine (VFRM) is analyzed using the loop-based HAM to compare NRM and FPM. Both dc- and ac-field excitations are present in its stator [9]. Hence, it is possible to increase the generated torque by increasing the dc-field excitation which results in operating in the nonlinear region of the magnetization curve of the electrical steel. What is more, the identical flux path for each phase provides a sinusoidal flux-linkage even under heavy magnetic saturation [9].

In this paper, a new loop-based HAM is developed to overcome the limitations of the node-based formulation which fails to converge. A 12/10 VFRM is investigated using the loop-based HAM with both FPM and NRM. After explaining the modeling method and the nonlinear solvers in Section II and III, the comparison of both solvers is discussed in terms of computational cost, convergence rate, and accuracy.

II. LOOP-BASED HAM

The concept of the proposed loop-based HAM is illustrated in Fig. 1 with the considered benchmark. The loop-based HAM is a combination of the loop-based MEC and FA. The lumped circuit parameters of MEC given in Fig. 1 are reluctances and magneto-motive force sources in both r - and θ -directions. The calculations of these parameters are explained in [3]. Pseudo flux loops ($\Phi_{i,j}^{\text{rotor}}$, $\Phi_{i,j}^{\text{stator}}$) in MEC regions and Fourier coefficients (a_n , b_n , c_n , d_n) in the FA region are concatenated in the unknown vector (X):

$$X = [\Phi_{i,j}^{\text{rotor}}, \Phi_{i,j}^{\text{stator}}, a_n, b_n, c_n, d_n]. \quad (1)$$

Fig. 1. The 12/10 VFRM benchmark and schematic of the loop-based HAM.

Since expressing the magnetic flux density in the airgap in terms of Fourier coefficients is discussed in [10], these expressions are not included in this study. The magnetic flux density in the rotor and stator is derived in terms of pseudo flux loops. For each loop, Kirchhoff's voltage law is applied as in [7]. Boundary conditions for the new loop-based HAM are derived by coupling the models explained in [10] and [7] to enforce the continuity of the following quantities at the interface:

- the magnetic flux in the r -direction:

$$\Phi_{i,j} - \Phi_{i,j+1} - \Phi_r^{FA}(r, \theta) = 0, \quad (2)$$

- the magnetic field strength in the θ -direction:

$$\frac{\gamma(\Phi_{i,j} + \Phi_{i,j+1} - \Phi_{i-1,j} - \Phi_{i-1,j+1})}{2S_\theta(i,j)\mu_r(i,j)\mu_0} - \frac{\Phi_\theta^{FA}(r, \theta)}{S_\theta(i,j)\mu_0} = 0, \quad (3)$$

where Φ^{FA} is the Fourier expression of the magnetic flux at the interface, S_θ is the cross-sectional area of the MEC element perpendicular to the θ -direction, μ_r is the relative permeability, and μ_0 is the permeability of free space. In (3), γ is 1 for the rotor/airgap interface, while it is -1 for the stator/airgap interface. The residual function $f(x)$ of nonlinear equations representing the loop-based HAM is written as

$$f(X) = C \left(R \left(\mu(B(\Phi_{i,j}^{\text{rotor}}, \Phi_{i,j}^{\text{stator}})) \right) \right) X - P \rightarrow 0, \quad (4)$$

where C is the coefficient matrix of the system of equations and P is the source term independent of X . Moreover, C depends on the reluctance (R) values in MEC regions. Furthermore, R is a function of the permeability (μ), while μ depends on the flux density (B) respecting the nonlinear magnetic saturation curve of the soft-magnetic material. Solving (4) for X is only possible using a nonlinear solver. After finding the flux loops, the magnetic flux density is obtained using

the geometric parameters of the MEC elements. In the next section, both NRM and FPM are investigated as candidates for the nonlinear loop-based HAM solver.

III. NONLINEAR SOLVERS

A. Newton-Raphson Method

NRM is one of the most commonly used iterative methods, especially for nonlinear finite element problems. Due to its quadratic convergence, NRM can converge to the correct solution in a few iterations with a proper evaluation of the Jacobian and a good initial guess [11]. However, a strong nonlinearity and ill-conditioned Jacobian may cause a large number of Newton-Raphson iterations or even fail to converge as indicated in [7] and [12]. The classical Newton-Raphson iteration is expressed as

$$X^{k+1} = X^k - J_f^{-1}(X^k)f(X^k), \quad (5)$$

where k is the iteration number and J_f is the Jacobian matrix of the residuals consisting of the partial derivatives of (4) with respect to X_k . The analytical derivation of the Jacobian matrix is carried out for the loop-based HAM to minimize the numerical noise and to speed up the Jacobian calculation. For the MEC equations, the product and chain rules are applied:

$$J_f(X^k) = C(X^k) + X \frac{\partial C(R)}{\partial R(\mu)} \frac{\partial R(\mu)}{\partial \mu(B)} \frac{\partial \mu(B)}{\partial B(X^k)} \frac{\partial B(X^k)}{\partial X^k}, \quad (6)$$

where the third term in the chain expression is computed using the saturation curve and the other terms are calculated analytically as in [7]. Besides, (6) is solely applied to the flux-loops of the MEC regions. Unlike MEC equations, the Jacobian of coupling equations (2) and (3) have non-zero terms for both flux loops located at the coupling interfaces and Fourier coefficients. However, since there is no reluctance term in the coupling equations, the chain rule consists of only the derivative of the μ with respect to flux loops which still requires computations involving the saturation curve. Moreover, the initial permeability of the iron regions is selected as 1000 for both FPM and NRM. Since the convergence of NRM is highly dependent on the initial point as discussed in [6], an optimum initial permeability selection algorithm could be used to improve the convergence.

B. Fixed-Point Method

The other candidate for the nonlinear solver of the loop-based HAM is FPM with its robust structure and stable convergence [13]. It is considered as advantageous compared to NRM for magnetic anisotropy and hysteresis problems [14]. Instead of using a Jacobian matrix, the FPM updates iteratively the reluctance based on the local incremental permeability in C and enforces an equivalent magnetization source term in P . The authors have discussed FPM in detail in [3]. Since it takes the magnetic flux density from HAM as input and outputs the relative permeability and the remanent flux density, its implementation with the loop-based HAM is thoroughly the same as in [3].

Fig. 2. a) Convergence comparison of both nonlinear solvers for a fixed saturation level and mesh refinement, b) effect of the increased saturation level, and c) the mesh refinement on the computation time and the convergence rate of both solvers.

IV. RESULTS

As in Fig. 1, only half of the benchmark is modeled with the periodic boundary condition using the symmetry argument. In addition, the Dirichlet boundary condition is introduced at the outer stator and inner rotor radii so that the leakage flux is neglected. The geometric parameters given in Fig. 1: h_{sy} , h_{st} , h_{rt} , h_{ry} , h_{rs} , τ_{rs} , τ_{rt} , τ_{ss} , and τ_{st} are respectively 10 mm, 15 mm, 7 mm, 10 mm, 20 mm, 0.25 rad, 0.37 rad, 0.31 rad, and 0.21 rad. Moreover, the stack length is 0.2 m, while the single-valued nonlinear magnetic saturation curve of the Cogent M800 material [15] is used in each iron region.

The magnetostatic analysis is carried out for the rotor position pictured in Fig. 1. The generalized minimal residual method (GMRES) is used to solve the linear system of equations at each nonlinear iteration. The Jacobian of NRM and the coefficient matrix of FPM are permuted and re-scaled to increase the diagonal dominance for the improved conditioning at each iteration which provides faster convergence and higher accuracy. Both solvers are employed to solve the nonlinear loop-based HAM with 15 A/mm² dc current density and 4.5, -10, 5.5 A/mm² ac current densities for phase A, B, and C, respectively for the commutation relative to the position. The total number of degrees-of-freedom (dof) is fixed to 1750 with 1350 MEC elements and 100 harmonics. The convergence of the unknowns, in terms of the norm of the solution vector, and the corresponding magnetic flux density calculated by FPM and NRM are presented in Fig. 2(a). The flux loops defined in MEC exhibit similar convergence behavior with the magnetic flux density, while Fourier coefficients converge faster than flux loops for both methods. It is also indicated that the flux density converges 3.5 times faster with FPM than NRM for the selected parameters. Moreover, the torque is derived as

$$T = \frac{2\pi l_s}{\mu_0} \sum_{n=1}^{N_h} (b_n c_n - a_n d_n), \quad (7)$$

and is used to compare performances of both solvers for different saturation levels and mesh refinements. The Maxwell stress tensor is employed in (7), where l_s is the stack length and N_h is the number of harmonics considered in the airgap. It is observed that the convergence of the torque is similar to the convergence of Fourier coefficients due to (7). Moreover,

the elapsed time per iteration and the torque convergence rate of both nonlinear solvers are compared in Fig. 2(b) and (c) for increasing saturation levels and mesh refinements. To compare the convergence rates, the normalized relative difference between successive torque values ($\varepsilon_T^{(k)}$) is considered:

$$\varepsilon_T^{(k)} = \frac{|T^{(k)} - T^{(k-1)}|}{|T^{(k)}|}, \quad \alpha = \frac{\log_{10}(\varepsilon_T^{(1)}) - \log_{10}(\varepsilon_T^{(20)})}{20-1}, \quad (8)$$

where k is the iteration number and α is the average convergence rate for 20 nonlinear iterations.

To ensure a fair comparison between the nonlinear solvers, HAM should be investigated under different saturation levels with different mesh and harmonic refinements. The dc-field current density is varied from 1 to 20 A/mm² to observe the performance of the nonlinear solvers under different saturation levels. Fig. 2(b) shows that the computation time per iteration is almost not affected by the saturation level. It is also observed that both solvers have a reduced convergence rate after 10 A/mm², which results in magnetic flux density values beyond the knee point of the nonlinear saturation curve, corresponding to 1.4 T for Cogent M800. Moreover, it is concluded that FPM exhibits a faster convergence rate for all analyzed saturation levels while the average convergence rate difference between NRM and FPM is indicated as 0.4. Furthermore, Fig. 2(c), where the mesh and number of harmonics are refined uniformly, shows that the computation time per iteration increases with the number of dof for both methods. The rate of increase in computation time is calculated as 1.5. Additionally, it is observed that NRM needs more time than FPM for the finest refinement. Fig. 2(c) also compares the convergence rates of both solvers where a quadratic decrease with respect to the number of dof is observed. The average convergence rate difference between the solvers remains the same. Also, the convergence rate of NRM is dramatically impacted at the coarsest mesh as observed in [13].

In addition to the computation time per iteration and the convergence rate, the accuracy and the number of required iterations are other critical parameters for the nonlinear solvers. To compare the converged solutions of NRM and FPM, the hybrid analytical model with FPM is solved with 5460 dof as the reference. The resultant torque values for 5, 10, and 15 A/mm²

Fig. 3. a) The magnetic flux density distribution by FPM, b) the discrepancy between FPM and FEM, and b) the discrepancy between NRM and FEM.

TABLE I

THE RELATIVE TORQUE DISCREPANCY WITH RESPECT TO THE FPM REFERENCE IN PERCENTAGE, FOR BOTH NONLINEAR SOLVER, UNDER THREE DIFFERENT MESH REFINEMENTS AND DC EXCITATION LEVELS.

	5 A/mm ²		10 A/mm ²		15 A/mm ²	
	FPM	NRM	FPM	NRM	FPM	NRM
Coarse mesh:	27	27	24	26	28	44
Medium mesh:	18	17	16	17	17	35
Fine mesh:	8	9	9	7	6	31

TABLE II

THE NUMBER OF REQUIRED ITERATIONS TO REACH CONVERGENCE FOR BOTH NONLINEAR SOLVERS, UNDER THREE DIFFERENT MESH REFINEMENTS AND DC-FIELD EXCITATION LEVELS.

	5 A/mm ²		10 A/mm ²		15 A/mm ²	
	FPM	NRM	FPM	NRM	FPM	NRM
Coarse mesh:	2	3	2	3	4	3
Medium mesh:	2	4	3	11	4	25
Fine mesh:	2	9	3	125	5	494

dc current densities with the same ac currents and rotor position are calculated as 5.6, 10.5, and 16.7 Nm respectively. The discrepancy between these references and the results of FPM and NRM are calculated for three different refinements: coarse, medium, and fine meshes which exhibit 874, 1750, and 2190 dof, respectively. The results are presented in Table I in terms of relative differences expressed in percentage. It shows that NRM loses its accuracy at the highest saturation level. Moreover, Table II presents the number of iterations required for both methods with the same refinements and saturation levels until ε_T becomes smaller than 1%. It is observed that FPM needs less than six iterations for all cases, while the number of iterations required for the convergence of NRM increases with both the refinement and saturation levels.

Moreover, the distribution of the magnitude of the magnetic flux density calculated by HAM coupled with FPM is shown in Fig. 3(a) for the same excitation and mesh parameters as Fig. 2(a). The local discrepancy distribution between HAM and FEM is obtained for both FPM and NRM as in Fig. 3(b) and Fig. 3(c), respectively. It is observed that FPM converges to a more accurate flux density than NRM. Even if NRM achieves to converge as in Fig. 2(a), its ill-conditioned Jacobian decreases its accuracy especially in the iron regions.

V. CONCLUSION

A new loop-based formulation for HAM is proposed in this paper. The developed modeling approach is coupled with both NRM and FPM to compare their convergence rate, computation time, and accuracy. The results show that both methods have a reduced convergence rate under the deep mag-

netic saturation and their convergence rates show a quadratic decrease with the increase in the number of dof. However, FPM exhibits a 0.4 larger convergence rate than NRM for all analyzed saturation levels and mesh refinements. Also, FPM is able to converge for all analyzed refinements and excitation levels with less than six iterations.

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