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Time-optimal Control of Electric Race Cars under Thermal Constraints

Alessandro Locatello, Mouleeswar Konda, Olaf Borsboom, Theo Hofman, Mauro Salazar

Abstract—This paper presents a quasi-convex optimization framework to compute the minimum-lap-time control strategies of electric race cars, accurately accounting for the thermal limitations of the electric motor (EM). To this end, we leverage a previously developed thermally-unconstrained framework and extend it as follows: First, we identify a thermal network model of an interior permanent magnet EM comprising its shaft, rotor, magnets, stator, windings and end-windings, including their individual loss-models. Second, we devise a convex battery model capturing the impact of the state of energy on the battery losses. Third, in order to cope with the nonlinearities stemming from the transcription of the problem from time-domain to a position-dependent representation, we leverage an iterative algorithm based on second-order conic programming to efficiently compute the solution. Finally, we showcase our framework on the Le Mans racetrack. A comparison with high-fidelity simulations in Motor-CAD reveals that our proposed model can accurately capture the temperature dynamics of the EM, revealing the end-windings and the magnets to be the limiting components in a cold-start and a long-run operation scenario, respectively. Furthermore, our numerical results underline the considerable impact of the EM thermal dynamics on lap time, while suggesting that using a continuously variable transmission could significantly improve lap time with respect to a fixed-gear transmission.

I. INTRODUCTION

ELECTRIC racing has received increasing attention over the past decade, for instance, with the emergence of the fully-electric Formula E racing championship and various student electric racing competitions. As in any other class of motorsport, the most important performance indicator is the lap time: the time needed to complete one lap around the race track. Whilst more conventional race cars rely on thermally robust internal combustion engines as their prime mover, electric race vehicles are solely propelled by electric motors (EMs), which are more sensitive to the heating effects during high power operation [1], [2]. Thus, when determining the optimal control strategies of an electric race car, the thermal behavior of the EM has to be accurately accounted for in order to avoid detrimental damages compromising its performance.

To this end, this paper presents a quasi-convex modeling and optimization framework to compute the minimum-lap-time control strategies for the battery electric race car shown in Fig. 1, accounting for the thermal strain on the EM. Specifically, the proposed framework allows to accurately characterize the impact of EM heating on the achievable lap time of electric race cars, and to compare the performance achievable with different transmission technologies.

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Fig. 1. Schematic layout of the electric powertrain. It consists of a battery pack (BT), an electric motor (EM), a transmission (GB) which is either a fixed-gear transmission (FGT) or a continuously variable transmission (CVT); and a final drive reduction gear (FD) connected to the wheels (W). The arrows indicate the power flows between the components.

Related literature: The problem studied in this paper pertains to two main research areas. The first research stream is devoted to the time-optimal control of (hybrid) electric race cars. A drawback of simultaneously optimizing the velocity profile and the driving trajectory [2], [3] is, that the problem becomes highly nonlinear, hence an effective simplification consists of condensing the vehicular dynamics and the expert driver’s feeling into a maximum speed profile. This way, the resulting time-optimal energy management problem can be solved with a wide variety of methods, such as convex optimization [4], [5], PMP [6], or a combination of approaches [7], [8]. However, these methods lack accurate thermal models on the component-level, or do not explicitly capture the impact of the transmission on the achievable performance. The second stream of research is devoted to the thermal modeling of electric motors. Typically, this problem is addressed with lumped-parameter thermal networks (LPTNs), computational fluid dynamics (CFD), or finite-element analysis (FEA) [9]. While CFD and FEA can achieve very accurate results at the expense of computational complexity [10], [11], LPTN-based models are the most widely used, as they leverage equivalent thermal network representations to significantly improve the overall computational tractability [12], [13]. Nonetheless, LPTN-based methods have not yet been applied to racing. In conclusion, to the best of the authors’ knowledge, there are no optimization methods to compute the time-optimal control strategies for battery electric race cars, while accurately capturing the temperature dynamics of the EM and characterizing the impact of the transmission on the thermal behavior and overall performance.

Statement of contribution: Against this backdrop, this paper presents an extension of the quasi-convex optimization framework outlined in [14] to compute the time-optimal control strategies of a battery electric race car, explicitly accounting for the thermal constraints on the EM and the impact of the transmission technology on the overall performance. In particular, we first derive an almost-convex model of the race car, including an LPTN capturing the temperature dynamics of the EM and a state-of-energy-dependent battery model.

Second, we formulate the time-optimal control problem and solve it using an iterative solution algorithm based on convex programming. Third, we validate our models with the high-fidelity EM simulation software Motor-CAD [15]. Finally, we showcase our framework with case studies on the Le Mans track, whereby we compare the performance of a fixed-gear transmission (FGT) and a continuously variable transmission (CVT) in cold-start and long-run racing scenarios.

Organization: The remainder of this paper is structured as follows: Section II presents the almost-convex electric powertrain model including an LPTN capturing the EM thermal dynamics, the time-optimal control problem and an iterative algorithm to effectively solve it. We analyze the numerical results stemming from different scenarios in Section III, alongside a validation of the models. We draw the conclusions and discuss future research directions in Section IV.

II. METHODOLOGY

This section presents an almost-convex model of the race car shown in Fig. 1, frames the time-optimal control problem and proposes an iterative solution algorithm to effectively solve it. The electro-mechanical modeling of the powertrain and the proposed iterative optimization approach are inspired by our previous work [14]. In this paper, we identify and validate a thermal network meticulously characterizing the temperature dynamics of the EM's components and an accurate state-of-energy dependent battery model, which we include in the minimum-lap-time control problem.

As shown in Fig. 1, the battery converts chemical energy to electrical energy which is supplied to the EM. In turn, the EM converts it into mechanical energy and heat losses. The mechanical energy flows through the transmission and the final drive before reaching the wheels, whilst the losses heat up the EM. In this application, we consider an FGT and a CVT for the transmission. The optimization of the driving path is split from the powertrain control problem, and the characteristics of the track and the vehicular dynamics are condensed into the maximum speed profile $v_{\max}(s)$ as a function of the position on the racetrack s , which can be either measured or pre-computed [4], [5]. For the CVT-equipped vehicle, the input variables are the motor force $F_m(s)$ and the transmission ratio $\gamma(s)$, whilst we design its maximum ratio $\gamma_{\max}$. For the FGT-equipped vehicle, the transmission ratio γ_1 is a design variable and $F_m(s)$ is the only input. The state variables of the problem are the kinetic energy of the vehicle $E_{\text{kin}}(s)$, the battery state-of-energy $E_b(s)$, and the temperatures of the nodes of the thermal network, specifically—considering an interior permanent magnet EM— of the shaft $\vartheta_{\text{sf}}(s)$, the permanent magnets $\vartheta_{\text{pm}}(s)$, the rotor $\vartheta_{\text{rt}}(s)$, the windings $\vartheta_{\text{wd}}(s)$, the stator $\vartheta_{\text{st}}(s)$, and the end-windings $\vartheta_{\text{ew}}(s)$. The remainder of the variables are lifting variables linked to the input and state variables via the constraints presented in this section.

A. Minimum-lap-time Objective

In line with [4], [14], we formulate the time-optimal control problem in space domain, so that it becomes a finite-

horizon optimal control problem accommodating position-dependent parameters such as the maximum speed profile. This way, our framework needs to be defined in terms of forces instead of power. The translation to power can be performed in post-processing, using the relation between force and power $F = P/v$ to obtain the power from the optimal solution P^* , using the force F^* and speed v^* .

Our objective is to minimize the lap time T :

$$\min T = \min \int_0^S \frac{dt}{ds}(s) ds, \quad (1)$$

where S is the length of the track and $\frac{dt}{ds}(s)$ is the lethargy, i.e., the inverse of speed $v(s)$. Since speed and lethargy are both optimization variables, it is necessary to establish a convex connection between the variables, defined as

$$\frac{dt}{ds}(s) \cdot v(s) \geq 1, \quad (2)$$

which can be written as a second-order conic constraint that will hold with equality at the optimum [4].

B. Longitudinal Vehicle Dynamics

This section presents a convex model of the longitudinal vehicle dynamics in space domain taken from [4], [14]. The kinetic energy is related to speed in a relaxed convex form as

$$E_{\text{kin}}(s) \geq m_{\text{tot}} \cdot v(s)^2/2, \quad (3)$$

where m_{tot} is the total mass of the vehicle. The kinetic energy's dynamics are given by

$$\frac{d}{ds} E_{\text{kin}}(s) = F_p(s) - F_d(s), \quad (4)$$

where $F_p(s)$ is the propulsion force and $F_d(s)$ is the drag force. The drag force is defined as

$$F_d(s) = c_d \cdot A_f \cdot \rho \cdot E_{\text{kin}}(s)/m_{\text{tot}} + m_{\text{tot}} \cdot g \cdot (\sin(\alpha(s)) + c_r \cdot \cos(\alpha(s))), \quad (5)$$

where c_d is the drag coefficient, A_f is the frontal area of the vehicle, ρ is the air density, g is the Earth's gravitational constant, $\alpha(s)$ is the inclination of the track, and c_r is the rolling friction coefficient. The propulsive force is modeled in a relaxed convex form as

$$F_p(s) \leq \min(\eta_{\text{fd}} \cdot F_{\text{gb}}(s), F_{\text{gb}}(s)/\eta_{\text{fd}}), \quad (6)$$

where $F_{\text{gb}}(s)$ is the force on the secondary axle of the transmission. We condense the racetrack characteristics, the vehicular dynamics and the expert driver's feeling into a maximum speed profile which we enforce as a maximum kinetic energy constraint:

$$E_{\text{kin}}(s) \leq E_{\text{kin,max}}(s) = m_{\text{tot}} \cdot v_{\max}^2(s)/2. \quad (7)$$

Finally, considering a free-flow racing lap, we impose identical speed at the start/finish line as

$$E_{\text{kin}}(0) = E_{\text{kin}}(S). \quad (8)$$

C. Transmission

In this section, we derive a model of the transmission. The speed of the electric motor is expressed by

$$\omega_m(s) = \gamma(s) \cdot \gamma_{fd} / r_w \cdot v(s), \quad (9)$$

where $\gamma(s)$ is the transmission ratio, γ_{fd} is the fixed final drive transmission ratio, and r_w is the radius of the wheels. This constraint is convex if the speed $v(s)$ is given, a property we will leverage throughout this section to devise an iterative convex solution algorithm. The transmission ratio is defined as

$$\gamma(s) \begin{cases} = \gamma_1 & \text{if FGT} \\ \in [\gamma_{\min}, \gamma_{\max}] & \text{if CVT,} \end{cases}$$

where $\gamma_1 > 0$ is the ratio of the FGT, and $\gamma_{\min} > 0$ and $\gamma_{\max} > 0$ are the lower and upper limit of the CVT ratio, respectively. Considering $\gamma_{\max}$ as a design variable with a given constant ratio coverage $c_\gamma = \frac{\gamma_{\max}}{\gamma_{\min}} > 1$, we rewrite the constraint above as

$$\gamma(s) \begin{cases} = \gamma_1 & \text{if FGT} \\ \in \left[\frac{\gamma_{\max}}{c_\gamma}, \gamma_{\max} \right] & \text{if CVT.} \end{cases} \quad (10)$$

We assume the transmission efficiency η_{gb} to be constant, and define the force on the secondary axle of the transmission similar to (6):

$$F_{gb}(s) \leq \min(\eta_{gb} \cdot F_m(s), F_m(s) / \eta_{gb}). \quad (11)$$

D. Electric Motor

In this section, we derive two models of the EM: a speed-independent convex model and a speed-dependent model including a thermal network describing the temperature dynamics of its components. The former model will be instrumental to compute an initial guess of the speed profile, which will be iteratively optimized using the latter model, as presented in more detail in Section II-F below. Given the high-performance application, we consider an interior permanent magnet EM. The nonlinear data that is used to identify and validate the EM model is obtained from the high-fidelity EM simulation software Motor-CAD. For both models, we enforce torque and power limits in space-domain as

$$F_m(s) \in \left[-\frac{\gamma(s) \cdot T_{\max}}{r_w}, \frac{\gamma(s) \cdot T_{\max}}{r_w} \right], \quad (12)$$

and

$$F_m(s) \in [-1, 1] \cdot \left(\frac{c_{m,1} \cdot \gamma(s)}{r_w} + c_{m,2} \cdot \frac{dt}{ds}(s) \right), \quad (13)$$

where $T_{\max}$, $c_{m,1}$ and $c_{m,2}$ are subject to identification. The limit on the rotational speed of the motor is expressed as

$$\gamma(s) \leq \omega_{m,\max} \cdot r_w \cdot \frac{dt}{ds}(s) \cdot \frac{1}{\gamma_{fd}}, \quad (14)$$

where $\omega_{m,\max}$ is the EM maximum speed.

The EM speed-independent model approximates the losses with a quadratic function as

$$P_{dc}(s) = \alpha_m \cdot P_m(s)^2 + P_m(s),$$

Fig. 2. The Motor-CAD interior permanent magnet motor model with the thermal network. We identify seven main components of the EM. A schematic representation of the thermal network is shown in Fig. 3.

Fig. 3. The thermal network of the motor presented in Fig. 2. Every component of the EM is represented by a node in the thermal network.

where $\alpha_m \geq 0$ is an efficiency parameter, subject to identification. We relax the quadratic power approximation and convert it to forces as

$$\frac{dt}{ds}(s) (F_{dc}(s) - F_m(s)) \geq \alpha_m \cdot F_m(s)^2. \quad (15)$$

This relation can be written as a second-order conic constraint which will hold with equality in the case where the solver converges to a time-optimal solution with limited battery energy [4].

In order to capture the behavior of the EM more accurately, we model it in a speed-dependent fashion. Moreover, we include the thermal dynamics as an LPTN, due to its computational tractability and usability. In contrast to the state of the art [16], where empirical models based on continuous and peak torque operation are used, we explicitly model the temperatures of the EM's individual components. The model is based on the following assumptions: The heat

flow in the circumferential direction is neglected, and it is independent in the radial and axial directions. Moreover, we lump the thermal properties of a component into one single node. Fig. 2 shows the Motor-CAD model of the EM and Fig. 3 depicts the thermal network. The EM components under consideration are the shaft (sf), the rotor (rt), the permanent magnets (pm), the stator (st), the windings (wd) and the end-windings (ew), i.e., the overhanging copper cables connecting the windings, which usually represent the most critical component at high-performance operation [17]. Considering the thermal dynamics of the EM, the following energy balance equations describe the heat flows between the components:

$$\begin{cases} P_{sf} = c_{sf} \dot{\vartheta}_{sf} + k_{sf,rt}(\vartheta_{sf} - \vartheta_{rt}) \\ P_{rt} = c_{rt} \dot{\vartheta}_{rt} + k_{sf,rt}(\vartheta_{rt} - \vartheta_{sf}) + k_{rt,pm}(\vartheta_{rt} - \vartheta_{pm}) \\ \quad + k_{rt,st}(\vartheta_{rt} - \vartheta_{st}) \\ P_{pm} = c_{pm} \dot{\vartheta}_{pm} + k_{rt,pm}(\vartheta_{pm} - \vartheta_{rt}) \\ P_{st} = c_{st} \dot{\vartheta}_{st} + k_{rt,st}(\vartheta_{st} - \vartheta_{rt}) + k_{st,wd}(\vartheta_{st} - \vartheta_{wd}) \\ \quad + k_{st,\infty}(\vartheta_{st} - \vartheta_{\infty}) \\ P_{wd} = c_{wd} \dot{\vartheta}_{wd} + k_{st,wd}(\vartheta_{wd} - \vartheta_{st}) + k_{wd,ew}(\vartheta_{wd} - \vartheta_{ew}) \\ P_{ew} = c_{ew} \dot{\vartheta}_{ew} + k_{wd,ew}(\vartheta_{ew} - \vartheta_{wd}), \end{cases}$$

where P_i is the power loss in node i —whereby the index i indicates the EM component and is an element of the set of strings $\{sf, rt, pm, st, wd, ew\}$ — ϑ_i its temperature, $\dot{\vartheta}_i$ its temperature's rate of change in time domain, c_i its heat capacity, and $k_{i,j}$ represents the overall heat transfer coefficient between two neighboring nodes i and j . Hereby, the coefficients $k_{i,j}$ and c_i are subject to identification, whilst, in line with the boundary conditions of Motor-CAD, the coolant's temperature ϑ_{∞} is a known constant.

We estimate the power losses in the individual nodes by fitting a set of experimental data produced by Motor-CAD as a function of speed and mechanical power. We can approximate the nonlinear model in a convex manner using convex quadratic functions. In particular, the losses of the nodes are set equal to $P_i(s) = x_i(s)^\top Q_i x_i(s)$, whereby Q_i is a symmetric and positive semi-definite matrix, and $x_i(s)$ is defined for each component based on the dependence of its losses on the EM speed and power, and the components' temperature. In order to preserve convexity, the power loss equations are relaxed in the set of constraints

$$P_i(s) \geq x_i(s)^\top Q_i x_i(s). \quad (16)$$

Given the racing application, we expect the motor to operate at high power levels. Therefore, we build a reference EM duty cycle to simulate the power losses in Motor-CAD and the results are used as fitting data for the semi-definite matrices. This method allows to minimize the error in the region where the motor is most likely to operate. As shown by the reference data in Fig. 4, the mechanical losses are power-independent and only affect the shaft, i.e., $x_{sf} = [1 \ \omega_m(s)]^\top$. Conversely, windings and end-windings are subject to copper losses, which are strongly influenced by the temperature of the components, in addition to the EM speed and power. Therefore, we define $x_i(s) = [1 \ \omega_m(s) \ P_m(s) \ \vartheta_i(s)]^\top$, for $i \in \{wd, ew\}$.

Fig. 4. The power loss models of the different components of the EM. Speed-dependent mechanical losses with $RMSE_{sf} = 1.3\%$ (top left); speed-, power- and temperature-dependent copper losses representing the sum of the windings' and end-windings' losses with $RMSE_{wd} = 5.9\%$ and $RMSE_{ew} = 7.6\%$, respectively (top right); speed- and power-dependent magnet losses with $RMSE_{pm} = 5.9\%$ (center left); speed- and power-dependent rotor losses $RMSE_{rt} = 4.8\%$ (center right); speed- and power-dependent stator losses $RMSE_{st} = 3.7\%$ (bottom left); and speed- and power-dependent total losses $RMSE_{st} = 7.1\%$ (bottom right).

The permanent magnets and the iron paths of the rotor and stator do not show a relevant thermal dependency and are therefore identified with $x_i(s) = [1 \ \omega_m(s) \ P_m(s)]^\top$, for $i \in \{pm, rt, st\}$. Similarly, we identify a total loss model dependent on speed and power [14], in order to evaluate the thermally-unconstrained case and compare it with the thermally-constrained scenario. The resulting models are shown in Fig. 4.

We convert the set of energy balance equations to space-domain, yielding the following constraints:

$$\begin{cases} c_{sf} \cdot \frac{d}{ds} \vartheta_{sf}(s) \cdot v(s) = P_{sf}(s) + k_{sf,rt}(\vartheta_{rt}(s) - \vartheta_{sf}(s)) \\ c_{rt} \cdot \frac{d}{ds} \vartheta_{rt}(s) \cdot v(s) = P_{rt}(s) + k_{sf,rt}(\vartheta_{sf}(s) - \vartheta_{rt}(s)) \\ \quad + k_{sf,rt}(\vartheta_{pm}(s) - \vartheta_{rt}(s)) \\ \quad + k_{rt,st}(\vartheta_{st}(s) - \vartheta_{rt}(s)) \\ c_{pm} \cdot \frac{d}{ds} \vartheta_{pm}(s) \cdot v(s) = P_{pm}(s) + k_{rt,pm}(\vartheta_{rt}(s) - \vartheta_{pm}(s)) \\ c_{st} \cdot \frac{d}{ds} \vartheta_{st}(s) \cdot v(s) = P_{st}(s) + k_{rt,st}(\vartheta_{rt}(s) - \vartheta_{st}(s)) \\ \quad + k_{st,wd}(\vartheta_{wd}(s) - \vartheta_{st}(s)) \\ \quad + k_{st,\infty}(\vartheta_{\infty} - \vartheta_{st}(s)) \\ c_{wd} \cdot \frac{d}{ds} \vartheta_{wd}(s) \cdot v(s) = P_{wd}(s) + k_{st,wd}(\vartheta_{st}(s) - \vartheta_{wd}(s)) \\ \quad + k_{wd,ew}(\vartheta_{ew}(s) - \vartheta_{wd}(s)) \\ c_{ew} \cdot \frac{d}{ds} \vartheta_{ew}(s) \cdot v(s) = P_{ew}(s) + k_{wd,ew}(\vartheta_{wd}(s) - \vartheta_{ew}(s)), \end{cases} \quad (17)$$

where $P_i(s)$ is the power loss given by (16). Again, this set of equations is convex whenever speed is a given parameter. Depending on the material's characteristics, we constrain the temperature of each node as

$$\vartheta_i(s) \leq \vartheta_{i,max}, \quad (18)$$

in order not to damage the EM.

Finally, we consider two thermal scenarios: a cold-start lap, in which the motor temperature starts at the temperature of the coolant ϑ_∞ , and a lap representing a long-run operation at steady-state. Mathematically, we characterize the scenarios as

$$\begin{cases} \vartheta_i(0) = \vartheta_\infty & \text{if cold start} \\ \vartheta_i(0) = \vartheta_i(S) & \text{if long run.} \end{cases} \quad (19)$$

E. Battery Pack

In this section, we derive an energy-independent and a more accurate energy-dependent model of the battery dynamics. The power at the terminals is $P_b(s) = P_{dc}(s) + P_{aux}$, where P_{aux} is a constant auxiliary power. Converting this constraint to forces yields

$$F_b(s) = F_{dc}(s) + P_{aux} \cdot \frac{dt}{ds}(s). \quad (20)$$

The internal battery power $P_i(s)$, which causes the actual change in the battery state of energy $E_b(s)$, is first approximated by $P_i(s) = \alpha_b \cdot P_b(s)^2 + P_b(s)$, where the efficiency parameter α_b is determined by a quadratic regression of the measurement data with a normalized RMSE of 0.96% [14]. This constraint can be translated into forces and relaxed as

$$(F_i(s) - F_b(s)) \cdot \frac{dt}{ds}(s) \geq \alpha_b \cdot F_b(s)^2, \quad (21)$$

which can be expressed as a second-order conic constraint [4].

We include state-of-energy-dependency in the model of the battery by modeling the internal power to the terminal power through the open-circuit voltage U_{oc} and the internal resistance R as

$$P_i(s) = P_b(s) + \frac{R}{U_{oc}^2} \cdot P_i(s)^2,$$

which can be rewritten with the open-circuit power $P_{oc} = \frac{U_{oc}^2}{R}$ as

$$(P_i(s) - P_b(s)) \cdot P_{oc}(s) = P_i(s)^2. \quad (22)$$

Using a similar reasoning as in [18], the open-circuit power can be expressed by the piecewise affine approximation

$$P_{oc}(E_b) = a_b^k \cdot E_b + b_b^k \cdot E_{b,\max} \quad \text{if } E_b \in [E_b^{k-1}, E_b^k] \quad \forall k \in [1, \dots, K],$$

where $a_b^k \geq a_b^{k+1} \forall k \in [1, \dots, K-1]$ and $b_b^k \leq b_b^{k+1} \forall k \in [1, \dots, K-1]$ are parameters subject to identification, and K is the number of affine functions in the approximation. The proposed open-circuit power model is fitted in Fig. 5 with three affine functions. To secure convexity, we relax the open-circuit power to

$$P_{oc}(E_b) \leq a_b^k \cdot E_b + b_b^k \cdot E_{b,\max} \quad \forall k \in [1, \dots, K]. \quad (23)$$

Finally, relaxing (22) and converting it to forces yields

$$(F_i(s) - F_b(s)) \cdot \frac{P_{oc}(s)}{v(s)} \geq F_i(s)^2,$$

which can be written as the second order conic constraint

$$F_i(s) - F_b(s) + \frac{P_{oc}(s)}{v(s)} \geq \left\| F_i(s) - F_b(s) - \frac{P_{oc}(s)}{v(s)} \right\|_2. \quad (24)$$

Fig. 5. A piecewise affine approximation of the battery open-circuit power P_{oc} as a function of the battery state-of-energy E_b .

Fig. 6. The battery efficiency model using a piecewise affine approximation of the open-circuit power. The normalized RMSE is 0.08%.

The resulting energy-dependent battery model is shown in Fig. 6 and can approximate the nonlinear model with a normalized RMSE of 0.08%. The constraint (24) above is not fully convex, considering the quotient in the optimization variables $P_{oc}(s)$ and $v(s)$. Yet again, we can recover convexity if speed is a given parameter.

The change in battery energy $E_b(s)$ during the lap is defined as

$$\frac{d}{ds} E_b(s) = -F_i(s), \quad (25)$$

and is bounded as

$$E_b(0) = E_{b,0}, \quad E_b(S) \geq 0, \quad (26)$$

with $E_{b,0}$ as the energy available at the beginning of the lap.

F. Minimum-lap-time Optimization Problem

In this section, we present an iterative algorithm used to solve the quasi-convex time-optimal control problem. The state variables for both the FGT and the CVT race car are $x = (E_{kin}, E_b, \vartheta_{sf}, \vartheta_{rt}, \vartheta_{pm}, \vartheta_{st}, \vartheta_{wd}, \vartheta_{ew})$. The control variables are $u = (F_m, \gamma)$, where $\gamma(s)$ is present for the CVT only. The design variables for the FGT and CVT are $p_{FGT} = \gamma_1$ and $p_{CVT} = \gamma_{\max}$, respectively, whilst the remainder of the variables are lifting variables. We state the time-optimal control problem as follows:

Algorithm 1 Iterative Solving Procedure

$\bar{v}(s) \leftarrow \text{Solve Problem 2}$
while $\frac{1}{S} \cdot \sqrt{\int_0^S (v(s) - \bar{v}(s))^2 ds} \geq \varepsilon_v$ **do**
 $\bar{v}(s) = v(s)$
 $v(s) \leftarrow \text{Solve Problem 3}$

Problem 1 (Full Nonlinear Problem). *The minimum-lap-time control strategies are the solution of*

$$\min \int_0^S \frac{dt}{ds}(s) ds$$

s.t. (2) – (21), (23) – (26).

Due to the speed-dependency of the EM model, the battery model, and the thermal network, Problem 1 above is not entirely convex. In order to circumvent these non-convexities, we leverage the iterative Algorithm 1 based on the two following problems:

Problem 2 (Simplified Convex Problem). *The minimum-lap-time control strategies for the thermally-unconstrained and speed-independent EM model and energy-independent battery model are the solution of the following second-order conic program (SOCP):*

$$\min \int_0^S \frac{dt}{ds}(s) ds$$

s.t. (2) – (8), (10) – (15), (20), (21), (25), (26).

Problem 3 (Thermal Speed-dependent Convex Problem). *Given a velocity profile $\bar{v}(s)$, the minimum-lap-time control strategies are the solution of the following SOCP:*

$$\min \int_0^S \frac{dt}{ds}(s) ds$$

s.t. (2) – (8), (10) – (14), (16), (18) – (20), (23) – (26),
and (9), (17), (24) with $v(s) = \bar{v}(s)$.

Algorithm 1 leverages the speed profile of the solution of speed-independent Problem 2 as an initial guess for $\bar{v}(s)$ to solve speed-dependent Problem 3, iterating on it until two consecutive speed profiles coincide up to a certain tolerance $\varepsilon_v > 0$.

G. Discussion

A few comments are in order. First, we model both the FGT and the CVT with a constant efficiency—a common approach for high-level modeling purposes—since the focus of this paper is not on the transmission modeling itself, and refer readers interested in this topic to [5]. Second, we assume the battery to be able to accommodate any EM power request. This assumption is acceptable for high-performance batteries designed for fast-charging applications. Finally, similar to [14], we solve the almost-convex optimization problem using an iterative algorithm based on convex optimization. Although the nonlinear problem form is hindering us from guaranteeing global optimality of the solution, the robustness of the speed profile on which we iterate (see the first plot in Fig. 7) and the consistency of the results are promising. We leave a validation with standard nonlinear programming solvers to a journal extension.

III. NUMERICAL RESULTS

This section studies the performance achievable by an electric race car equipped with two identically operated EMs connected to an FGT and a CVT in different operating scenarios on the Le Mans race track. We first discuss the results obtained for an FGT-equipped car in a cold-start scenario and compare it with the thermally-unconstrained solution. Thereby, we also validate our results with the high-fidelity EM software Motor-CAD and a nonlinear battery model. Finally, we study the time-optimal strategies under a long-run steady-state operation, comparing the FGT- and CVT-equipped vehicles.

We set the weight of the FGT- and CVT-equipped vehicles to 1342 kg and 1393 kg, respectively. Furthermore, we set the EM-to-wheels efficiency of the FGT-equipped car to 98 %, whilst assuming a lower average value of 96 % for the CVT-equipped car [5], [14]. Finally, the temperature limits are 200 °C for windings and end-windings, 170 °C for stator and rotor iron paths, and 120 °C for the permanent magnets, whilst we set the coolant's temperature ϑ_∞ to 65 °C.

In line with [4], we discretize the model presented in Section II with a step-size of 10 m (resulting in sampling times well below 1 s) using the trapezoidal method in order to avoid numerical instabilities stemming from the LPTN. We parse the problem using YALMIP [19] and solve it with the second-order conic solver ECOS [20]. One iteration takes about 53 s to parse and 34 s to solve when using a 2.3 GHz Quad-Core Intel Core i5 processor with 8 GB of RAM. Given a tolerance $\varepsilon_v = 0.05$ 1/s, the solution Algorithm 1 typically takes four to six iterations to converge, resulting in an average total computation time of 7-8 min, including overhead.

A. FGT Cold Start

First, we optimize the FGT-equipped powertrain for a cold-start scenario and we compare the thermally-constrained and unconstrained solutions. Fig. 7 shows the results, whereby the lap time of the thermally-constrained solution is 2.1 s slower than the lap time of the thermally-unconstrained solution. From the motor power trajectory, we observe that the thermal constraints do not hinder the motor from operating at maximum power. However, the optimal gear ratio γ_1 decreases from 4.54 for the thermally-unconstrained problem to 3.83 for the thermally-constrained problem.

It can also be observed that the EM is using regenerative braking in both cases, although the thermally-constrained operation shows a more gradual approach to the negative power region. The thermal constraints attempt to limit the absolute value of the output power because of its dependency with the power losses. As a result, the low-power operations of the EM are extended to reduce the temperature rise or to briefly cool down the EM components.

The lower subfigure of Fig. 7 shows the temperature evolution of the motor components. As can be seen, the end-windings are the component limiting the operation of the motor by reaching the thermal limit towards the end of the lap. Fig. 7 also indicates the temperature trajectory of the EM simulated in Motor-CAD while subjected to the EM duty

Fig. 7. The speed, the motor power and the EM components' temperature of an FGT-equipped vehicle in a cold-start scenario. The bottom graph additionally shows the overheating temperature limits and a validation by comparing it with a Motor-CAD simulation. The results are presented by the solid lines and the Motor-CAD validation data is presented by the dashed lines.

Fig. 8. The validation of the battery energy consumption obtained by the solution, represented in continuous lines, and in dashed lines the resulting battery energy consumption using a Motor-CAD loss model and a nonlinear battery model. The relative drift in energy at the end of lap is equal to 1.1%.

cycle of the thermally-constrained solution. We can observe that our models can closely reproduce the results stemming from a high-fidelity simulator, resulting into a cumulative drift below 2°C for all the EM components except the stator, which achieves an error of 5°C while staying well below its limit of 150°C .

We validate the power loss model and the battery model in Fig. 8, showing the state-of-energy trajectory of our model compared to the one resulting from Motor-CAD and a nonlinear battery model. As can be seen, the small deviation between the two solutions results in a total drift of 1.1%.

B. FGT Long Run

The trends observed in the cold-start scenario are much more clearly emphasized in a long-run operation where the EM is at a higher temperature. Fig. 9 shows the performance and the temperature trajectory of the FGT-equipped car during the lap, whereby the lap time is more than 10 s longer than in the unconstrained case, highlighting the prominent

Fig. 9. The speed, the motor power, and the EM components' temperature of an FGT-equipped race car during a long-run operation.

impact of the thermal constraints on the time-optimal strategies. Hereby, the more gradual transition into regenerative braking is emphasized to the extent that energy regeneration is almost never allowed in order to reduce the EM losses and, in turn, the components' overheating. Interestingly, and in contrast to the cold-start scenario, in this case the EM power is limited by the temperature of the permanent magnets, mildly oscillating in the vicinity of their thermal boundary.

C. CVT Long Run

Finally, we showcase our models for a CVT-equipped race car to study the impact of different transmission technologies on achievable performance during a long-run operation. Fig. 10 shows the numerical results of this scenario, where it is possible to observe a similar response as in the FGT-equipped car. Similar to the FGT, the optimal maximum CVT gear ratio $\gamma_{\max}$ is reduced from 6.5 in the thermally-unconstrained case to 5.5 for the thermally-constrained lap. Moreover, when coupled with a CVT, the EM shows intense regenerative braking in the thermally-unconstrained case, and almost no regeneration when the temperature constraints are active, whilst also in this case, the permanent magnets are the limiting factor.

With respect to lap times, the thermally-unconstrained solution shows that the EM coupled with an FGT would be 1.12 s faster, since the CVT cannot compensate its higher weight with its ability to control the EM in a more efficient fashion. However, under a thermally-constrained long-run scenario, a more efficient EM operation not only improves energy consumption, but also reduces overheating, so that the CVT-equipped car can significantly outperform the FGT-equipped car with a 1.89 s faster lap time.

Fig. 10. The speed, the motor power and the EM components' temperature for a CVT-equipped race car during a long-run operation.

IV. CONCLUSION

In this paper, we studied the impact of thermal limits in electric racing. To this end, we devised a quasi-convex optimization framework, accurately capturing the temperature dynamics of an interior permanent magnet electric motor (EM) using a thermal network including the stator, the rotor, the permanent magnets, the windings and the end-windings. We validated our models with the high-fidelity EM simulation software Motor-CAD and a nonlinear battery model, showing a cumulative drift of about 1% for both the temperatures and the battery energy. When showcasing our framework on the Le Mans racetrack for a car equipped with a fixed-gear transmission and different operating scenarios, we concluded that the main components limiting the EM performance are the end-windings in a cold-start scenario and the permanent magnets during a long-run operation. Finally, we observed that thermal limitations can result in a significant lap time loss which can, however, be partially salvaged by equipping the race car with a continuously variable transmission.

This work opens the field for several research directions. First, our promising preliminary results prompt a more detailed analysis based on high-fidelity models. Second, we would like to perform a more theoretical analysis to examine whether the almost-convex problem structure could be leveraged in order to guarantee global optimality of the solutions found by the proposed iterative algorithm. Third, we are interested in exploring the application of multi-speed gearbox transmissions, as they combine a low mass and high efficiency with the ability to operate at multiple transmission ratios. Finally, we would like to leverage the proposed offline framework to devise temperature-aware minimum-lap-time control algorithms that can be implemented in real-time.

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